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TECHNICAL NOTE

Reliability Analysis of Gravity Retaining Wall System using Response Surface Methodology

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Key words
Response surface, reliability analysis, retaining wall, MCS, FORM

Abstract: The conventional factor of safety approach does not take into consideration the variability and uncertainty in the soil parameters in an appropriate manner and decision on selection of factor of safety is mostly based upon engineering judgment and past experience. Probabilistic approach can handle it but the complexities involved in mathematical calculations combined with the complex functional relationship between input soil parameters and output response restrict its applications in routine geotechnical analysis and design. The present study illustrates the applicability of response surface methodology (*RSM*) in establishing an approximate and simple functional relationships between input and output variables, which can be used in the reliability index calculations. The procedure is described using an example application of gravity retaining wall system. The results of the analysis are compared with other methods of reliability analysis i.e. *FORM* (first order reliability method), *FOSM* (first order second moment method), *SORM* (second order reliability method), *MCS* (Monte Carlo simulation), and Subsim (*MATLAB* code). The advantage of the proposed approach is established in terms of reduced computational efforts.

Introduction

The reliability based design procedures for the retaining walls have been used since 1970s. The advantage of the probabilistic approach is a direct linkage between uncertainty in the design parameters and the risk (probability) of failure (Whitman 2000). Considering the case of a retaining wall system, Studies in the past demonstrated that the consideration of variability in the input soil properties may result in high failure probabilities in spite of having high deterministic factor of safety (Hoeg and Muruka 1974). In contrast to the conventional factor of safety approach in which soil parameters are taken as deterministic, the probabilistic analysis is performed considering these soil parameters as random variables and the associated safety is expressed in terms of an index known as reliability index (β). USACE (1997) established guidelines for the acceptable performance level of the geotechnical systems in terms of reliability index values and it is stated that for the good performance of the system, the calculated reliability index values should be at least 3.0.

Reliability analysis requires a functional relationship between input and output variables, which is sometimes difficult to establish due to complexity in the problem definition. Even the established functional relationship is sometimes too complicated to perform reliability analysis using conventional approaches like integration or first order reliability method using Taylor's series expansion. Under such circumstances approach like Monte Carlo simulation (*MCS*) is widely used (Baecher and Christian, 2003). *MCS* requires considerable time and computational efforts especially in a situation where the required output for probabilistic analysis needs numerical modeling procedures.

In the present study, authors propose to use the concept of response surface methodology (*RSM*) in establishing an approximate functional relationship between input and output variables that can be used in the reliability analysis using *FORM*. The concept of response surface methodology (*RSM*) has been used in many branches of engineering, specially chemical and manufacturing engineering. Response surface method (*RSM*) is a set of techniques used in the systematic study of relationships (Cornell, 1990). It is the process of identifying and fitting an approximate response surface model from input and output data obtained from experimental studies or from the numerical analysis where each run can be regarded as an experiment. Myers and Montgomery (2002) provided excellent literature on the topic and can be referred for more detailed discussion.

Wong (1985) performed reliability analysis of soil slopes using response surface method. Humphreys and Armstrong (1993) analyzed a slope stability problem using results of finite difference method and regression analysis. Tandjiria et al., (2000) used response surface method for reliability analysis of laterally loaded piles. Sivakumar Babu and Srivastava (2000) presented a study on the analysis of allowable bearing pressures on shallow foundation using response surface method and showed that a comparative study of the results of the analysis from conventional solution and numerical analysis in terms of reliability indices enables rational choice of allowable loads.

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Objectives of the Present Study

The objective of the present study is to demonstrate the applicability of concept of response surface methodology (*RSM*) in the reliability index calculations considering the case of a gravity retaining wall system. The advantage of the approach lies in the fact that a simple linear relationship between input variables and output response developed through *RSM* replace the originally more complicated functional relationship and therefore it reduces the computational efforts in the reliability index calculations.

The results of the reliability analysis are compared with other methods of reliability analysis i.e. *FORM* (first order reliability method), *SORM* (second order reliability method), *MCS* (Monte Carlo simulation), Subsim (MATLAB code).

Probabilistic Analysis

Response Surface Methodology

The approximate explicit functional relationship between input variables (x_1, x_2, x_3 …) and output response (*y*) through regression analysis is established for the range of expected variations in the input parameters.

$$y = f(x_1, x_2, x_3 ...) + e \tag{1}$$

In the above relationship, '*e*' represents other sources of uncertainty not accounted for in '*f*', such as measurement error on the output response, other sources of variation inherent in the process or the system, effect of other variables and so on. It is treated as statistical error with mean zero and variance σ^2. In the present study, the above relationships between input and output variables are considered as linear.

For the development of the *RSM*, 2^n factorial design is often used where "*n*" is the number of input variables. The procedure involves determination of output response (*y*) for the combination of input parameters (sample points) and regression analysis is performed based on least square error approach to fit a linear or non-linear regression model. The selection of sample points are based on combination of upper ($\mu + m\sigma$) and lower ($\mu - m\sigma$) limit value of each input parameters; where μ, σ are mean value and standard deviation of the input parameters, respectively. In this way, the number of sample points for 2, and 3 input variables will be 4, and 8, respectively.

Becker (1996) and Orr (2000) suggest that selection of characteristic values of geotechnical parameters and corresponding confidence intervals should be incorporated in reliability based designs. Hence, the value of *m* is taken as *1.65* with the assumption that the input soil parameters follow normal distribution and upper and lower limit values have probabilities of 5% and 95% being exceeded. The output response corresponding to each sample point can either be obtained from the established deterministic functional relationship between input and output or through numerical analysis.

The adequacy of the fitted model is examined to ensure that it provides an adequate approximation of the true system and none of the least square assumptions are violated. The computed values of coefficient of multiple determinations (R^2) and adjusted R^2 (R^2_{adj}) also give information on the adequacy of the fitted model. The non-dimensional quantity R^2 is calculated as below:

$$R^2 = \sum_{i=1}^{n}\left(\hat{y}_i - \bar{y}\right)^2 \Big/ \sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2 \tag{2}$$

$$R^2_{adj} = 1 - \frac{k-1}{k-p}\left(1 - R^2\right) \tag{3}$$

Where *k* is the total number of observations and *p* is the number of regression coefficients. The value of R^2 lies between 0 and 1 and a value very close to 1 indicates that most of the variability in y is explained by regression model. For a good fitted model, a difference between R^2 and (R^2_{adj}) should be small. After getting the response surface model for the output variable, reliability index (β) is calculated, this is discussed in the next section.

Reliability Analysis

Reliability is defined as probability of success of system in a given environment and loading conditions. For the given performance function, *g() = C-D*, where *C* is the capacity and *D* is the demand of the system, *g()* > 0 is safe and *g()* < 0 is unsafe. The limit state equation *g()* = 0 separates safe and unsafe region. For uncorrelated normally distributed *C* and *D* the reliability index (β) value is calculated using the following expression (Baecher and Christian, 2003):

$$\beta = \frac{C - D}{\sqrt{\sigma_C^2 + \sigma_D^2}} \tag{4}$$

Reliability Analysis of Gravity Retaining Wall System

As shown in Figure 1, a gravity retaining wall with

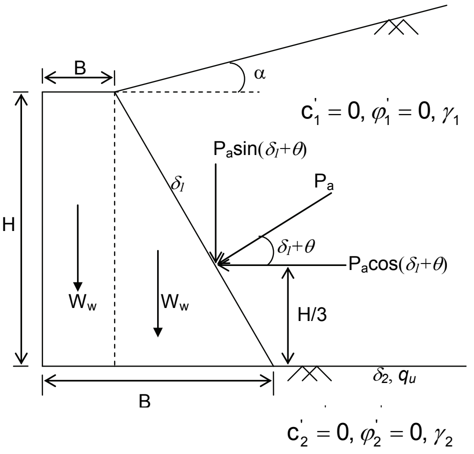

Fig. 1 Gravity Retaining Wall with Sloping Backfill

sloping backfill at and angle (α) is considered. Table 1 provides statistics of the list of input variables involved in deciding the stability of the retaining wall system. It can be noted that there are four independent input variables i.e. γ_1, γ_2, ϕ_1, and ϕ_2.

For the reliability analysis, the performance function can be defined as g() = $(FS)_i - 1$; where $(FS)_i$ is the factor of safety of the retaining wall system with respect to different modes of failure i.e. (i) $[(FS)_1]$ from sliding failure, (ii) $[(FS)_2]$ from middle third rule, (iii) $[(FS)_3]$ from bearing capacity failure, and (iv) $[(FS)_4]$ from overturning failure. Following are the expressions for the factors of safety from different failure criteria:

$$FS_1 = \frac{\left[P_a \sin(\delta_1+\theta) + W_w + W_R\right]\tan\delta_2}{P_a\cos(\delta_1+\theta)} \qquad (5)$$

$$FS_2 = \frac{L/6}{e_L} \qquad (6)$$

$$FS_3 = \frac{q_u \bar{L}}{W_w + W_R + P_a\sin(\delta_1+\theta)} \qquad (7)$$

$$FS_4 = \frac{M_R}{M_O} \qquad (8)$$

Where; B, H, L, α, γ_1, γ_2, ϕ_1, ϕ_2 are defined in Table 1, and

δ_1 = friction angle between the backfill and back of the wall ($2/3\phi_1$)

δ_2 = friction angle between the foundation soil and base of the wall ($2/3\phi_2$)

P_a = active force; $P_a = \frac{1}{2}K_a\gamma_1H^2$; K_a is the active earth pressure coefficient.

q_u = bearing capacity of the foundation soil; $q_u = \frac{1}{2}\gamma_2 L N_\gamma F_{\gamma i}$

$$N_\gamma = 2(N_q + 1)\tan\varphi_2;\ N_q = e^{\pi\tan\varphi_2}\tan^2(45+\varphi_2/2)$$

$F_{\gamma i} = (1-\beta/\varphi_2)^2$; Hanna and Meyerhof (1981)

$$\beta = \tan^{-1}[P_a\cos(\delta_1+\theta)/(P_a\sin(\delta_1+\theta)+W_w+W_R)] \qquad (9)$$

MR= resisting moment, $M_R = W_w\frac{B}{2} + P_a\sin(\delta_1+\theta).\left(B+\frac{2}{3}(L-B)\right) + W_R\left(B+\frac{1}{3}(L-B)\right)$

M_O = overturning moment, $M_O = P_a\cos(\delta_1+\theta).H/3$

$W_w + W_R$ = weight of the retaining wall.

Table 1 Statistics of the Input Variables

Sr.No.	Variables	Description	Distribution	Mean (μ)	CoV (%)
1	B (m)	Top width	Deterministic	1.45	-
2	H (m)	Height	Deterministic	6.0	-
3	L (m)	Bottom width	Deterministic	4.0	-
4	α (degree)	Backfill slope	Deterministic	5°	-
5	γ_1; kN/m^3	Backfill soil	Gaussian	19	10
6	γ_2; kN/m^3	Foundation soil	Gaussian	17	10
7	ϕ_1	Backfill soil	Gaussian	35°	10
8	ϕ_2	Foundation soil	Gaussian	35°	10

θ is the back angle of the retaining wall which is calculated as follows.

$$\theta = \tan^{-1}\left(\frac{L-B}{H}\right)$$

$$\bar{L} = L - 2e_L$$

$$e_L = \left(\bar{x} - L/2\right)$$

$$\bar{x} = \frac{(M_R - M_O)}{\left(W_W + W_R + P_a\sin(\delta_1+\theta)\right)}$$

Results of Analysis and Discussion

The functional relationships between four input variables γ_1, γ_2, ϕ_1, ϕ_2 and output responses i.e. FS_1, FS_2, FS_3, and FS_4 can be replaced by simple functional relationship between input and output variables using the concept of response surface methodology (RSM). As discussed earlier, Table 2 shows the output responses for the combinations of input variables i.e. 16 sample points for four input variables, the values of factor of safety are calculated using the above equations and regression analysis is performed to obtain a linear response surface model for each output response as given below:

$$FS_1 = -0.880 - 0.078\gamma_1 + 0.061\varphi_1 + 0.0620\varphi_2 \qquad (10)$$

$R^2 = 0.986$, $R^2_{adj} = 0.973$

$$FS_2 = 0.450 - 0.0034\gamma_1 + 0.0213\varphi_1 \qquad (11)$$

$R^2 = 0.994$, $R^2_{adj} = 0.989$

$$\begin{aligned} FS_3 = &-29.96 - 0.0864\gamma_1 + 0.352\gamma_2 \\ &+ 0.0640\varphi_1 + 0.840\varphi_2 \end{aligned} \qquad (12)$$

$R^2 = 0.984$, $R^2_{adj} = 0.969$

Table 2 Output Responses for Each Combination of Input Parameters

Sr.No.	γ_1	γ_2	ϕ_1	ϕ_2	γ_1	γ_2	ϕ_1	ϕ_2	FS_1	FS_2	FS_3	FS_4
1	+	+	+	+	22.14	19.81	40.78	40.78	2.370	1.253	12.891	4.386
2	+	+	+	-	22.14	19.81	40.78	29.23	1.633	1.253	1.361	4.386
3	+	+	-	+	22.14	19.81	29.23	40.78	1.649	0.982	11.205	3.120
4	+	+	-	-	22.14	19.81	29.23	29.23	1.136	0.982	1.183	3.120
5	+	-	+	+	22.14	14.20	40.78	40.78	2.370	1.253	9.240	4.386
6	+	-	+	-	22.14	14.20	40.78	29.23	1.633	1.253	0.976	4.386
7	+	-	-	+	22.14	14.20	29.23	40.78	1.649	0.982	8.032	3.120
8	+	-	-	-	22.14	14.20	29.23	29.23	1.136	0.982	0.848	3.120
9	-	+	+	+	15.87	19.81	40.78	40.78	3.063	1.249	13.904	5.372
10	-	+	+	-	15.87	19.81	40.78	29.23	2.110	1.249	1.468	5.372
11	-	+	-	+	15.87	19.81	29.23	40.78	2.114	1.029	12.477	3.783
12	-	+	-	-	15.87	19.81	29.23	29.23	1.457	1.029	1.317	3.783
13	-	-	+	+	15.87	14.20	40.78	40.78	3.063	1.249	9.966	5.372
14	-	-	+	-	15.87	14.20	40.78	29.23	2.110	1.249	1.052	5.372
15	-	-	-	+	15.87	14.20	29.23	40.78	2.114	1.029	8.943	3.783
16	-	-	-	-	15.87	14.20	29.23	29.23	1.457	1.029	0.944	3.783

$$\mathrm{FS}_4 = 2.34 - 0.132\gamma_1 + 0.124\varphi_1 \qquad (13)$$

$R^2 = 0.995$, $R^2_{\mathrm{adj}} = 0.990$

It can be observed that values of R^2, and R^2_{adj} are close to 1.0 and hence, it the developed response surface model is adequate.

Using the above response surface models, for the given mean and variance of the input parameters, the mean (*C*) and variance (σ_C^2) of the output response (i.e. capacity) are evaluated using simple statistical calculations. The demand (*D*) is unity i.e. 1.0 which is ensured so that the calculated factor of safety values should not fall below 1.0. The variance in demand (*D*) is not taken into consideration. Now, with the help of equation [4], reliability index (β) values for all the failure modes of the retaining wall are calculated and minimum reliability index (β_{min}) value dominates the system reliability of the gravity wall. Table 3 provides the results of the analysis (deterministic factor of safety and reliability index values) from different failure modes. It is interesting to note that the deterministic factor of safety from bearing capacity failure mode is quite high but the corresponding reliability index (β) value is quite low. It clearly demonstrates the effect of variation in the input parameters on the system performance which is not well captured in the deterministic solutions. Table [4] compares the results of the reliability analysis of the present study with the other methods of reliability analysis. It can be observed that the values are quite comparable.

Table 3 Reliability Index Values for all the Four Modes of Failure of Gravity Retaining Wall

Sr. No.	*Failure modes*	*Deterministic* $(FS)_i$	*Reliability index* (β)
1	Sliding failure mode	1.855	2.780
2	Middle third rule	1.127	1.720
3	Bearing capacity failure	3.536	1.658
4	Overturning failure	4.028	6.340

Table 4 Comparing the Results of the Reliability Analysis from Different Approaches

Solution Methods	*FORM*$^{(a)}$	*SORM*$^{(b)}$	*Subsim*$^{(c)}$	*Monte Carlo simulation*	*Present Study*
β	1.6704 (lower) 1.7066 (Upper)	1.6819 1.6894	1.3799	1.3772	1.658
P. of failure (p_f)	0.0474 (lower bound β value) 0.0440$^{(1)}$ 0.0807 (for upper bound β value) 0.0862$^{(2)}$	0.0463 0.0456	0.0838$^{(3)}$	0.0842	-

(a) Using Gradient projection algorithm; (b) Algorithm by Der Kiureghian and Stefano (1991)[12]; (c) MATLAB code

(1), (2) Ang and Tang (1984)[13]; (3) Average of 100 runs of Subsim (MATLAB code),
Values in (a), (b), (c) are reported in Jianye Ching & Yi-Hung Hsieh (2008)

Conclusions

It can be concluded that the concept of response surface methodology can be efficiently applied for the generation of simple approximate polynomial function between input random variables and output response that can be used in the reliability index calculations. The advantage of *RSM* is that it is computationally less demanding.

The results of the reliability analysis of retaining wall system indicated the influence of extent of variation of input parameters on the performance assessment and it is clearly shown that a reliability based approach is essential in the decision making process.

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