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Dynamic Analysis of Earth Dam and Embankments under Earthquake Force

Indrajit Chowdhury[1] and Shambhu P. Dasgupta[2]

Key words

Clay core, dynamic response, earth dam, factor of safety, modes of vibration, variable shear modulus.

Abstract: This paper presents a generalized solution for dynamic analysis of earth dam based on modal response technique. Formulation for dynamic response takes care of both homogenous dams and dams with internal clay core. A technique based on distribution of static stiffness has been evolved to evaluate factor of safety (FOS) for a dam with internal clay core.

It also furnishes formulation for dynamic analysis when the dynamic shear modulus varies with depth of the dam and proposes a modified method for stability analysis that tries to combine the pseudo static method of slip circle and sliding block technique to arrive at a rational factor of safety under earthquake loading.

The method being analytic in nature does not require a sophisticated software development, a simple spread sheet or a MATHCAD shell would suffice for the same. Wherever possible, results have been compared with existing solutions.

Introduction

Earth dams and embankments play an important role in irrigation, flood control and development of rail road infrastructure. Considering their functional requirement and importance in serving the community, it becomes imperative that they remain operational even after a strong motion earthquake with minimum damage.

The major analytical techniques that are popular for earthquake analysis of such dams are those developed by Mononobe (1936), Makdisi & Seed (1977), Gazetas (1982) and finally numerical analysis, based on Finite Element Method (FEM) as proposed by Clough & Chopra (1966) to name only the pioneering few.

Of all these techniques the first three methods are analytical in nature, where the dam is considered as a triangular homogenous shear beam and a solution is sought, based on its equation of motion in one dimension under seismic force. Gazetas's analytical formulation is usually considered to be most rigorous (Kramer 1984), while Seed's method can cater to non-linear deformation of dam based on hyperbolic stiffness degradation of soil which is a function of induced strain level due to vibration (Seed & Idriss, 1971).

Finite element method (FEM) obviously gives the most exhaustive solution, but is not without its limitation. For other than exhaustive data preparation and input requirement – that makes the analysis expensive; if proper constitutive model cannot be fitted into the mathematical model, it can often yield unrealistic result. In many cases it is difficult to fit an earth dam problem into a general purpose finite element software thus necessitating development of special purpose software which is not always readily available commercially.

It is for this, analytical methods as mentioned above have remained more popular around the world because of their simplicity in application, though they suffer from deficiencies in many cases. For instance, all methods cited above assume the dam to be a perfect triangle (i.e. crest area is zero) while in reality it is not. As a matter fact United States Bureau of Reclamation of small dams suggests an empirical formula for crest width (B_t) and height H as B_t= H/5+10(in FPS unit).A dam always has some finite dimension at crest for maintenance and inspection. How much this data affects its dynamic response is not very apparent.

Both Mononobe (1936), Makdisi & Seed(1877) gave solution assuming the dynamic shear modulus (G) constant with depth, while this would hardly be the case in real field condition. It has been found that G varies with height (H) of the dam. Gazetas (1982) proposed a formulation where he considered G to vary with depth (z) as G $(z/H)^{\alpha}$ where α =0,1, 2 etc depicting variation of G as constant, linear and parabolic with depth respectively. However, many designers feel this could be unrealistic, especially for linear and parabolic variation as because it depicts G = 0 at the crest of dam.

Finally, none of the analytical methods takes care of the practical situation of impermeable clay core which is often deployed in an earth dam of large magnitude to control the seepage through it.

1 Head of the Department, (Civil and Structural Engineering), Petrofac International Limited, Sharjah U.A.E. , Email Indrajit.Chowdary@petrofac.com

2 Professor & Head, Department of Civil Engineering, Indian Institute of Technology, Kharagpur, Kharagpur 721 302,India, Email dasgupta@civil.iitkgp.ernet.in

Present paper is thus an attempt to furnish a solution to number of such deficiencies as cited above and arrive at a more realistic solution.

The Proposed Method

Shown in Figure 1 is a typical triangular shear beam usually considered for a conventional linear analysis of a homogeneous earth dam.

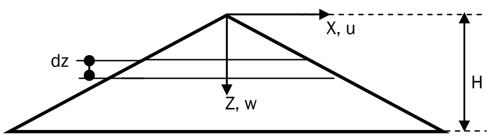

Fig. 1 Typical Triangular Shear Beam as Considered for Earth Dam

The equation of motion of such dams are usually expressed as

$$\frac{\partial^2 u}{\partial t^2}=\frac{G}{\rho}\left[\frac{\partial^2 u}{\partial z^2}+\frac{1}{z}\frac{\partial u}{\partial z}\right] \tag{1}$$

in which, soil properties are: G = dynamic shear modulus; ρ= mass density (γ/g); γ= unit weight; and g = acceleration due to gravity; u = displacement amplitude of the soil in x-direction.

For the above type of earth dams, boundary conditions considered are

At z = 0, $\partial u/\partial z$ = 0 for all values of t;

At z = H, u = 0 for all values of t.

Imposing the above boundary conditions, eqn. (1) may be written as

$$u(z,t)=\sum_{n=1}^{\infty}\left[A_n sin\,\omega_n t+B_n cos\,\omega_n t\right]J_0(\beta_n z/H) \tag{2}$$

in which, A_n = B_n = constants that are functions of the initial conditions; J_0= Bessel's function of first kind of order 0; β_n = a constant having values β_1 = 2.404, β_2 = 5.52 β_3 = 8.65 etc. (Jeffrey, 2005).

Based on above it may be stated that the generalized shape function of a triangular shear beam vibrating under its own inertia can be expressed as

$$N_j=J_0\left(\beta_j z/H\right) \tag{3}$$

where j =1,2,3,4,…. are the number modes considered for analysis.

For a two dimensional plane strain body under deformation, the shear strain energy (Timoshenko & Young, 1982) may be expressed as

$$V=\frac{\nu G}{1-2\nu}\left(\varepsilon_x+\varepsilon_z\right)^2+G\left(\varepsilon_x{}^2+\varepsilon_z{}^2\right)+\frac{G}{2}\left(\gamma_{xz}{}^2\right) \tag{4}$$

in which, V = strain energy density; ν = Poisson's ratio; ε_x,ε_z and γ_{xz} = strain in the body.

Considering u(x,z,t) = N_r(x,z).q(t) we have

$$\frac{\partial V}{\partial q_r}=\frac{2G(1-\nu)}{1-2\nu}\frac{\partial N_i}{\partial x}\frac{\partial N_r}{\partial x}q_i q_r+G\frac{\partial N_i}{\partial z}\frac{\partial N_r}{\partial z}q_i q_r \tag{5}$$

where, q(t) is exclusively a time function.

From the above, it can be shown (Hurty & Rubenstein, 1967) that for a body of height H and uniform width B, the stiffness and mass matrices can be expressed as

$$K_{ir}=\int_0^H\int_0^B\left[\frac{2G(1-\nu)}{1-2\nu}\frac{\partial N_i}{\partial x}\frac{\partial N_r}{\partial x}+G\frac{\partial N_i}{\partial z}\frac{\partial N_r}{\partial z}\right]dxdz \tag{6}$$

and

$$M_{ir}=\frac{\gamma}{g}\int_0^H\int_0^B N_i N_r dxdz \tag{7}$$

For shear waves propagating in the vertical direction, it has been shown by Chowdhury & Dasgupta (2007) that in one dimension, a soil body of depth H and extending to infinity in horizontal direction, eqns. (6) and (7) can be expressed as

$$K_{ir}=\int_0^H\left[G\frac{\partial N_i}{\partial z}\frac{\partial N_r}{\partial z}\right]dz \tag{8}$$

$$M_{ir}=\frac{\gamma}{g}\int_0^H N_i N_r dz \tag{9}$$

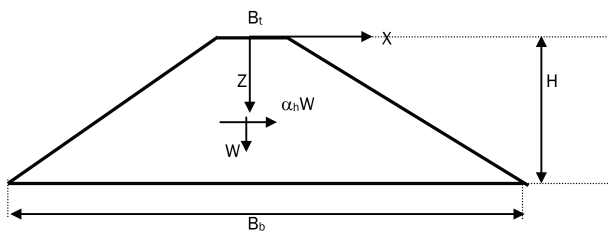

Fig. 2 A typical Trapezoidal Section of an Earth Dam

For a typical trapezoidal section of an earth dam shown in Figure 2, eqns. (8) and (9) can be modified and expressed as

$$K_{ir}=GA_t\int_0^H\left[\left(1+\psi\frac{z}{H}\right)\frac{\partial N_i}{\partial z}\frac{\partial N_r}{\partial z}\right]dz \tag{10}$$

and

$$M_{ir}=\frac{\gamma A_t}{g}\int_0^H\left(1+\psi\frac{z}{H}\right)N_i N_r dz \tag{11}$$

where, A_t= cross sectional area of the dam at z = 0 and for 1.0m depth (perpendicular to the plane of the paper) is equal to B_t; ψ is dimensionless constant expressed as $\psi = (B_b - B_t)/B_t$;

N_i and N_r are expressed in eqn. (3).

Considering $\xi = z/H$ when $z \to 0$ $\xi \to 0$ and when $z \to H$, $\xi \to 1$, eqns. (10) and (11) can be expanded and expressed for the first three modes as

$$[K] = \frac{GA_t}{H}\begin{bmatrix} \beta_1^2\int_0^1(1+\psi\xi)J_1^2(\beta_1\xi)d\xi & & \text{Symmetrical} \\ \beta_1\beta_2\int_0^1(1+\psi\xi)J_1(\beta_1\xi)J_1(\beta_2\xi)d\xi & \beta_2^2\int_0^1(1+\psi\xi)J_1^2(\beta_2\xi)d\xi & \\ \beta_1\beta_3\int_0^1(1+\psi\xi)J_1(\beta_1\xi)J_1(\beta_3\xi)d\xi & \beta_2\beta_3\int_0^1(1+\psi\xi)J_1(\beta_2\xi)J_1(\beta_3\xi)d\xi & \beta_3^2\int_0^1(1+\psi\xi)J_1^2(\beta_3\xi)d\xi \end{bmatrix} \quad (12)$$

$$[M] = \frac{\gamma A_t H}{g}\begin{bmatrix} \int_0^1(1+\psi\xi)J_0^2(\beta_1\xi)d\xi & & \text{Symmetrical} \\ \int_0^1(1+\psi\xi)J_0(\beta_1\xi)J_0(\beta_2\xi)d\xi & \int_0^1(1+\psi\xi)J_0^2(\beta_2\xi)d\xi & \\ \int_0^1(1+\psi\xi)J_0(\beta_1\xi)J_0(\beta_3\xi)d\xi & \int_0^1(1+\psi\xi)J_0(\beta_2\xi)J_0(\beta_3\xi)d\xi & \int_0^1(1+\psi\xi)J_0^2(\beta_3\xi)d\xi \end{bmatrix} \quad (13)$$

Eqns. (12) and (13) are generalized expression for stiffness and mass matrices for a trapezoidal dam shown in Figure 2, having G invariant with depth. Individual elements of [K] and [M] matrices are obtained by numerical integration of the respective terms between limits 0-1.

For the particular case of trapezoidal section is a triangle i.e. limit $A_t \to 0$ and $\psi \to \infty$, eqns. (12) and (13) can be expressed as

$$[K] = \frac{GA_t}{H}\begin{bmatrix} 3.894\times10^6 & -736.854 & -6.425\times10^3 \\ -736.854 & 8.82\times10^6 & 1.582\times10^4 \\ -6.425\times10^3 & 1.582\times10^4 & 1.378\times10^7 \end{bmatrix} \quad (14)$$

And

$$[M] = \frac{\gamma A_t H}{g}\begin{bmatrix} 6.742\times10^5 & -156.128 & -18.743 \\ -156.128 & 2.895\times10^5 & 207.419 \\ -18.473 & 207.419 & 1.844\times10^5 \end{bmatrix} \quad (15)$$

Considering the eigen value problem of the above (Bathe, 1996) as

$$[K][\varphi] = \lambda[M][\varphi] \quad (16)$$

where λ and φ are the eigen values and eigen vectors of matrix [K] and [M] respectively.

We have then

$$[\lambda] = \begin{bmatrix} 5.775 & 0 & 0 \\ 0 & 30.469 & 0 \\ 0 & 0 & 74.758 \end{bmatrix}\frac{V_s^2}{H^2} \quad (17)$$

where $V_s = \sqrt{Gg/\gamma}$, the shear wave velocity of soil.

Since $\omega^2=\lambda$, one can write

$$[\omega] = \begin{bmatrix} 2.403 & 0 & 0 \\ 0 & 5.52 & 0 \\ 0 & 0 & 8.646 \end{bmatrix}\frac{V_s}{H} \Rightarrow [C_{\omega i}]\frac{V_s}{H} \quad (18)$$

Values of $C_{\omega\iota}$ obtained as above, is compared with other established methods in Table-1.

Table 1 Comparison of $C_{\omega\iota}$ by Various Methods

Mode	Mononobe	Makdisi & Seed	Gazetas	Proposed
1	2.409	2.4	2.404	2.403
2	5.521	5.52	5.52	5.52
3	8.654	8.65	8.654	8.646

It is observed that $C_{\omega\iota}$ based on proposed method is in excellent agreement with established methods for limit $A_t \to 0$.

Having established the correctness of basic condition of derivation for the particular case of limit $A_t \to 0$, we present variance of time period of the dam for various values of ψ considering $T=2\pi/\omega$. This is given in Table-2. The time period is expressed as $T_i = C_{Ti}(H/V_s)$.

Table 2 Variation of C_T for First Three Modes

ψ	C_{T1}	C_{T2}	C_{T3}
∞	2.615	1.138	0.727
99	2.64	1.149	0.733
49	2.666	1.159	0.739
24	2.715	1.178	0.749
15	2.763	1.193	0.756
12	2.809	1.207	0.762
9	2.854	1.219	0.766

It is observed that time periods gets elongated with increase in crest width though the variation for fundamental time period(which is most critical) is not much and is of the order of 9% only (from $\psi \rightarrow \infty$ to 9). As such, the proposed method though more realistic, does not give any significant advantage over the established methods using triangular homogeneous sections. However, distinct advantage of the proposed method will emerge subsequently when we take up the case of dam with impermeable cores which is more a reality in field than a homogenous section assumed for dynamic analysis of such earth dams.

The first three mode shapes/ eigenvectors for the dam based on eqn. (3) are shown in Figure 3.

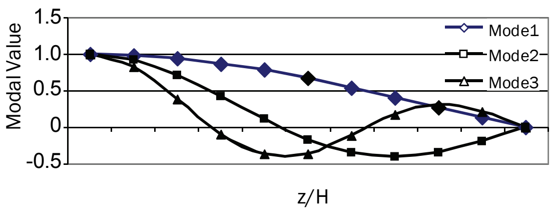

Fig. 3 Eigen Vectors for First Three Modes for the Dam

Effect of Variation of G with Depth

Gazetas (1982) derived solution of dynamic response of an earth dam considering $G(z)=G(z/H)^{\alpha}$. It is apparent from above that while it gives a G value constant with depth (α=0), for linear and parabolic variation it gives G = 0 at the crest of the dam. Since this will invariably have a finite value we propose to consider G(z) as

$$G(z) = G\left(1 + z / H\right)^{\alpha} \tag{19}$$

The variation of G for this type of curve is shown in Figure 4.

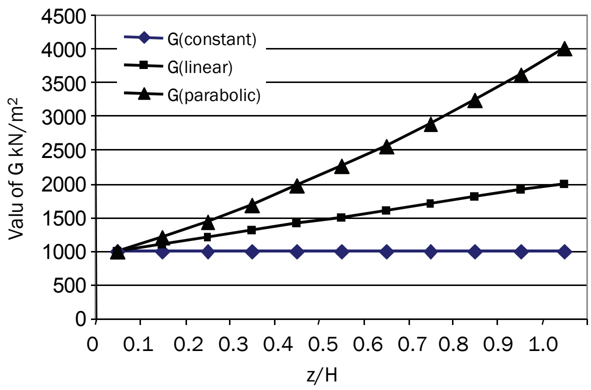

Fig. 4 Proposed Variation of G With Depth

For solution to this end, that has no established answers, eqn. (12) gets modified to

$$[K]=\frac{GA_t}{H}\begin{bmatrix} \beta_1^2\int_0^1(1+\xi)^{\alpha}(1+\psi\xi)J_1^2(\beta_1\xi)d\xi & & \text{Symmetrical} \\ \beta_1\beta_2\int_0^1(1+\xi)^{\alpha}(1+\psi\xi)J_1(\beta_1\xi)J_1(\beta_2\xi)d\xi & \beta_2^2\int_0^1(1+\xi)^{\alpha}(1+\psi\xi)J_1^2(\beta_2\xi)d\xi & \\ \beta_1\beta_3\int_0^1(1+\xi)^{\alpha}(1+\psi\xi)J_1(\beta_1\xi)J_1(\beta_3\xi)d\xi & \beta_2\beta_3\int_0^1(1+\xi)^{\alpha}(1+\psi\xi)J_1(\beta_2\xi)J_1(\beta_3\xi)d\xi & \beta_3^2\int_0^1(1+\xi)^{\alpha}(1+\psi\xi)J_1^2(\beta_3\xi)d\xi \end{bmatrix} \tag{20}$$

where α =1 and 2 respectively for linear and parabolic variation.

Based on above variation of C_T for linear and parabolic profile for different values of ψ are as presented in Table-3.

Intermediate values can well be linearly interpolated considering the variation of C_T with respect to ψ is almost linear as shown in Figure 5.

Table 3 Values of C_T With Respect to Linear and Parabolic Variation of Soil for First Three Modes

ψ	Linear Variation			Parabolic Variation		
	C_{T1}	C_{T2}	C_{T3}	C_{T1}	C_{T2}	C_{T3}
∞	2.005	0.924	0.585	1.554	0.754	0.460
99	2.026	0.934	0.59	1.573	0.763	0.463
49	2.048	0.943	0.594	1.591	0.776	0.466
24	2.089	0.959	0.601	1.626	0.785	0.470
15	2.134	0.975	0.607	1.664	0.798	0.475
12	2.163	0.984	0.610	1.688	0.806	0.477
9	2.203	0.996	0.614	1.722	0.816	0.480

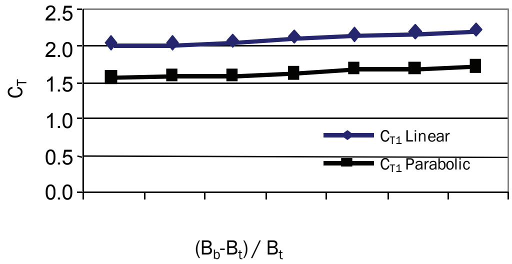

Fig. 5 Variation of Factor C_{T1} for Linear and Parabolic Distribution of Soil

It is again observed here that while the variation in time period is about 25% for linear variation of G, with respect to G constant with depth, the variation due to tapering of the section is of the order of 10% only.

Effect of Inner Core on Dynamic Response of the Dam

No analytical methods exist till date for this case in terms of dynamic response. It is in such a situation one resorts to the finite element method (FEM) for a comprehensive analysis. However, considering the dynamic response remaining linear, the problem can be approached as given hereunder.

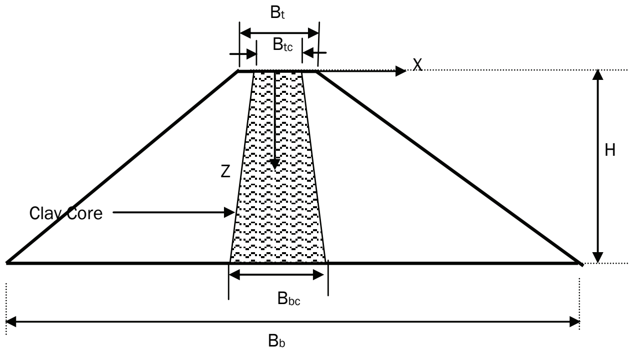

Fig. 6 An Earth Dam with Clay Core to Restrict the Seepage

Shown in Figure 6 is a typical earth dam with an internal clay core. Let width of the core at crest be B_{tc}, and B_{bc} be the width at bottom. Here suffix c denotes clay core.

Maintaining mathematical similarity let us define,

$$\psi_c = (B_{bc} - B_{tc}) / B_{tc} \tag{21}$$

The stiffness matrix for the dam section can be expressed as

$$[K] = GA_t \int_0^H \left[\left(1+\psi \frac{z}{H}\right) \frac{\partial N_i}{\partial z} \frac{\partial N_r}{\partial z} \right] dz \; - \; GA_{tc} \int_0^H \left[\left(1+\psi_c \frac{z}{H}\right) \frac{\partial N_i}{\partial z} \frac{\partial N_r}{\partial z} \right] dz \; + \; G_c A_{tc} \int_0^H \left[\left(1+\psi_c \frac{z}{H}\right) \frac{\partial N_i}{\partial z} \frac{\partial N_r}{\partial z} \right] dz \tag{22}$$

$$\Rightarrow [K] = GA_t \int_0^H \left[\left(1+\psi \frac{z}{H}\right) \frac{\partial N_i}{\partial z} \frac{\partial N_r}{\partial z} + (m-1)\left(\frac{A_{tc}}{A_t}\right) \int_0^H \left(1+\psi_c \frac{z}{H}\right) \frac{\partial N_i}{\partial z} \frac{\partial N_r}{\partial z} \right] dz \tag{23}$$

in which, G_c= dynamic shear modulus of clay core and let m = G_c/G the modular ratio of clay core and outer material, A_{tc} is area of inner core at the crest of dam @ $B_{tc} \times 1$.

Substituting eqn. (3), eqn. (23) can be expanded for the first three modes as,

$$[K] = \begin{bmatrix} k_{11} & \text{Symm.} & \\ k_{12} & k_{22} & \\ k_{13} & k_{32} & k_{33} \end{bmatrix} \tag{24}$$

where,

$$k_{11} = \frac{GA_t}{H} \left[\int_0^1 (1+\psi\xi) \beta_1^2 J_1^2(\beta_1 \xi) + (m-1)\left(\frac{A_{tc}}{A_t}\right) \int_0^1 (1+\psi_c \xi) \beta_1^2 J_1^2(\beta_1 \xi) \right] d\xi \tag{25}$$

$$k_{12} = \frac{GA_t}{H}\left[\int_0^1 (1+\psi\xi)\beta_1\beta_2 J_1(\beta_1\xi)J_1(\beta_2\xi) + (m-1)\left(\frac{A_{tc}}{A_t}\right)\int_0^1 (1+\psi_c\xi)\beta_1\beta_2 J_1(\beta_1\xi)J_1(\beta_2\xi)\right]d\xi \tag{26}$$

$$k_{13} = \frac{GA_t}{H}\left[\int_0^1 (1+\psi\xi)\beta_1\beta_3 J_1(\beta_1\xi)J_1(\beta_3\xi) + (m-1)\left(\frac{A_{tc}}{A_t}\right)\int_0^1 (1+\psi_c\xi)\beta_1\beta_3 J_1(\beta_1\xi)J_1(\beta_3\xi)\right]d\xi \tag{27}$$

$$k_{22} = \frac{GA_t}{H}\left[\int_0^1 (1+\psi\xi)\beta_2^{\,2} J_1^{\,2}(\beta_2\xi) + (m-1)\left(\frac{A_{tc}}{A_t}\right)\int_0^1 (1+\psi_c\xi)\beta_2^{\,2} J_1^{\,2}(\beta_2\xi)\right]d\xi \tag{28}$$

$$k_{23} = \frac{GA_t}{H}\left[\int_0^1 (1+\psi\xi)\beta_3\beta_2 J_1(\beta_3\xi)J_1(\beta_2\xi) + (m-1)\left(\frac{A_{tc}}{A_t}\right)\int_0^1 (1+\psi_c\xi)\beta_3\beta_2 J_1(\beta_3\xi)J_1(\beta_2\xi)\right]d\xi \tag{29}$$

$$k_{33} = \frac{GA_t}{H}\left[\int_0^1 (1+\psi\xi)\beta_3^{\,2} J_1^{\,2}(\beta_3\xi) + (m-1)\left(\frac{A_{tc}}{A_t}\right)\int_0^1 (1+\psi_c\xi)\beta_3^{\,2} J_1^{\,2}(\beta_3\xi)\right]d\xi \tag{30}$$

Similarly the mass matrix is given by the expression

$$[M] = \begin{bmatrix} m_{11} & \text{Symmetrical} & \\ m_{12} & m_{22} & \\ m_{13} & m_{23} & m_{33} \end{bmatrix} \tag{31}$$

Considering p $=\gamma_c/\gamma$ the unit weight ratio of core soil and the outer soil, mass matrix coefficients can be expressed as

$$m_{11} = \frac{\gamma A_t H}{g}\int_0^1\left[(1+\psi\xi)J_0^{\,2}(\beta_1\xi) + (p-1)\left(\frac{A_{tc}}{A_t}\right)(1+\psi_c\xi)J_0^{\,2}(\beta_1\xi)\right]d\xi \tag{32}$$

$$m_{12} = \frac{\gamma A_t H}{g}\int_0^1\left[(1+\psi\xi)J_0(\beta_1\xi)J_0(\beta_2\xi) + (p-1)\left(\frac{A_{tc}}{A_t}\right)(1+\psi_c\xi)J_0(\beta_1\xi)J_0(\beta_2\xi)\right]d\xi \tag{33}$$

$$m_{13} = \frac{\gamma A_t H}{g}\int_0^1\left[(1+\psi\xi)J_0(\beta_1\xi)J_0(\beta_3\xi) + (p-1)\left(\frac{A_{tc}}{A_t}\right)(1+\psi_c\xi)J_0(\beta_1\xi)J_0(\beta_3\xi)\right]d\xi \tag{34}$$

$$m_{22} = \frac{\gamma A_t H}{g}\int_0^1\left[(1+\psi\xi)J_0^{\,2}(\beta_2\xi) + (p-1)\left(\frac{A_{tc}}{A_t}\right)(1+\psi_c\xi)J_0^{\,2}(\beta_2\xi)\right]d\xi \tag{35}$$

$$m_{23} = \frac{\gamma A_t H}{g}\int_0^1\left[(1+\psi\xi)J_0(\beta_2\xi)J_0(\beta_3\xi) + (p-1)\left(\frac{A_{tc}}{A_t}\right)(1+\psi_c\xi)J_0(\beta_2\xi)J_0(\beta_3\xi)\right]d\xi \tag{36}$$

$$m_{33} = \frac{\gamma A_t H}{g}\int_0^1\left[(1+\psi\xi)J_0^{\,2}(\beta_3\xi) + (p-1)\left(\frac{A_{tc}}{A_t}\right)(1+\psi_c\xi)J_0^{\,2}(\beta_3\xi)\right]d\xi \tag{37}$$

Once the stiffness and mass matrices are obtained based on above formulation one can follow eqn. (16) for standard eigen value solution of the problem to derive the time periods and vibration modes. Stiffness coefficients derived through eqns. (25) to (30) is based on soil having G constant with depth. However, considering generality of the solution it is not difficult to incorporate the variation of G with depth. For instance if we have a case where the outer shell has a soil that varies linearly with depth while inner core has stiff clay whose dynamic shear modulus is constant with depth the stiffness matrix gets modified as follows.

$$k_{11} = \frac{GA_t}{H}\left[\int_0^1 (1+\psi\xi)(1+\xi)\beta_1^{\,2} J_1^{\,2}(\beta_1\xi) + (m-1)\left(\frac{A_{tc}}{A_t}\right)\int_0^1 (1+\psi_c\xi)\beta_1^{\,2} J_1^{\,2}(\beta_1\xi)\right]d\xi \tag{38}$$

and so on.

It is apparent from above that where established methods do not provide an answer to this problem in terms of *propagation of waves through a homogeneous shear beam of varying cross section,* the proposed method comes up with a rational solution of this practical problem.

Estimation of Dynamic Amplitude and Stress

For a body subjected to earthquake force, maximum amplitude is expressed as (Clough & Penzien, 1982)

$$S_d = S_a / \omega^2 \tag{39}$$

In terms of provision as furnished in IS code, eqn. (39) can be finally expressed as

$$S_{di} = \kappa_i \left(\frac{ZI}{2R} \right) \frac{S_{ai}}{\omega^2} \tag{40}$$

where, Z = zone factor; I = importance factor; R= ductility factor and, κ_i = modal mass participation factor usually expressed as $\sum_{i=1}^{n} m_i \phi_i \Big/ \sum_{i=1}^{n} m_i \phi_i^2$.

It is to be noted that IS 1893(2002) [draft code for earthen dam] does not have any provision for R the ductility factor in terms of earthen dam , though it implicitly assumes a value of R=1.5 in a semi empirical approach proposed there in. However, it has been shown by (Chowdhury & Dasgupta 2007) that R=2 to 3 will mostly give a reasonable result. Surely, more research is needed to define this value with more clarity for earth dams.

The modal mass participation factor based on above formulation can be expressed in this case as

$$\kappa_i = \int_0^1 (1+\psi\xi) J_0(\beta_i \xi) \Big/ \int_0^1 (1+\psi\xi) J_0(\beta_i \xi)^2 \tag{41}$$

It is observed here that instead of a unique value as expressed by Makdisi & Seed (1977) modal participation factor is a function of the top and bottom width in terms of ψ. Different values of κ_i with respect to ψ are furnished hereafter in Table- 4.

Table 4 Variation of
Modal Participation Factor κ_i with ψ

ψ	κ_1	κ_2	κ_3
∞	1.602	-1.065	0.851
99	1.591	-1.00	0.85
49	1.581	-0.94	0.849
24	1.562	-0.827	0.847
15	1.543	-0.718	0.845
12	1.532	-0.655	0.844
9	1.515	-0.561	0.843

It will be observed from the table that modal participation factor varies significantly especially for the second mode with respect to the top and bottom width of the dam. The value of κ_i as obtained for $\psi \to \infty$ matches exactly the value as proposed by Makdisi & Seed (1977).

The displacement of the dam over the height H can be thus expressed as

$$u_i(z,t) = \kappa_i \left(\frac{ZI}{2R} \right) \frac{S_{ai}}{\omega_i^2} J_0 \left(\beta_i \frac{z}{H} \right) \tag{42}$$

Substituting $T = 2\pi/\omega$, eqn. (42) after some simplification, can be expressed as

$$u_i(z,t) = \kappa_i \left(\frac{ZI}{2R} \right) \frac{C_{Ti}{}^2 \gamma H^2}{4\pi^2 G} \left(\frac{S_{ai}}{g} \right) J_0 \left(\beta_i \frac{z}{H} \right) \tag{43}$$

The shear strain is expressed as

$$\gamma_{xz\,i} = \frac{\partial u}{\partial z} = \kappa_i \left(\frac{ZI}{2R} \right) \frac{C_{Ti}{}^2 \beta_i \gamma H}{4\pi^2 G} \left(\frac{S_{ai}}{g} \right) J_1 \left(\beta_i \frac{z}{H} \right) \tag{44}$$

The shear stress that varies along the depth is finally expressed as

$$\tau_{xz\,i} = \kappa_i \left(\frac{ZI}{2R} \right) \frac{C_{Ti}{}^2 \beta_i \gamma H}{4\pi^2} \left(\frac{S_{ai}}{g} \right) J_1 \left(\beta_i \frac{z}{H} \right) \tag{45}$$

Earthquake Stability and Estimation of Lateral Seismic Coefficient (α_h)

Estimation of lateral stability of earthen dams under earthquake is still marred by some uncertainties, and a number of theories have been proposed e.g.

- Slip Circle analysis based on pseudo static method.
- Sliding block method
- Dynamic finite element analysis.

Of all the above methods, slip circle method based on pseudo static analysis is most popular though it has been shown that the method can at times gives unsatisfactory results. Terzaghi (1950) who first proposed it for stability of embankments under earthquake conceded the method to be crude, while Seed (1979) showed a number of dams in USA that had undergone failure in earthquake, had a factor of safety (FOS) greater than 1.0 based on pseudo static analysis.

Newmark (1965) proposed a sliding block technique, though found to be theoretically sound, essentially requires time history response of a site which is always not available. Moreover the movement of failed mass as a rigid block may not be always realistic when the soil is not reasonably stiff.

Dynamic finite element analysis (Chopra &

Clough 1966, Duncan 1992) though gives the most comprehensive result, requires significant engineering effort and very careful evaluation of in-situ soil parameters of dam which is surely an expensive exercise and can possibly be justified only for very large dam of utmost importance.

Thus, in-spite of its limitation, pseudo static analysis based on slip circle method because of its simplicity in application continues to be the most popular method in practice where the earthquake force is simulated by an equivalent lateral seismic coefficient α_h (Figure 2).

One major difficulty and limitation that arises in estimation of α_h for dams is that unlike a normal structure, where the lateral seismic coefficient can be considered as constant over the height of a structure, an earth dam is a far more flexible system and as such α_h varies with height. Hence in many cases what would be the effective lateral seismic coefficient becomes a matter of judgment.

As per IS-1893(2002) [draft code for earthen dam], for instance, α_h is expressed by an empirical formula of

$$\alpha_h = \frac{1}{3} \times Z \times I \times S \tag{46}$$

where Z and I are as explained earlier and S is soil factor that varies from max 2.0 for soft soil to 1.0 minimum.

The peak horizontal ground acceleration (PHGA) is considered as $a_{\max} = Z \times I \times S$, while the net lateral force due to earthquake is given by

$$F = \frac{1}{3} \times Z \times I \times S \times W \tag{47}$$

It is obvious that irrespective of the time period of the dam this value remains constant for a particular zone(II, III, IV etc) though it is well known that how much acceleration a body is subjected to is dependent on time periods of the body in different modes.

Since value of α_h proposed is empirical in nature, there could be cases when response could be overestimated and vice versa and matching with above mentioned codal provision vide Equation (47) can just be a coincidence.

Based on the proposed dynamic theory, lateral coefficient α_h can be estimated as mentioned hereafter. Typical slope failure profile of an earth dam, based on wedge theory, is shown in Figure 7.

Following Makdisi & Seed (1977), the total horizontal force acting on this wedge is expressed as

$$F(z,t) = \text{Mass of triangular wedge } \times \ddot{u}_{av} \tag{48}$$

were $\ddot{u}_{av} =$ average acceleration over the dam height.

Based on Figure 7, eqn. (48) can be expressed as

$$\ddot{u}_{av} = \frac{2F(z,t)g}{B_z \times z \times \gamma} = \frac{2\tau_{xz}g}{z\gamma} \tag{49}$$

or, $\alpha_{hi} = \dfrac{\ddot{u}_{av}}{g} = \dfrac{2\tau_{xzi}}{z\gamma} =$

$$2\kappa_i\left(\frac{ZI}{2R}\right)\frac{C_{Ti}{}^2\beta_i H}{4\pi^2 z}\left(\frac{S_{ai}}{g}\right)J_1\left(\beta_i\frac{z}{H}\right) \tag{50}$$

where i =1,2,3,....is the number of modes.

It is evident from eqn. (50) that α_h is a function of the depth z and shape function as described in eqn. (3).

Now let as assume that this lateral seismic coefficient is effective over the whole cross section of dam.

Thus over full height H of the dam, combining eqns. (50) and (11), we have

$$F_{ei} = \alpha_{hi}W =$$

$$2\kappa_i\left(\frac{ZI}{2R}\right)\frac{C_{Ti}{}^2\beta_i H\gamma A_t}{4\pi^2}\left(\frac{S_{ai}}{g}\right)\int_0^H\frac{1}{z}\left(1+\psi\frac{z}{H}\right)J_1\left(\beta_i\frac{z}{H}\right)dz \tag{51}$$

where

$$W = \gamma A_t\int_0^1(1+\psi\frac{z}{H})dz \tag{52}$$

Thus average lateral seismic coefficient over the full height of the dam can be expressed as

$$\alpha_{hi} = \frac{F_{ei}}{W} =$$

$$\frac{2\kappa_i\left(\frac{ZI}{2R}\right)\frac{C_{Ti}{}^2\beta_i}{4\pi^2}\left(\frac{S_{ai}}{g}\right)\int_0^1\frac{1}{\xi}(1+\psi\xi)J_1(\beta_i\xi)d\xi}{\int_0^1(1+\psi\xi)d\xi} \tag{53}$$

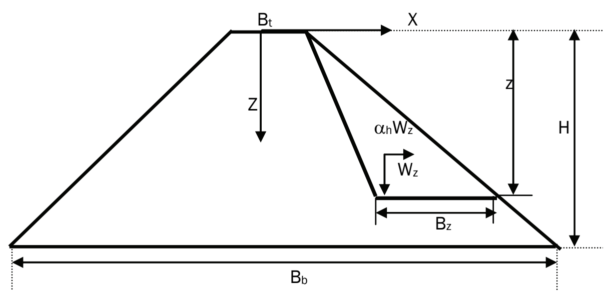

Fig.7 Assumed Failure Profile of an Earth Dam

It may be observed that α_{hi} obtained using eqn. (53) as expected is a dimensionless quantity.

Eqn. (53) can be further simplified to

$$\alpha_{hi} = \mathrm{Coeff}_i \times \left(\frac{ZI}{2R}\right)\left(\frac{S_{ai}}{g}\right) \quad (54)$$

where, Coeff_i

$$= \left[2\kappa_i \frac{C_{Ti}^{\ 2}\beta_i}{4\pi^2}\int_0^1 \frac{1}{\xi}(1+\psi\xi)J_1(\beta_i\xi)d\xi\right] \Big/ \int_0^1 (1+\psi\xi)d\xi \quad (55)$$

Values of the coefficient in eqn. (55) for various values of ψ are furnished in Table-5.

The design lateral coefficient based on eqn. (54) can finally be considered for first three modes as

$$\alpha_{hr} = \sqrt{\alpha_{h1}^{\ 2} + \alpha_{h2}^{\ 2} + \alpha_{h3}^{\ 2}} \quad (56)$$

Table 5 Values of Seismic Coefficient (coeff_i) for Different Values of ψ, for the First Three Modes

Ψ	G constant with depth Modes			G linear with depth Modes			G parabolic with depth Modes		
	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd
∞	1.11	0.14	0.05	0.65	0.09	0.03	0.39	0.06	0.02
99	1.13	0.14	0.05	0.66	0.09	0.03	0.40	0.06	0.02
49	1.14	0.14	0.05	0.68	0.09	0.03	0.41	0.06	0.02
24	1.18	0.14	0.06	0.70	0.09	0.04	0.42	0.06	0.02
15	1.21	0.14	0.07	0.72	0.09	0.04	0.44	0.06	0.03
9	1.26	0.14	0.08	0.74	0.09	0.05	0.45	0.06	0.03

Stability Analysis of Dam

As mentioned earlier the most popular method for stability analysis of dams under static load is based on slip circle methods like Swedish slip circle method, Bishops simplified analysis, etc. That soil fails along an arc was observed in a number of land slides in Alps from where Fellenius (1936) originally developed the method. In spite of its sound scientific background it has been observed to overestimate FOS under an actual earthquake Seed(1979). One of the possible reasons for this could be that for a homogenous isotropic medium under transient dynamic load the failure surface could be different from a circular arc as assumed under static load.

Without going into detailed discussion, it can be argued that under a transient shock if a body fails, the failure profile would follow the path of least resistance. Since resisting force within soil body is developed by friction and cohesion in a particular plane it is apparent that for least resistance this surface should be a minimum. In other words it should be a straight line rather than a circular arc. Failure of dams along a straight line path is not uncommon and Whitman & Lambe (1979) points out that a number of dams resting on firm ground have been observed to undergo failure along a straight line even under static loads. Since most of the earthen dams are built on relatively firm ground it would thus not be illogical to assume a straight line failure profile of a dam surface under an earthquake. Finally under earthquake a number of frames which are essentially treated as shear frames the infill panels have been observed to undergo cracks that follow a straight line rather than a circular arc (Agarwal & Shrikhande 2004). Similarly deep beams and squat shear walls (where shear strain energy dominates) failing under earthquake has been observed to follow more or less a straight line (Park & Pauley 1980).

Based on above logic we propose to evaluate the dynamic stability of the dam as mentioned hereunder.

Shown in Figure 8 is an earth dam subjected to lateral earthquake force. It is evident that we can draw infinite number of planes having an angle θ that can vary from 0 to angle α (the slope of the dam) through which the dam can potentially slide. Among all these planes there exists one unique plane at an angle θ which is the weakest and has the maximum probability of failure along that plane. In other words FOS against failure on this plane is a minimum.

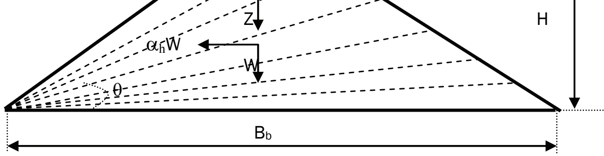

Fig. 8 An Earth Dam with Various Failure Planes

Shown in Figure 9 is an earth dam section where the critical failure profile plane is at an angle θ with the horizontal.

Based on geometry it can then be shown that area of triangle PQR (Δ) is given by

$$\Delta = \frac{1}{2} B_b{}^2 \frac{\sin\alpha \sin\theta}{\sin(\alpha+\theta)} \quad (57)$$

Thus weight of the dam above failure plane PQ can be expressed as

$$W_{cr} = \frac{\gamma}{2}\left[\left(B_b + B_t\right) H - B_b{}^2 \frac{\sin\alpha \sin\theta}{\sin(\alpha+\theta)} \right] \quad (58)$$

Weight W_{cr} is then expected to slide down the plane PQ which is the critical plane of failure.

As shown in Figure 10, the weight W_{cr} slides down a critical plane at an angle θ with horizontal.

The resistive force R may be expressed as

$$R = \mu N + c \times PQ \quad (59)$$

where, μ= Angle of internal friction of soil usually expressed as tan ϕ; c = cohesion of soil, and PQ= length of the failure plane $= B_b\left[\sin\alpha / \sin(\alpha+\theta)\right]$.

From free body diagram one can write

$$N = W_{cr}(\cos\theta - \alpha_h \sin\theta) \quad (60)$$

The force which drives the body down, is given by, $F_D = W_{cr}(\sin\theta + \alpha_h \cos\theta)$.

The resisting force is thus given by

$$F_R = \mu W_{cr}(\cos\theta - \alpha_h \sin\theta) + c \times B_b \frac{\sin\alpha}{\sin(\alpha+\theta)} \quad (61)$$

The factor of safety (FOS) is then expressed as

$$FOS = \frac{F_R}{F_D} = \frac{\mu\left(\cos\theta - \alpha_h \sin\theta\right) + \frac{cB_b}{W_{cr}} \frac{\sin\alpha}{\sin(\alpha+\theta)}}{\sin\theta + \alpha_h \cos\theta} \quad (62)$$

where W_{cr} is the weight expressed by eqn. (58).

To determine minimum factor of safety (FOS) one can now vary θ from 0 to α at an increment of ,say, 1 degree and evaluate the plane which is critical and FOS is minimum.

Effect of Vertical Seismic Coefficient α_v on FOS

Normal perception is that vertical component of an earthquake has no effect on overall stability of dam. Though Mononobe (1924) showed that, a combination of horizontal and vertical acceleration led to severe damage in earth retaining structures in Kanto earthquake in Japan as early as 1920. Chopra (1966) through Finite Element Analysis showed vertical acceleration affects seismic response of earth dams. Ling & Mohri (1997) did an extensive study of this effect on earth dams and strongly advocates its consideration for estimating the stability. As per IS-1893 (2002), α_v is usually considered as 0.5 of α_h. This can then be incorporated in eqn. (62) as follows

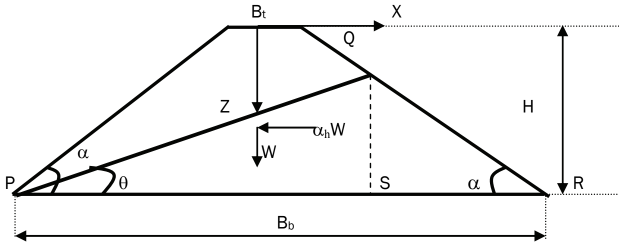

Fig. 9 A Typical Trapezoidal Section of an Earth Dam with its Failure Plane

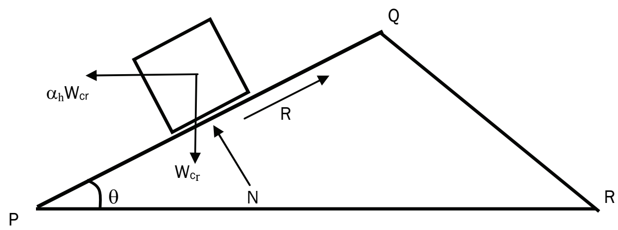

Fig. 10 Sliding of Weight W_{cr} along a Sloped Surface

$$FOS = \frac{\mu\left(\left(1 \pm \alpha_v\right)\cos\theta - \alpha_h \sin\theta\right) + \frac{cB_b}{W_{cr}} \frac{\sin\alpha}{\sin(\alpha+\theta)}}{\left(1 \pm \alpha_v\right)\sin\theta + \alpha_h \cos\theta} \quad (63)$$

Stability of Dam with Impermeable Core

Stability of dam including the impermeable core induces significant computational difficulties. Firstly due to heterogeneity of the medium, shear stress changes between outer and inner shell material. Moreover, as there is a significant variation in stiffness between the outer and inner core material, distribution of forces between the two parts also varies. Usual practice is to use FEM for estimation of stability, where uncertainties prevail for if the meshing of the soil material is not sufficiently refined can provide upper bound results.

The problem is approached as given hereafter.

Normally the core constitute of stiff compacted silty clay/clayey silt (little friction is preferable for it significantly enhances shear strength with depth along with cohesion c) having minimum void ratio to reduce seepage. From strength point of view also they are usually stiffer than the outer shell material by about 3 to 4 times. Thus, if one looks at the overall response of the heterogeneous earth dam under earthquake, it can be concluded that the system acts analogous to a shear wall framed structure where the inner core acts as a stiff shear wall and outer shell acts like a connected frame.

In other words distribution of the lateral force among outer and inner shell material is dependent on the relative stiffness of these two materials.

For a homogeneous isotropic body of area A and height H, static stiffness of soil is simply expressed as

$$K_{st} = GA/H \tag{64}$$

For earth dam with variable cross section having clay core eqn. (64) can be expressed as

$$K_{st} = \frac{GA_t}{H}\int_0^1 (1+\psi\xi)d\xi - \frac{GA_{tc}}{H}\int_0^1 (1+\psi_c\xi)d\xi + \frac{G_cA_{tc}}{H}\int_0^1 (1+\psi_c\xi)d\xi \tag{65}$$

On simplification, eqn. (65) can be expressed as

$$K_{st} = \frac{GA_t}{H}\left[\left(1+\frac{\psi}{2}\right)+(m-1)r_c\left(1+\frac{\psi_c}{2}\right)\right] \tag{66}$$

where $m = G_c/G$ and $r_c = A_{tc}/A_t$.

The stiffness of the inner core may be expressed as

$$K_c = \frac{G_cA_{tc}}{H}\left(1+\frac{\psi_c}{2}\right) \tag{67}$$

Thus, if $\alpha_h W$ is the total force on the dam, force resisted by inner core is given by

$$F_{hc} = \frac{K_c}{K_{st}}(\alpha_h W)$$

$$\Rightarrow F_{hc} = \left[\frac{mr_c\left(1+\frac{\psi_c}{2}\right)}{1+\frac{\psi}{2}+(m-1)r_c\left(1+\frac{\psi_c}{2}\right)}\right](\alpha_h W) \tag{68}$$

$\Rightarrow F_{hc} = (\alpha_{hc}W)$ where

$$\alpha_{hc} = \alpha_h\left[\frac{mr_c\left(1+\frac{\psi_c}{2}\right)}{1+\frac{\psi}{2}+(m-1)r_c\left(1+\frac{\psi_c}{2}\right)}\right] \tag{69}$$

Proceeding in identical manner it can be shown that the force induced in outer shell can be expressed as

$$F_{hS} = \left(\frac{\alpha_h W}{2}\right)\left[\frac{1+\frac{\psi}{2}-r_c\left(1+\frac{\psi_c}{2}\right)}{1+\frac{\psi}{2}+(m-1)r_c\left(1+\frac{\psi_c}{2}\right)}\right] \tag{70}$$

Eqn. (70) can be rewritten as

$$F_{hS} = \alpha'_{hs}W$$

where,

$$\alpha'_{hs} = \frac{\alpha_h}{2}\left[\frac{1+\frac{\psi}{2}-r_c\left(1+\frac{\psi_c}{2}\right)}{1+\frac{\psi}{2}+(m-1)r_c\left(1+\frac{\psi_c}{2}\right)}\right] \tag{71}$$

Here the term ½ comes because of the fact that outer shell constitutes of two parts, and as such the total force on outer shell is thus divided equally into two sections (provided of course the outer shell is symmetric to the core).

Based on time period obtained from eqns. (24) and (31), the seismic lateral force coefficient α_h is obtained using eqn. (56).

For a composite dam the total weight W is expressed as

$$W = \gamma A_t H\left[1+\frac{\psi}{2}+r_c(p-1)\left(1+\frac{\psi_c}{2}\right)\right] \tag{72}$$

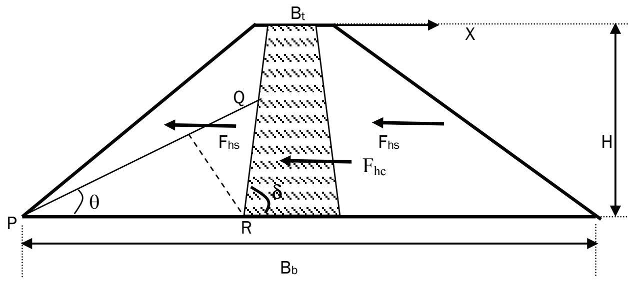

Fig. 11 Lateral Seismic Force on an Earth Dam with Clay Core to Restrict the Seepage

For calculation of FOS let us consider Figure 11.

It is evident that there can be two failures within the dam section:

1) Slope failure of the outer shell at a critical angle θ (the PQ plane).

2) Shearing off of the inner core due to the force F_{hc}, vide eqn. (68).

Proceeding in the same line as explained earlier if θ be the critical plane at which the FOS is minimum,

Based on Figure 11, the weight of soil, W_b below the critical PQ plane is expressed as

$$W_b = \frac{\gamma(B_b - B_{bc})^2}{8}\frac{\sin\theta\sin\delta}{\sin(\delta-\theta)} \tag{73}$$

The gross weight of one outer shell is given by

$$W_s = \frac{\gamma H}{2}\left[B_{bs} + B_{ts}\right] \tag{74}$$

where B_{bs}=0.5(B_b-B_{bc}) and B_{ts}=0.5(B_t-B_{tc})

Subtracting eqn. (73) from eqn. (74), we get the critical weight W_{cr} that slides along plane PQ.

The factor of safety for outer shell is now expressed as

$$FOS = \frac{\mu\left(\left(1 \pm \alpha'_v\right)\cos\theta - \alpha'_h \sin\theta\right) + \frac{c(B_b - B_{bc})}{2W_{cr}} \frac{\sin\delta}{\sin(\delta - \theta)}}{\left(1 \pm \alpha'_v\right)\sin\theta + \alpha'_h \cos\theta} \tag{75}$$

where α'_{hs} is as expressed by eqn. (71) and α'_v is 0.5 times α'_{hs}.

To arrive at minimum FOS we now vary θ from 0 to α at some predefined increment of say 1 degree to obtain the minimum value.

For inner core, force F_{hc} as per eqn. (68) basically acts as the base shear like in a continuous shear wall. Then, this can be distributed over full height of the dam as

$$Q_i = F_{hc} \frac{W_i \times h_i}{\sum W_i \times h_i^2} \tag{76}$$

where

W_i= Weight of core at a particular depth h_i given by $\gamma_c A_t (1 + \psi_c \frac{z_i}{H})(z_i - z_{i-1})$; Q_i= Nodal force at depth h_i.

Summing up of all these nodal forces Q_i up to a particular depth h_i gives the shear force acting at that particular level. Thus,

$$V_{i(z=hi)} = \sum_0^{h_i} Q_i \tag{77}$$

The shear stress developed at any section is given by

$$\tau_{zi} = \frac{V_{zi}}{B_{tc}\left(1 + \psi_c \frac{z}{H}\right) \times 1} \le c + \gamma_c \times z \tan\varphi_c \tag{78}$$

where $c + \gamma_c \times z \tan\varphi_c$ is the allowable shear stress at a particular depth z from crest and c is cohesion of soil in core and ϕ_c is its internal angle of friction.

Effect of Hydrodynamic Pressure on Stability

Hydrodynamic pressure on the dam will be most critical when it creates a suction pressure on sloped surface.

It has been shown by Chowdhury & Dasgupta (2008) that hydrodynamic pressure due to earthquake on a vertical surface for a fluid medium extending to infinity can be expressed as

$$p = \frac{12}{\pi(\pi + 2)} \frac{S_a \gamma_w H_w}{g} \sin\frac{\pi z}{2H_w} \tag{79}$$

in which, H_w= vertical height of water in dam; S_{aw}= acceleration corresponding to free field time period of the water expressed as $4H_w/v_w$; v_w = velocity of sound in water @ 1149 m/sec, and γ_w= unit weight of water.

Thus for a surface at an inclination α the pressure normal (p_n) to the surface can be expressed as

$$p_n = \frac{12}{\pi(\pi + 2)} \frac{S_{aw} \gamma_w H_w}{g} \sin\frac{\pi z}{2H_w} \sin\alpha \tag{80}$$

Thus the suction force V_w which reduces the normal force N is given by

$$V_w = \int_0^{Hw} p_n dz = 0.473 \frac{\gamma_w S_{aw} H_w^2}{g} \sin\alpha \tag{81}$$

The factor of safety against stability considering the suction effect of hydrodynamic pressure can then be expressed as

$$FOS = \frac{\mu\left(\left(1 \pm \alpha_v\right)\cos\theta - \alpha_h \sin\theta - \frac{V_w}{W_{cr}}\right) + \frac{cB_b}{W_{cr}} \frac{\sin\alpha}{\sin(\alpha + \theta)}}{\left(1 \pm \alpha_v\right)\sin\theta + \alpha_h \cos\theta} \tag{82}$$

For clay core dam replace α_v and α_h by α'_{vs} and α'_{hs} respectively and shall affect the outer shell material only.

Results and Discussions

To compare the results, typical data of a dam shown in Figure 12 is considered.

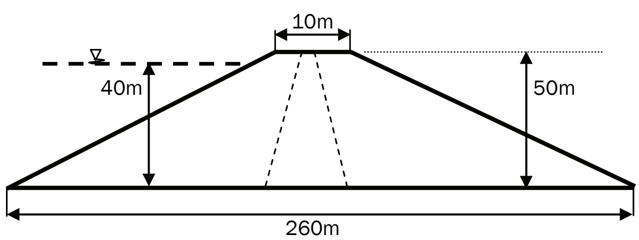

Fig. 12 Basic Dimension of Earth Dam

The dam is analyzed for two cases, visavis.

1) Homogeneous section having v_s =125 m/sec ϕ=32°, c= 50 kN/m^2, unit weight γ = 20 kN/m^3, height of water = 40m.

2) The same dam with a silty-clay core (shown by dotted line) the core having width at crest of 4.0 meter and width at base of 65m having unit weight of γ_c=21 kN/m^3 and shear wave velocity of 220 m/sec, c=85 kN/m^2 and ϕ_c=12°.

Maximum height of water in the reservoir = 40m.

The dam is in Zone IV as per IS-1893 (2002) having importance factor I as 2.0 and R is taken as 2.0 for this case.

Comparison of time periods (in seconds) is shown below:

Mode	IS-1893 (1984)	Mononobe	Seed & Makdisi	Gazetas	Proposed B_t=0.0	Proposed B_t=10.0	With clay core
1	1.044	1.044	1.046	1.046	1.046	1.085	0.783
2	-	0.455	0.455	0.455	0.455	0.471	0.338
3	-	0.291	0.300	0.290	0.300	0.299	0.214

It is observed that by proposed method time periods are in excellent agreement with other established methods for B_t = 0. However top width having finite dimension (B_t=10.0m) elongates the time period by about 4% for the fundamental mode- which would be most critical. The clay core being much stiffer than the outer shell has an overall stiffening effect on the dam response when time period gets reduced by 25% than established expression- this is logical.

Accelerations induced in different modes in dam in terms of g is given hereunder.

Mode	S_a(homogeneous dam) proposed	S_a with clay core(proposed)	S_a as per IS-1893(2002)
1	0.15g	0.21g	0.19g
2	0.30g	0.30g	NA
3	0.30g	0.30g	NA

Acceleration induced for first three modes by proposed method is as shown above, it is evident that depending on time period value the acceleration varies- unlike recommendation of IS code that suggests a constant value. Here it is again observed that the clay core stiffening the overall response- attracts more acceleration than a homogeneous dam.

Modal displacement of dam crest is given below:

Mode	u(mm): homogeneous dam	u(mm): with clay core
1	675	683
2	-135	-98
3	55	40
SRSS value	690	692

The modal response is found to be quite high though within acceptable limit. Here it is observed that stiffening effect of impermeable core induces more acceleration and also induces a higher displacement in fundamental mode though the SRSS values are almost same.

The lateral seismic coefficient for the dam is given hereunder.

Dam type	α_h
Homogeneous section	0.183
With clay Core	0.250
As per IS code	0.190

Following the same trend as expressed earlier, the lateral seismic coefficient for the heterogeneous dam is higher- this reflects the importance of considering the heterogeneous effect rather than ignoring it – as is the practice- as of now. It may however be noted that the values were obtained based on damping value D = 5% for all cases .For clay core with high plasticity damping is usually high thus considering 7% damping for the homogeneous dam and 10% for the dam with clay core value of α_h will come to 0.16 and 0.20 respectively.

Force transmitted to outer and inner shell based on stiffness distribution is outlined below:

Dam section	Lateral Seismic coeff(α_h)	Force(kN)
Outer core	0.053	7268
Inner Core	0.118	16228

The minimum factor of safety of dam under various load combinations is given below:

Load Case	FOS (Reservoir full)	FOS (Reservoir empty)
Under Static load	-	3.47**
Under Earthquake- Homogenous Dam	1.60	2.01
Under earthquake- with clay core	1.71	1.97

Variation of shear stress in the internal core is given in Figure 13.

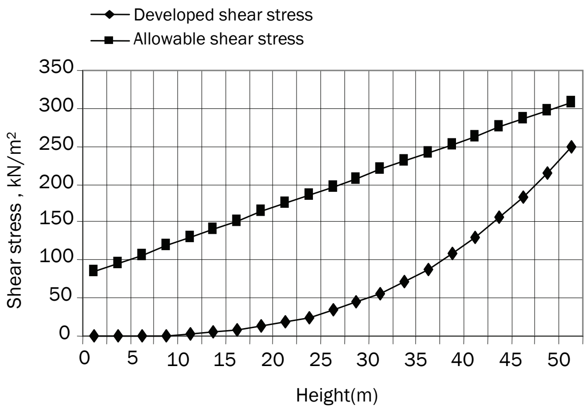

Fig. 13 Variation of Shear Stress in Inner Core for the Dam with C= 85 kN/m² and ϕ_c=12°

Conclusions

Based on above, it can be concluded that the present paper proposes a formulation that is more generic than the established methods for dynamic analysis of earth dams based on modal response technique. The proposed method can well cater to the dynamic response of

> Homogeneous section with finite crest width.

> Variation in soil property along the depth of the dam.

> Heterogeneous section with impermeable clay core.

> A rational formulation of lateral seismic coefficient α_h

The paper also proposes a modified method for stability analysis of dam that tries to combine the pseudo static slip circle method and sliding block technique to arrive at a rational FOS under earthquake. The technique for evaluation FOS for clay cored dam based on distribution of their static stiffness is novel but have a logical basis.

The analysis being analytical in nature does not require sophisticated software, a simple spread sheet or a MATHCAD shell would suffice for the same.

Effect of stiffness degradation due to large strain and dynamic pore pressure due to transient shock could further enhance this study.

Notations

The following symbols are used in this paper:

A_n = integration constant which is a function of time

A_t = area of dam at crest

A_{tc} = area of clay core at crest

B_b = base width of dam

B_{bc} = width of core at base

B_n = integration constant which is a function of time

B_{tc} = width of core at crest

B_t = width of crest

C_ω = coefficient to determine natural frequency of dam

C_T = coefficient to determine time period of dam

c = cohesion of soil

F = lateral seismic force

F_D = driving force along critical plane

F_{hc} = lateral seismic force on clay core

F_{hs} = lateral seismic force on outer shell

F_R = resistive force along critical plane

G = dynamic shear modulus of soil

G_c = dynamic shear modulus of core material

g = acceleration due to gravity

H = height of dam

H_w = height of water in the reservoir

i = mode number i= 1,2,3,....

I = importance factor as per IS code

j = mode number 1,2,3,...

J_0 = Bessel's function of first kind of order 0

J_1 = first derivative of Bessel's function J_0

K_{ir} = stiffness matrix of dam

K_{st} = static stiffness of dam with clay core

K_c = static stiffness of clay core

m = modular ratio G_c/G

m_{ij} = modal mass coefficient

M_{ir} = mass matrix of dam

N = normal reaction on the critical plane of failure

N_i, N_r = modal shape functions

P = ratio of core and outer shell weight density

p = hydrodynamic pressure

p_n = normal hydrodynamic pressure of dam slope.

Q_i = modal shear force

q(t) = generalized time function

r_c = ratio of core area at top to crest area at top

R = response reduction factor as per IS code

S_{ai} = modal spectral acceleration

S_{aw} = spectral acceleration of free field water

S_{di} = modal displacement

T = time period of dam

t = time

u = displacement amplitude of dam

$\ddot{u}_{av}$ = average seismic acceleration

V = strain energy density

V_s = shear wave velocity

V_w = force due to hydrodynamic pressure

v_w =velocity of sound in water

W_{cr} = critical weight of soil mass above the failure plane

X = co-ordinate axes in horizontal direction

Z = zone factor as per IS code

z = co-ordinate axes in vertical direction

α = a parameter defining variation of dynamic modulus of soil with depth

α = angle of inclination of the dam slope with horizontal axes

α_h = lateral seismic coefficient

α_V = vertical seismic coefficient

α_{hc} = lateral seismic coefficient for clay core

α_{hs} = lateral seismic coefficient for outer shell material

β_n = constants of Bessel's function J_0

δ = angle of inclination of clay core with horizontal axes

Δ = area of the triangular wedge of failure

$\varepsilon_x, \varepsilon_z, \gamma_{xz}$ = strain in soil medium

ϕ = angle of internal friction of soil

ϕ_c = angle of internal friction of clay core

φ = eigen vector of the dynamic system

γ = unit weight of soil

γ_c = weight density of core material

γ_w = weight density of water

κ_i = modal mass participation factor

λ = eigen value of the system

μ = coefficient of friction for soil mass

ν = Poisson's ratio of soil

θ = critical angle of failure

ρ = mass density of soil

τ_{xz} = dynamic shear stress induced in soil

ω = natural frequency of dam

ξ = dimensionless parameter z/H

ψ = dimensionless variable B_b-B_t/B_t

ψ_c = dimensionless variable B_{bc}-B_{tc}/B_{tc}.

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