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Slope Stability Modelling for Embankments using Particle Swarm Optimization

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Key words

Slope stability, Earthen embankments, modelling, optimization, Particle swarm optimization, Factor of safety

Abstract: Many studies concerning slope stability analysis for the earthen embankments have used limit equilibrium based approaches such as slip circle method to estimate the critical factor of safety. The slope stability estimation is a non-linear optimization problem and recently evolutionary algorithms were proposed for locating global optimal solutions. In using those search algorithms, appropriate selection of range of decision variables is very important to obtain faster global optimal solutions. In this paper, a new analytical procedure is proposed for locating the best possible feasible region for the centre of slip circle. An efficient global optimization method namely particle swarm optimization (PSO) is used for solving the slope stability problem and demonstrated its effectiveness for different soil conditions. It is found that the proposed PSO based approach performs equally well when compared with the existing classical search methods and genetic algorithm solutions.

Introduction

Earthen embankments are widely used for roads, dams, river training works and canals. The economy and safety of these works are one of the major concerns for civil engineers. The failure of these structures may result in considerable loss of life and property. There are several methods available for stability analysis of earthen embankments, such as limit equilibrium methods (Fellenius 1936; Bishop 1955; Janbu 1973; Sarma, 1979), variational calculus based methods (Baker and Graber 1978; Baker 1980; Baker 2005) and finite element methods (Griffiths and Lane 1999; Lane and Griffiths 2000; Griffiths and Fenton 2004; Li 2007; Zheng et al. 2009). Eventhough finite element methods may help for rigorous slope stability analysis of complex soil profiles, they are not that popular as compared to limit equilibrium methods in practical design of embankments. This may be because of, in addition to higher computational cost and effort, the finite element methods require definition of initial conditions, stress-strain relations, properties and the construction of loading sequence etc (Duncan 1996). In limit equilibrium method of slope stability analysis, the ratio of restoring forces to the disturbing forces gives the factor of safety. The method of slices (Felleneus 1936), modified Bishop's method (Bishop 1955), Janbu's method (Janbu 1973), Sarma's method (Sarma 1979) etc are some of the popular limit equilibrium methods used for slope stability analysis. Some other studies (Bhattacharya and Basudhar, 2000; 2001) also attempted to use varied approaches for slope stability analysis. Research studies also suggested that with little inaccuracy that the critical slip surface can be assumed as circular unless there are geological controls that constrain the slip surface to be non-circular in shape (Duncan 1996). Thus the limit equilibrium based approach such as slip circle method can be used effectively to estimate critical factor of safety of soil slopes.

Since the slope stability analysis is framed on the principle of estimation of minimal factor of safety among random selection of trial surfaces, the problem can also be solved in an optimization framework. In the past, classical optimization methods were widely used to calculate the factor of safety of the critical slip surface (Basudhar et al., 1979; Nguyen 1985; Li and White 1987; Chen and Shao 1988; Bardet and Kapuskar 1989). The slope stability problem is highly non-linear in nature with multi-modal behaviour. While solving slope stability problems, the classical optimization methods were found to be more susceptible to trapping at local optima due to the irregular response surface. The gradient based approach often causes difficulties in finding the derivative of highly non-linear objective functions like slope stability models. Moreover, these methods are highly sensitive to the initial solutions supplied. Greco (1996) and Malkawi et al. (2001) used Monte Carlo method for locating the critical slip surface. This method also has some limitations, as they do not guarantee the global optimum within the search space. Since they are highly dependent on search space, the computational complexity is also quite high in such

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methods. While solving the slope stability problems knowledge about the possible location of centre of critical slip surface is very important to obtain faster global optimal solutions.

In recent past, evolutionary algorithms are introduced as global optimization methods and are gaining much popularity in solving non-linear optimization problems. Genetic Algorithms (GA) is the most popular method in this class of problem solvers and was also used in the past for solving the slope stability problems (Goh 1999; Goh 2000; McCombie and Wilkinson 2002; Sabhahit et al. 2002; Zolfaghri et al. 2005). Bhattachrjya and Satish (2007) applied genetic algorithm for computing the factor of safety of canal slope and suggested that the stability of canals can be assured on linking this GA based slope stability model internally with the optimization model for the least cost design of trapezoidal canals. Recently, Particle Swarm Optimization (PSO) is evolving as a powerful tool for solving non-linear optimization problems. The PSO method was successfully applied for solving real world optimization problems in civil engineering such as slope stability modelling, reservoir operation, pipe network analysis, and calibration of geotechnical parameters (Cheng et al. 2007; Nagesh Kumar and Janga Reddy 2007; Janga Reddy and Nagesh Kumar 2007; Jung and Karney 2006). In this paper, a PSO based approach is adopted for the estimation of critical factor of safety of earthen embankments.

In the solution of the slope stability problem, all search algorithms start with initialization of random selection of trial surfaces. The convergence rate of any search algorithm (i.e., in locating global optimum) can be improved by properly defining the feasible range of decision variables. In the case of toe failure type slope stability problems, coordinates of centre of slip surface forms the decision variables.

At present there exists no procedure which will define the feasible region for finding the position of the centre of slip circle. Most of the earlier works follows the selection of a sufficiently large rectangular search space during the stability analysis (Malkawi et al. 2001; McCombie and Wilkinson 2002). The method of generation of trial slip surfaces was addressed in the past by (Greco 1996; Malkawi et al. 2001; Cheng et al. 2007). But again in such methods the bounds of *X*-coordinate of centre of slip surface is fixed based on engineering experience alone (Cheng et al. 2007). The selection of search space based on some analytical expressions related to slope geometry will be more convenient and easy to use for the practitioners while designing the stable slopes for different earthen embankments. In this study a new procedure is developed for finding the narrow feasible region of the coordinates of centre of slip circle for toe failure case of earthen embankments.

Estimation of Critical Factor of Safety of Embankments

The side slopes of the earthen embankments should be safe against shear failure. The natural slope of the embankment will develop actuating or driving forces which inturn results in the movement of soil mass. This movement is resisted by the stabilizing forces at the potential failure surface due to the shear strength of soil. The side slope will be stable if the sum of resisting forces on every possible surface of failure is greater than the sum of all the actuating forces. The stability of slope is indicated by the ratio of stabilizing force to actuating force and this ratio is known as factor of safety. In slip circle method, the process of estimation of factor of safety is to be repeated for a large number of trial surfaces and the trial surface which gives the minimum factor of safety is the critical surface.

By considering a circular failure surface the critical factor of safety can be computed using an optimization technique. In this study, the development of optimization model for the estimation of critical factor of safety is based on the following considerations: (i) the soil is homogeneous and isotropic; (ii) the surface of rupture is assumed as cylindrical; (iii) failure arc passes through the toe of the slope (iv) pore pressure estimation is based on method suggested by Bishop and Morgenstern (1960).

Figure 1 shows a homogeneous earthen slope with a trial slip circle AB of radius R. The toe of embankment (point A) is considered as the origin of coordinates i.e., A (0, 0). Let O (X, Y) be the centre of the slip circle; β be the inclination of the slope; H be the height of the slope in m; and δ be the angle subtended at the centre by the assumed slip circle.

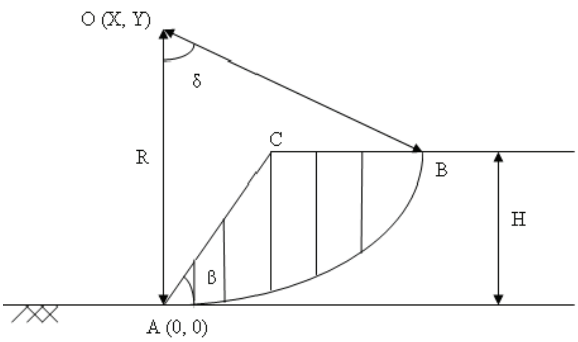

Fig. 1 Definition Sketch of an Earthen Slope with Circular Failure Surface

The optimization model applicable for a toe failure case can be formulated as:

Minimize:

$$F(X,Y) \quad (1)$$

Subject to

$$X_l < X < X_u \quad (2)$$

$$Y_l < Y < Y_u \quad (3)$$

where, F = factor of safety; X_l and X_u are lower and upper limit on X; Y_l and Y_u are lower and upper limit on Y.

Assume a trial slip surface and divide the wedge above the slip surface into a convenient number of slices. The accuracy of the results depends on number of slices (n). According to the method of slices (Felleneus 1936) the factor of safety of slopes can be expressed as:

$$F = \frac{C + \tan\phi \sum_{i=1}^{n} (W_i \mathrm{Cos}\,\theta_i - U_i)}{\sum_{i=1}^{n} W_i \mathrm{Sin}\,\theta_i} \quad (4)$$

where C = total cohesive force in kN; ϕ = angle of internal friction; U_i is the force developed within the i^{th} slice due to pore water in kN; W_i is the weight of i^{th} slice in kN ; θ_i is the angle between the normal component of the weight of i^{th} slice and vertical.

The model takes the soil parameters like cohesion (c) and angle of friction (ϕ), unit weight of soil (γ), slope parameters like inclination (β) and height (H) and the pore pressure coefficient (r_u) as input parameters. The normal force, tangential force, cohesive force and force due to pore pressure (if accounted) can be expressed in terms of the coordinates of the centre of slip circle (X, Y) and its radius R. In the absence of flow net and accurate pore pressure estimation measures, the method given by Bishop and Morgenstern (1960) can be used for determination of the force due to pore water pressure (Michalowski 2002). Bishop and Morgenstern (1960) defined the pore pressure coefficient (r_u) as:

$$r_u = \frac{U_i}{\gamma h_i} \quad (5)$$

where h_i is the depth of middle ordinate of i^{th} slice upto the failure surface measured from the surface of the embankment.

Estimation of Feasible Region for Coordinates of The Centre of Slip Circle

The slope stability problem has several local and alternate optimal solutions. Random search optimization techniques such as PSO have the capability in finding the global optimal solutions. However, in the slope stability problems the inherent necessity of elimination of non-possible failure surfaces may delay the search in locating the optima. Thus information regarding an approximate range of decision variables may speed-up the search process towards the global optima. While handling practical design problems of embankments such information will be very useful for the designers in achieving faster global optimal solutions.

In this study, analytical expressions are developed to identify the narrow feasible region of the coordinates of critical slip circle passing through toe. These expressions are obtained after conducting several experiments through geometrical constructions. The method gives a narrow range of decision variables (i.e., bringing down the wider search space closer to the centre of slip circle) in terms of slope geometry (height of the slope, inclination of the slope) and top width of the embankment. Such information can be available at sites for designing the embankments. The step-by-step procedure is illustrated below with the help of Figure 2.

Let S be the mid point of the sloping face and CD be the top width of the embankment (T).

1. The locus of lower limit on X is a line drawn at the

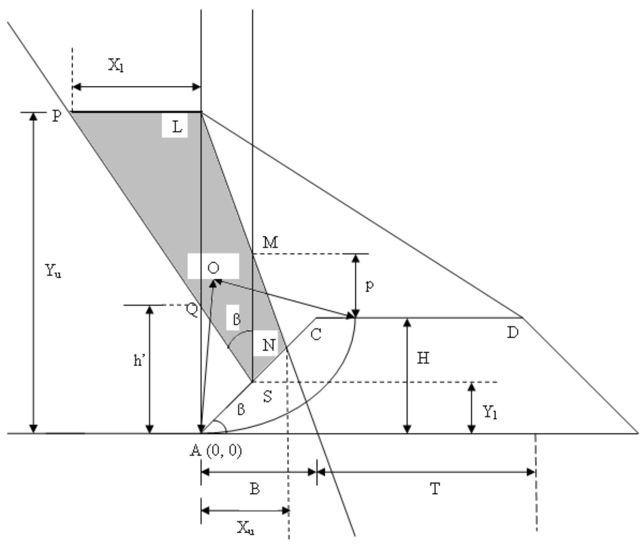

Fig. 2 Illustration Sketch for the Determination of Feasible Region for Coordinates of Centre of Slip Circle

midpoint of the sloping face AC, which bears an angle equal to the angle of slope (β) with the vertical drawn at S. This locus is the line segment PS. It may also have interdependence with *Y.*

2. The above mentioned locus crosses the vertical line (at the point Q) drawn at the toe of the slope at a height h' measured from the base , where h' is given by :

$$h' = \left(\frac{H}{2} + \frac{H\cot^2\beta}{2}\right) \quad (6)$$

3. Since the lower bound of *X* is interdependent on upper bound of *Y* (defined by the point L ($0, Y_u$), upper bound of *Y* (i.e., Y_u) is found to be:

$$Y_u = \frac{(B+T)^2 + H^2}{2H} \quad (7)$$

where, B is the base width of the slope.

4. The lower bound on *X* (denoted as X_l) is found to be:

$(X_l) = (-h \tan \beta)$ (8)

where, $h = Y_u - h'$

5. Find a point M along the vertical line drawn at S at a height of *p* above the top of the embankment, which is defined as:

$$p = \frac{(T^2 + T.B - H^2)}{2H} \quad (9)$$

6. The extension of LM will meet the sloping face of embankment (i.e., AC) at the point N. The horizontal equivalent of distance AN will be the upper bound of *X* (denoted as X_u).
7. From the geometrical constructions the lower bound of *Y* (denoted as Y_l) is found to be *H/2.*

The truncated cone PSNL will define the actual feasible region for the *X* and *Y* coordinates of the considered slip circle.

Particle Swarm Optimization for Slope Stability Modelling

Particle swarm optimization (PSO) is a population based random search technique inspired by social behaviour of bird flocking and other insect colonies. PSO was proposed by Kennedy and Eberhart in 1995 (Kennedy and Eberhart, 1995; Eberhart and Kennedy 1995). The social sharing of information among individuals is the fundamental principle of PSO. PSO shows the benefits of co-operation where there is no global knowledge about an environment. By using PSO, it will be easier to handle nonlinearity and non-convexity of the problem domain; the search does not depend on initial population, and it may overcome the problem of trapping to local optima. Similar to an evolutionary algorithm, PSO algorithm is also initialized with random solutions. The random individual solution within a search space is called 'particle' and the entire set of particles is called as the 'swarm'. Particles fly through search space with velocities which are dynamically adjusted according to their historical behaviours. Hence the particle has a tendency to fly over their entire search space over the course of search process (Nagesh Kumar and Janga Reddy 2007).

The general procedure of PSO involves the following steps:

1. The first step is to initialize the particles with random positions and velocities within the predefined range of decision space. The dimension of the problem will be the number of decision variables involved. In a *D* dimensional search space, the i^{th} particle can be represented as a *D* dimensional vector $X_i = (x_{i1}, x_{i2}, \ldots, x_{iD})^T$. The velocity (position change) of this particle is designated as $V_i = (v_{i1}, v_{i2}, \ldots, v_{iD})^T$. The best previously visited position of the i^{th} particle is given by $P_i = (p_{i1}, p_{i2}, \ldots, p_{iD})^T$.
2. For each particle, the desired objective function in *D* variables is evaluated.
3. The current fitness value of each particle is compared with its best fitness achieved in the past (called as personal best or '*pbest*'). If the current fitness value is better than old '*pbest*', set '*pbest*' value equal to the current value.
4. The current '*pbest*' fitness value of each particle in the population is compared with the population's overall best value of fitness (called as global best or '*gbest*'). If any fitness value is better than '*gbest*', reset the '*gbest*' position to the current array index value of that particle.
5. The velocity and position vectors are updated as follows

$$V_i^{n+1} = \chi\left[\omega V_i^n + c_1 r_1 \frac{(P_i^n - X_i^n)}{\Delta t} + c_2 r_2 \frac{(P_g^n - X_i^n)}{\Delta t}\right] \quad (10)$$

$$X_i^{n+1} = X_i^n + (\Delta t) V_i^{n+1} \quad (11)$$

where $i = 1,2,3,\ldots,N,$ the index for swarm population; N is the size of the swarm; ω is inertia weight; χ is constriction coefficient ; the symbol g represents the index of best particle among all particles in the population; c_1 and c_2 are called acceleration coefficients; r_1 and r_2 are

D-dimensional vectors of uniform random numbers generated between 0 and 1; *n* is the iteration number; Δt is time step (which is considered as unity).

6. Loop to step 2 until the imposed termination criteria is met. That can be either a pre-defined maximum number of iterations is reached or the desired convergence of fitness is achieved.

Equation 10 is the equation describing the flying trajectory of a population of particles. It describes how the velocity is dynamically updated and Equation 11 is the position update of particles. The updating is symbolically represented in Figure 3. The updating can be achieved because of current motion influence, particle memory influence and swarm influence. The first part of Equation 10 is the momentum part (represent current motion influence), the second term is the cognitive component (experiential knowledge component which represent the particle memory influence) and third term is the social component (represent the swarm influence).

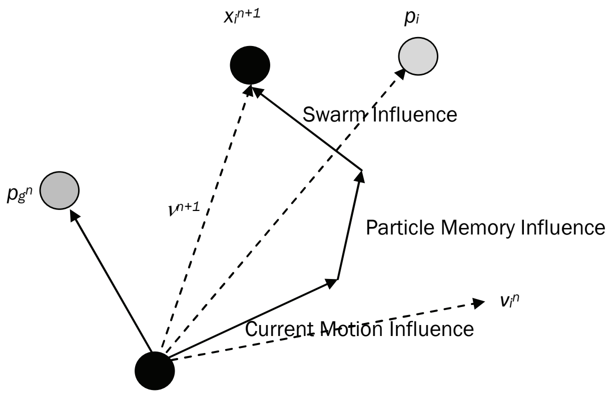

Fig. 3 Depiction of Population Dynamics in PSO

c_1 and c_2 are used to scale the cognitive and social components of the particles. Appropriate selection of the inertia weight and constriction coefficient can help the algorithm to achieve faster convergence towards the global optimal solution. A pseudo code of basic PSO algorithm is given in Figure 4.

The slope stability model is implemented in MATLAB environment and PSO is used to solve the problem. The coordinates of the centre of assumed slip circle are the decision variables. Each of the possible solution vectors of decision variables is called as a 'particle'. Number of such vectors considered for a trial run indicates the swarm (population) size. The swarm size and PSO control parameters are fixed through a sensitivity analysis. The factor of safety is the fitness of an individual. If a non acceptable failure surface is generated during the search process, a suitable penalty is given for the fitness.

```
For (i=1,....,N)
        Initialize Xi (0) and Vi (0)
End For
Set n=0
While (termination criteria not met) do
        For (i=1,......,N)
                Compute fitness function
                Compute (pbest)i
        End For
                Compute gbest
        For (i=1,.....,N)
         Calculate Vi (n+1) and Xi (n+1) using Equation 10 and 11
        End For
 n=n+1
End While
```

Fig. 4 Pseudo Code of the PSO Algorithm

Validation of PSO Based Slope Stability Model

By considering suggestions from earlier studies (Nagesh Kumar and Janga Reddy 2007; Janga Reddy and Nagesh Kumar 2007) and after performing a thorough sensitivity analysis for PSO, the following control parameters are adopted for solving the models: swarm size (*N*) = 30; maximum number of iterations = 100; inertia weight (ω) = linearly varying from 1.2 to 0.4; acceleration coefficients, c_1 = 1 and c_2 = 0.5; constriction coefficient (χ) = 0.9. The developed slope stability model was validated by applying it to two example problems that were reported in the literature.

Example Problem 1

The first problem has been taken from Michalowski (2002). The slope has an inclination of 30°. The height of the embankment is 10 m. The soil parameters are cohesion, c = 10 kN/m^2 and angle of shearing resistance, ϕ = 20°. The unit weight of soil is 17 kN/m^3. The top width of embankment is assumed as 5 m. By using the developed methodology for finding the bounds of coordinates of centre of slip circle, the bounds for *X*- coordinate is fixed as (-5.72, 10.65) and the bounds for *Y*- coordinate is fixed as (5.00, 29.91). A sample computational procedure for fixing the decision space for the problem is given in Appendix-1. The centre of the critical slip circle for the present study is found as O (4.033, 17.543) for r_u= 0 case and O (4.26, 14.00) for r_u= 0.25 case. The centre of critical slip surface is found to be within the obtained search space.

The results are compared with the stability chart provided by Michalowski (2002) and GA based solution reported by Bhattacharjya and Satish (2007). The results for pore pressure coefficient, r_u= 0 and r_u= 0.25

cases are given in Table 1. The results for r_u= 0.25 are presented to show the potential applicability of the model in handling the pore pressure effect. It can be seen that the PSO based solution gives a better factor of safety (F=1.244) as compared to GA based solution (F=1.256) and the conventional approach (F=1.300). On incorporating the pore pressure effect, the solutions are found to be comparable with the previous solution methods. A qualitative comparison cannot be made as the exact procedure adopted for pore pressure estimation in these methods is not known.

Table 1 Comparison of PSO Results with GA and Stability Chart for Problem 1

Method	*F (r_u = 0)*	*F (r_u = 0.25)*
PSO	1.243	1.129
GA#	1.256	0.982
Stability Chart$	1.300	1.068

Note: # Bhattacharjya and Satish (2007);
$ Michalowski (2002)

Example Problem 2

The second problem is taken from Greco (1996). The slope inclination (β) is 26.56°, height (H) is 5 m; bulk density (γ) is 17.64 kN/m³; c = 9.8 kN/m² and ϕ = 10°.

Assuming top width of embankment as 5 m and by using the developed methodology for locating the bounds of coordinates of centre of slip circle, the bounds for X- coordinate is computed as (-6.25, 7.14) and the bounds for Y- coordinate is computed as (2.50, 25.00). A comparison of the factor of safety value obtained by PSO algorithm with different existing solution methods is provided in Table 2. This model results also shows that the factor of safety found by PSO based model is better than that obtained by the GA model. The PSO model resulted in a smaller factor of safety of 1.268 at the centre of critical slip circle O (3.846, 7.297). By incorporating the pore pressure effect, the factor of safety was found to be 1.151 at the centre of critical slip circle O (4.00, 6.93). In both cases the centre of slip circle is found to be within the obtained region. For these two problems, the exact centre of the slip circle is not reported in the past studies. The statistical properties for the accepted trial of PSO model for r_u=0 case for both the example problems are given in Table 3.

Table 2 Comparison of PSO Results with GA and other Methods for Problem 2

Method	*F (r_u = 0)*
PSO	1.268
Genetic Algorithm#	1.276
Gradient Search #	1.335
Pattern search $	1.327-1.33
Monte Carlo $	1.327-1.333

Note: #Bhattacharjya and Satish (2007); $Greco (1996)

Table 3 Statistical Properties of Factor of Safety for the Accepted Trial

Property	*Problem 1 (r_u=0)*	*Problem 2 (r_u=0)*
Maximum F	1.307	1.360
Minimum F	1.243	1.268
Average F	1.244	1.268
Standard deviation	0.005	0.006

Discussion

The field applicability of the developed model is also evaluated for 30 more combinations of soil slopes taken from Bhattacharjya and Satish (2007). The input for the models is given in col. (2) to (6) in Table 4. The centre of slip circle and critical factor of safety are estimated by solving the optimization formulation involving Equation 1 to 3 using PSO algorithm. The results are presented in col. (7) and (8) of Table 4. The design factor of safety values reported by (Bhattacharjya and Satish 2007) are presented in col. (9) of Table 4. In all cases, PSO based methodology provides more critical estimation for factor of safety as that of results reported by (Bhattacharjya and Satish 2007). From the obtained results, it can be inferred that for slopes steeper than 45°, there is a general trend of the upper bound of X-coordinate can be fixed as '0', therefore the feasible region for the centre of slip circle is falling to the left side of toe.

Incorporating Earthquake Effect in Slope Stability Modelling

The earthquake effect can also be taken into account, while performing the stability analysis of slopes. The pseudo-static approach is adopted to incorporate this feature which involves the computation of minimum factor of safety by including the static horizontal and vertical earthquake forces. The vertical or horizontal earthquake forces are usually expressed as a product of respective seismic coefficients and the

Table 4 Computation of Critical Factor of Safety for Different Soil Conditions using PSO Based Model and Comparison with GA Solutions

Sl No (1)	*c* (kPa) (2)	ϕ(deg) (3)	γ(kN/m³) (4)	β(deg) (5)	*H*(m) (6)	PSO Model# O (*X, Y*) (7)	PSO Model# *F*# (8)	GA* *F** (9)
1	20	20	18	75.96	4.55	(-3.922, 7.375)	1.529	1.542
2	15	15	18	54.25	4.41	(-0.234, 6.288)	1.477	1.499
3	10	15	18	34.78	4.05	(1.849, 5.937)	1.488	1.509
4	20	5	18	57.38	4.44	(0.068, 6.613)	1.486	1.502
5	10	20	18	42.02	4.21	(0.738, 5.622)	1.482	1.500
6	10	10	18	27.27	3.81	(3.270, 6.192)	1.490	1.499
7	5	20	18	26.34	3.78	(2.310, 7.073)	1.485	1.499
8	5	10	18	14.78	3.23	(5.690, 8.354)	1.497	1.499
9	10	10	20	24.15	3.69	(3.411, 5.598)	1.497	1.499
10	5	10	16	15.94	3.30	(5.138, 7.739)	1.498	1.500
11	20	28	18	74.89	5.58	(-5.351, 8.573)	1.485	1.518
12	15	28	18	61.19	5.51	(-2.265, 7.531)	1.477	1.500
13	10	28	18	45.87	5.28	(-0.220, 7.341)	1.477	1.499
14	5	28	18	32.33	4.89	(1.153, 7.576)	1.485	1.499
15	20	20	18	64.83	5.54	(-2.372, 8.262)	1.499	1.500
16	20	10	18	51.34	5.39	(0.526, 7.631)	1.472	1.500
17	20	5	18	41.51	5.18	(2.145, 6.689)	1.480	1.500
18	15	5	18	23.96	4.53	(5.179, 7.117)	1.495	1.499
19	5	5	18	7.360	3.24	(11.89, 12.290)	1.489	1.499
20	20	10	22	39.11	5.12	(2.216, 7.386)	1.481	1.499
21	20	28	18	61.20	7.30	(-3.003, 0.004)	1.483	1.499
22	30	28	18	73.30	7.38	(-6.549, 2.014)	1.652	1.674
23	25	28	18	73.30	7.38	(-6.689,11.604)	1.464	1.499
24	15	28	18	50.99	7.06	(-0.927, 8.650)	1.456	1.500
25	10	28	18	38.66	6.77	(0.765, 9.580)	1.481	1.500
26	5	28	18	29.33	6.33	(1.781, 10.937)	1.482	1.499
27	20	18	18	21.49	5.82	(4.320,12.162)	1.480	1.500
28	10	15	20	21.96	5.85	(5.580,11.539)	1.486	1.500
29	10	10	20	15.82	5.33	(8.665, 12.617)	1.497	1.500
30	10	5	20	9.625	4.55	(14.667,14.824)	1.514	1.500

Note: #Present study; * Bhattacharjya and Satish (2007)

weight of the potential sliding mass. Since the vertical pseudo-static earthquake force has the same effect on driving and resisting force, it can be ignored. Hence in this case the factor of safety of a slope primarily depends on the value of horizontal seismic coefficient (α_h). A typical method of estimation of factor of safety of embankments (BIS 7894-2002) is given below:

$$F = \frac{\left(C + \sum_{i=1}^{n}[(W_i Cos\theta_i - U_i)\tan\phi\right) - \left(\sum_{i=1}^{n}\alpha_h W_{si} Sin\theta_i \tan\phi\right)}{\left(\sum_{i=1}^{n} W_i Sin\theta_i + \sum_{i=1}^{n} W_{si}\cos\theta_i\right)} \quad (12)$$

where, W_{si} is the weight of i^{th} slice under seismic force.

In the absence of accurate estimation of induced pore pressure under earthquake loading, the total stress analysis is a suitable option capable in giving fairly accurate results (Choudhary et al. 2004) and hence this approach is used for the estimation of factor of safety.

The same example problems considered for model validation are solved incorporating the seismic effect. In the absence of field observations, an approximate estimation is made with three different horizontal seismic coefficient values to show the potential of the model to account the seismic effect. A saturated unit weight value of soil γ =19.6 kN/m³ is used in the study. The results of the model are presented in Table 5. The comparison of factor of safety values presented in Tables 1 and 2 with those given in Table 5 shows a reduction of factor of safety values on accounting the seismic effect, which is an expected outcome. Thus these results also show that the proposed method can effectively handle the seismic effect in slope stability analysis.

Table 5 Results of PSO Based Slope Stability Model after Incorporating the Seismic Effect

Horizontal seismic Coefficient (α_h)	Coordinates of centre of critical slip circle O (X, Y)	Factor of safety (F)
Problem 1		
0.10	(4.732, 16.119)	0.960
0.15	(5.006, 15.559)	0.856
0.20	(5.238, 15.087)	0.769
Problem 2		
0.10	(4.761, 7.586)	0.939
0.15	(4.928, 7.141)	0.825
0.20	(5.932, 7.929)	0.730

Summary and Conclusions

Rigorous slope stability analysis is essential for the safer construction of earthen embankments. In this study, a particle swarm optimization (PSO) algorithm based method is presented for slope stability modelling of earthen embankments. While using slip circle method to compute the factor of safety of earthen slopes, the knowledge about the narrow feasible region of centre of slip circle will give faster solutions as it significantly reduces the search space of the problem. To identify the feasible region, analytical expressions are developed and efficiency of the methodology is tested for different soil and slope conditions. The PSO based solutions are found to be efficient as compared to classical search techniques and genetic algorithms. Thus the methodology presented for the slope stability modelling is very useful in analyzing slope failure problems and the slope stability criteria can be incorporated effectively for the safe design of embankments of canals, roads and earth dams.

The main conclusions of the study are: (1) PSO is a derivative free optimization technique and is very effective in solving slope stability problems. (2) analytical expressions provided for determination of feasible region for centre of slip circle and the PSO based estimation of critical factor of safety are very much useful to achieve good quality solutions for slope stability problems.

Appendix-1

Sample calculations for estimation of feasible region for coordinates of centre of slip circle

The procedure for the determination of feasible region for coordinates of centre of slip circle O(*X, Y*) can be illustrated with the help of Figure 2 and Equation. 6 to Equation 9. For the case of example problem-1, the sample calculations are presented below:

Let slope geometry for problem 1: H = 10 m; β = 30°; B= 17.32 m.

Using Equation 6, h' = 20 m.

Assuming a minimum required top width of 5 m for canal embankments, from Equation 7, upper bound of Y (i.e., Y_u) = 29.91 m.

The lower bound of Y (i.e., Y_l) = ($H/2$) = 5.00 m.

From Equation 8, the lower bound on X (i.e., X_l) = -5.72 m.

As per Equation 9, p =0.58 m.

Thus the range of decision variable Y can be fixed as: (Y_l, Y_u) = (5.00, 29.91).

In Figure 2, The coordinates of point L $(x_1, y_1) = (0.00, 29.91)$. The co-ordinates of point M $(x_2, y_2) = \left(\frac{B}{2}, (H+p)\right)$.

The equation of the straight line joining L and M can be represented as:

$$\frac{(x - x_1)}{(x_2 - x_1)} = \frac{(y - y_1)}{(y_2 - y_1)}$$

Substituting the coordinate values the straight line can be represented as $y = 29.91 - 2.232x$

The coordinates of the intersection point N (between line LM and sloping face AC) is $N(x_3, y_3) = (X_u, X_u \tan\beta)$.

Since N is a point on the extension of line LM, X_u = 10.65 m.

Mathematically, the lower bound of X_u can be given as follows:

$$X_u = \frac{Y_u}{\left(\tan\beta - \frac{2}{B}\left(H + p - Y_u\right)\right)}$$

Hence the range of decision variable *X* can be selected as: (X_l, X_u) = (-5.72, 10.65).

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