Indian Geotechnical Journal, 40(2), 2010, 135-144

TECHNICAL NOTE

Reliability Based LRFD for Shallow Foundations

Vikas Pratap Singh[1] **and Sivakumar Babu G. L.**[2]

Key words
Conventional design; Reliability; LRFD; Shallow foundations; Bearing capacity

Abstract: The bearing capacity of soil for the design of shallow foundations is conventionally determined using allowable stress design (ASD) method. Commonly used ASD approach requires a single factor of safety to account for the uncertainties involved in: (a) the expected foundation loads, and (b) the evaluation of geotechnical parameters of the foundation soil. In the present article, another popular design methodology namely 'load and resistance factor design' (LRFD) is discussed in context of shallow foundations. Procedure to calibrate the available margin of safety for the allowable bearing capacity determined using ASD approach for different target reliability index (representing probability of failure) and variability in soil parameters by the use of reliability based resistance factors is discussed and illustrated. The article also illustrates the advantage of LRFD approach over conventional ASD approach. In general, the article highlights the necessity of reviewing existing standard practice (with reference to Indian code) for the determination of bearing capacity for shallow foundations in purview of LRFD approach.

Introduction

Shallow foundations are designed to ensure that the risk of bearing capacity failure is minimal. The allowable bearing capacity of a shallow foundation must account for the uncertainties involved in the applied loads and variability in the assessment of the foundation soil properties. Several researchers (e.g. Sivakumar Babu et al. 2006; Sivakumar Babu and Srivastava 2007; Massih et al 2008; Massih and Soubra 2008) have shown that a reliability based analysis can provide better insight into the above aspect. In the conventional shallow foundation design based on the allowable stress design (ASD) approach, a factor of safety of 3.0 is commonly applied on the ultimate bearing capacity to account for various uncertainties (Bowles 1996; Murthy 2003; Das 2007). Since, the factor of safety chosen in ASD is usually based on experience and judgement, quantitative measures of risk cannot be determined for ASD (FHWA 2001).

Load and resistance factor design (LRFD) methodology is very popular worldwide and recent trends are toward its increased use. In foundation design, use of LRFD allows a better control of design uncertainties and provides a more consistent level of reliability than the use of conventional ASD (Becker 1996a-b; FHWA 2001; Foye et al. 2006a-b; Fenton et al. 2007).

In India, IS: 6403 (1998) is the prevalent code of practice for the determination of bearing capacity of shallow foundations, and is based on the ASD approach. In the present article, influence of the variability in the assessment of foundation soil parameters on the reliability of the bearing capacity of shallow footings has been highlighted. Further, using the concept of reliability analysis, the allowable bearing capacity determined in accordance with IS: 6403 (1998) is calibrated to the anticipated level of the probability of failure (expressed in terms of target reliability index). Subsequently, reliability based LRFD of shallow foundations is discussed and illustrated with reference to a typical example. In general, the present article encourages the use of LRFD format for the design of shallow foundations over conventional ASD approach.

Allowable Bearing Capacity

For the purpose of better understanding and illustration of the objectives of the present article, an isolated square footing is considered for the study. According to IS 6403 (1998), for footing subjected to the vertical load, the ultimate net bearing capacity q_d for $c-\phi$ soil assuming a general shear failure is given by

$$q_d = cN_c s_c d_c + \gamma D_f \left(N_q - 1\right) s_q d_q + 0.5B\gamma N_\gamma s_\gamma d_\gamma \tag{1}$$

where c, ϕ and γ are the foundation soil cohesion, angle of internal friction and unit weight respectively; D_f is the depth of footing below ground surface; N_c, N_q, and N_γ are the bearing capacity factors given by equation 1a; s_c, s_q, and s_γ are the shape factors equal to 1.3, 1.2 and 0.8 respectively for the square footing; and d_c, d_q, and d_γ are the depth factors given by equation 1b.

$$N_c = \left(N_q - 1\right)\cot\phi;\quad N_q = e^{\pi\tan\phi}\tan^2\left(45 + \phi/2\right);\quad N_\gamma = 2\left(N_q + 1\right)\tan\phi \tag{1a}$$

---

1 Department of Civil Engineering, Indian Institute of Science, Bangalore – 560 012 Email: vps.vikas@gmail.com

2 Department of Civil Engineering, Indian Institute of Science, Bangalore – 560 012 E-mail: gls@civil.iisc.ernet.in

$$\left.\begin{array}{l} d_c = 1+0.2\dfrac{D_f}{B}\tan\left(45+\phi/2\right) \\ d_q = d_\gamma = 1+0.1\dfrac{D_f}{B}\tan\left(45+\phi/2\right) \end{array}\right\}\text{for } \phi > 10^0 \qquad (1b)$$

Further, the allowable bearing capacity q_{all} can be obtained by dividing the net ultimate bearing capacity q_d obtained using equation 1 by a suitable factor of safety FS, i.e.

$$q_{all} = \frac{q_d}{FS} \qquad (2)$$

Equation 2 represents the ASD expression for the allowable bearing capacity determination. The general range of the factor of safety used for shallow foundation against bearing capacity failure (i.e. shear failure) is 2.0-3.0 (Becker 1996a; FHWA 2001). However, as mentioned earlier, a factor of safety FS of 3.0 is commonly adopted for all kind of footings (Bowles 1996; Murthy 2003; Das 2007). Knowing the allowable bearing capacity, allowable load Q_{all} on an isolated square footing of size B x B can be obtained as

$$Q_{all} = q_{all}B^2 \qquad (3)$$

Further, for the known value ultimate net bearing capacity q_d, the margin of safety M against bearing capacity failure with reference to the allowable bearing capacity q_{all} may be defined as

$$M = q_d - q_{all} \qquad (4)$$

From equation 4, it is evident that use of a higher factor of safety results in a greater margin of safety.

LRFD for Shallow Foundations

The following sub-sections presents: (a) an overview of the general LRFD format and its comparison with ASD format, (b) proposed LRFD format for shallow foundations, and (c) procedure to determine reliability based resistance factors.

General Introduction

The basic difference between ASD and the LRFD approaches (e.g., Becker 1996a-b; FHWA 2001) is illustrated in Figure 1. As mentioned earlier, in the ASD approach all uncertainties in the variation of applied loads and the ultimate capacityof the structure to support the loads are incorporated in a factor of safety FS. The general format of the ASD is given by

$$\frac{R_n}{FS} \geq \sum Q \qquad (5)$$

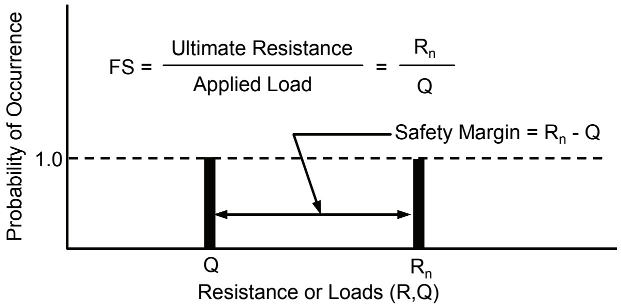
where R_n and $\sum Q$ are the nominal (ultimate) resistance and the summation of load effects, respectively.

Figure 1a illustrates one of the principal limitations of ASD, wherein, the values of Q and R_n are assumed to be unique such that they both have a probability of occurrence equal to unity. In addition, selection of FS is subjective that depends on the design models used and material parameters chosen, and is not inherently related to the probability of the component failure (FHWA 2001). ASD approach does not explicitly consider the uncertainties associated with the variability and the reliability of engineering properties of the various strength and load variables.

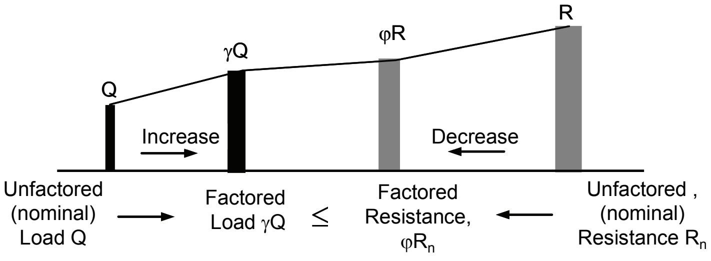
On the other hand, LRFD (see, Figure 1b) represents a more rational approach by which significant uncertainties in loads and material resistance can be incorporated quantitatively into the design process. The basic LRFD relationship is defined by

$$\varphi R_n \geq \sum \gamma_i Q_i \qquad (6)$$

where φ is the statistically determined dimensionless multiplicative factor called the resistance factor (usually less than one), which on multiplication with the nominal resistance R_n accounts for the uncertainties in resistances; γ_i are the dimensionless multiplicative factors called as load factors (usually more than one), which on multiplication with load component Q_i accounts for the uncertainties in loads derived from the load type, variability, and model error associated with a particular limit state. Thus, left hand side of equation 6 is the factored (reduced) resistance term which should always be equal to or greater than its right hand side representing factored (increased) load effects.



(a) ASD Approach



(b) LRFD Approach

Fig.1 Basic Difference between ASD and LRFD Approaches (Becker 1996a-b; FHWA 2001)

The common practice is to use a single resistance factor φ in equation 6 to reduce the ultimate resistance R_n which is usually determined based on the in-situ test data (FHWA 2001). In case of bearing capacity of shallow foundations, the resistance factor φ is determined based on the statistical analysis (using statistical parameters such as mean, standard deviation, and coefficient of variation) of the bearing capacity obtained directly from the results of in-situ standard penetration testing (SPT) and/or cone penetration testing (CPT). The resistance factor φ is then usually calibrated for a target reliability index β_T selected based on the margin of safety used for ASD coupled with the experience and judgment (FHWA 2001; Foye et al 2006b; Fenton et al. 2007).

On the other hand, selection of the load factor(s) γ_i to be used is a function of the type of load, loading combinations, limit state (strength or serviceability) being evaluated and the local codal provisions (FHWA 2001; Scott et al. 2003). In order to have foundation design to be consistent with current structural design practice, the use of the same loads, load factors, and load combinations would be required (Scott et al. 2003). Using a reliability analysis, Scott et al. (2003) reviewed the load factors presented in various LRFD codes from the United States, Canada, and Europe. In the present study, only two types of loads are considered, namely dead load (DL) and the live load (LL), and the corresponding load factors are denoted as γ_D (maximum range 1.25-1.95) and γ_L (maximum range 1.35-1.75) respectively. The above maximum ranges for load factors γ_i have been adopted from the comparison of various load factors presented by Scott et al. (2003).

Proposed LRFD format

From the general bearing capacity equation (i.e. equation 1) and the fundamentals of the soil mechanics, it is obvious that the bearing capacity of the shallow foundations is predominantly a function of its shear strength parameters (i.e. cohesion c and angle of internal friction ϕ) and the unit weight γ of soil. Further, reliable statistical information about these basic soil properties (i.e. c, ϕ and γ) is more readily available in the literature (Phoon and Kulhawy 1999a&b; Duncan 2000). Therefore, it is logical to use separate multiplicative resistance factor φ_i (where i = c, ϕ , γ) on each soil property as an alternative to the single resistance factor φ used in equation 6 to reduce the ultimate resistance R_n. Thus, equation 6 can be modified as

$$\sum \varphi_i R_i \geq \sum \gamma_i Q_i \tag{7}$$

where resistance factor φ is to be applied on the resisting component R_i dependant on 'i'. The LRFD formulation for bearing capacity of square footing (with reference to equation 1) considering only dead and live loads can be expressed as

$$Q'_{all} \geq Q_{act} \tag{8}$$

where

$$\left.\begin{aligned} Q'_{all} &= q'_{all}B^2 \\ Q_{act} &= \left(\gamma_D DL + \gamma_L LL\right) \end{aligned}\right\} \tag{8a}$$

$$q'_{all} = \frac{1}{FS}\left[1.3c'N'_c d'_c + 1.2\gamma' D_f\left(N'_q - 1\right)d'_q + 0.4B\gamma' N'_\gamma d'_\gamma\right] \tag{8b}$$

Where Q'_{all} is the allowable load obtained using factored soil parameters; q'_{all} is the allowable bearing capacity obtained using factored soil parameters; Q_{act} is the actual load acting on the footing; $c' = \varphi_c c$; $\gamma' = \varphi_\gamma \gamma$; the superscripted bearing capacity factors (N'_c, N'_q, N'_γ) and depth factors (d'_c, d'_q, d'_γ) are obtained by using $\phi' = \varphi_\phi \phi$ instead of ϕ in equations 1a and 1b respectively. The soil properties c', ϕ' and γ' are the factored (reduced) resistance components with corresponding resistance factors as φ_c, φ_ϕ and φ_γ respectively. As discussed in the following sub-section, the resistance factors φ_c, φ_ϕ and φ_γ can be determined considering the desired level of structural stability for a target reliability index value and accounting for the variability in the assessment of foundation soil parameters.

Reliability Based Resistance Factors

In the present article, reliability analysis is performed using the spreadsheet based Hasofer-Lind reliability method (Hasofer and Lind 1974) discussed in detail in Low and Tang (1997) and Low (2005). Figure 2 illustrates the definition and the intuitive interpretation of the Hasofer-Lind reliability index for the case of two random variables. As shown in Figure 2, the Hasofer-Lind reliability index is obtained by minimising its quadratic form (i.e. ellipsoid) subject to the constraint that it becomes tangential to the failure surface g(X). This point of tangency is defined as the design point (or most probable failure point) on the limit state surface (as indicated by x^* in Figure 2). The matrix formulation (Low and Tang 1997; Low 2005) of the Hasofer-Lind reliability index β is given by

$$\beta = \min_{x \in F} \sqrt{\left[\frac{x_i - \mu_i^N}{\sigma_i^N}\right]^T [R]^{-1} \left[\frac{x_i - \mu_i^N}{\sigma_i^N}\right]} \tag{9}$$

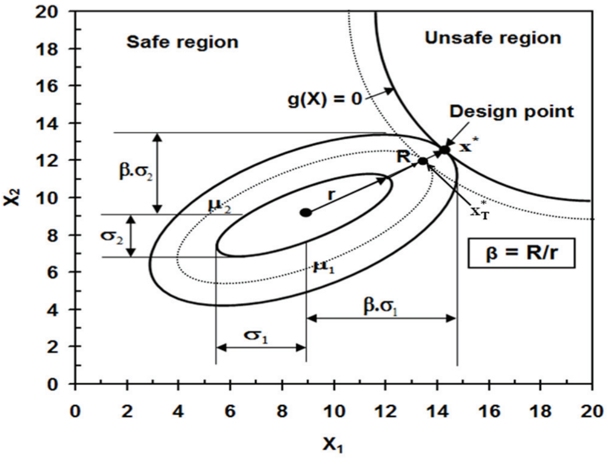


Fig. 2 Definition and the Intuitive Interpretation of the Hasofer-Lind Reliability Index

where x_i is a vector representing the set of random variables, μ_i^N is the vector of equivalent normal mean values (for non-normal random variables), R is the correlation matrix, σ_i^N is the equivalent normal standard deviation, and F is the failure domain. In the present study, foundation soil properties c, ϕ and γ are considered as the three lognormally distributed uncorrelated random variables for the reliability analysis. The performance function (i.e. limit state function or failure surface) g(X) for the bearing capacity failure mode of shallow foundation can be obtained by equating the margin of safety M (defined by equation 4) to zero, i.e.

$$g(X) = M = q_d - q_{all} = 0 \tag{10}$$

where q_d and q_{all} are same as defined earlier. However, it is be to noted that for reliability analysis, q_{all} obtained from equation 2 is considered as a constant value determined from the q_d value computed using the mean soil properties in equation 1 and a suitable factor of safety FS.

Knowing the most probable failure point (or design point x^*), the resistance factors φ_i (Ayyub and White 1987; Baecher and Christian 2003; Gupta and Choi 2003) can be obtained as

$$\varphi_i = \frac{(x^*)_i}{\mu_{x_i}} \tag{11}$$

where: 'i' represents the i^{th} random variable (i.e. cohesion c or friction angle ϕ or unit weight γ); (X*) is the value of i^{th} random variable at design point x*, and μ_i is the mean value of i^{th} random variable. For a given limit state, Hasofer-Lind reliability index and the design point x* can be obtained using Microsoft Excel spreadsheet based constrained optimisation tool SOLVER.

As mentioned earlier, the resistance factors defined by equation 11 can be obtained for a target value of reliability index β_T by limiting the automatic iterative procedure in SOLVER. In such a case, design point x^* changes to x_T^* (as shown in Figure 2), and the resistance factors corresponding to a target reliability index β_T can be obtained using this design point x_T^* instead of x* in equation 11. A suitable value of target reliability index β_T can be adopted depending upon the probability of failure, importance and the stability requirements (Phoon 2004). Table 1 can be used to select a suitable value of target reliability index β_T. The probability of failure for foundations lies in the range of 10^{-2} to 10^{-3} (Becker 1996a). Therefore, in the present study, the target reliability index β_T values equal to 2.0, 2.5 and 3.0 are adopted for the purpose of the illustration of the proposed LRFD methodology.

Table 1 Relationship between Reliability Index β and Probability of Failure P_f (Phoon 2004)

Reliability index β	*Probability of failure Pf*	*Stability of geotechnical structure*
1.0	≈ 0.16	Hazardous
1.5	≈ 0.07	Unsatisfactory
2.0	≈ 0.023	Poor
2.5	≈ 6 x 10^{-3}	Below average
3.0	≈ 1 x 10^{-3}	Above average
4.0	≈ 3 x 10^{-5}	Good
5.0	≈ 3 x 10^{-7}	High

General Procedure

In the present sub-section, the general procedure is provided for the determination of reliability based resistance factors for shallow foundations calibrated with reference to the ASD factor of safety and different values of the target reliability indices. Development of reliability based resistance factors requires following main steps:

i. Identify the resistance parameters that are to be considered as the random variables in the reliability based analysis and estimate the corresponding statistical properties (usually from the existing literature) such as mean, standard deviation, coefficient of variation (COV), probability distribution function and correlation coefficient (if desired) for the chosen random variables.

ii. Depending upon the shape of the footing (e.g. strip, square, rectangular and circular), choose the appropriate (may be from the prevalent code of practice) bearing capacity equation for the determination of the ultimate net bearing capacity.

iii. For the adopted value of the factor of safety FS, determine the allowable load q_{all} using equation 2 corresponding to the mean values of the random variables adopted for the study. This value of q_{all} shall be considered constant for the reliability analysis.

iv. For chosen level of variability in the random variables (in terms of COVs) and FS value, determine the maximum value of reliability index β using equations 9 and 10.

v. Chose a target reliability index β_T depending upon the desired level of structural safety (see Table 1), and such that it is less than the maximum value of reliability index β obtained in step 4 (i.e. $\beta_T < \beta_{max}$).

vi. For the target reliability index β_T chosen in step 5, compute the corresponding resistance factors φ_i using equation 11 as discused in previous section.

vii. Determine the allowable load Q'_{all} computed using factored random variables and the actual load on footing Q_{act} (such as obtained in equation 8a), and check the condition given by equation 8 (i.e. $Q'_{all} \geq Q_{act}$).

For all cases satisfying the condition $Q'_{all} \geq Q_{act}$, the actual load on footing Q_{act} have the desired factor of safety FS and the target reliability index β_T which can be interpreted in terms of the probability of failure (as given in Table 1). Result and discussions presented in the following section illustrate the procedure to obtain and use calibrated resistance factors with reference to a typical square shallow footing.

Results and Discussions

In the present article, square shallow footing is considered for the study. The depth of the footing D_f, is kept constant at 1.2 m. Foundation soil parameters (i.e. c, ϕ and γ) are considered as the three un-correlated random variables for the reliability analysis. The requisite statistical properties of the adopted random variables are given in Table 2. Variability in the estimation of the random variables (i.e. c, ϕ and γ) is considered in three levels: (a) low level with COVs of c, ϕ and γ equal to 10%,2% and 3% (i.e. lower limit values of corresponding COV range) respectively, (b) medium level with COVs of c, ϕ and γ equal to 20% , 6% and 5% (i.e. average values of corresponding COV range) respectively, and (c) high level with COVs of c, ϕ and γ equal to 30% , 10% and 7% (i.e. upper limit values of corresponding COV range) respectively. The results of the analysis are discussed under following sub-headings.

Significance of Reliability Based Analysis

The Hasofer-Lind reliability indices for the margin of safety defined by equation 4 are determined by varying the COV of foundation soil friction angle ϕ from 2%-10%. The COVs of the other two random variables c and γ are kept constant at 20% and 5% respectively. Reliability indices are determined using equations 9 and 10. Figure 3 shows the influence of variability in ϕ on the reliability index value of the margin of safety. From Figure 3, it is apparent that with the increase in the COV of ϕ, reliability index values for the margin of safety decrease considerably. In other words, the margin of safety with respect to the allowable bearing capacity obtained using a factor of safety value (say FS = 3.0) depends significantly on the variability in the estimation of the mean values of the random variables (i.e. soil parameters c, ϕ and γ). For example, when COV of ϕ is equal to 2%, the reliability index value corresponding to FS = 3 is nearly equal to 12.0, however, when COV of ϕ is increased to 10% the corresponding reliability index value is nearly 2.5 (see Figure 3). Thus, an increase in the variability in estimation of the soil parameters results in the decreased margin of safety and increased probability of the failure p_f (Phoon 2004). Likewise, consideration of variability in other (or many) design parameters may have substantial influence on the available margin of safety. Such a quantification of the influence of variability in the adopted mean design parameters on the available margin of safety and the probability of failure is not possible by the use of a conventional factor of safety FS alone. Thus, reliability analysis provides a significant insight into the study of shallow foundations.

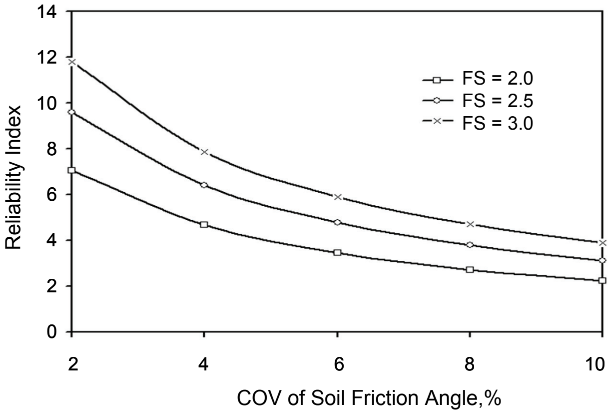


Fig. 3 Influence of Variability in Foundation Soil Internal Friction Angle on Margin of Safety for Bearing Capacity Failure

Influence of D_f/B Ratios on Reliability Based Analysis

Figures 4 and 5 show the variation in the reliability indices determined for the margin of safety with respect to allowable bearing capacity for varying D_f/B ratios computed for different FS values and variability levels, respectively. From Figure 4, it can be noted that the reliability index values (computed for medium level variability) are not influenced by the D_f/B ratio. The reliability index values corresponding to the margin of safety computed using FS equal to 2.0, 2.5 and 3.0 are 3.45, 4.75 and 5.87 respectively (see Figure 4).

Table 2 Statistical Details of the Un-correlated Random Variables

Random variable	*Mean value* μ	*Coefficient of variation COV (%)*		*Probability distribution*	*Reference(s)*
		Range	*Average*		
Foundation soil cohesion c (kPa)	10.0	10-30	20	Lognormal	Phoon and Kulhawy (1999a-b) Duncan (2000)
Foundation soil angle of internal friction ϕ (degrees)	30.0	2-10	6	Lognormal	
Foundation soil unit weight γ (kN/m^3)	17.5	3-7	5	Lognormal	

Further, Figure 5 shows that an increase in the variability level results in the decrease of the reliability index values for margin of safety. However, for a given variability level, reliability indices are again independent of D_f/B ratios. The reliability index values for low, medium and high levels of variability in soil parameters are 12.73, 4.75 and 2.86 respectively (see Figure 5).

Thus, observations from Figures 4 and 5 indicate that the margin of safety is least influenced by the D_f/B ratio; however, is extremely dependant on the variability level in the estimation of the mean values of soil parameters and the factor of safety FS used for the computation of allowable bearing capacity.

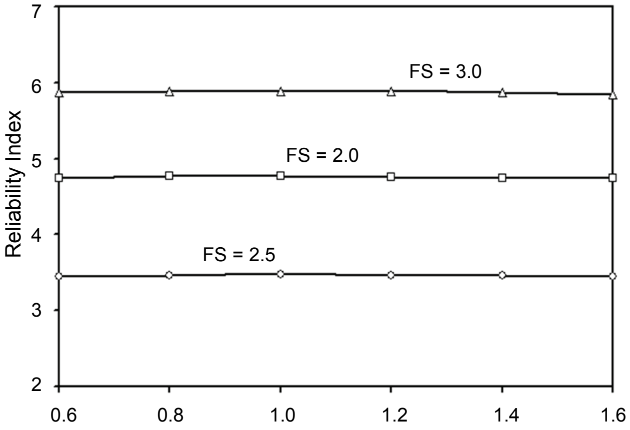


Fig. 4 Variation of Reliability Index with D_f/B Ratios for Different FS Values (Medium Variability)

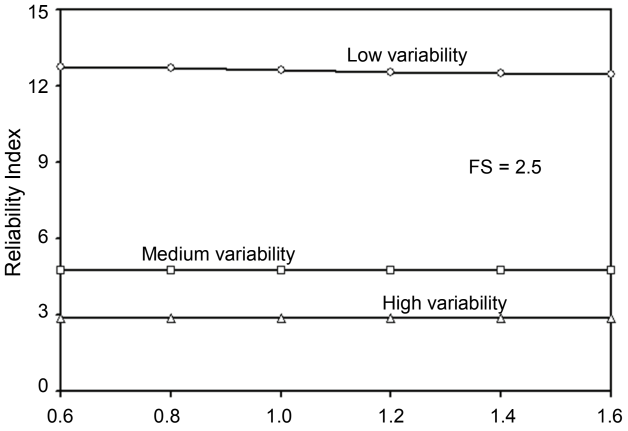


Fig. 5 Variation of Reliability Index with D_f/B Ratios for Different Variability Levels (for FS = 2.5)

Development of Reliability Based Resistance factors

Following the detailed discussion and the methodology presented in earlier sections, reliability based resistance factors φ_i are determined for the allowable bearing capacity of square shaped shallow footings founded on the soil with statistical properties given in Table 2. An important step in the determination of the reliability based resistance factors is to assess the maximum value of the reliability index (i.e. β) for the margin of safety M using the performance function defined by the equation 10. The adopted value of the target reliability index β_T should be less than β (since a value of β_T more than β will represent an unrealistic case). Table 3 shows the maximum value of the reliability index β for different variability levels and factor of safety values. For example, for FS equal to 3.0, the maximum values of β_T that can be adopted for low, medium and high levels of variability are 15.54, 5.89 and 3.58 respectively (see Table 3). Practically, as stated earlier, a β_T value in the range 2.0-3.0 is sufficient (Becker 1996a; Phoon 2004) for foundations.

It is to be noted that for a given level of variability and for any $\beta_T < \beta$, the resistance factors φ_i are independent of the FS values. For example, for medium variability, β values corresponding to FS equal to 2.0, 2.5 and 3.0 are 3.46, 4.77 and 5.89 respectively (see Table 3). Now, if a β_T value of 3.0 (which is less than β value 3.46 for FS equal to 2.0) is adopted for the determination of resistance factors φ_i for medium variability, resistance factors φ_i are same for FS equal to 2.0, 2.5 and 3.0.

Table 4 shows the resistance factors φ_i for different variability levels and corresponding to the FS value equal to 3.0. Resistance factors are tabulated for different values of $\beta_T < \beta$. From Table 4, it can be noted that as the adopted variability level and the β_T increases, resistance factors φ_i decreases. Dashes in Table 4 indicate $\beta_T > \beta$ (see Table 3 for β) for which resistance factors cannot be determined. Practically, to obtain a design with a target reliability index β_T, the mean values of the random variables shall be reduced using the resistance factors corresponding to that β_T. In the following sub-section, influence of variability in individual random variable on all resistance factors φ_i is discussed.

Table 3. Evaluation of Target Reliability Index β_T

	Low variability	*Medium variability*	*High variability*
Factor of safety FS	COV_c = 10% COV_ϕ = 2% COV_γ = 3%	COV_c = 20% COV_ϕ = 6% COV_γ = 5%	COV_c = 30% COV_ϕ = 10% COV_γ = 7%
	β_{max}	β_{max}	β_{max}
2.0	9.34	3.46	2.07
2.5	12.69	4.77	3.00
3.0	15.54	5.89	3.58

Note: (a) For the adopted variability level and FS, resistance factors can be determined for any $\beta_T < \beta_{max}$.

(b) c ,ϕ and γ in the subscripts are the soil cohesion, friction angle and unit weight, respectively.

Influence of Variability in Individual Random Variable on φ_i

As indicated from Table 4, the resistance factors φ_i are considerably influenced by variability in the random variables. Therefore, it is desirable to assess the influence of variability in individual random variable on the values of resistance factors corresponding to all random variables.

Table 4. Resistance Factors φ_i for FS = 3.0

Target reliability index β_T	*Low variability*			*Medium variability*			*High variability*		
	COV_c = 10% COV_ϕ = 2% COV_γ = 3%			COV_c = 20% COV_ϕ = 6% COV_γ = 5%			COV_c = 30% COV_ϕ = 10% COV_γ = 7%		
	φ_c	φ_ϕ	φ_γ	φ_c	φ_ϕ	φ_γ	φ_c	φ_ϕ	φ_γ
2.0	0.90	0.97	0.99	0.84	0.90	0.98	0.76	0.83	0.98
2.5	0.88	0.96	0.98	0.80	0.87	0.98	0.71	0.80	0.97
3.0	0.86	0.95	0.98	0.76	0.85	0.97	0.66	0.76	0.96
4.0	0.82	0.94	0.97	0.69	0.81	0.96	--	--	--
5.0	0.77	0.92	0.96	0.63	0.77	0.95	--	--	--

Note: (a) Factors to be multiplied with corresponding mean values.
(b) c , ϕ and γ in the subscripts are the soil cohesion, friction angle and unit weight, respectively.

In order to illustrate and understand the influence of the variability in a given random variable on the values of all the three resistance factors, resistance factors for the margin of safety corresponding to the factor of safety FS and target reliability index β_T values equal to 3.0 are evaluated. The COV of one random variable is varied at a time from its lower limit to the upper limit value of the corresponding COV range (see Table 2), and the other random variables are kept at the average value of their corresponding COV range. For example, for studying the influence of variability in in-situ soil cohesion c on the resistance factors, the COV of c is varied from 10% to 30%, and other random variables ϕ and γ are kept constant at 6% and 5% respectively. Figures 6a, 6b, and 6c show the influence of the variability in random variables c, ϕ and γ on the resistance factors, respectively. Dashed line in Figures 6a to c indicates the resistance factors at medium variability.

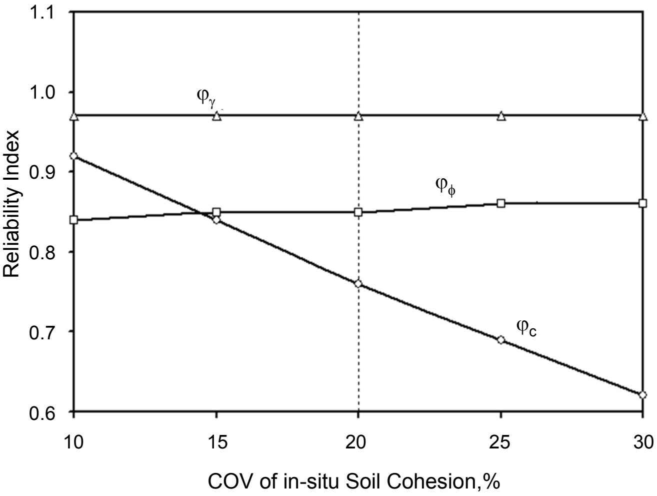


Fig. 6a Influence of Variability in Foundation Soil Cohesion on Resistance Factors (for FS = 3.0; β_T = 3.0)

From Figure 6a, it can be seen that as the COV of cohesion c is increased from 10% to 20%, resistance factor for cohesion i.e. φ_c is significantly decreased; whereas, the resistance factor for friction angle i.e. φ_ϕ is marginally increased and for unit weight φ_γ remained unaltered. Further from Figure 6b, it can be seen that as the COV of friction angle ϕ is increased from 2% to 10%, resistance factor for friction angle i.e. φ_ϕ is considerably decreased; whereas, the resistance factor for soil cohesion i.e. φ_c is slightly increased and for unit weight φ_γ is negligibly influenced. Similarly, from Figure 6c it can be seen that as the COV of unit weight γ is increased from 3% to 7%, resistance factor for unit weight i.e. φ_γ is decreased; whereas, the resistance factors for soil cohesion i.e. φ_c and friction angle φ_ϕ remained unaltered.

Based on the observations from Figure 6a-c, it is apparent that the value of a resistance factor is considerably dependant on the variability in the corresponding random variable (in the present case it decreases with increasing variability but in general depends upon the limit state in consideration); and at the same time, it may or may not have significant influence on the other resistance factors. Also, from Figure 6a-c, it is evident that variability in soil unit weight and the resistance factor φ_γ are not predominant. The study presented in this section is similar to a sensitivity analysis and is significant in adopting appropriate variability for the chosen random variables for the determination of reliability based resistance factors.

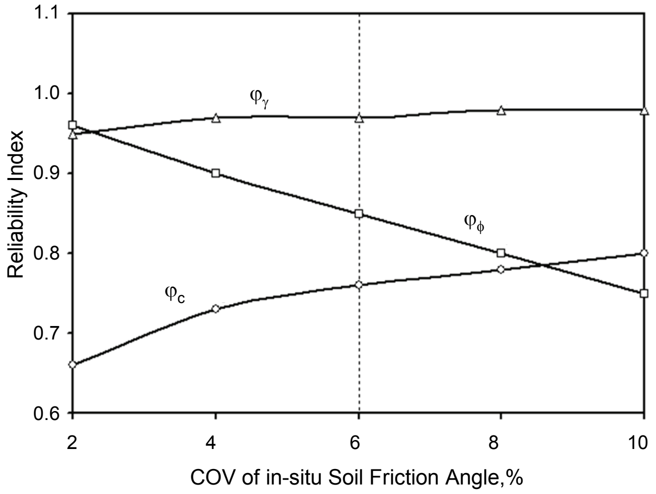


Fig. 6b Influence of Variability in Foundation Soil Friction Angle on Resistance Factors (for FS = 3.0; β_T = 3.0)

Illustrative Application Example

The present example illustrates the application of the proposed LRFD format and reliability based resistance factors in context of the shallow foundations. Assuming that a square shallow footing with D_f/B ratio equal to 0.8 (i.e. B =1.5 m for D_f =1.2) is to be founded on the soil with properties as given in Table 2. The footing is arbitrarily assumed to be carrying a dead load (DL) equal to 50 kN and live load (LL) equal to 300 kN from the superstructure. The variability level is assumed to medium and the ASD factor of safety FS is adopted equal to 3.0. For the above information, it is desired to: (a) compare the ASD allowable load with that determined using resistance factors corresponding to the target reliability index β_T equal to 2.0, 2.5 and 3.0, and (b) check the safety of foundation for various loading combinations using different load factors for DL and LL for both ASD and proposed LRFD (for β_T =3.0) methods.

The allowable load Q_{all} for FS equal to 3.0 for ASD method can be obtained using equations 1-3. Further, the resistance factors corresponding to the medium variability, FS equal to 3.0 and β_T for equal to 2.0, 2.5, and 3.0 given in Table 4 can be used to obtain the allowable load using factored soil parameters Q'_{all} from equations 8a and 8b. Table 5a summarises the corresponding results. From Table 5a, it can be noted that, allowable load for ASD method is equal to 956.38 kN; whereas, for β_T values equal to 2.0, 2.5, and 3.0, the values of the allowable load Q'_{all} are 634.48 kN, 565.40 kN and 516.94 kN respectively. This example suggests that though a load of 956.38 kN can yield a factor of safety of 3.0 against bearing capacity failure, but the maximum load that can be applied to achieve a β_T say equal to 3.0 (and the associated probability of failure, see Table 1) is only 516.94 kN.

Table 5a. Illustrative Example: Evaluation of Allowable Load using ASD and LRFD

ASD		LRFD with medium variability (COV_c = 20%; COV_ϕ = 6%; COV_γ = 5%)				
FS	Q_{all} (kN)	β_T	φ_c	φ_ϕ	φ_γ	Q'_{all} (kN)
3.0	956.38	2.0	0.84	0.90	0.98	634.48
3.0	956.38	2.5	0.80	0.87	0.98	565.40
3.0	956.38	3.0	0.76	0.85	0.97	516.94

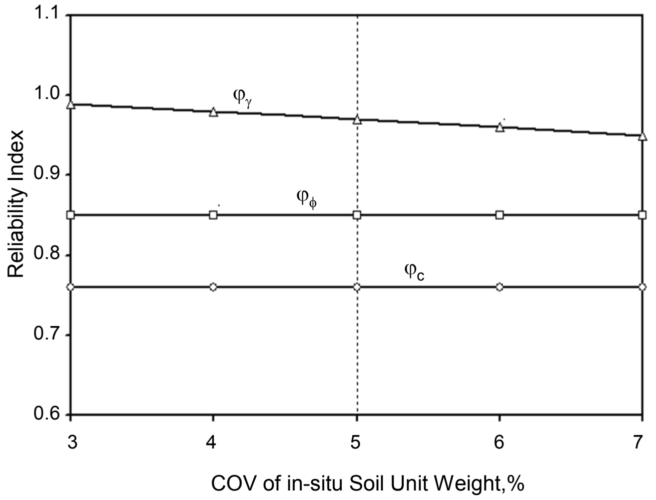


Fig. 6c Influence of Variability in Foundation Soil Unit Weight on Resistance Factors (for FS = 3.0; β_T = 3.0)

Scott et al. (2003) summarised the various loading combinations for dead loads and live loads with different corresponding load factors adopted by various international codes. In the present example, a few of the loading combinations are adopted from Scott et al. (2003). Table 5b shows the actual footing load Q_{act} obtained for various loading combinations and the checks for the footing stability for both ASD and LRFD methods. From Table 5b, it can be seen that ASD method shows that the actual load on footing Q_{act} are 'Ok' for all loading combinations; whereas, LRFD method show that only two loading combination fulfils the desired stability criteria (i.e. FS =3.0 and β_T =3.0). Thus, the above example shows that the LRFD is more rational than the conventional ASD method for shallow foundations.

Concluding Remarks

The present article highlighted the importance of LRFD methodology in context of the allowable bearing capacity evaluation of shallow foundations. The article demonstrated that the ASD approach using a factor the safety can be further rationalised to quantify the probability of failure (expressed in terms of target reliability index) associated with available margin of safety on allowable bearing capacity. The procedure to calibrate allowable bearing capacity of shallow

Table 5b. A Comparison between ASD and LRFD Approach (for FS = 3.0 and β_T = 3.0)

Loading			*ASD*		*Proposed LRFD*	
Dead load factor γ_D	Live load factor γ_L	Actual footing load Q_{act} (kN)	Q_{all} (kN)	Check Is $Q_{all} > Q_{act}$?	Q'_{all} (kN)	Check Is $Q'_{all} > Q_{act}$?
1.00	1.00	350.00	956.38	Ok	516.94	Ok
1.25	1.50	512.50	956.38	Ok	516.94	Ok
1.25	1.75	587.50	956.38	Ok	516.94	Not Ok
1.35	1.50	517.50	956.38	Ok	516.94	Not Ok
1.40	1.70	580.00	956.38	Ok	516.94	Not Ok

foundations obtained using ASD approach with respect to target reliability and variability levels in the soil parameters is discussed and illustrated. Target reliability based resistance factors so obtained are used in the proposed LRFD format for shallow foundations. In essence, the article brings out the necessity of reviewing the existing code of practice for bearing capacity determination of shallow foundations in the purview of LRFD approach which is a popular methodology worldwide and the recent trends are towards its increasing use.

Notations

Following notations are used in this paper.

β	maximum reliability index
β_T	target reliability index
γ	foundation soil unit weight
γ_i	dimensionless multiplicative load factors
γ_D	load factor for dead load
γ_L	load factor for live load
γ'	factored (reduced) foundation soil unit weight
ϕ	foundation soil angle of internal friction
ϕ'	factored (reduced) foundation soil angle of internal friction
$\sum Q$	summation of load effects
φ	dimensionless multiplicative resistance
φ_i	multiplicative resistance factor (where i = c, ϕ and γ)
φ_c, φ_ϕ, φ_γ	resistance factors for soil parameters c, ϕ and γ, respectively
μ_i^N	vector of equivalent normal mean values
σ_i^N	equivalent normal standard deviation
B	width of square footing
c	foundation soil cohesion
c'	factored (reduced) foundation soil cohesion
COV	coefficient of variation
d_c, d_q, d_γ	depth factors
d'_c, d'_q, d'_γ	depth factors obtained using ϕ'
D_f	depth of footing below ground surface
DL	dead load
F	failure domain
FS	factor of safety applied on q_d
g(X)	failure surface or performance (limit state) function
LL	live load
M	margin of safety against bearing capacity failure
N'_c, N'_q, N'_γ	bearing capacity factors obtained using ϕ'
N_c, N_q, N_γ	bearing capacity factors
p_f	probability of failure
q'_{all}	allowable bearing capacity using factored soil parameters
q_{all}	allowable bearing capacity using ASD
q_d	ultimate net bearing capacity
Q_i	load component
Q'_{all}	allowable load on footing using factored soil parameters
Q_{act}	actual load on footing
R	correlation matrix
R_n	nominal (ultimate) resistance
s_c, s_q, s_γ	shape factors
x^*	design point corresponding to reliability index β
x_i	vector representing the set of random variables
X^*_T	design point corresponding to target reliability index β_T

References

Ayyub, B.M. and White, G.J. (1987): 'Reliability-Conditioned Partial Safety Factors', *Journal of Structural Engineering*, ASCE, 113(2), pp. 279–294.

Baecher, G.B. and Christian, J.T. (2003): *Reliability and Statistics in Geotechnical Engineering*, Wiley, London, UK.

Becker, D.E. (1996a): 'Eighteenth Canadian Geotechnical Colloquium: Limit States Design for Foundations. Part I. An overview of the Foundation Design Process', *Canadian Geotechnical Journal*, 33(6), pp. 956–983.

Becker, D.E. (1996b): 'Eighteenth Canadian Geotechnical Colloquium: Limit States Design for Foundations. Part II. Development for the National Building Code of Canada', *Canadian Geotechnical Journal*, 33(6), pp. 984–1007.

Bowles, J.E. (1996): *Foundation Analysis and Design*. 5th Edition, McGraw-Hill Companies, Inc., Singapore.

Das, B.M. (2007): *Principles of Foundation Engineering*, 6th Edition (Indian Edition), Thomson, Canada.

Duncan, J.M. (2000): 'Factors of Safety and Reliability in Geotechnical Engineering', *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, 126(4), pp. 307–316.

Fenton, G.A., Zhang, X. and Griffiths, D.V. (2007): 'Reliability of Shallow Foundations Designed against Bearing Failure Using LRFD', *Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards*, 1(4), pp. 202-215.

FHWA (2001): *Load and Resistance Factor Design (LRFD) for Highway Bridge Structures*, Report No. FHWA HI-98-032, Federal Highway Administration, Washington D.C.

Foye, K.C., Salgado, R. and Scott, B. (2006a): ‘Assessment of Variable Uncertainties for Reliability-Based Design of Foundations’, *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, 132(9), pp. 1197-1207.

Foye, K.C., Salgado, R. and Scott, B. (2006b): ‘Resistance Factors for Use in Shallow Foundation LRFD’, *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, 132(9), pp. 1208-1218.

Gupta, A. and Choi, B. (2003): ‘Reliability Based Load and Resistance Factor Design for Piping: An Exploratory Case Study’, *Nuclear Engineering and Design*, 224(2), pp. 161–178.

Hasofer, A.M. and Lind, N.C. (1974): ‘Exact and Invariant Second Moment Code Format’, *Journal of Engineering Mechanics*, ASCE, 100(1), pp. 111–121.

IS 6403 (1998): *Code of Practice for Determination of Bearing Capacity of Shallow Foundations*, IS 6403: 1981 (Reaffirmed 1997, 6th Reprint), Bureau of Indian Standards, New Delhi.

Low, B.K. (2005): ‘Reliability-Based Design Applied to Retaining Walls’, *Geotechnique*, 55(1), pp. 63–75.

Low, B.K. and Tang, W.H. (1997): ‘Efficient Reliability Evaluation Using Spreadsheet. *Journal of Engineering Mechanics*, ASCE, 123(7), pp. 749–752.

Massih, D.S.Y.A. and Soubra, A.-H. (2008): ‘Reliability-Based Analysis of Strip Footings Using Response Surface Methodology’, *International Journal of Geomechanics*, ASCE, 8(2), pp. 134-143.

Massih, D.S.Y.A., Soubra, A.-H. and Low, B.K. (2008): ‘Reliability-Based Analysis and Design of Strip Footings against Bearing Capacity Failure’, Journal of *Geotechnical and Geoenvironmental Engineering*, ASCE, 134(7), pp. 917-928.

Murthy, V.N.S. (2003): Geotechnical Engineering - Principles and Practices of Soil Mechanics and Foundation Engineering, Marcel Dekker, Inc., New York.

Phoon, K.K. (2004): ‘Towards Reliability-Based Design for *Geotechnical Engineering’*, Special Lecture for Korean Geotechnical Society, Seoul, pp. 1–23.

Phoon, K.K. and Kulhawy, F.H. (1999a): ‘Characterization of Geotechnical Variability’, *Canadian Geotechnical Journal*, 36(4), pp. 612–624.

Phoon, K.K. and Kulhawy, F.H. (1999b): ‘Evaluation of Geotechnical Property Variability’, *Canadian Geotechnical Journal*, 36(4), pp. 625–639.

Scott, B., Kim, B.J. and Salgado, R. (2003): ‘Assessment of Current Load Factors for Use in Geotechnical Load and Resistance Factor Design’, *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, 129(4), pp. 287-295.

Sivakumar Babu, G.L. and Srivastava, A. (2007): ‘Reliability Analysis of Allowable Pressure on Shallow Foundation Using Response Surface Method’, *Computers and Geotechnics*, 34(3), pp.187–194.

Sivakumar Babu, G.L., Srivastava, A. and Murthy, D.S.N. (2006): ‘Reliability Analysis of the Bearing Capacity of a Shallow Foundation Resting on Cohesive Soil’, *Canadian Geotechnical Journal*, 43(2), pp.217-223.