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Kinematics and Bearing Capacity of Reinforced Foundation Bed on Soft Ground

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Key words

Reinforced foundation beds, Bearing Capacity Ratio (BCR), reinforcement, axial tension, transverse tension.

Abstract: This paper presents a simple method to estimate the bearing capacity of a strip footing on the surface of a reinforced foundation bed over soft homogeneous clay. The proposed model considers the effect of kinematics, i.e., the effect of the transverse resistance, in addition to the effect of axial resistance of the reinforcement together with shear resistance of the granular fill. The improvement in Bearing Capacity Ratio (BCR) with the consideration of transverse resistance of the reinforcement is significant over and above the effect of axial resistance of the reinforcement. Parametric studies quantify the improvement in bearing capacity.

Introduction

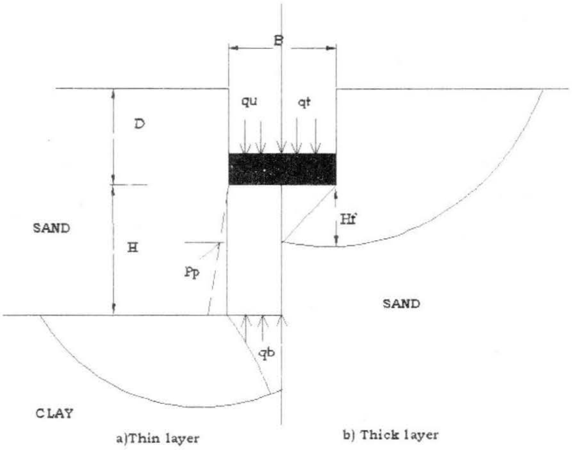
Low-lying coastal areas and deltas are often underlain by soft soils which hardly can support any structural or embankment loads. Similarly, construction of roads and highways on soils with low *CBR* is a very difficult and challenging task. One of the most commonly adopted alternatives to deal with these sites or situations, in recent times has been to provide a reinforced granular bed. The granular platform provides a strong base/layer and distributes the applied loads on to wider areas and facilitates increased loads to be applied on its top. Reinforcement of the granular layer with a geosynthetic further enhances the load carrying capacity of the system and reduces the required thickness of granular fill to achieve the same degree of effectiveness.

Literature Review

A compacted granular bed laid over soft soil improves the bearing capacity and settlement response of footings placed over the latter. The bearing capacity can be further improved by providing a geosynthetic layer at the interface of the soft clay and the fill. Meyerhof (1974) and Meyerhof and Hanna (1978) presented a punching mode of failure to estimate the bearing capacity of a footing on sand layer overlying clay (Figure 1) and compared the results with those of model tests on circular and strip footings. Yamanouchi and Gotoh (1979) proposed a formula for estimating the bearing capacity, q_u, of clay beds reinforced with polymer nets (Figure 2) as

$$q_u = \propto cN_c + T\left(\frac{2sin\theta}{B} + \frac{N_q}{R}\right) + \gamma' D_f N_q \qquad (1)$$

where α is the shape factor of the footing; c - undrained shear strength of clay; N_c & N_q - bearing capacity factors; T - mobilized tensile stress in the geosynthetic; B = width of the strip loading; γ' - unit weight of clay; D_f - depth of footing; θ - angle with the horizontal at which the tensile stress acts and R - radius of the imaginary circle.



a)Thin layer b) Thick layer

Fig. 1 Bearing capacity analysis for sand overlying clay (Meyerhof, 1974)

Burd & Frydman (1995) carried out a study using finite element and finite difference methods on the bearing capacity of sand overlying clay soils. The kinematic approach of limit analysis has been used by Michalowski & Shi (1995) to calculate the average limit pressure of footings over a two layer foundation soil. An upper bound solution to limit loads on strip footings over two-layer foundation soil has been presented by Michalowski (2002, 2004) considering two mechanisms

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of failure. Raghavendra et al. (1998) developed a simplified approach to the analysis of a reinforced soil bed as a two layered system. Lee et al. (1999) carried out numerical and model studies of strip footing on a reinforced granular fill-soft system with and without a layer of geotextile reinforcement at the interface. Kumar et al. (2006) suggested an approximate method to calculate the ultimate bearing capacity of a square footing resting on reinforced layered soil. Madhavi Latha and Somwanshi (2009) presented results from laboratory tests and numerical simulations on square footings resting on geosynthetic reinforced sand.

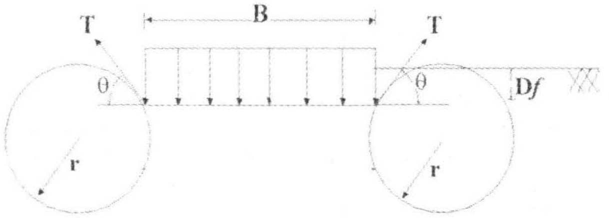


Fig. 2 Schematic diagram of the effect of geotextile (Yamanouchi et al., 1979)

Shivashankar et al. (1993) and Rethaliya and Verma (2009) consider the contributions to bearing capacity from stress distribution through the upper sand layer, shear layer effect (Figure 3a) and the membrane action of the reinforcement (Figure 3b). The ultimate bearing capacity, q_u, of a strip footing on sand overlying soft clay with geotextile reinforcement at the sand-clay interface is

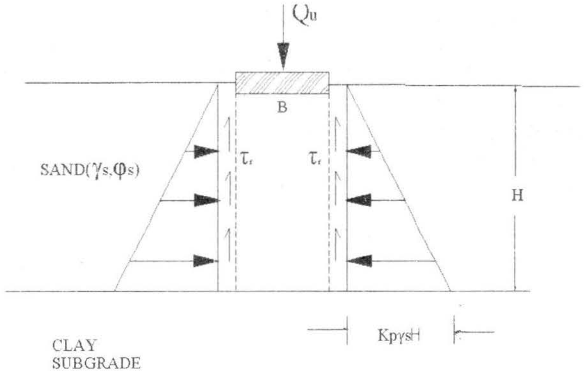
$$q_u + \frac{c_u N_c B_r}{B} + \frac{k_p \gamma_s H^2 \tan\varphi_s}{B} + \frac{2\gamma_s H \tan\varphi_r L_e \tan\varphi_s}{B} \tag{2}$$

where $B_r = B + 2\,H \tan\alpha$ is the increased width of loading on soft clay due to load spread; k_p - coefficient of passive earth pressure; γ_s - unit weight of sand; H - depth of sand layer; φ_s - angle of friction for sand; L_e - effective length of reinforcement.

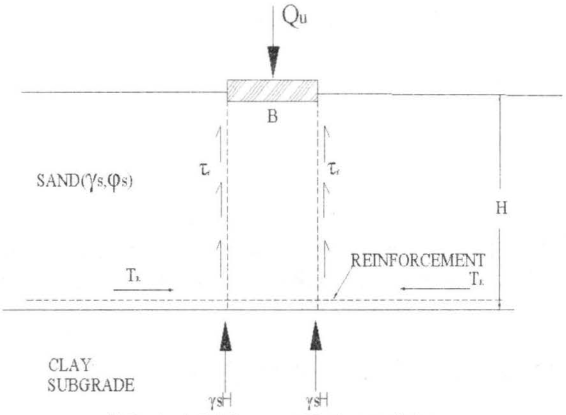
Bond Resistance of the Reinforcement

Axial Pullout

Figure 4 depicts the initial and deformed positions at the limit equilibrium state of the reinforcement layer of length, L_r during punching shear failure of a footing. The reinforcement originally laid out horizontally, gets deformed into the new position defined by *ABB'C'CD*.



a) Shear layer effect



b) Tension in Reinforcement (Membrane action)

Fig. 3 Mechanics of (a) Shear layer effect and (b) Membrane action of the reinforcement for a strip footing (Shivashankar et al. 1993 and Rethaliya and Verma, 2009)

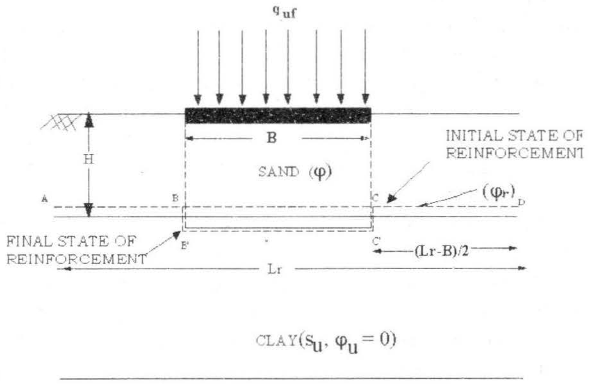


Fig. 4 Displacement of reinforcement due to punching shear failure of footing

Figures 5a and 5b show the stresses developed on the sand column and in the reinforcement due to the punching shear failure of the footing. The interface/bond resistance of the reinforcement layer is φ_r. It is assumed that axial tensile force gets developed in the reinforcement layer due to interface shear resistance at the interface. The axial tensile force developed in the reinforcement layer due to the frictional developed on either side of the reinforcement layer at the interface with the soil is

$$T_r = 2\frac{\gamma H}{B} tan\varphi_r \frac{(L_r - B)}{2} \qquad (3)$$

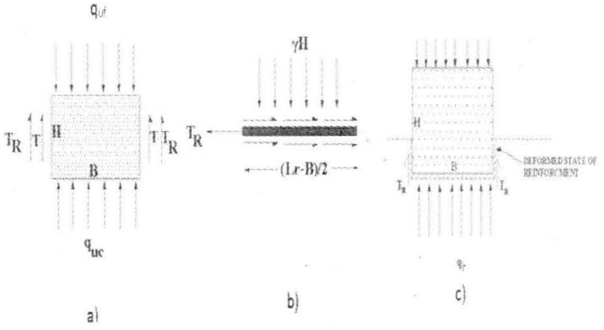


Fig. 5 Stresses on (a) Sand Column, (b) Reinforcement and (c) Additional Forces due to Kinematics (Transverse Displacement)

Kinematics and Transverse Pull

The kinematics of failure of a two-layered soil usually adopted for the estimation of bearing capacity considers punching type of failure. The column of granular material along with the strip footing moves down mobilizing shear resistance along its sides. The axial pull resulting from this movement is considered and accounted for by several researchers (Shivashankar et al. 1993, Rethaliya and Verma 2009, etc.). The effect of downward movement causes the geosynthetic reinforcement to be pulled down. Any transverse movement (Figure 5c) causes additional stresses to be mobilized underneath the reinforcement (Madhav and Umashankar 2003). The idealized deformed shape of the reinforcement layer and the additional stresses developed on the sand column due to the transverse displacement of the reinforcement are shown in Figure (5c). A transverse displacement, δ (= w_L), of the reinforcement layer at the edge of the footing is considered to estimate the additional resistance mobilized. A resisting force, P, gets mobilized as a result of the transverse displacement, δ, of the reinforcement. The pullout force in the reinforcement increases due to the transverse displacement. The tension mobilized in the reinforcement gets modified due to the additional normal force, P, as

$$T_a = 2\gamma H tan\varphi_r + P tan\varphi_r \qquad (4)$$

where P is the transverse force in the reinforcement developed due to transverse component of displacement, δ. The upward resisting force at the intersection, P is estimated from the relationship

$$P = \gamma H L_e P^* \qquad (5)$$

where P^* is the normalized transverse force in the reinforcement obtained from the analysis developed by Madhav and Umashankar (2003) for single inextensible sheet of reinforcement of length, L_e, embedded at a depth, H, in backfill of unit weight, γ. The interface shear resistance between the reinforcement and the soil is characterized by the angle, ϕ_r ($\leq \phi$, the angle of shearing resistance of the soil).

In a backfill of global relative stiffness, μ (= $k_s L/\gamma H$), the inextensible sheet reinforcement is subjected to transverse force, P, due to transverse displacement, w_L, in addition to the normal stresses acting on the top due to overburden pressure. The soil below the reinforcement generates additional normal stresses due to downward displacement, w_L. The response is represented by a set of Winkler type springs. The normalized tension, T^*_k, and normalized displacement, W_k, are evaluated by solving two coupled vertical and horizontal force equilibrium equations (Madhav and Umashankar, 2003), as

$$T^*_{k+1} = T^*_k + \frac{1}{2}\left(\mu \frac{w_0}{L} W_k + 2\right) \qquad (6)$$

$$W_k = \frac{T^*_k n^2 (W_{k+1} + W_{k-1})}{\left(2n^2 T^*_k + \frac{\mu}{2 tan\, \varphi_r}\right)} \qquad (7)$$

where k_s - modulus of subgrade reaction of backfill; n - the number of elements the reinforcement is divided into; W (= w/w_L) - the transverse displacement of reinforcement at any point normalized with w_L (the transverse displacement of reinforcement at free end); μ = relative subgrade stiffness factor; T^* (= T_d $/2\gamma D_e L \tan\phi_r$) - the normalised tension developed in the reinforcement and T_d - the ultimate tension developed in the reinforcement. The normalized transverse force, P^*, can be computed (Madhav and Umashankar, 2003), as

$$P^* = \frac{P}{\gamma H L_e} = \mu \frac{w_0}{L_e}\frac{1}{n}\left(\frac{w_1+1}{2} + \sum_{k=2}^{n} W_k\right) \qquad (8)$$

For a given displacement of the reinforcement, the response of the reinforcement is obtained by solving the coupled equations (6) and (7). The corresponding normalized transverse force mobilized due to a given displacement is obtained from Eq. (8).

Problem Definition and Formulation

A strip footing of width, B, rests on the surface of a sand stratum of thickness, H, overlying clay deposit with geosynthetic reinforcement laid at the clay-sand interface. The unit weight and the angle of shearing resistance of granular stratum are γ and φ respectively while s_u is the undrained shear strength of soft ground and φ_r is the interface/bond resistance between geosynthetic layer and the granular fill.

Method of Analysis

The bearing capacity, q_{ug}, of a footing at the surface of the granular bed of finite thickness, *H*, overlying soft soil (Meyerhof 1974) is

$$q_{ug} = s_u N_c + \frac{\gamma H^2}{B} K_s tan\varphi' \quad (9)$$

where S_u is the undrained shear strength of the soft soil; *H* - thickness of the granular layer; φ' - angle of shearing resistance of the granular layer; *D* - depth of the footing; *B* - width of the footing; γ - unit weight of the sand, and K_s - coefficient of punching shear. The bearing capacity of a footing on the above two-layered soil is limited by the ultimate bearing capacity of the granular layer of infinite extent as

$$q_{ug} = \gamma D N_q + 0.5\gamma B N_\gamma \quad (10)$$

where N_q and N_γ are the bearing capacity factors. Eq. (9) is normalised with undrained shear strength, 's_u' to get the equivalent bearing capacity factor, N_{cg}, for a two-layered soil as

$$N_{c,g} = N_c + \frac{\gamma B}{s_u}\left(\frac{H}{B}\right)^2 K_s\, tan\varphi' \quad (11)$$

$N_{c,g}$ combines the contributions of the two layers, the soft clay and the overlying granular fill to the bearing capacity of the footing.

Bearing Capacity of Reinforced Granular Bed on Soft Soil

The bearing axial capacity of the reinforced granular bed of soft soil can be obtained by summing the bearing capacity of an unreinforced granular bed overlying soft ground and the contribution of the tensile force mobilized in the reinforcement layer considering that only the lengths of reinforcement beyond the edge of the footing contribute, as

$$q_{ur}{}^* = s_u N_c + \frac{\gamma H^2}{B} K_s tan\varphi' + \frac{\gamma H}{B} tan\varphi_r (L_r - B) \quad (12)$$

It should be noted that even though the block of soil below the footing moves vertically down pulling the reinforcement with it, only the axial pullout force mobilized is considered by Shivashankar et al. (1993) and Rethaliya and Verma (2009). Normalising Eq. (12) by the undrained shear strength, 's_u' one gets

$$N_{cr}{}^* = N_c + \frac{\gamma B}{s_u}\left(\frac{H}{B}\right)^2 K_s tan\varphi' + \frac{\gamma B}{s_u}\frac{H}{B} tan\varphi_r\left(\frac{L_r}{B} - 1\right) \quad (13)$$

where $N_{cr}{}^* = q_{ur}{}^*/s_u$.

Consideration of Kinematics - Transverse Displacement

The mechanics of failure proposed by Meyerhof (1974) consists of a block of granular material with width equal to that of the footing and thickness equal to that of the granular layer, moving down as it punches in to the soft underlying soil. The reinforcement sheet is pulled inward and mobilizes interface shear resistance. In the currently available methods of estimating bearing capacity of the footing on reinforced granular bed, only the axial component of pullout resistance is considered and incorporated. However, importantly, the block of soil pulls the reinforcement downward along with it. This movement is transverse to the orientation of the reinforcement. Therefore, the analysis of Madhav and Umashankar (2003) is incorporated additionally.

The transverse force mobilized depends on the magnitude of transverse displacement, the stiffness of the ground, the effective length of reinforcement beyond the footing width and the interface frictional resistance. The additional force in the reinforcement due to transverse displacement is estimated based on Eq. (5). The bearing capacity of the reinforced granular bed overlying soft soil is then obtained by adding the shear resistance of the granular bed, the mobilized tensile force in the reinforcement layer and the additional transverse force developed because of the transverse displacement (combining Eqs. 4 and 5) as

$$\begin{aligned} q_{ur}{}^{**} = s_u N_c + \frac{\gamma H^2}{B} K_s tan\varphi' + \frac{\gamma H}{B} tan\varphi_r (L_r - B) \\ + \frac{\gamma H}{B} tan\varphi_r (L_r - B) T^* \\ + \frac{\gamma H}{B}\frac{(L_r - B)}{2} P^* \end{aligned} \quad (14)$$

Normalising Eq. (14) by undrained shear strength, 's_u' one gets

$$N_{cr}{}^{**} = N_c + \frac{\gamma B}{s_u}\left(\frac{H}{B}\right)^2 K_s tan\varphi' + \frac{\gamma B}{s_u}\frac{H}{B}\left(\frac{L_r}{B} - 1\right)\left\{tan\varphi_r(1 + T^*) + \frac{P^*}{2}\right\} \quad (15)$$

where $N_{cr}{}^{**} = q_{ur}{}^{**}/s_u$

The bearing capacity ratios, BCR, are defined as

$(BCR)_{ug} = N_{c,g}/N_c$ is the ratio of bearing capacity of the unreinforced two layered system to that of footing on clay alone. This ratio quantifies the contribution of the granular layer to the bearing capacity of the footing.

$(BCR)_{ax} = N_{cr}^*/N_c$ is the ratio of bearing capacity of the reinforced two layered system (considering the effect of axial tension only) to that of footing on clay alone. This ratio quantifies the contributions of the granular layer and the axial tension mobilized in the reinforcement to the overall bearing capacity of the footing.

$(BCR)_{tr} = N_{cr}^{**}/N_c$ is the ratio of bearing capacity of the reinforced two layered system (considering kinematics, i.e., the effect of transverse force in addition to axial tension) to that of footing on clay alone. $(BCR)_{tr}$ quantifies, in addition, the contribution of the transverse force mobilized as a consequence of kinematics, i.e., transverse displacement of the footing and the soil below, over and above the contributions of granular layer and the axial force mobilized in the reinforcement to the bearing capacity of the footing.

$(BCR)_{ax}^* = N_{cr}^*/N_{c,g}$ is the ratio of bearing capacity of the reinforced two layered system (considering the effect of axial tension only in the reinforcement) to that of an unreinforced two-layered system.

$(BCR)_{tr}^* = N_{cr}^{**}/N_{c,g}$ is the ratio of bearing capacity of the reinforced two-layered system (considering kinematics, i.e., the effect of transverse force mobilized in addition to axial tension) to that of footing on unreinforced system. $(BCR)_{ax}^*$ and $(BCR)_{ax}^{**}$ quantify the improvement of bearing capacities of the two-layered system due to axial force alone and the axial and transverse forces in the reinforcement.

The bearing capacity of a footing resting on reinforced granular bed overlying a homogeneous clay layer, depends on the angle of shearing resistance, φ, and H/B related to the granular layer, $\gamma B/s_u$, to the ratio of the unit weight of granular fill times the width of the footing to the undrained strength of the clay layer, the ratio φ_r/φ, related to the bond strength relative to the angle of shearing resistance of the granular layer, L_r/B, the relative length of the reinforcement for axial tension and transverse force mobilized in the reinforcement and W_O, the normalized transverse displacement.

Parametric study is carried out for the following ranges of parameters: $\gamma B/s_u$: 5 to 35; and H/B: 0 to 5.0. The computations are made for φ equal to 35^0, L_r/B of 3.0, φ_r/φ equal to 0.75 and for W_O equal to 0.003. The present paper highlights the contribution of transverse force in the reinforcement for a normalized transverse displacement of 0.003. For a footing of 1.0 m width with a reinforcement layer of length, 3.0 m, the actual displacement works out to be about 3 mm which is adequate for the mobilization of transverse shear stresses in the granular layer. The paper investigates the contributions of these parameters.

RESULTS AND DISCUSSION

Variation of $N_{c,g}$, N_{cr}^* and N_{cr}^{**} with H/B - Effect of $\gamma B/s_u$

Figure 6 presents the variation of normalized bearing capacity factor of an unreinforced two layer system, $N_{c,g}$, on a log scale with normalised granular layer thickness, H/B, for a granular fill with φ of 35^0, for values of $\gamma B/s_u$ increasing from 5.0 to 35.0. $N_{c,g}$ increases sharply with H/B, for different values of $\gamma B/s_u$, up to a critical value of H/B denoted by $(H/B)_{cr}$. The rate of increase of $N_{c,g}$ with H/B increases with increase of $\gamma B/s_u$. $N_{c,g}$ increases from 6.2 at H/B equal to 0.4 to 83.1, at H/B equal to 3.5, for $\gamma B/s_u$ equal to 5.0. $N_{c,g}$ attains the maximum value of 93 at H/B equal to 3.8. The corresponding values of $N_{c,g}$ at H/B equal to 0.4 and 3.5 for $\gamma B/s_u$ equal to 35 are respectively 12.1 and 445.2. The maximum value of $N_{c,g}$ increases to 650.8 at H/B equal to 4.25, for $\gamma B/s_u$ of 35.0 (Figure 6).

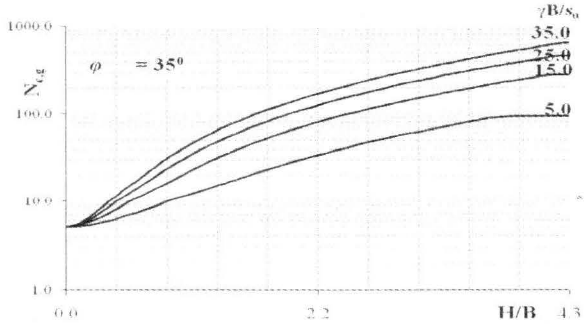


Fig. 6 $N_{c,g}$ versus H/B- Effect of $\gamma B/s_u$

While the increase in $N_{c,g}$ with $\gamma B/s_u$ is significant, the increase in $(H/B)_{cr}$ with $\gamma B/s_u$ is marginal. For $H/B > (H/B)_{cr}$, $N_{c,g}$ remains constant as it is limited by the ultimate bearing capacity of the footing on the granular layer. The shear layer contributes to the bearing capacity of an unreinforced two layer system in addition to that of clay alone. The maximum bearing capacity and the relative granular layer thickness required to attain the maximum bearing capacity is more for relatively soft clays or relatively wide footings, i.e., at higher values of $\gamma B/s_u$.

Figure 7 depicts the variation of normalized bearing capacity of a reinforced two layer system, considering axial resistance of reinforcement to pullout, N_{cr}^*, for a granular fill with φ of 35^0, φ_r/φ of 0.75 and L_r/B of 3.0, for values of $\gamma B/s_u$ increasing from 5.0 to 35.0 and Figure 8 illustrates the variation of normalized bearing capacity, N_{cr}**, of a reinforced two layer system considering both the axial and transverse resistances of reinforcement to pullout, with *H/B*, for a granular fill with φ of 35^0, φ_r/φ of 0.75, W_0 of 0.003 and L_r/B of 3.0, for values of $\gamma B/s_u$ increasing from 5.0 to 35.0. N_{cr}* and N_{cr}** values increase sharply with increase in *H/B*, for different values of $\gamma B/s_u$, up to a critical value of *H/B* denoted by $(H/B)_{cr}$ as it is limited by the ultimate bearing capacity of the granular layer. For $H/B > (H/B)_{cr}$, the values remain constant.

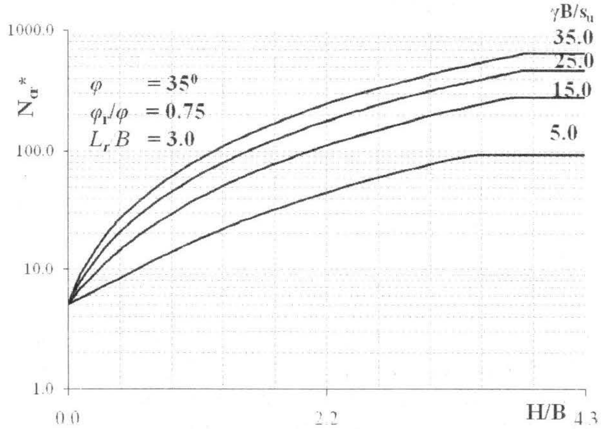


Fig. 7 N_{cr}* versus *H/B*- Effect of $\gamma B/s_u$

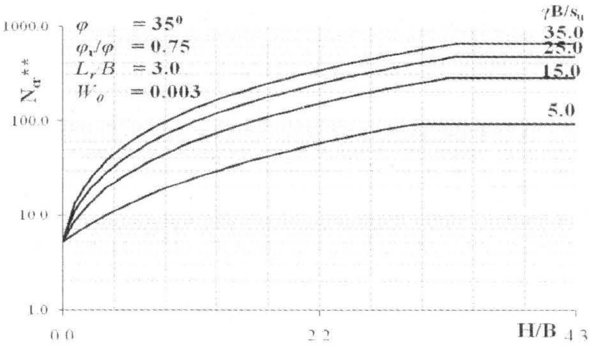


Fig. 8 N_{cr}** versus *H/B*- Effect of $\gamma B/s_u$

N_{cr}* increases from 8.1 at *H/B* equal to 0.4 to a maximum value of 93.0, at *H/B* equal to 3.4, for $\gamma B/s_u$ equal to 5.0. The corresponding values of N_{cr}* at *H/B* equal to 0.4 and 3.4 for $\gamma B/s_u$ equal to 35 are respectively 27.3 and 543.4. The maximum value of N_{cr}* increases to 650.8 at *H/B* equal to 3.8, for $\gamma B/s_u$ of 35.0 (Figure 7). While the increase in N_{cr}* with $\gamma B/s_u$ is significant, the increase in $(H/B)_{cr}$ with $\gamma B/s_u$ is marginal.

N_{cr}** increases from 10.5 at *H/B* equal to 0.4 to a maximum value of 93.0, at *H/B* equal to 3.0, for $\gamma B/s_u$ equal to 5.0. The corresponding values of N_{cr}** at *H/B* equal to 0.4 and 3.0, for $\gamma B/s_u$ equal to 35 are respectively 45.9 and 564.6. The maximum value of N_{cr}** increases to 650.8 at *H/B* equal to 3.4, for $\gamma B/s_u$ of 35.0 (Figure 8). While the increase in N_{cr}** with $\gamma B/s_u$ is significant, the increase in $(H/B)_{cr}$ with $\gamma B/s_u$ is marginal.

In a reinforced two layer system, the contribution from the resistance of reinforcement to pullout adds to that from the shear and clay layers and thus the relative thickness of granular layer required to attain the maximum bearing capacity becomes less than that in an unreinforced system. The granular layer thickness required to attain the maximum bearing capacity considering axial resistance is less than that considering an unreinforced system, due to the contribution of axial tension in reinforcement to pullout. The granular layer thickness required to attain the maximum bearing capacity considering transverse resistance of reinforcement to pullout in addition to the axial resistance, is much less than that considering axial resistance alone, due to the additional contribution of the transverse resistance of reinforcement.

Variation of $(BCR)_{ug}$, $(BCR)_{ax}$ and $(BCR)_{tr}$ with *H/B* - Effect of $\gamma B/s_u$

The *BCR* response in terms of $(BCR)_{ug}$ for φ of 35^0, for $\gamma B/s_u$ = 5.0, 15.0, 25.0 and 35.0, with normalized granular layer thickness *H/B*, is presented in Figure 9, to illustrate the effect of granular layer in an unreinforced system. The shear layer improves the *BCR* response of the footing significantly. The value of *BCR* increases sharply from 1.0, which is equal to the bearing capacity of clay layer alone, to a maximum value, due to the contribution of the granular layer, at a critical value of normalized granular layer thickness, referred as $(H/B)_{cr}$, beyond which *BCR* values are constant (Figure 9). $(H/B)_{cr}$ and *BCR* increase with increase in $\gamma B/s_u$, indicating an increase in *BCR* value with decreasing undrained shear strength of clay.

$(BCR)_{ug}$ increases from 1.45 at *H/B* equal to 0.6 to 12.1, at *H/B* equal to 3.0, for $\gamma B/s_u$ equal to 5.0. $(BCR)_{ug}$ attains the maximum value of 18.1, at $(H/B)_{cr}$ equal to 3.8. The corresponding values of $(BCR)_{ug}$ at *H/B* equal to 0.6 and 3.0, for $\gamma B/s_u$ equal to 35 are respectively 3.5 and 64.7. The maximum value of $(BCR)_{ug}$ increases to 126.6 at $(H/B)_{cr}$ equal to 4.25, for $\gamma B/s_u$ of 35.0 (Figure 9). While the increase in $(BCR)_{ug}$ with $\gamma B/s_u$ is significant, the increase in $(H/B)_{cr}$ with $\gamma B/s_u$ is marginal.

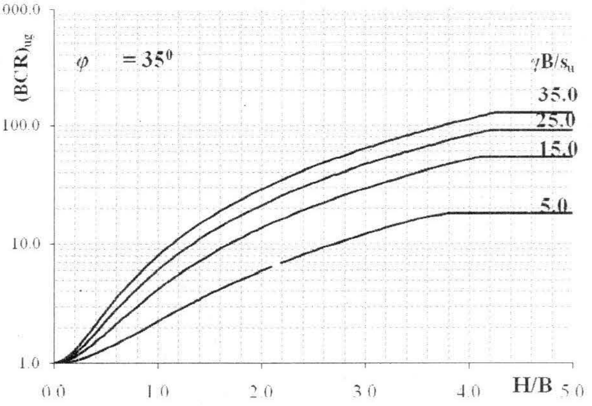


Fig.9 *(BCR)*$_{ug}$ versus *H/B*- Effect of $\gamma B/s_u$

The *BCR* response profiles in terms of $(BCR)_{ax}$ and $(BCR)_{tr}$, for φ of 35^0, φ_r/φ of 0.75 and L_r/B of 3.0, for $\gamma B/s_u$ = 5.0, 15.0, 25.0 and 35.0, with normalized granular layer thickness *H/B*, are presented in Figures 10 and 11 respectively to illustrate the effect of axial tension in reinforcement (Figure 10) and the effect of transverse coupled with axial tension in reinforcement (Figure 11) in a reinforced two layered system. The computations for $(BCR)_{tr}$ are made for a transverse displacement W_0 equal to 0.003. The reinforcement layer improves the *BCR* response further, due to axial tension in the reinforcement (Figure 10). The tensile forces in the reinforcement are dependent on the interfacial shear forces mobilized. The consideration of transverse tension in addition to the axial tension in reinforcement layer improves the *BCR* response beyond that obtained by considering axial tension in reinforcement alone (Figure 11).

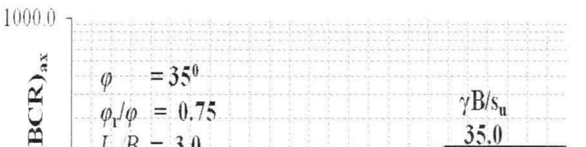


Fig. 10 $(BCR)_{ax}$ versus *H/B*- Effect of $\gamma B/s_u$

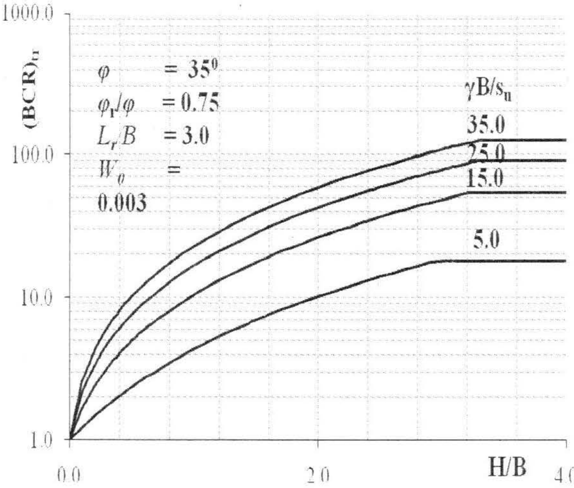


Fig. 11 $(BCR)_{tr}$ versus *H/B*- Effect of $\gamma B/s_u$

$(BCR)_{ax}$ increases from 2.0 at *H/B* equal to 0.6 to 15.0, at *H/B* equal to 3.0, for $\gamma B/s_u$ equal to 5.0. $(BCR)_{ax}$ attains the maximum value of 18.1 at $(H/B)_{cr}$ equal to 3.44. The corresponding values of $(BCR)_{ax}$ at *H/B* equal to 0.6 and 3.0 for $\gamma B/s_u$ equal to 35 are respectively 7.5 and 85.0. The maximum value of $(BCR)_{ax}$ increases to 126.6 at $(H/B)_{cr}$ equal to 3.8, for $\gamma B/s_u$ of 35.0 (Figure 10). While the increase in $(BCR)_{ax}$ with $\gamma B/s_u$ is significant, the increase in $(H/B)_{cr}$ with $\gamma B/s_u$ is marginal.

$(BCR)_{tr}$ increases from 2.7 at *H/B* equal to 0.6 to 13.3, at *H/B* equal to 2.4, for $\gamma B/s_u$ equal to 5.0. $(BCR)_{tr}$ attains the maximum value of 18.1 at $(H/B)_{cr}$ equal to 3.0. The corresponding values of $(BCR)_{tr}$ at *H/B* equal to 0.6 and 2.4 for $\gamma B/s_u$ equal to 35 are respectively 12.5 and 77.1. The maximum value of $(BCR)_{tr}$ increases to 126.6 at $(H/B)_{cr}$ equal to 3.3, for $\gamma B/s_u$ of 35.0 (Figure 10). While the increase in $(BCR)_{tr}$ with $\gamma B/s_u$ is significant, the increase in $(H/B)_{cr}$ with $\gamma B/s_u$ is marginal.

Figure 12 depicts the variation of $(BCR)_{ax}$ and $(BCR)_{tr}$ with *H/B* in comparision to that of $(BCR)_{ug}$, for $\gamma B/s_u$ of 15.0, φ of 35^0, φ_r/φ of 0.75, W_0 of 0.003 and L_r/B = 3.0. *BCR* values increase from 1.0, which represents the bearing capacity of clay alone, to a maximum value. The values of *BCR* equal 2.2, 4.0 and 6.4, at *H/B* equal to 0.6, for an unreinforced system, reinforced two layered system considering axial resistance of reinforcement to pullout only and that considering transverse and axial resistance of reinforcement to pullout repectively. The corresponding values at *H/B* equal to 2.5 are equal to 20.7, 27.9 and 36.7, for an unreinforced system, reinforced two layered system considering axial resistance of reinforcement to pullout only and that considering transverse and axial resistance of reinforcement to pullout repectively.

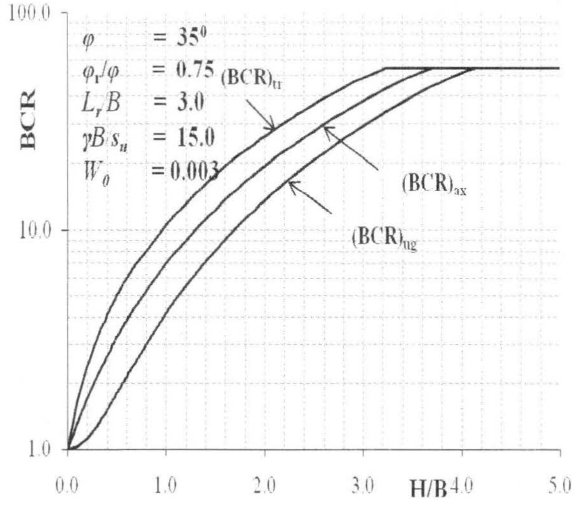


Fig. 12 ***BCR*** **versus** ***H/B*****- Effect of reinforcement**

At the maximum *BCR* value of 54.3, the value of $(H/B)_{cr}$ decreases from 4.1 for an unreinforced system, to 3.6 for a reinforced two layered system considering axial resistance of reinforcement to pullout only and 3.24 for a reinforced two layered system considering transverse and axial resistances of reinforcement to pullout. A thicker shear layer is needed to attain the maximum value of *BCR* in an unreinforced two layered system, while a relatively thinner granular layer is adequate in a reinforced two layered system, due to the contribution of axial tension in reinforcement. The thickness of shear layer required further lessens, when the contribution of transverse tension of reinforcement is considered in addition to that of axial tension.

Variation of $(BCR)_{ax}$* and $(BCR)_{tr}$ * with *H/B* - Effect of $\gamma B/s_u$

Figure 13 depicts the variation of normalized bearing capacity ratio profiles of a reinforced two layer system, considering axial resistance of reinforcement to pullout, $(BCR)_{ax}$* for a granular fill with φ of 35⁰, φ_r/φ of 0.75 and L_r/B of 3.0, for values of $\gamma B/s_u$ increasing from 5.0 to 35.0. $(BCR)_{ax}$* increases from 1.0 at $H/B = 0$ to a maximum value at a critica granular layer thickness, $(H/B)_{cr}$. $(BCR)_{ax}$* value equals 1.2, 1.5, 1.8 and 2.1 at $H/B = 0.2$, for $\gamma B/s_u$ equal to 5.0, 15.0, 25.0 and 35.0 respectively. The maximum value equals 1.43 at *H/B* equal to 0.9, for $\gamma B/s_u$ equal to 5.0 and 2.27 at *H/B* equal to 0.4, for $\gamma B/s_u$ equal to 35.0. Beyond $(H/B)_{cr}$, the *BCR* response shows a gradual drop, as the effect of reinforcement diminishes. At *H/B* values equal to 3.4 and 3.8, for $\gamma B/s_u$ equal to 5.0 and 35.0 respectively, a sharp decrease in *BCR* response is reflected as the effect of reinforcement further declines to zero (Figure 13). While the increase in $(BCR)_{ax}$* with increase in $\gamma B/s_u$ is significant, the decrease in $(H/B)_{cr}$ with increase in $\gamma B/s_u$ is marginal.

Figure 14 illustrates the variation of and normalized bearing capacity ratio profiles of a reinforced two layer system considering the axial and transverse resistance of reinforcement to pullout, $(BCR)_{tr}$* with *H/B*, for a granular fill with φ of 35⁰, φ_r/φ of 0.75, W_0 of 0.003 and L_r/B of 3.0, for values of $\gamma B/s_u$ increasing from 5.0 to 35.0.

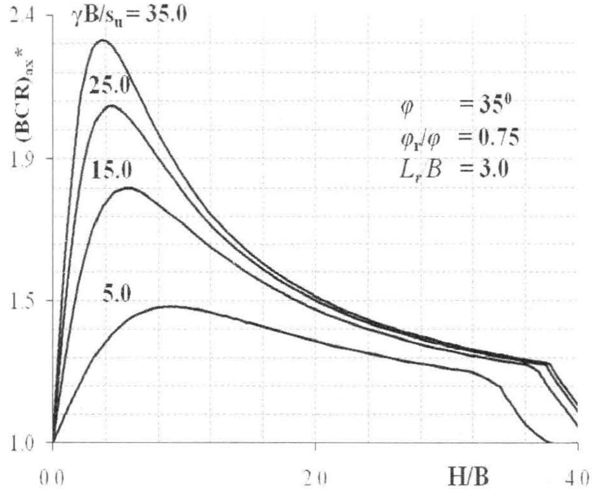


Fig. 13 $(BCR)_{ax}$*** versus** ***H/B*****- Effect of** $\gamma B/s_u$

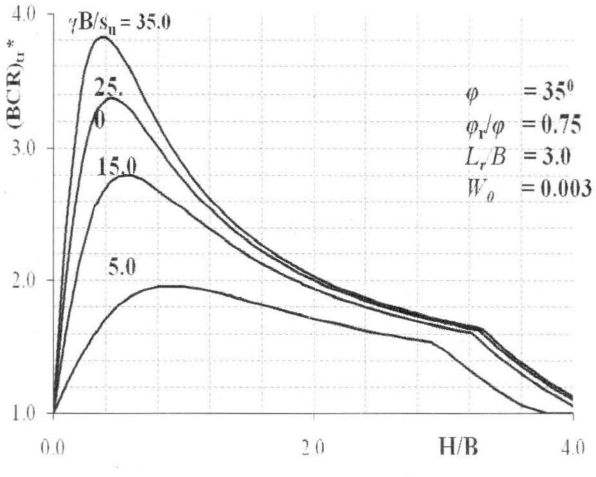


Fig. 14 $(BCR)_{tr}$*** versus** ***H/B*****- Effect of** $\gamma B/s_u$

$(BCR)_{tr}$* increases from 1.0 at $H/B = 0$ to a maximum value at a critica granular layer thickness, $(H/B)_{cr}$. $(BCR)_{tr}$* value equals 1.4, 2.1, 2.8 and 3.3 at $H/B = 0.2$, for $\gamma B/s_u$ equal to 5.0, 15.0, 25.0 and 35.0 respectively. The maximum value of $(BCR)_{tr}$* equals 1.96 at *H/B* equal to 0.9, for $\gamma B/s_u$ equal to 5.0 and 3.82 at *H/B* equal to 0.4, for $\gamma B/s_u$ equal to 35.0. Beyond $(H/B)_{cr}$, the *BCR* response shows a gradual drop, as the effect of reinforcement diminishes. At *H/B* values equal to 2.9 and 3.3, for $\gamma B/s_u$ equal to 5.0 and 35.0

respectively, a sharp decrease in *BCR* response is reflected as the effect of reinforcement further declines to zero (Figure 14). While the increase in $(BCR)_{tr}^*$ with increase in $\gamma B/s_u$ is significant, the decrease in $(H/B)_{cr}$ with increase in $\gamma B/s_u$ is marginal.

BCR values increase with increase in *H/B*, for specific values of $\gamma B/s_u$. The increase in *BCR* values is more at higher values of $\gamma B/s_u$ i.e., for relatively soft clays. The values reduce to unity with increase in *H/B*, beyond the critical value. An increase in granular layer thickness beyond the relative $(H/B)_{cr}$ values, results in development of thicker failure zone above the reinforcement layer, as a result of which the contribution of the axial and transverse tension in reinforcement to the ultimate bearing capacity gradually becomes relatively less leading to a decrease in bearing capacity ratio values. The contribution of reinforcement to bearing capacity of the footing becomes zero as the reinforced granular system functions as an unreinforced one.

Variation of $(H/B)_{cr}$ with $\gamma B/s_u$ - Effect of reinforcement

The variation of $(H/B)_{cr}$ with $\gamma B/s_u$ is presented in Figure 15. $(H/B)_{cr}$ value increases from 3.8 at $\gamma B/s_u$ equal to 5.0 to 4.25 at $\gamma B/s_u$ equal to 35.0, for an unreinforced system. $(H/B)_{cr}$ value increases from 3.4 at $\gamma B/s_u$ equal to 5.0 to 3.79 at $\gamma B/s_u$ equal to 35.0, for a reinforced system (considering axial tension in reinforcement) and from 3.0 at $\gamma B/s_u$ equal to 5.0, to 3.4 at $\gamma B/s_u$ equal to 35.0, for a reinforced system (considering transverse and axial tension in reinforcement). A decrease in value of $(H/B)_{cr}$ is observed in a reinforced system considering axial tension in reinforcement, when compared to that of an unreinforced one. Consideration of transverse tension in addition to the axial tension in reinforcement in a reinforced two layered system decreases the value of $(H/B)_{cr}$ further.

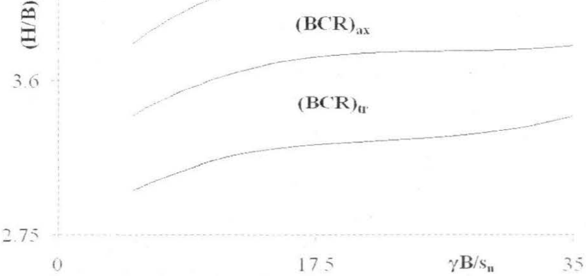


Fig. 15 $(H/B)_{cr}$ versus $\gamma B/s_u$ - Effect of reinforcement

CONCLUSIONS

Reinforced granular beds are finding common application for foundations and beneath embankments on soft ground. The paper presents a new method of estimating the bearing capacity of a footing founded on the reinforced foundation bed overlying soft ground incorporating kinematics of failure. The basis for the method is the punching mode of failure proposed by Meyerhof (1974) for dense sand overlying soft clay. A block of granular fill bounded by the footing is assumed to punch into soft ground. The reinforcement laid near the bottom of the granular fill moves down along with the block of granular fill punching through. Conventionally, only axial pull is assumed to get mobilized in the granular fill even though the movement of reinforcement is transverse to its alignment. In this paper, the theory proposed by Madhav and Umashankar (2003) is incorporated to account for the additional resistances mobilized in the reinforcement due to transverse displacement. The transverse force generated contributes to additional bond resistance in the reinforcement over and above the one due to axial pull. Thus the bearing capacity of the foundation on reinforced granular fill over soft ground is the sum of bearing capacity to undrained strength of soft ground, shear resistance mobilized in the dense granular fill, the axial resistance in the reinforcement and the additional resistances mobilized therein due to kinematics (transverse force and additional bond resistance in the reinforcement). A new normalized bearing capacity factor that incorporates the above mechanics and the corresponding bearing capacity ratios (*BCR*) have been defined and estimated for different cased and compared for a nominal displacement of 0.33% of footing width. Results indicate significant improvement in bearing capacity of the footing over the conventionally estimated values due to consideration of kinematics of failure, i.e., transverse displacement of the block of granular fill and the reinforcement.

A parametric study quantifies the contributions of relative thickness of granular fill, *(H/B)*, and the normalised parameter, $\gamma B/s_u$, on the normalised bearing capacity factors and *BCR*s. $(BCR)_{ax}$ and $(BCR)_{tr}$ increase with *H/B*, for different $\gamma B/s_u$ until a maximum value is attained, at $(H/B)_{cr}$. The rate of increase in values of $(BCR)_{ax}$ and $(BCR)_{tr}$ improves at higher values of $\gamma B/s_u$ i.e., for realtively soft clays or reatively wide footings. It is therefore established that reinforcement in dense granular fills subjected to an additional transverse pull generates pullout resistances larger than purely axial pull-out capacity in reinforced earth systems.

$(BCR)_{ax}^*$ and $(BCR)_{tr}^*$ values increase with *H/B*, for different values of $\gamma B/s_u$. $(H/B)_{cr}$ shows a decreasing trend with increase in $\gamma B/s_u$. Consideration of transverse tension results in an improved *BCR* response when compared to a reinforced two layered system, where only axial tension is considered. The increase in *BCR* is significant until a maximum value is attained at $(H/B)_{cr}$. Consideration of kinematics and transverse displacement of the reinforcement thus establishes that the bearing capacity of the reinforced foundation bed attains the maximum possible value corresponding to the failure mechanism wholly within the granular fill (according to Meyerhof, 1974) at a much smaller thicknees of the granular bed than that based on consideration of axial pull alone. Considerable savings in granular fill material can be achieved by the consideration of kinematics of failure.

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