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TECHNICAL NOTE

Active Earth Pressure on Retaining Wall Backfilled with $c-\phi$ Soil

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Key words

single critical wedge, $c-\phi$ backfill, seismic loads, active force, pseudo-static method

Abstract: This paper presents an analytical solution for total active force on the retaining wall backfilled by c-ϕ soil considering both horizontal and vertical seismic coefficients. Pseudo-static force based concept is used for this particular analysis. The effect of tension crack is also taken into account. The analysis is done in such a way that for the simultaneous action of weight, cohesion and surcharge a single critical wedge surface is generated. The results are presented in tabular form. Variation of different parameters like ϕ, δ, c, k_h, k_v are studied on the variation of seismic active earth pressure coefficient which are presented in graphical form.

Introduction

Mononobe- Okabe analysis (Okabe 1924; Mononobe and Matsuo 1929) was the first to modify Coulomb's solution (Coulomb, 1776) to evaluate the seismic active earth pressure on the back of a retaining wall supporting cohesionless backfill. Using Mononobe – Okabe analysis, Seed and Whitman (1970) have provided the solution for dynamic active earth pressure considering that the retaining wall is backfilled only with ϕ soil. The Mononobe-Okabe concept was extended for c-ϕ backfill by Saran and Prakash (1968), Das and Puri (1996), Saran and Gupta (2003), Ghosh and Saran (2007) and Ghosh et al. (2008). In their analytical approach, pressure due to weight, surcharge and cohesion were optimized separately. But in real situation, for the simultaneous action of weight, surcharge and cohesion, a single critical wedge surface will be generated. Shukla et al. (2009) has given a solution for single critical wedge surface, but without considering the roughness of the wall and also without taking into account of the effect of tension crack. Ghosh (2010) has given a solution for active earth pressure in which the same coefficient is used for weight and surcharge but a separate coefficient used for cohesion. Therefore, in this analysis, an attempt is made to find out the dynamic active earth pressure in such a way that a single critical wedge surface is generated for the simultaneous action of weight, surcharge and cohesion.

Analysis

The analysis of retaining wall is made in the following steps.

Consider a vertical rigid retaining wall retaining a levelled backfill of height H, angle of internal friction ϕ, cohesion c and unit weight γ. A surcharge load of intensity q per unit length is assumed to act on the backfill. At any time during active state of equilibrium, if θ is the wedge angle then Figure 1a shows the forces acting on the retaining wall – soil wedge system. In the assumed model (Figure 1b), weight of the backfill soil in tension crack zone (height H_0) is considered to act along with the weight of the surcharge. Cohesive and adhesive resistances of the tension crack zone are neglected.

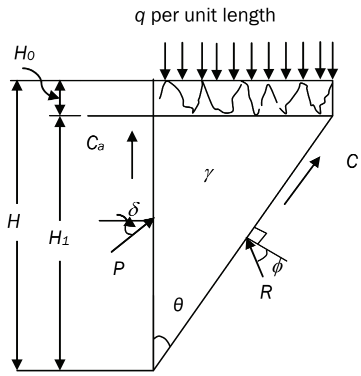

Fig.1a Original Retaining wall – Soil Mass System

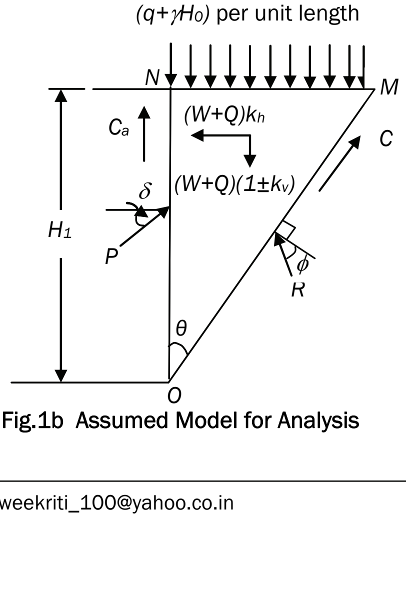

Fig.1b Assumed Model for Analysis

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P is the force acting on retaining wall at an angle δ to the normal of the wall and *R* is the reaction due to retained soil on the sliding soil whereas *W* and *Q* are the weight due to wedge soil and surcharge respectively. *C* is the cohesive force acting on the wedge surface and C_a is the adhesive force acting on the wall surface.

Derivation of Formulations during Active State of Equilibrium

Geometrical Parameters and Various Force Components

Depth of the tension crack zone, $H_0 = \frac{2c}{\gamma}\sqrt{N_\phi}$ (1)

where

$$N_\phi = \frac{1+\sin\phi}{1-\sin\phi} \text{ (Lambe and Whitman,1979)} \quad (2)$$

So, $H_1 = H - H_0$ (3)

For sliding triangular wedge OMN,

$$MN = H_1 \tan\theta \quad (4)$$

$$W = \frac{1}{2}\gamma H_1^2 \tan\theta \quad (5)$$

$$Q = (q + \gamma H_0) H_1 \tan\theta \quad (6)$$

$(W+Q)k_h$ and $(W+Q)(1 \pm k_v)$ are horizontal and vertical inertia forces due to earthquake where k_h and k_v are horizontal and vertical seismic coefficients.

$$OM = H_1 \sec\theta \quad (7)$$

$C = cH_1 \sec\theta$ and $C_a = c_a H_1$ (8)

Application of Equilibrium Conditions and Evaluation of Active Force

Applying the force equilibrium conditions $\Sigma H = 0$, $\Sigma V = 0$

$$C_a + C\cos\theta + R\sin(\phi+\theta) + P\sin\delta - (W+Q)(1 \pm k_v) = 0 \quad (9)$$

$$C\sin\theta - R\cos(\phi+\theta) + P\cos\delta - (W+Q)k_h = 0 \quad (10)$$

$$P_a \sin(\phi+\delta+\theta) + cH\sec\theta\cos\phi + c_a H\cos(\phi+\theta) = \frac{W+Q}{1 \pm k_v}\left\{\cos(\phi+\theta) + \frac{k_h}{1 \pm k_v}\sin(\phi+\theta)\right\} \quad (11)$$

Substituting the values of *W* and *Q* as per Eqn 5 and 6 respectively and considering $\psi = \tan^{-1}\frac{k_h}{(1 \pm k_v)}$ and $c = c_a$, we get,

$$P\sin(\phi+\delta+\theta) = \frac{[\gamma H_1 + 2(q+\gamma H_0)](1 \pm k_v)}{2}\left[\frac{\tan\theta}{\cos\psi}\{\cos(\phi-\psi+\theta)\}\right]H_1 - cH_1\frac{\cos\phi}{\cos\theta} \quad (12)$$

$$P = \frac{\gamma H_1 + 2(q+\gamma H_0)(1 \pm k_v)}{2} x$$

Or, $$\left[\frac{\sin\theta\cos(\phi-\psi+\theta) - m\cos\phi\cos\psi}{\cos\psi\cos\theta}\right] x \frac{H_1}{\sin(\phi+\delta+\theta)} \quad (13)$$

where, $m = \frac{2c}{[\gamma H_1 + 2(q+\gamma H_0)](1 \pm k_v)}$

It can be seen from Eqn 13 that for a particular retaining wall backfill system, all the terms are constant except θ. So, for different values of θ will produce different values of *P* and the optimum value is given by $(dP/d\theta) = 0$.

The Eqn (13) can also be written as

$$P = \frac{[\gamma H_1 + 2(q+\gamma H_0)](1 \pm k_v)}{2} kH_1 \quad (14)$$

where $k = \frac{\sin\theta\cos(\phi-\psi+\theta) - m\cos\phi\cos\psi}{\cos\psi\cos\theta\sin(\phi+\theta+\delta)}$

And at critical satisfying the condition, $(dP/d\theta)=0$, Eqn. 14 becomes

$$P_{ae} = \frac{\gamma H_1 + 2(q+\gamma H_0)}{2} k_{ae} H_1 \quad (15)$$

Where P_{ae} = Total seismic active force on the back of a retaining wall,

k_{ae}= Coefficient of active earth pressure.

Results

The coefficients for seismic active earth pressure are obtained optimizing k with respect to θ. To optimize k, a computer program using "c" programming was developed that would find the wedge angle θ at which corresponding coefficient of active earth pressure k_{ae} is also calculated. The results obtained from the analysis are presented in tabular form in Table 1 for the following variation of parameters.

k_h = 0, 0.1 and 0.2

k_v = 0, k_h/2 and k_h

ϕ = 20°, 30° and 40°

δ = -ϕ/2, 0, ϕ/2 and ϕ

m= 0,0.1 and 0.2

Table 1 Values of Seismic Active Earth Pressure Coefficients (k_{ae})

k_h, k_v	ϕ	m = 0				m = 0.1				m = 0.2			
		δ=-ϕ/2	δ=0	δ=ϕ/2	δ=ϕ	δ=-ϕ/2	δ=0	δ=ϕ/2	δ=ϕ	δ=-ϕ/2	δ=0	δ=ϕ/2	δ=ϕ
k_h=0, k_v=0	20	0.578	0.49	0.447	0.427	0.326	0.286	0.263	0.252	0.097	0.086	0.08	0.077
	30	0.416	0.333	0.302	0.293	0.197	0.164	0.15	0.149	0	0	0	0
	40	0.286	0.218	0.199	0.21	0.101	0.081	0.075	0.08	-0.062	-0.05	-0.048	-0.05
k_h=0.1, k_v=0	20	0.649	0.569	0.532	0.52	0.418	0.374	0.35	0.341	0.199	0.18	0.169	0.164
	30	0.48	0.397	0.368	0.372	0.276	0.235	0.22	0.222	0.088	0.076	0.072	0.073
	40	0.339	0.268	0.253	0.275	0.169	0.138	0.132	0.143	0.014	0.011	0.011	0.012
k_h=0.2, k_v=0	20	0.738	0.672	0.648	0.653	0.524	0.48	0.46	0.458	0.314	0.288	0.275	0.272
	30	0.553	0.473	0.452	0.474	0.364	0.318	0.304	0.315	0.184	0.163	0.156	0.161
	40	0.401	0.329	0.319	0.359	0.243	0.204	0.2	0.224	0.095	0.082	0.08	0.09
k_h=0.1, k_v=0.05	20	0.677	0.593	0.553	0.54	0.445	0.393	0.371	0.362	0.225	0.203	0.191	0.185
	30	0.5	0.413	0.383	0.386	0.295	0.251	0.234	0.237	0.106	0.092	0.086	0.087
	40	0.353	0.279	0.262	0.285	0.181	0.148	0.141	0.153	0.025	0.021	0.02	0.022
k_h=0.2, k_v=0.1	20	0.792	0.715	0.685	0.686	0.567	0.524	0.5	0.496	0.363	0.332	0.316	0.312
	30	0.592	0.504	0.479	0.496	0.4	0.347	0.331	0.341	0.219	0.192	0.183	0.188
	40	0.428	0.348	0.336	0.376	0.266	0.223	0.217	0.242	0.117	0.1	0.097	0.108
k_h=0.1, k_v=0.1	20	0.706	0.617	0.575	0.561	0.472	0.421	0.394	0.383	0.252	0.226	0.212	0.207
	30	0.521	0.429	0.397	0.401	0.315	0.267	0.249	0.251	0.125	0.107	0.101	0.102
	40	0.367	0.289	0.272	0.295	0.195	0.159	0.151	0.163	0.037	0.031	0.03	0.032
k_h=0.2, k_v=0.2	20	0.847	0.761	0.725	0.722	0.627	0.569	0.541	0.535	0.413	0.377	0.358	0.352
	30	0.632	0.535	0.506	0.522	0.437	0.378	0.358	0.368	0.253	0.221	0.211	0.216
	40	0.455	0.369	0.354	0.394	0.291	0.242	0.234	0.26	0.139	0.118	0.115	0.127

Discussion on Results

The variation of seismic active earth pressure coefficient (k_{ae}) with variation of different parameters ϕ, δ, m, k_h and k_v are discussed in the following sections.

Effect of ϕ on Seismic Active Earth Pressure Coefficient

Figure 2 shows the variation of seismic active earth pressure coefficient (k_{ae}) with k_h for different value of ϕ at m=0.1, $\delta = \phi/2$, $k_v = k_h/2$. From the study, it is seen that for a particular value of ϕ, k_{ae} increases with increase in k_h whereas, for a particular value of k_h, k_{ae} decreases with increase in ϕ. Due to increase in k_h, disturbance in the soil increases and due to decrease in ϕ, internal resistance of the soil decreases. Due to these reasons, k_{ae} increases with increase in k_h and decrease in ϕ.

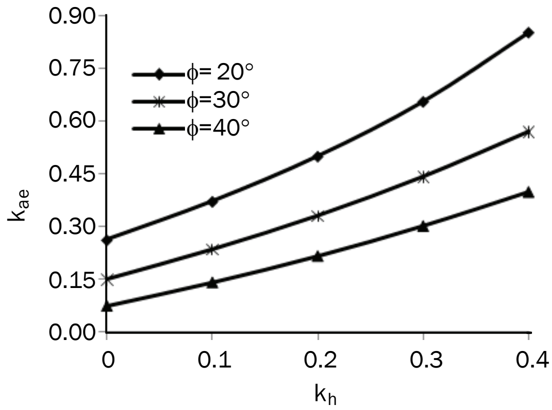

Fig. 2 Variation of k_{ae} with k_h for different values of ϕ (m=0.1, δ=ϕ/2, k_v=k_h/2)

Effect of m on Seismic Active Earth Pressure Coefficient

Figure 3 shows the variation of k_{ae} with k_h for different value of m at ϕ =20°, δ =ϕ/2, k_v=k_h/2. From the plot, it is seen that k_{ae} decreases with increase in m. Increase in m means increase in cohesion.

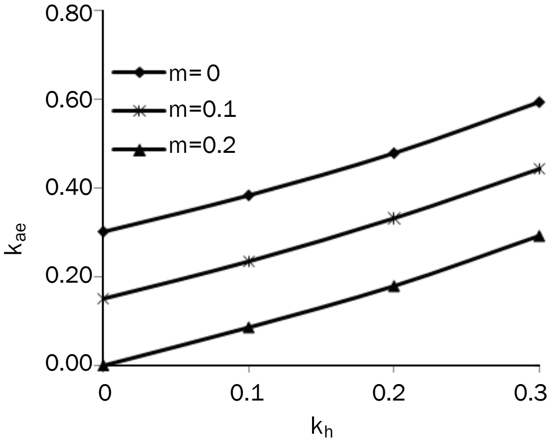

Fig. 3 Variation of k_{ae} with k_h for different values of m (ϕ=20°, δ=ϕ/2, k_v=k_h/2)

Effect of δ on Seismic Active Earth Pressure Coefficient

Figure 4 shows the variation of k_{ae} with k_h for different value of δ at ϕ =30°, m=0.1, k_v=k_h/2. From the plot, it is seen that initially for change in δ from $-\phi/2$ to 0, k_{ae} decreases with increase in δ. No significant change is noticed in k_{ae} due to further increase in δ.

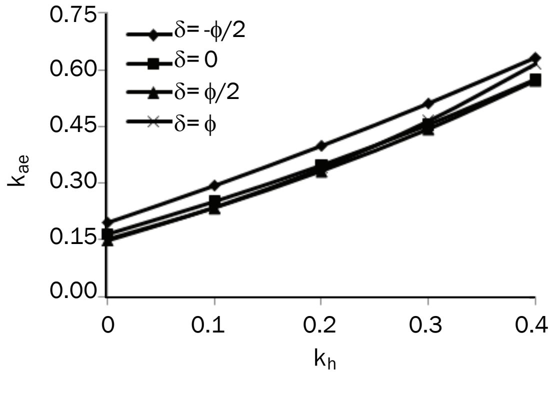

Fig. 4 Variation of k_{ae} with k_h for different values of δ (ϕ=30°, m=0.1, k_v=k_h/2)

Effect of K_v/K_h Ratio on Seismic Active Earth Pressure Coefficient

Figure 5 shows the variation of k_{ae} with k_h for different ratio of k_v/k_h. From the plot, it is seen that k_{ae} increases with increase in k_v/k_h ratio. Higher value of k_v/k_h ratio means more disturbances in the backfill material which results increase in k_{ae}.

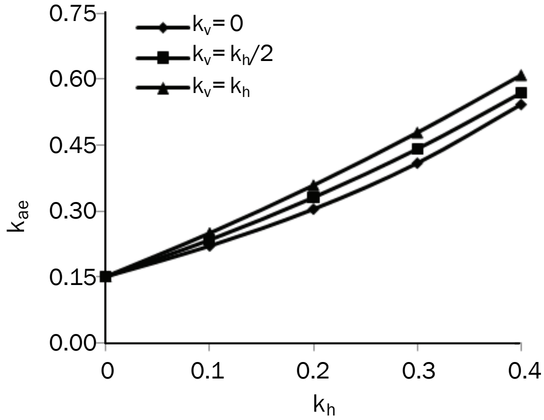

Fig.5 Variation of k_{ae} with k_h for different values of k_v (ϕ=30°, m=0.1, δ=ϕ/2)

Comparison of Results from Present Study with Shukla et al. (2009)

Figure 6 shows the variation of seismic active earth pressure with k_h obtained from the present study and the results obtained from Shukla et al. (2009). From the comparative study, it is seen that seismic active earth pressure as calculated from present methodology are of lesser magnitude in comparison to Shukla et al. (2009). The reason may be due to the fact that Shukla et al. (2009) do not consider the tension crack zone in the backfill side.

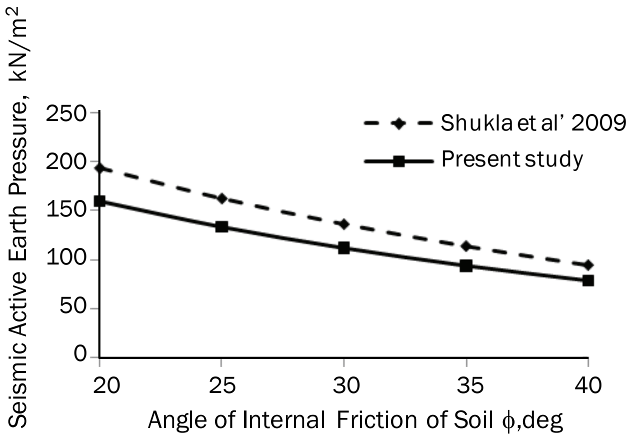

Fig.6 Comparison of Seismic Active Earth Pressure obtained from Present Study with Earlier Analytical Work (H=6.0m, γ=18 kN/m², c=6.0kN/m², k_h=0.2, k_v=0.1)

Conclusion

A methodology developed for the determination of seismic active earth pressure on the back of a retaining wall supporting c-ϕ backfill in such a way that for the simultaneous action of weight, surcharge and cohesion, a single critical wedge angle will develop. The results are presented in tabular form and for intermediate portion, a linear interpolation is suggested. The general conclusions of the present study are summarized as follows:

> The present study provides an analytical expression for the total active force on the back of a retaining wall supporting *c*-ϕ backfill considering both horizontal and vertical seismic coefficients based on Coulomb's sliding wedge mechanism of total force equilibrium.

> For a particular retaining wall backfill system, the coefficient of active force increases with increase in the value of seismic coefficients indicating higher value of active force and more participation of backfill soil at active state during seismic condition.

> The magnitude of active force coefficient decreases with increase in the magnitude of *c* and ϕ of the backfill and for a particular combination, it may be negative. At negative values of active force coefficients, we can say that there should not be any force acting on the retaining wall during active state.

> This generalized solution is more realistic as compared to the other analysis as here we are getting only single wedge angle for the simultaneous action of weight, surcharge and cohesion, which simulates the practical situation.

Notations

c, c_a	cohesion and adhesion of the backfill material
ϕ, δ	Angle of internal friction of soil and angle of wall friction
k_h, k_v	horizontal and vertical seismic coefficient imposed in the backfill material
γ	unit weight of the backfill material
P	seismic force at the back of retaining wall at any time of earthquake during active state of equilibrium
P_{ae}	seismic active force (optimum) at the back of a retaining wall
k_{ae}	seismic active force coefficient at the back of a retaining wall
θ	inclination of the wedge surface with vertical
q	intensity of surcharge loading acting over the top of the backfill

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