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Excavation Induced Building Response by Laminate Beam Method

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Key words

Cracking of building, Differential settlement, Strain energy of shear, Laminate beam, Permissible deflection ratio, Angular distortion, Case study

Abstract: Prediction of building damage or cracking due to ground settlement during braced excavation is important in urban areas. Reasonably good amount of work on numerical analysis and empirical methods on the prediction of building damage potential have been reported in the literature. However, theoretical studies or mathematical modeling have not been well addressed in the literature. Here laminate beam method, as available in the literature for estimating building response induced by excavation, is modified using strain energy concept. Building is considered as simply supported beam with load concentrated at centre of the beam. A deformation profile of the ground surface near excavation is generated which is hogging type in nature. Two equations relating bending and shear stiffness of a building to critical deflection ratio are derived. Deflection of beam due to shear is calculated by using the strain energy of shear. Conventionally deflection due to shear is calculated considering deflection curve of beam where it is assumed that the beam is free to warp everywhere. This may not be valid for neighbourhood of plane middle section. But in strain energy approach such assumptions are not required. The proposed method is used to estimate the response of three multi-storied buildings adjacent to northern stretches of Kolkata Metro Construction.

Introduction

Building damage adjacent to any kind of excavation (Figure 1) is a major design consideration in urban areas. In spite of support system, excavation leads to some ground movements and any building within the zone of influence is likely to be affected. It is hence necessary to predict building damage for preventing any adverse effect due to excavation induced ground movement. A number of methods are used for calculating building damage potential associated with ground movement. Most of these methods consist of estimating critical differential settlement of a structure due to self weight (Skempton and McDonald, 1956; Polshin and Tolkar, 1957). Burland and Wroth (1975) proposed deep beam method and modeled a building as a deep isotropic beam to relate strains in the building to imposed deformations. Boscardin and Cording (1989) extended the deep beam model and considered horizontal extension strains (ε_h) for buildings with load-bearing brick walls caused by lateral ground movements due to adjacent excavation and tunnelling. Boone (1996) presented another approach to evaluate building damage due to differential ground movement caused by adjacent excavation considering structure geometry & design, strain superposition and the critical strains of building materials. Voss (2002) extended Burland and Wroth (1975) equation assuming building as a simply supported beam with load concentrated at mid point and related limiting deflection ratio with bending strain at the top and bottom of the beam. Voss (2003) and Finno et. al. (2005) used a complimentary virtual work approach to determine the strain deflection relationship of a laminate beam in terms of bending strain to deflection ratio (δ/L) and shear strain to deflection ratio (δ/L). In this method deflection due to shear stress is derived considering general deflection curve assuming that all cross sections are free to warp. But from the condition of symmetry, middle section must remain plane while adjacent sections carrying a shear force $P/2$. From continuity of deformation the abrupt change from plain middle section to warped adjacent section is unlikely (Timoshenko and Young, 1968). From this consideration neighbourhoods of the plane middle section cannot be free to warp and so the normal stress distribution at plane middle section cannot be predicted by elementary beam theory.

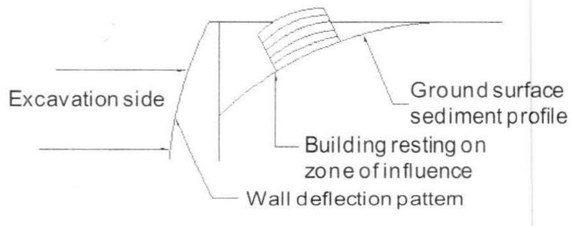

Fig. 1 Building Resting on Deformed Profile Adjacent to the Excavation

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In this paper, permissible deflection ratio of a building lying within the influence zone of a braced excavation is estimated using laminate beam method considering ground movement profile suggested by Peck (1969) for large wall movements. Deflection due to shear is calculated by the strain energy of shear which does not require the above mentioned assumptions. The calculated deflection ratio is compared with the reported case studies for three buildings (Som, 2000) subjected to angular distortion due to excavation during Kolkata Metro Construction.

General Considerations

Any structure or building located on the ground surface adjacent to an excavation is tilted following the deformed profile of the ground. But this tilt of building has two components. These are rigid body rotation and differential settlement. Rigid body rotation of the structure causes no stress or strain in the building. So, the cracks in the building may develop only due to differential settlement. For single mode of deformation (Figure 2(a)) slope of the deformed profile is same as the rigid body rotation. But if a settlement profile is such that a building experiences multiple mode of deformation (Figure 2(b)), then slope of each mode is not equal to rigid body rotation and additional shearing strain may arise.

The permissible deflection ratio is expressed in terms of critical bending and shear strain. This critical bending strain or shear strain varies from one material to another. It mainly depends on the material properties. Boone (1996) summarized the critical strain that causes failure in common building materials.

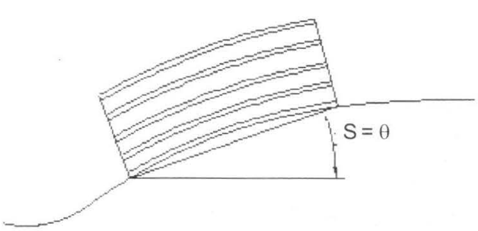


Fig. 2(a) Settlement Profile of Ground Surface with Single Mode of Deformation

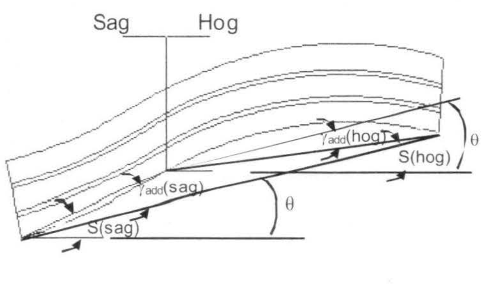


Fig. 2(b) Settlement Profile of Ground Surface with Two Mode of Deformation

In laminate beam method, building is considered as a beam with unit thickness. EI/GA_V is considered as parameter to account for the variation in bending and shear stiffness of structure. Here, deformation due to bending is proportional to the bending stiffness EI, where 'I' is the moment of inertia of the beam while that due to shear is proportional to the shear modulus times the area contributing to shear resistance GA_V.

In the buildings with large area of floors or slabs provides resistance against in plane deformation or bending deformation and load bearing wall or column provide shear transfer from floor to wall.

Derivation of Deflection Ratio

Deflection Ratio in terms of Bending Strain

Considering building as a simply supported beam with concentrated load at mid section (Figure 3), then maximum deflection (at centre) is $\delta_1 = \dfrac{PL^3}{48EI}$

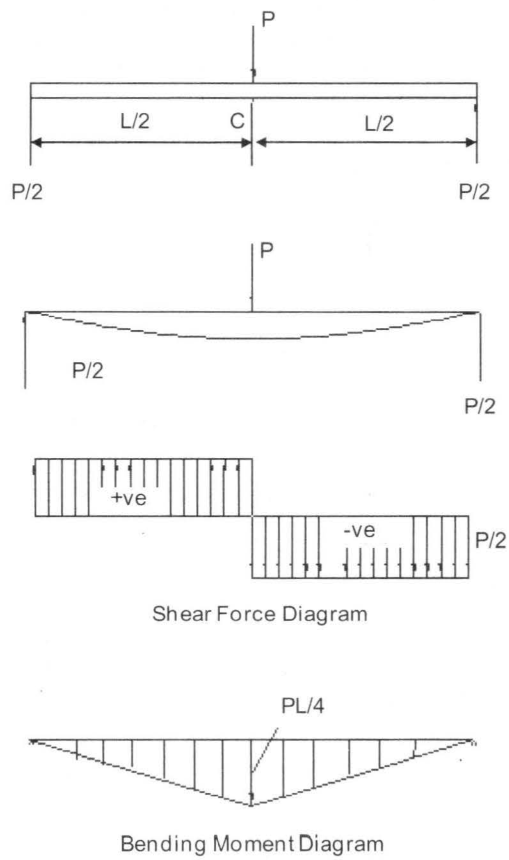


Fig. 3 Deflection, Shear Force & Bending Moment Diagram of a Simply Supported Beam with Concentrated Load at Middle

From elementary theory of bending for the cross section at mid span bending stress (σ) will be $\sigma = \frac{PL}{4} \times \frac{h}{2I}$ (For rectangular beam neutral axis is at mid section)

$$\frac{\delta_1}{L} = \frac{L}{12\lambda h}\varepsilon_b \tag{1}$$

where, λh = Distance of neutral axis from bottom and equal to h/2 for beam with rectangular cross section

Now, additional deflection occurs due to shear stress where there is non- uniform bending. These shear stresses are not uniformly distributed for a beam with rectangular cross-section. Slope of the deflection curve due to shear at any cross section is equal to shear strain γ at neutral axis. If δ_2 is deflection due to shear then,

$$\frac{d\delta_2}{dx} = \frac{\tau_{max}}{G} = \frac{kV_x}{G} \tag{2}$$

But, here shear deformation is calculated by using the strain energy of shear as mentioned earlier.

The shear force at any section of beam is P/2, where, P is the concentrated load at the centre of the beam (Figure 3). Shear stress at any element situated at the distance 'y' from neutral surface is $\tau = \frac{VQ}{Ib}$, where Q is the static moment at that section. Now,

$$\tau = \frac{V}{2I} \times \left(\frac{h^2}{4} - y^2\right) \tag{3}$$

Total strain energy in the entire beam is obtained by integrating strain energy of any element,

$$U = \frac{P^2Lh^2}{80GI} \quad \text{(For beam of unit thickness, b=1)}$$

Equating the total strain energy to the work done, $P\delta_2/2$,

$$\delta_2 = \frac{PLh^2}{40GI} \tag{4}$$

Now, it can be written as $\delta_2 = \frac{PL^3}{48EI} \times \frac{1.2h^2E}{L^2G}$ $= \frac{0.2AE}{G} \times \varepsilon_b$ and as the thickness of the beam is unit so h = A,

$$\delta_2 = \frac{0.2EI}{Gr^2} \times \varepsilon_b = \frac{0.8EI}{GA_v\lambda h}\varepsilon_b \tag{5}$$

where, $I = Ar^2$ and r = radius of gyration, A_v= Area contributing the shear resistance = $\frac{2}{3}A$. So, total deflection ratio,

$$\frac{\delta}{L} = \frac{\delta_1}{L} + \frac{\delta_2}{L} = \frac{L}{12\lambda h}\varepsilon_b + \frac{0.8EI}{GA_vL\lambda h}\varepsilon_b \tag{6}$$

Deflection Ratio in Terms of Shear Strain

From the pure bending consideration, deflection at centre $\frac{\delta_1}{L} = \frac{PL^2}{48EI}$. Now at the centre shear force is V= P/2 or,

$$\frac{\delta_1}{L} = \frac{VL^2}{24EI} = \frac{L^2GA_v}{24EI}\gamma \tag{7}$$

For the multi-storied building if γ_i is the shear strain at each storey and γ is the total shear strain of building then, for each storey

$$\frac{\delta_1}{L} = \frac{L^2GA_v}{24EI}\gamma = \frac{L^2(GA_v)_i}{24EI(V_i/V)}\gamma_i \tag{8}$$

Additional deflection due to shear force

$$\frac{\delta_2}{L} = \frac{PL^2}{48EI} \times \frac{1.2h^2E}{L^2G} = \frac{V}{2.5GA_v} \tag{9}$$

For multi-storied building additional deflection ratio for shear of each storey is $= \frac{(GA_v)_i}{2.5(V_i/V)GA_v}\gamma_i$

So, total deflection ratio due to combined bending and shear for each storey

$$\frac{\delta}{L} = \frac{\delta_1}{L} + \frac{\delta_2}{L} = \frac{L^2(GA_v)_i}{24EI(V_i/V)}\gamma_i + \frac{(GA_v)_i}{2.5(V_i/V)GA_v}\gamma_i \tag{10}$$

Parameters Estimation

In laminate beam method parameters are estimated using the procedure given by Finno et al (2005) and Voss (2003). First, the distance of the neutral axis (Figure 4) from the bottom of the building (λh) is estimated and then moment of inertia of whole laminate beam is calculated. The moment of inertia of each floor slab about its centroidal axis is ignored because thickness of each floor is small compared to overall height of building. If the building is considered as beam the equivalent shear stiffness,

$$GA_v = \frac{V}{\gamma} = \frac{VH}{\sum_{i=1}^{n}\gamma_i y_i} = \frac{1}{\sum_{i=0}^{n}\frac{V_i}{V}\frac{y_i}{h}\frac{1}{(GA_v)_i}} \tag{11}$$

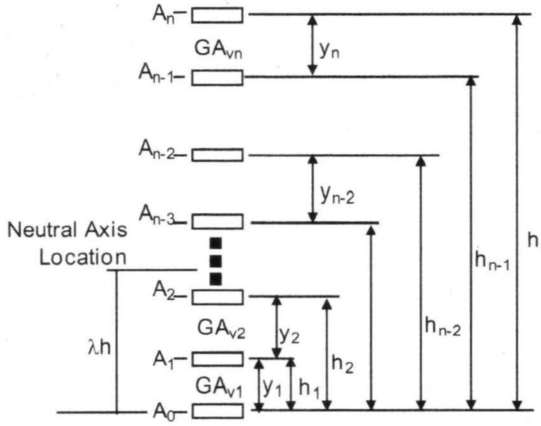


Fig. 4 Neutral Axis Location for a Multi- Storied Building

Now these parameters are used to calculate the critical deflection ratio of a building. The minimum value of δ/L as obtained from the equations (1) and (2) is the permissible deflection ratio and if the actual deflection ratio of the building is more than the permissible value, then cracks may develop in the building.

Here ground surface deflection profile (Figure 5) is hogging type in nature as given by Peck (1969) for large wall movements of the braced excavation. Any structure which is situated on this surface is deformed following the ground surface. The actual deflection ratio of a building is calculated by dividing the maximum deviation of the deformed profile from the straight line joining two extreme points of the building by its length.

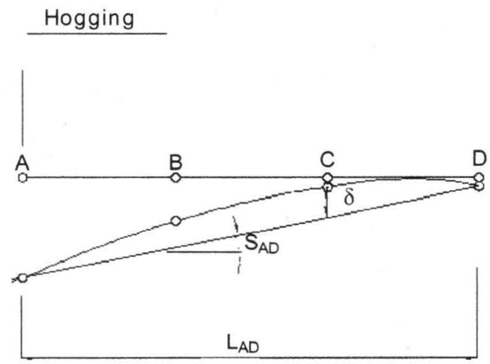


Fig. 5 Deflection Quantities for a Hogging Type Deflection Curve

Comparison with Case Study

Deflection ratio is calculated for three buildings of the northern stretches of Kolkata Metro Excavation. These buildings are 161, C. R. Avenue, 164, C.R. Avenue and 180A, C.R. Avenue. Details of the buildings are given in Table 1. Cross-sectional views of three buildings modeled as laminate beam are shown in Figure 6.

Taking a section of building the permissible deflection ratio for combined bending and shear are calculated considering the critical strain given below.

Critical bending strain is taken as 0.067% (Burland and Wroth, 1975) and Critical shear strain is taken as 0.15% (Boone, 1996).

Table 1 Deflection Ratio in terms of Bending and Shear Strain

Building	161, C.R. Avenue	164, C.R. Avenue	180A, C.R. Avenue
A_i (m^2)	0.90	0.90	0.75
h_i (m)	3.5	3.5	3.5
λh (m)	10.50	10.50	8.75
n	6	5	5
λ	0.5	0.5	0.5
$I_{building}$ (m^4)	308.7	192.9	160.8
I_{wall} (m^4)	1.786	1.786	1.786
$(GA_V)_{wall}$ (kN)	0.93×10^6	0.586×10^6	7.382×10^6
$(GA_V)_l$ (kN)	0.65×10^6	0.41×10^6	5.167×10^6
GA_V (kN)	3.9×10^6	2.064×10^6	25.796×10^6
EI/GA_V (m^2)	1963.0	2318.2	154.6
δ/L(in terms of bending)	6×10^{-3}	2.26×10^{-3}	1.608×10^{-3}
δ/L(in terms of shear)	3.994×10^{-3}	3.2454×10^{-3}	3.2975×10^{-3}
δ/L(permissible)	3.994×10^{-3}	2.26×10^{-3}	1.608×10^{-3}
δ/L(observed)	3.69×10^{-4}	1.174×10^{-3}	6×10^{-5}

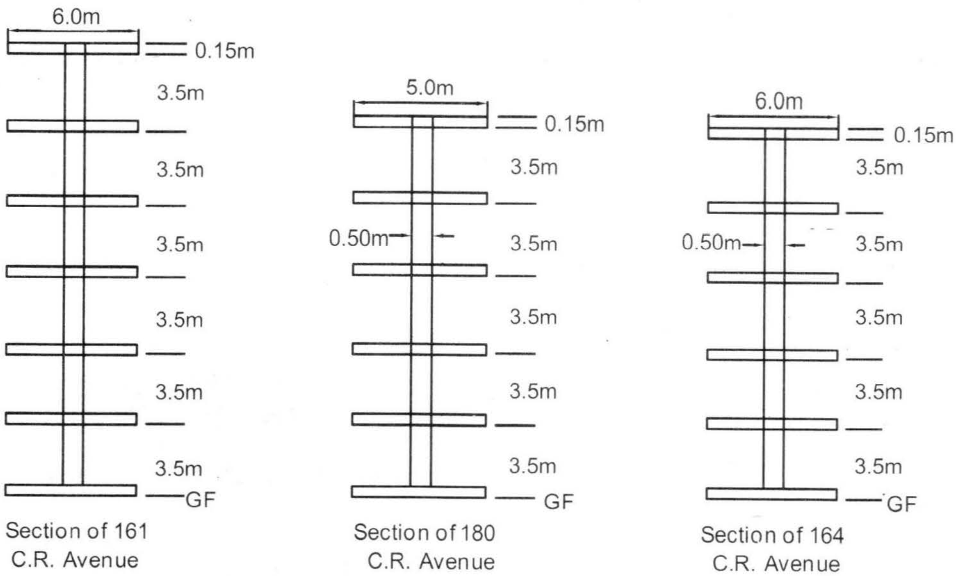


Fig. 6 Cross Sectional View of three Buildings Modeled as Laminate Beam

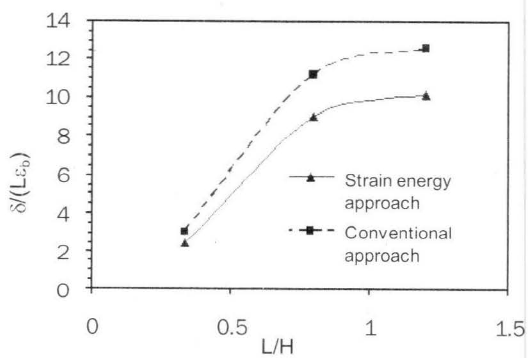
The permissible deflection ratio is nearly 1/300 which is the well accepted permissible value for a concrete building (McDonald and Skempton (1955)).

Discussion

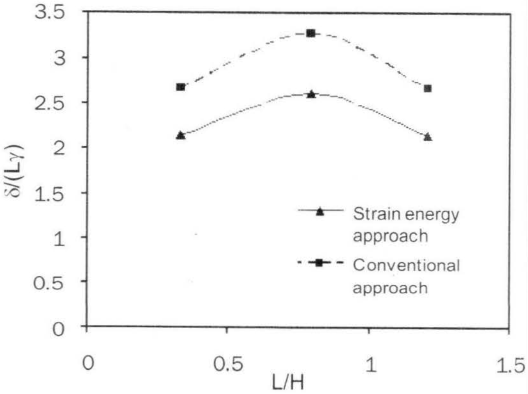
The permissible values of deflection ratio given in Table 1 for three buildings at Northern Stretches of Kolkata Metro Construction are compared with the actual deflection ratio obtained from deformation profile of observed data. Now, comparing the actual values with the permissible deflection ratio given in Table 1 it can be said that the buildings were safe in both bending and shear during excavation. Further, it may be noted that the observed values of deflection ratio for those three buildings are well within the permissible limits.

Relationship between Critical Deflection Ratio and L/H of the Building

From the Figure 7 (a) and 7(b) it is clear that when deflection ratio is expressed in terms of bending strain, the limiting value of δ/L is initially directly proportional to L/H which matches with Burland's approach of deep beam method. Both the curves give critical condition for lesser L/H. But when deflection ratio is expressed in terms of shear strain then such relationship is not observed.



(a) Deflection Ratio in terms of Bending Strain



(b) Deflection Ratio in terms of Shear Strain

Fig. 7 Relation between Deflection Ratio & L/H

Conclusions

1. Calculation of permissible deflection ratio from strain energy approach is more accurate as it does not require any assumption.
2. The assumption of building as a simply supported beam is adequate for building of small length because it experiences single mode of deformation.
3. Permissible deflection ratio for three buildings of Kolkata Metro Construction is nearly equal to the conventional permissible value for a concrete building.
4. A building will be more critical if L/H ratio is lesser.
5. The deflection ratio of the three buildings in Kolkata Metro Construction is within the permissible deflection ratio. So, proper prediction of building damage near any deep excavation may be done.

Symbols and Notations

s	Ground slope in each mode of deformation
θ	Rigid body rotation of the building near excavation
E	Young modulus of building component
G	Shear modulus of building component
δ_1	Maximum deflection of the beam due to pure bending
δ_2	Maximum deflection of the beam due to shear
λh	Distance of neutral axis from the bottom of laminate beam
U	Strain energy of beam due to shear
A_v	Area of the building contributing to shear resistance
EI	Equivalent bending stiffness when building modeled as laminate beam
GA_v	Equivalent shear stiffness when building modeled as laminate beam
V_i/V	Percentage shear in storey by laminate beam model
y_i and V_i	Height and shear force of 'i' th storey of building
γ_i and γ	Shear strain in 'i'th storey and total shear strain of building
δ/L	Permissible Deflection ratio of building

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