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Discussion and Closure

Discussion on "Seismic Active Earth Pressure Coefficients on Battered Retaining Wall Supporting Inclined c-ϕ Backfill: Sima Ghosh[1] in Indian Geotechnical Journal 40(1), 2010, 79-83"

Sanjay Kumar Shukla[2]

Discussion by Sanjay Kumar Shukla

The author is appreciated for deriving an analytical expression (Eqn. (14)) for total seismic active earth pressure *P* from c-ϕ soil backfill by defining coefficients K_γ and K_c (Eqns. (15) and (16)) associated with unit weight and cohesion, respectively. The attempt at deriving the expression for a generalised case with an inclined back face of the wall and a sloping top surface of the backfill under surcharge and seismic loadings has been made following the analytical approach presented by Shukla et al. (2009). The analytical expression derived by Shukla et al. (2009) and improved recently by Shukla and Zahid (2010) to incorporate the effect of surcharge pressure *q* is given as

$$P_{ae} = (1 \pm k_v)(q + \frac{1}{2}\gamma H)HK_{ae\gamma} - cHK_{aec} \quad (1)$$

where P_{ae} is the total seismic active force, k_v is the vertical seismic coefficient, *q* is the surcharge pressure at the top of the backfill, *H* is the height of the retaining wall, c is the cohesion, and $K_{ae\gamma}$ and K_{aec} are the seismic active earth pressure coefficients associated with unit weight and cohesion, respectively. Earth pressure coefficients $K_{ae\gamma}$ and K_{aec} are defined, respectively as

$$K_{ae\gamma} = \frac{\cos(\varphi - \theta) - \dfrac{\sin(\varphi - \theta)}{\tan\alpha_c}}{\cos\theta\left(\cos\varphi + \sin\varphi\tan\alpha_c\right)} \quad (2)$$

and

$$K_{aec} = \frac{\cos\varphi(1 + \tan^2\alpha_c)}{(\cos\varphi + \sin\varphi\tan\alpha_c)\tan\alpha_c} \quad (3)$$

where

$$\theta = \tan^{-1}\left(\frac{k_h}{1 \pm k_v}\right) \quad (4)$$

and

$$\alpha_c = \tan^{-1}\left(\frac{\sin\varphi\sin(\varphi-\theta) + m\sin 2\varphi + \sqrt{\sin\varphi\sin(\varphi-\theta)\cos\theta + 4m^2\cos^2\varphi + 2m\cos\varphi\{\sin\varphi\cos\theta + \sin(\varphi-\theta)\}}}{\sin\varphi\cos(\varphi-\theta) + 2m\cos^2\varphi}\right) \quad (5)$$

with

$$m = \frac{c\cos\theta}{\gamma H(1 \pm k_v)} \quad (6)$$

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In Eqns. (2) and (3), ϕ is the angle of shearing resistance of the soil backfill, in Eqn. (4), k_h is the horizontal seismic coefficient, and in Eqn. (5), α_c is the critical value of inclination to the horizontal of the failure plane within the soil backfill. One can get the total seismic active earth pressure from c-ϕ soil backfills using the above equations with

$$k_h \leq (1 \pm k_v) \tan\phi + \frac{2c}{2q + \gamma h}.$$

To consider the effect of tension cracks, Lambe and Whitman's approach as explained by Shukla et al (2009) can be adopted.

It is noticed that Eqns. (14), (15) and (16) presented by the author correspond to Eqns. (1), (2) and (3) respectively. No attempt was made to present an expression for α_c corresponding to Eqn. (5). In the discusser's opinion, deriving an expression for critical value of inclination to the horizontal/vertical of the failure plane within the soil backfill is the real expected task, which has not been done from the readers' point of view so that the Eqns. (14), (15) and (16) presented by the author can be used conveniently.

The discusser would like to point out that presenting an expression for the critical value of inclination to the horizontal/vertical of the failure plane within the soil backfill has been the most important aspect in the analysis of static/dynamic active earth pressure since the development of classical Rankine, Coulomb, and Mononobe-Okabe expressions for the active earth pressures (Okabe 1926; Mononobe 1929, Lambe and Whitman 1979; Terzaghi et al. 1996; Das and Ramana 2010). An analytical derivation for the critical value of inclination of the failure plane for a generalised case considering both cohesion c and angle of shearing resistance ϕ of the soil backfill under surcharge and seismic loading conditions has been a challenging problem for the researchers. To avoid this difficulty in the analytical formulation, Saran and Prakash (1968) maximized the pressure due to weight of soil wedge and cohesion separately, resulting in different failure planes, which cannot be observed in field situations. This pseudo-static analysis was extended further by Saran and Gupta (2003) following exactly the same approach for a generalised field situation as considered by the author.

It has been experienced by the discusser that the analytical derivation in generalized explicit form with the parameters considered in this note under seismic loading condition requires a great effort; this has probably been the reason for unavailability of an explicit analytical expression for the c-ϕ soil backfill case until recently; although the Mononobe-Okabe equation for cohesionless soil (ϕ soil) backfills has been reported without a detailed derivation in some books since 1930. Shukla's equation (Shukla et al. 2009; Shukla and Zahid 2010) provides an explicit analytical expression for the total seismic active force on a vertical wall from the c-ϕ soil backfills. This equation corresponds to the Mononobe-Okabe equation for cohesionless soil backfills, and is in fact an extension of Rankine equation for total static active force on a vertical wall as noticed correctly by the author.

In addition to the above-mentioned limitations of the work, the technical note contains some general and technical mistakes. Eqns. (2) of the note requires correction as

$$N_\phi = \frac{1 + \sin\phi}{1 - \sin\phi} \tag{7}$$

The discusser has reported Eqns. (1) and (2) without any reference. The readers should note that Eqns. (1) and (2) are based on the Rankine's analysis of active earth pressure from the c-ϕ soil backfill under static condition (Lambe and Whitman 1979; Terzaghi et al. 1996; Das 2008). Alternatively, one can use the value of depth of tension cracks obtained from the field observations.

Eqns. (6), (8), (9) and (10) of the note require corrections, respectively, as

$$SN(\perp to\ OM) = H_1 \sec\alpha \sin b \tag{8}$$

$$Q = (q + \gamma H_0) H_1 \frac{\sec\alpha}{\sin d} \sin b \tag{9}$$

$$OM = H_1 \frac{\sec\alpha}{\sin d} \sin a \tag{10}$$

$$C = cH_1 \frac{\sec\alpha}{\sin d} \sin a \tag{11}$$

The discusser has checked the complete derivation and found that the mistakes in Eqns. (6), (8), (9) and (10) do not result in any changes in the final equations (14), (15) and (16). Therefore, it appears that these are typographical mistakes.

The readers should note that that Eqn. (3) of the technical note as H_1 H – $H_0$is not correct technically. Its correct form is given as (Saran and Gupta 2003):

$$H_1 = H - \left\{\frac{\cos\alpha \cos i}{\cos(\alpha - i)}\right\} H_0 \tag{12}$$

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In view of the above technical mistake, a correction is required in Fig. 1 too. In the figure, H_0 as labelled should be replaced by $\left\{\frac{\cos\alpha\cos i}{\cos(\alpha - i)}\right\}H_0$, and it should be noted that the weight of the tension crack zone as γH_0 per unit length has been correctly labelled in the figure.

In the discusser's opinion, the steps of analysis for dynamic earth pressure as presented in the note on page 80 have not been explained logically from the readers' point of view. Are these steps suggested by others? If yes, the reference should have been provided. If no, a very clear and logical explanation is expected because the steps consist of calculating the static and seismic earth pressures separately, which does not meet the basic objective of the note as claimed by the author.

In view of limitations and technical mistakes as described above, the author's derivation of the analytical expression remains incomplete from the point of view of its application as generally required for convenience in design practice. However, the discusser appreciates the author for presenting the research effort made in this regard.

Closure by Sima Ghosh

The well-known solution for the determination of seismic active earth pressure on the back of a retaining wall supporting c-Φ backfill is presented by Saran and Prakash (1968). This is an extension of Mononobe-Okabe (1929) solution to take into account the c-Φ nature of the backfill. Saran and Prakash (1968) considered horizontal backfill surface and only horizontal seismic acceleration. The work of Saran and Prakash (1968) is again further extended by Saran and Gupta (2003) and Ghosh et al. (2008) to take into account, the inclination of the backfill surface and both horizontal and vertical seismic acceleration. Either in Saran and Prakash (1968) or in Saran and Gupta (2003), the seismic active earth pressure co-efficient is obtained considering three separate failure mechanisms. These three failure mechanisms are for backfill weight, surcharge and cohesion. On the basis of the optimization of these three failure mechanisms, three separate coefficients are introduced for backfill weight, surcharge and cohesion in tabular form and non-dimensional charts.

Through this technical note, attempt is made to simplify the solution reducing the required number of coefficients. For that assumed analytical model of retaining wall (height H, inclined at an angle α with vertical) supporting c-Φ backfill is shown in Figure 1.

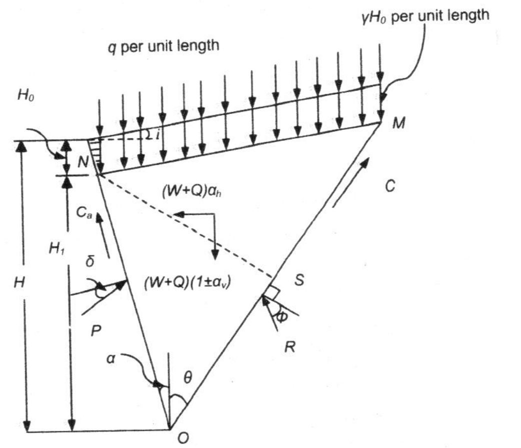

Fig.1 Forces acting on retaining wall – soil mass system during active state of equilibrium

Here, i = inclination of the backfill surface with horizontal, W=weight of the backfill excluding cracked zone, Q=total surcharge load along with the weight of cracked zone, C_a = total adhesion acting upto length ON,c = total cohesion acting upto length OM,α_h and α_v = horizontal and vertical seismic acceleration co-efficient respectively, P = force acting on retaining wall during active state of equilibrium, R = reaction offered by the retained soil over the wedge and θ = inclination of the wedge surface with vertical.

In this model, similar to Saran and Gupta (2003), it is considered that due to c-Φ nature of the backfill, at the top of the backfill, tension cracks are present. Depth of the tension crack zone (H_0) is given by Rankine (1857) as follows:

Depth of the tension crack zone

$$(H_0) = \frac{2c}{\gamma}\sqrt{N_\phi} \tag{1}$$

$$\text{where } N_\phi = \frac{1+\sin\phi}{1-\sin\phi} \tag{2}$$

This assumption is approximate as this is the depth of tension crack zone given by Rankine (1857) under static loading condition considering the wall as vertical retaining wall supporting horizontal backfill (Lambe and Whitman 1979; Terzaghi et al. 1996). One can also use the value of depth of tension crack obtained from field observation. It is also assumed in the study under consideration that the weight of the tension crack zone acts along with the weight of surcharge at the bottom surface of tension crack zone. It is considered in the study that the bottom of the tension crack surface is plane and smooth which is lying parallel to backfill surface. Due to that assumption, a little portion of the tension crack zone (shaded) is neglected. Though, it is not mentioned in the study, weight of this portion in comparison to the whole weight of backfill and surcharge is negligible.

The analysis suffers a few typographical mistakes: one in Eqn 6 of the technical note. In front of to, $\perp$ sign is missing. Another typing mistake is in Eqn 9 and 10 of the technical note. In these Eqns, 'θ' should be replaced by 'α'. To present the steps of analysis for dynamic active earth pressure, author has followed Saran and Gupta (2003) recommendation. To present step 6, '(iii)' should be replaced by '3' and 'in' should be replaced by 'with' in the second line of step 6. So, following the steps as given in the technical note and co-efficient of seismic active earth pressures as presented in the tabular form in the technical note, one can calculate the seismic active earth pressure on the back of a retaining wall supporting c-Φ backfill. To use this methodology, there is no need to calculate the critical wedge angle or such terms. But for intermediate portions author suggests linear interpolation. On the basis of the analysis under consideration of discussion, results show that the values of seismic active earth pressure coefficients do not depend on unit weight of backfill, surcharge loading, cohesion and height of retaining wall.

The depth of tension crack zone is assumed using Rankine (1857). Sharma and Ghosh (2010) has given a solution for the determination of seismic active earth pressure supporting c-Φ backfill in such a way that for a particular retaining wall backfill system, single active earth pressure coefficient is introduced for the simultaneous action of unit weight, surcharge and cohesion. Under any seismic loading condition, supporting c-Φ backfill, the analysis itself gives the depth of tension crack zone. In Sharma and Ghosh (2010), it is considered that unit cohesion is equal to unit adhesion. So, again to remove this limitation, following sections present a pseudo-static procedure in which a formulation is developed for the determination of seismic active earth pressure on the back of a vertical retaining wall where we can use different values of cohesion and adhesion. This solution is applicable for any value of δ, instead of limiting the value of $\delta \leq 2\Phi/3$. The detailed analysis is as follows:

Let us consider a rigid retaining wall of height 'H' supporting a c-Φ backfill of unit weight 'γ', unit cohesion 'c', unit adhesion 'c_a', angle of wall friction 'δ', angle of soil friction 'Φ'. On the top of the backfill a surcharge load of intensity 'q' per unit length is acting.

At any stage of earthquake (having seismic acceleration coefficients α_h and α_v) during active state of equilibrium, if the planer wedge surface BD generates an angle 'θ' with the vertical, then the forces acting on the wedge system as shown in Fig.2, 'P_a' and 'R' being the force on the retaining wall and reaction offered by the retained earth on the sliding wedge ABD at the face AB and BD respectively. The other forces are total cohesion $C = cH\sec\theta$, total adhesion $C_a = c_aH$, weight of wedge, $W = \gamma H^2\tan\theta/2$, surcharge load, $Q = qH\tan\theta$, horizontal inertia force $= (W+Q)\alpha_h$ and vertical inertia force $= (W+Q)\,\alpha_v$.

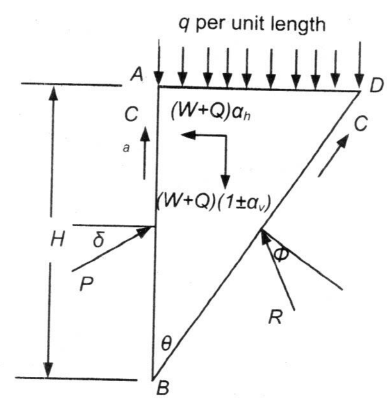

Fig.2. Forces acting on retaining wall – soil wedge system during active state of equilibrium

Applying the equilibrium conditions, $\Sigma H = 0$ and $\Sigma V = 0$ we get respectively,

$$P_a \cos\delta - R\cos(\varphi+\theta) + C\sin\theta = (W+Q)\alpha_h \tag{3}$$

$$P_a \sin\delta + R\sin(\varphi+\theta) + C\sin\theta + C_a = (W+Q)(1\pm\alpha_v) \tag{4}$$

On simplification of Eqn 3 and 4 substituting the values of C, Q, W etc. we get,

$$P_a \sin(\varphi+\delta+\theta) = \left(\gamma + \frac{2q}{H}\right)\frac{H^2}{2}(1\pm\alpha_v)$$
$$\frac{\tan\theta}{\cos\psi}\cos(\varphi+\theta-\psi) - cH\sec\theta\cos\varphi$$
$$-c_aH\cos(\varphi+\theta) \tag{5}$$

Where $\psi = \tan^{-1}\frac{\alpha_h}{1\pm\alpha_v}$ (6)

Replacing $(\gamma+2q/H)$ by γ_e, Eqn 5 can be written as

$$P_a = \gamma_e \frac{H^2}{2}(1\pm\alpha_v)\frac{\left[\frac{\tan\theta}{\cos\psi}\cos(\varphi+\theta-\psi) - \frac{2c}{\gamma_e H(1\pm\alpha_v)}\sec\theta\cos\varphi - \frac{2c_a}{\gamma_e H(1\pm\alpha_v)}\cos(\varphi+\theta)\right]}{\sin(\varphi+\delta+\theta)} \tag{7}$$

Substituting $\frac{2c}{\gamma_e H(1\pm\alpha_v)} = n_c$ and

$$\frac{2c_a}{\gamma_e H(1\pm\alpha_v)} = m_c$$

$$P_a = \gamma_e \frac{H^2}{2}(1\pm\alpha_v)\frac{[\sin\theta\cos(\phi+\theta-\psi) - n_c\cos\psi\cos\phi - m_c\cos\theta\cos(\phi+\theta)]}{\sin(\phi+\delta+\theta)\cos\theta\cos\psi} \tag{8}$$

which can also be written as

$$P_a = \gamma_e \frac{H^2}{2}(1\pm\alpha_v)k_a \tag{9}$$

Where $$k_a = \frac{[\sin\theta\cos(\phi+\theta-\psi) - n_c\cos\psi\cos\phi - m_c\cos\theta\cos(\phi+\theta)]}{\sin(\phi+\delta+\theta)\cos\theta\cos\psi} \tag{10}$$

In Eqn 10, all the terms are constant except θ. On optimizing this coefficient for seismic active earth pressure we get the value of θ which is represented here as θ_c and given by

$$\theta_c = \cos^{-1}\sqrt{\frac{(r+s)s+u^2+t\sqrt{s^2+u^2-r^2}}{2(s^2+u^2)}} \tag{11}$$

where

$$r = -\sin(\psi+\delta) - m_c\cos\psi\cos\delta \tag{12}$$

$$s = \sin(2\phi+\delta-\psi) + 2n_c\cos\phi\cos\psi\cos(\phi+\delta) + m_c\cos\psi\cos\delta \tag{13}$$

$$u = 2\sin(\phi + \delta)[\sin(\phi - \psi) + n_c \cos\phi \cos\psi] \quad (14)$$

Putting this value of θ_c in Eqn 10, we get the coefficient of seismic active earth pressure k_{ae} and substitution this value of k_{ae} in Eqn 8 gives the magnitude of total seismic active force (P_{ae}) on the back of a retaining wall supporting c-Φ backfill.

Comparison of results

Table 1 shows the comparison of results as obtained from present approach with *Sharma and Ghosh (2010)*. From the comparative study, it can be concluded that the results as obtained from both the studies are comparable and suggests the acceptability of the methods.

Conclusion

A reply has been given over the discussion on the paper entitled "Seismic Active Earth Pressure Coefficient on Battered Retaining Wall Supporting Inclined c-Φ Backfill" by Sima Ghosh: Indian Geotechnical J., 40(1), 2010, 78-83. Earlier analysis suffers a few typographical errors. The methodology suggested in that paper simplifies the way of determination of seismic active earth pressure supporting c-Φ backfill. Further, simplification is made by Sharma and Ghosh (2010) introducing single active earth pressure co-efficient for the simultaneous action of weight, surcharge and cohesion. These are presented in tabular forms in the original paper. In the reply, author is presented another methodology which again simplifies the way of evaluation of seismic active earth pressure.

Table 1: Comparison of the seismic active earth pressure coefficients obtained from present study with Sharma and Ghosh (2010) [Φ = 30°, δ = 2Φ/3, γ = 18kN/m³, q = 0, H = 10 m]

	$\alpha_h = \alpha_v = 0$		$\alpha_h = 0.1,\ \alpha_v = 0.05$		$\alpha_h = 0.2,\ \alpha_v = 0.0$	
Value of cohesion and adhesion	Present study	*Sharma and Ghosh (2010)*	Present study	*Sharma and Ghosh (2010)*	Present study	*Sharma and Ghosh (2010)*
$c = c_a$ = 4 .5kN/m²	0.215	0.223	0.283	0.278	0.37	0.379
$c = c_a$ =8 .5kN/m²	0.148	0.149	0.215	0.204	0.308	0.305
$c = c_a$ =12 .5kN/m²	0.066	0.074	0.129	0.131	0.222	0.23
$c = c_a$ =16 .5kN/m²	0.0001	0.0001	0.06	0.057	0.156	0.156

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