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Factor of Safety of a Consolidating Slope by Finite Element Method

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Key words
Consolidating slope, Time dependent analysis; Slope stability, Factor of safety, Finite element analysis

Abstract: Factor of safety of a consolidating slope is obtained at various time intervals using finite element method. Four noded isoparametric plane strain element with two translational degrees of freedom is used to model the soil deformation. Pore pressure in soil is also modeled using four noded isoparametric element. Displacement and pore pressure in soil are coupled and the resulting equations are solved to obtain the displacement and pore pressure in soil at various time intervals from the beginning of consolidation up to the end of consolidation. Effective stresses in soil are obtained using finite element method whereas the critical slip surface and factor of safety of a slope is obtained using a Monte Carlo technique. The analysis is used to obtain the factor of safety, pore pressure and effective stresses in soil at various time intervals. It is concluded from the study that the method of combining the finite element method to determine effective stresses and Monte Carlo technique to determine the critical slip surface is simple and can be used to obtain the factor of safety of a consolidating slope at various time interval.

Introduction

Analysis of slopes in terms of factor of safety is important to define the stability of a slope. Most commonly adopted methods for slope stability analysis are the limit equilibrium methods and finite element method of analysis. Limit equilibrium methods have been widely adopted for slope stability analysis mainly due to their simplicity and applicability. A potential sliding surface is assumed prior to the analysis and a limit equilibrium analysis is then performed with respect to the soil mass above the probable slip surface. [Bishop (1955), Janbu (1957), Morgenstern and Price (1965), Spencer (1967), Sarma (1979)]. Although, the approach is straight forward, these methods may not give reliable results for nonhomogeneous and anisotropic soil mass. They do not consider the stress – strain behaviour of the soil mass while calculating the stresses. Finite element method has been used for the analysis of deformation and stress distribution. In contrast to the simplified techniques, finite element method can deal with complex boundary and loading conditions. In general slope stability analysis using finite element method is carried out in two steps. The first step is to calculate the stresses using finite element method and in the second step factor of safety is defined by joining either the points of local failure or by using limit equilibrium methods. Duncan and Dunlop (1969), Donald and Giam (1988) derived factor of safety using the nodal displacements obtained from the finite element method. Scott et al. (1993) obtained the stresses in soil using finite element method and a critical slip surface by joining the points of local failure in a slope. Delwyn et al. (1999) combined finite element analysis and a limit equilibrium method to obtain the stresses in soil and the factor of safety.

To search for the critical surface many optimization techniques have also been used. Baker (1980) developed a dynamic programming technique coupled with Spencer's limit equilibrium method. Bardet and Kapuskar (1989) employed the simplex method. Monte Carlo technique was employed by Greco (1996). Venanzio (1996) proposed a Monte Carlo method of the random walking type for locating critical slip surface of the general shape for slope stability analysis.

In general, the slope has been analysed either for long term stability (drained condition) or at the end-of-construction stability (undrained condition). It is well known that the factor of safety of a saturated slope is controlled by the pore pressure and the amount of settlement expected after construction. The factor of safety of such slopes varies with time due to dissipation of excess pore pressure after construction. In some situations, it is necessary to obtain the factor of safety of slope at various time intervals after the construction. Hence, the stability analysis considering the effect of pore pressure at various time intervals is essential for obtaining realistic results on stability of slopes. In this paper, a time dependent analysis to obtain the factor of safety of a slope at various time intervals from the end-of-construction up to the complete dissipation of pore pressure is carried out. The analysis is used

> to obtain the factor of safety of a slope at various time interval when it is constructed with saturated soil of low coefficient of permeability. In this case, it is assumed that pore pressure will not dissipate during construction and the excess pore pressure developed at any point immediately after construction is due to the saturated weight of the soil above this point.

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- To obtain the factor of safety of a slope at various time interval when it is constructed with pervious soil on a soft saturated foundation. In this case it is assumed that pore pressure will not develop in the slope during or after the construction whereas the pore pressure will develop in the foundation soil after the construction.
- To obtain the factor of safety of the saturated and fully consolidated slope at various time interval after a uniformly distributed surcharge load is applied. In this case it is assumed that the slope and the foundation soil is completely consolidated (excess pore pressure is zero) before the application of surcharge load but pore pressure will develop in the slope and foundation soil due to surcharge.

Studies on time dependent behaviour of soil due to dissipation of excess pore pressure (consolidation) has received greater attention after Terzaghi published his consolidation theory and principle of effective stress. Consolidation settlements were obtained in most cases using Terzaghi's one dimensional consolidation theory. Biot (1941) developed a more general theory for three dimensional consolidation coupling the soil deformation and the pore pressure. Finite element method is also proved to be an extremely powerful analytical tool for the solution of consolidation problems. Sandhu and Wilson (1969), Zienkiewicz (1977) formulated analysis for coupled consolidation problem using finite element method. Lewis et al. (1976) and Manoharan and Dasgupta (1995) studied the time dependent behaviour of strip footing using finite element method. In the proposed analysis, displacement and pore pressure in soil are coupled using the theory proposed by Zienkiewicz (1977).

Method of Analysis

The factor of safety of a slope is obtained using a combination of finite element analysis to obtain the stresses and pore pressure and a Monte Carlo technique proposed by Venanzio (1996) to obtain the critical slip surface.

The analysis consists of two parts

1. Finding the effective stresses at required points using finite element method.
2. Locating the critical slip surface and finding the factor of safety using these stresses.

Determination of Pore Pressure and Effective Stresses in Soil by Finite Element Method

Displacement u and excess pore pressure p within the finite element can be related to nodal displacement vector $\{u_n\}$ and the nodal pore pressure vector $\{p_n\}$ as

$$u = [N_s]\{u_n\} \qquad (1)$$

$$p = [N_f]\{p_n\} \qquad (2)$$

N_s is the shape functions defining the displacement of the soil element while N_f is the shape functions defining the pore pressure distribution.

The elemental equation of consolidation proposed by Zienkiewicz (1977) can be expressed in matrix form as

$$\begin{bmatrix} K_s & L \\ 0 & H \end{bmatrix} \begin{Bmatrix} \dot{u}_n \\ \dot{p}_n \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ L^T & 0 \end{bmatrix} \begin{Bmatrix} \dot{u}_n \\ \dot{p}_n \end{Bmatrix} = \begin{Bmatrix} f \\ 0 \end{Bmatrix} \qquad (3)$$

K_s is the soil stiffness matrix and H is the flow matrix. f is the vector of nodal forces. L is the coupling matrix which is formed from the equation

$$\int_s {N_s}^T \begin{Bmatrix} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial y} \end{Bmatrix} N_f \, ds \qquad (4)$$

Equation 3 in incremental form can be written as

$$\begin{bmatrix} K_s & L \\ 0 & H \end{bmatrix} \begin{Bmatrix} \Delta\dot{u}_n \\ \Delta\dot{p}_n \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ L^T & 0 \end{bmatrix} \begin{Bmatrix} \Delta\dot{u}_n \\ \Delta\dot{p}_n \end{Bmatrix} = \begin{Bmatrix} \Delta f \\ 0 \end{Bmatrix} \qquad (5)$$

Δu_n and Δp_n are the resulting changes in displacement and excess pore pressure respectively from time t to $t+\Delta t$. The displacement u_{ni} and excess pore pressure p_{ni} at the end of i^{th} time step is

$$u_{ni} = u_{n(i-1)} + \Delta u_n$$

$$p_{ni} = p_n(i-1) + \Delta p_n \qquad (6)$$

Saturated soil medium is discritized into number of finite elements. For each element, the stiffness matrix k_s, coupling matrix l and fluid conduction matrix h is obtained and added by direct stiffness method to obtain overall stiffness matrix K_s, coupling matrix L and fluid conductivity matrix H. The resulting equation is solved in the incremental form using Newmark's method to obtain change in displacement Δu_n and change in pore pressure Δp_n .

Determination of Factor of Safety

Venanzio (1996) proposed a new Monte Carlo method of the random walking type, for locating critical slip surface. In this method, each search stage is articulated in two phases: exploration phase and extrapolation phase. In the present method, a critical slip surface is obtained using the procedure of exploration phase proposed by Venanzio (1996). Also, in the present method, the width of search step is kept constant. The procedure for determining the critical slip surface and factor of safety is explained below.

Nonlinear programming procedures for searching for the minimum of a function of several variables start from a trial slip surface S^o and proceed towards the minimum in an iterative way, generating sequence of feasible slip surfaces S^1, S^2, S^3, …… S^k,S^{k+1} so that the sequence of the associated safety factors decreases.

ie. $F(S^0) > F(S^1) > F(S^2) > F(S^3) > \ldots\ldots > F(S^k) > F(S^{k+1})$

Where $S^K = \{x_1^k, y_i^k, x_2^k, y_2^k, x_3^k, y_3^k, \ldots.\ x_n^k, y_n^k\}$

$S^{K+1} = \{x_1^{k+!}, y_i^{k+1}, x_2^{k+1}, y_2^{k+1}, x_3^{k+!}, y_3^{k+1}, \ldots\ldots\ x_n^{k+1}, y_n^{k+1}\}$

A trial slip surface with six or seven trial points is selected. Figure 1 presents one such trial surface with point A, point B, point C, point D and point E as trial points. Each of these trial points is then shifted to a new position in eight directions. A, B',C, D and E is one such surface obtained after shifting B to B'. For each shift the factor of safety is computed. If the factor of safety of the new slip surface (A, B', C, D, E) is lesser than the factor of safety of the previous slip surface (A, B, C, D, E), then the point B is redefined to this new position (B'). Otherwise, it is returned to the previous position (B). The process of shifting the trial points to new position based on comparison of old and new factor of safety is repeated for all trial points A, B, C, D and E. As trial points are shifted to new position a new slip surface is obtained. The factor of safety of the new slip surface, thus obtained is compared with the factor of safety of the old slip surface. If the factor of safety of the new surface is less than the factor of safety of the old surface then the procedure is repeated for the new surface. Thus shifting of trial points of the slip surface in eight direction as explained above is repeated until the factor of safety of new slip surface and previous slip surface are in best agreement. Three trial slip surfaces considered for the analysis are shown in Figure 2.

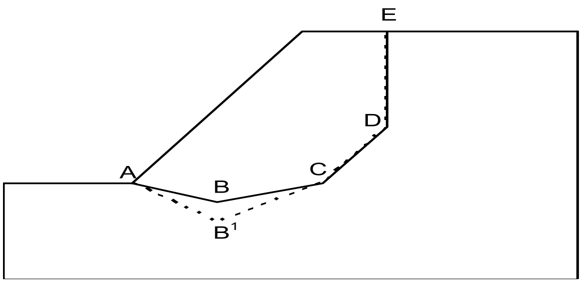

Fig. 1 Trial Slip Circle ABCDE

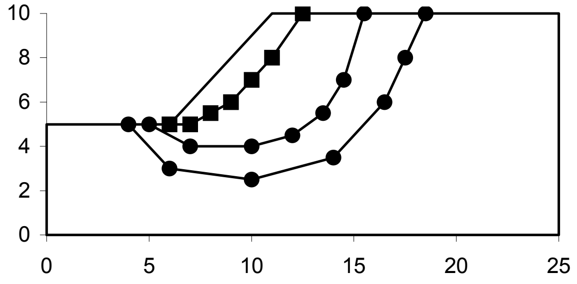

Fig. 2 Trial Slip Circles Considered for the Analysis

Equation for Factor of Safety

The trial slip surface is divided into 'n' number of segments each of length ΔL . The overall factor of safety for a particular slip surface is obtained using the equation

$$F.S = \frac{\sum \tau_f \Delta Li}{\sum \tau_i \Delta Li} \tag{7}$$

where τ_i is the mobilized shear stress and τ_f is the shear strength of the material. ΔLi is the length of ith segment. For the ith segment on a particular slip surface, the values of τ_i and τ_f may be expressed as

$$\tau_f = c + \sigma_{ni} \tan \phi \tag{8}$$

$$\tau_i = 0.5\ (\sigma_{yi} - \sigma_{xi})\ \sin 2\alpha_i + \tau_{xyi} \cos 2\alpha_I \tag{9}$$

$$\sigma_{ni} = 0.5\ (\sigma_{yi} + \sigma_{xi}) + 0.5(\sigma_{yi} - \sigma_{xi}) \cos 2\alpha_i - \tau_{xyi} \sin 2\alpha_I \tag{10}$$

where c and ϕ are the effective cohesion and effective angle of internal friction of the soil. σ_{ni} is the normal stress acting on segment i. σ_{xi} , $\sigma_{y\ i}$ and τ_{xyi} are the effective stresses on ith segment. α_i is the inclination of the ith segment with horizontal.

Criteria for Moving the Trial Points

Each trial point of the current slip surface is randomly moved to a new position to obtain the reduction in safety factor. The point i is moved from point (x_i^k, y_i^k) to point (x_i^{k+1}, y_i^{k+1}) where

$x_1^{k+1} = x_1^k + \xi_i$ and $y_1^{k+1} = y_1^k + \eta_i$

$\xi_i = N_x Dx_i$ and $\eta_i = N_y Dy_i$, Dx_i and Dy_i are the widths of the search steps in directions x and y for various points i. Eight combinations are given for parameters N_x and N_y . In this way eight random displacements are tried for every trial points of the slip surface. In the present analysis, the values of Dx_i and Dy_i are taken as 0.5 m.

Numerical Applications

Following four problems are considered to demonstrate the applicability of the analysis explained above

i. Comparison of the factor of safety obtained from the present analysis with available results
ii. Stability analysis of slope constructed with saturated soil of low permeability
iii. Stability analysis of slope constructed with pervious soil on soft saturated foundation
iv. Stability analysis due to surcharge on saturated slope

Comparison of Factor of Safety Obtained from the Present Analysis with Available Results

Factor of safety obtained from the proposed method of combining the finite element method to obtain stresses and a Monte Carlo technique to find critical slip surface is compared with the factor of safety obtained from available results. A homogeneous 1:1 slope with unit weight 18.5 kN/m^3, modulus of elasticity 5000 kN/m^2 and Poisson's ratio 0.3 is analysed. The height of the slope is equal to 8.0 m. The slope is discretised using four noded quadrilateral element of size 0.5 m x 0.5m and triangular element as shown in Figure 3. The displacement along X and Y direction is restrained at the bottom surface of the slope (along BC) and displacements in X direction is restrained along the side CD and AB as shown in the figure. Various combinations of cohesion c and angle of internal friction ϕ is considered. Factor of safety obtained from the analysis are tabulated in table 1. The factor of safety obtained by Scott et al. (1993) based on the local minimum factor of safety method and by using Bishop's (1952) method of analysis for the same slope are also tabulated in table 1 for various values of c and ϕ. It can be observed from the table that the results obtained from the present analysis and that presented by Scott et al. (1993) based on local minimum factor of safety method and by using Bishop's (1952) method agree very well for all values of c and ϕ considered. Kim and Lee (1997) analysed a 1:1 slope of height 5.0 m using finite element method. The unit weight and modulus of elasticity of soil are 20 kN/m^3 and 1500 kN/m^2 respectively. The same slope is also analysed by the proposed method. Finite element discritization of this slope is shown in Figure 4. Factor of safety and critical slip surface obtained by the proposed method and by Kim and Lee (1997) for various combinations of c and ø are shown in Figure 5. There is a close agreement between the factor of safety and slip surface obtained by the present analysis and Kim and Lee (1997).

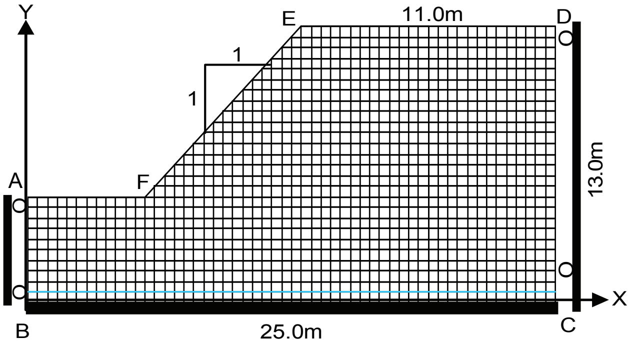

Fig. 3 Finite Element Discretisation of 8.0 m Slope

Table 1 Factor of Safety Obtained by the Present Method and by Scott et al. (1993)

c (Mpa)	*ϕ (degrees)*	*F.S by proposed method*	*F.S by local minimum FOS method*	*FOS by Bishop's method*
25	20	1.89	1.87	1.71
20	20	1.67	1.68	1.50
15	20	1.39	1.46	1.29
10	20	1.08	1.0	1.05
30	15	1.88	1.85	1.75
25	15	1.65	1.65	1.53
20	15	1.43	1.45	1.32
15	15	1.21	1.24	1.11
10	15	0.94	1.0	0.89
25	10	1.46	1.42	1.35
20	10	1.25	1.23	1.15
15	10	1.02	1.0	0.97

Hence, the proposed method of combining the finite element method to obtain stresses and a Monte Carlo method to find the factor of safety can be used to analyse the embankment slopes. Also, the method is simple since it requires few trial surfaces to determine the factor of safety.

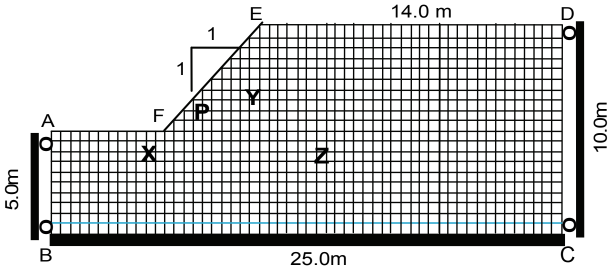

Fig. 4 Finite Element Discretisation of 5.0 m Slope

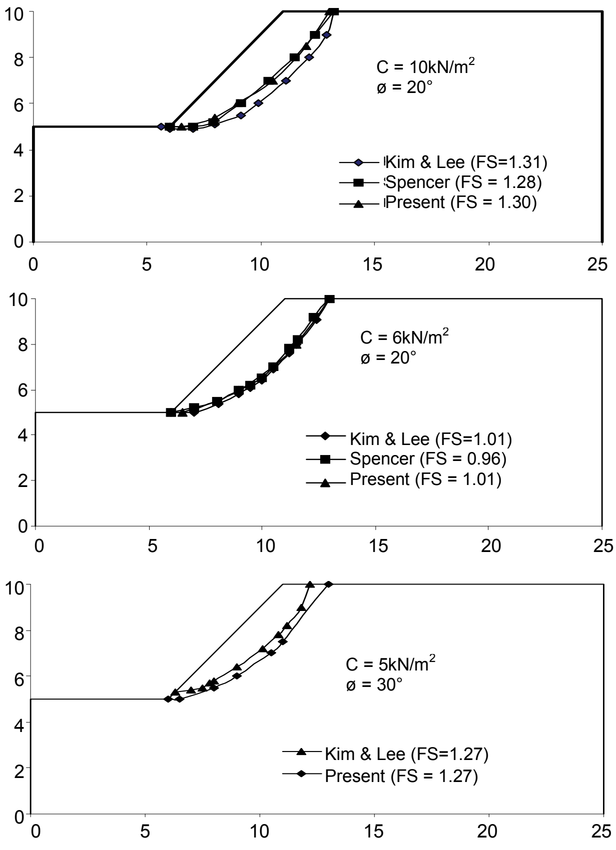

Fig. 5 Comparison of Factor of Safety and Slip Circle Obtained from Present Analysis with Available Results

Analysis of a Consolidating Slope

Applicability of the proposed method is used to analyse a consolidating slope in which the pore pressure and effective stresses at any point changes with time. A 1:1 slope of height 5.0 m shown in Figure 4 is considered for the analysis. As shown in figure, the displacement along X and Y direction is restrained at the bottom surface of the slope (along BC) and displacements in X direction is restrained along the sides CD and AB. Seepage is permitted along the surface AF, FE and ED and the bottom surface BC and sides CD and AB are considered as impervious. Pore pressure, effective stresses and factor of safety is obtained at various time intervals from the beginning of consolidation (T=0) till the end of consolidation process. Factor of safety, pore pressure and effective stresses obtained from the analysis at various time interval is discussed below for the various cases.

Analysis of a Slope Constructed with Soil of Low Permeability

In this case, slope is constructed suddenly (without allowing pore pressure to dissipate) with saturated soil. Pore pressure at any point immediately after construction is due to the saturated weight of the soil above this point. In this case, the similar properties are considered for slope and foundation.

Unit weight = 20 kN/m^3
Modulus of elasticity = 5000 kN/m^2
Poissons ratio = 0.30
Effective cohesion = 20 kN/m^2
Effective angle of internal friction = 30°
Coefficient of permeability = 8.67×10^{-4} m/day

Figure 6 shows the variation of factor of safety with time. Pore pressure and effective stresses at point P and point Y obtained from the analysis at various time intervals is also shown in Figure 6. Factor of safety of the slope at the end - of - construction (at T=0, undrained condition) is equal to 1.89 and it increases with time due to dissipation of excess pore pressure and increase in effective stresses in soil mass. The factor of safety of the slope after complete dissipation of excess pore pressure (at T = 350 days, drained condition) is equal to 2.37. Pore pressure immediately after construction is more at point Y as compared to point P. Pore pressure dissipates with time and becomes zero after 350 days. Effective horizontal stress immediately after construction is positive at point P whereas it is negative at point Y. Effective horizontal and effective vertical stresses increases with time at points P and Y where as shear stresses will not change much with time. It can be observed from the figure that, as the pore pressure decreases from 25.0 kN/m^2 to zero at point P,

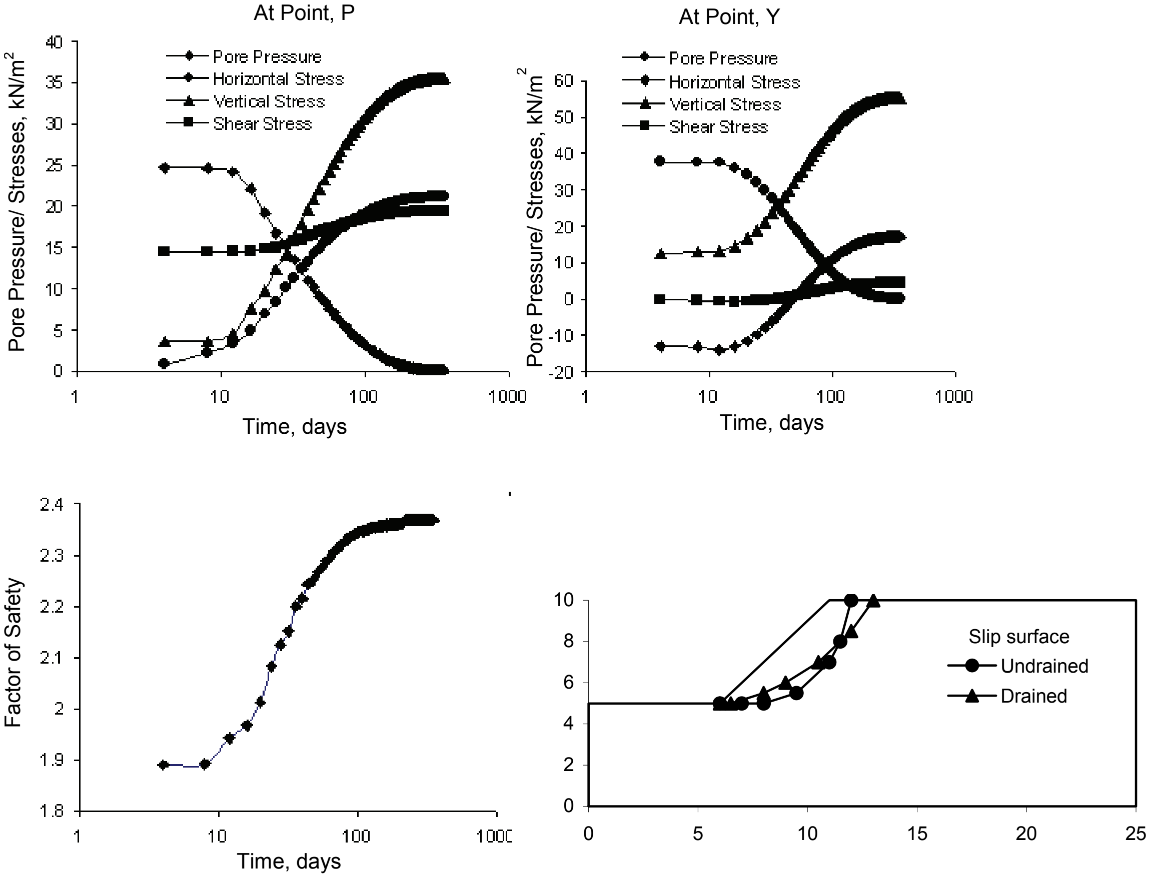

Fig. 6 Results Obtained from the Analysis of Slope Constructed Suddenly with Soil of Low Permeability

the vertical stress increases from 3.5 kN/m^2 to 35 kN/m^2, horizontal stress increases from 0.7 kN/m^2 to 21.0 kN/m^{2}and shear stress increases from 14.0 kN/m^2 to 19.0 kN/m^2. Similarly as the pore pressure decreases from 37.0 kN/m^2 to zero at point Y, the effective vertical stress increases from 12.50 kN/m^2 to 55.0 kN/m^2, horizontal stress increases from -13.0 kN/m^2 to 17.0 kN/m^2 and shear stress increases from 0 to 4.0 kN/m^2. The critical slip surfaces obtained from the analysis for undrained condition (T = 0) and completely drained condition (T = 365 days) is shown in Figure 6. As seen in figure, the failure surface completely passes through the slope for both drained and undrained condition. However, the slip surface corresponding to drained condition is slightly different from the slip surface corresponding to undrained condition.

Analysis of a Slope Constructed with Pervious Soil on Soft Foundation

In this case, slope is constructed with pervious soil on soft and saturated foundation of low coefficient of permeability. Since, the slope is pervious; pore pressure will not develop in the slope during and after construction. However, pore pressure will develop in the foundation soil and consolidate due to dissipation of pore pressure. Properties of slope and foundation soil are as follows:

Slope

Unit weight = 20 kN/m^3
Modulus of elasticity = 5000 kN/m^2
Poissons ratio = 0.3
Effective cohesion = 20 kN/m^2
Effective angle of internal friction = $30°$

Foundation

Unit weight = 20 kN/m^3
Modulus of elasticity = 5000 kN/m^2
Poissons ratio = 0.3
Effective cohesion = 10 kN/m^2
Effective angle of internal friction = $10°$
Coefficient of permeability = 8.67x10^{-4} m/day

Factor of safety, pore pressure and effective stresses are obtained at various time intervals from the end - of - construction of the slope till the complete dissipation of excess pore pressure in foundation soil. Figure 7 shows the variation of factor of safety, pore pressure and effective stresses at points X, Y and Z with time. Since the point Y is situated in the pervious slope, the pore pressure is zero at all time intervals. As seen in

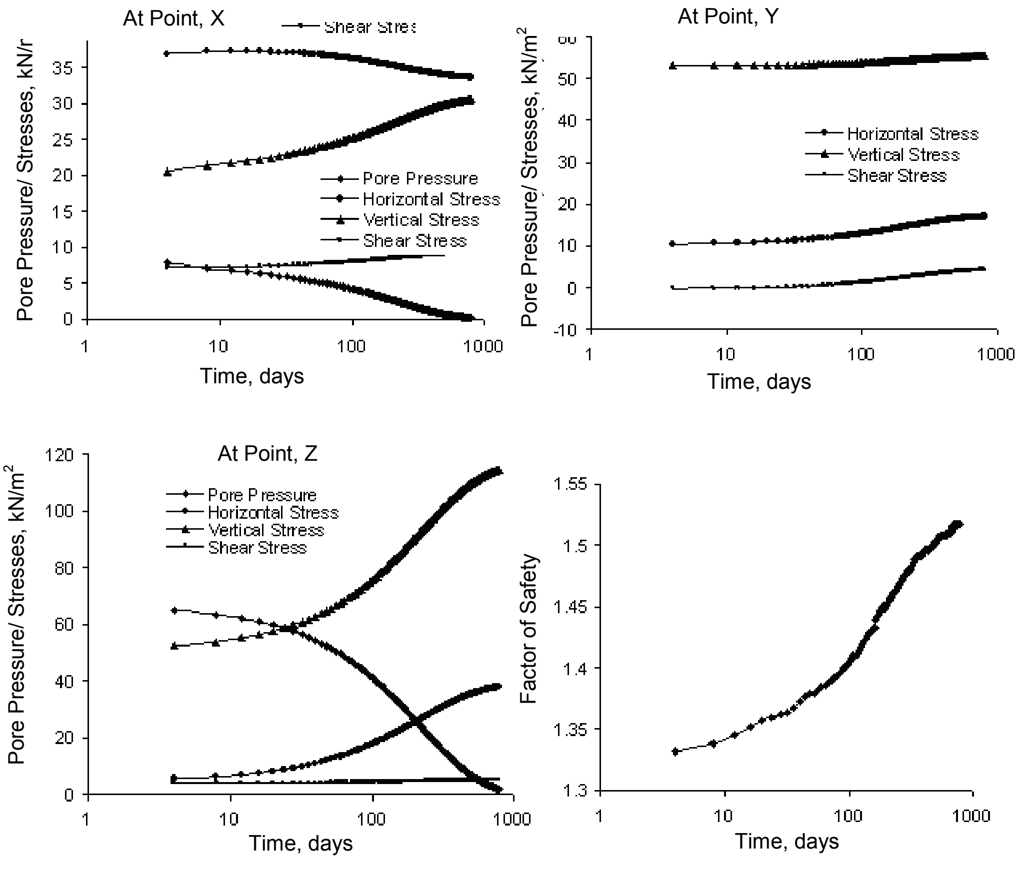

Fig. 7 Results Obtained from the Analysis of Pervious Slope on Soft Foundation

Figure 4, point Z is below the slope where as point X is at a horizontal distance of 1.0 m from the toe of the slope. It can be observed from the figure that the factor of safety of the slope immediately after construction is equal to 1.33 and it increases with time. Factor of safety equal to 1.52 at 792 days after the construction of the slope, when pore pressure in foundation is completely dissipated. Thus the factor of safety increases from 1.33 to 1.52 due to the consolidation of the foundation soil. It can be observed from the figure that the pore pressure immediately after construction is equal to 7.88 kN/m^2 at X and 65.0 kN/m^2 at Z. Due to dissipation of pore pressure, horizontal stresses at point X decreases where as it increases at point Y and Z. Vertical stress increases at point X and Z and will not change much at point Y with time. Shear stresses will not change much with time at all the three points X,Y and Z. Thus, it can be observed that even though there is no change in pore pressure at point Y in the slope, all the three effective stresses will change slightly with time due to dissipation of excess pore pressure in foundation soil. This is due to the settlement of the soil due to dissipation of pore pressure in foundation. As seen in Figure 8, the critical slip surface in this case passes through the slope and foundation soil for both the drained and undrained condition.

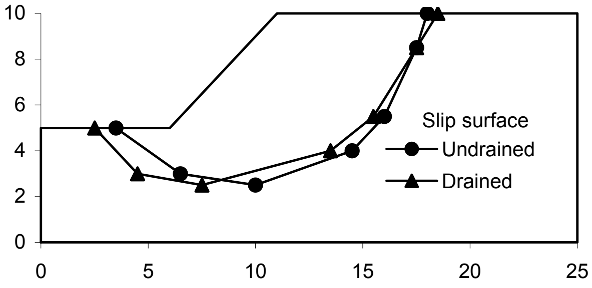

Fig. 8 Critical Slip Circles of Slope on Soft Foundation

Analysis of a Saturated Slope due to Surcharge

In this case, a uniformly distributed surcharge load of intensity 50.0 kN/m^2 is applied after the slope and foundation soil are completely consolidated. ie. the excess pore pressure is zero in the slope and foundation soil before the application of the load. However, it is assumed that the slope and foundation soil are saturated. Properties of slope and foundation soil are as follows:

Unit weight	= 20 kN/m^3
Modulus of elasticity	= 5000 kN/m^2
Poissons ratio	= 0.3
Effective cohesion	= 20 kN/m^2
Effective angle of internal friction	= 30°
Coefficient of permeability	= 8.67x10^{-4} m/day

The variation of factor of safety, pore pressure and effective stresses (at point P and Y) with time due to the surcharge load is shown in Figure 9. It can be observed from Figure 7 (for case 1) that, before the application of the surcharge, the factor of safety is equal to 2.37 and the effective horizontal, vertical and shear

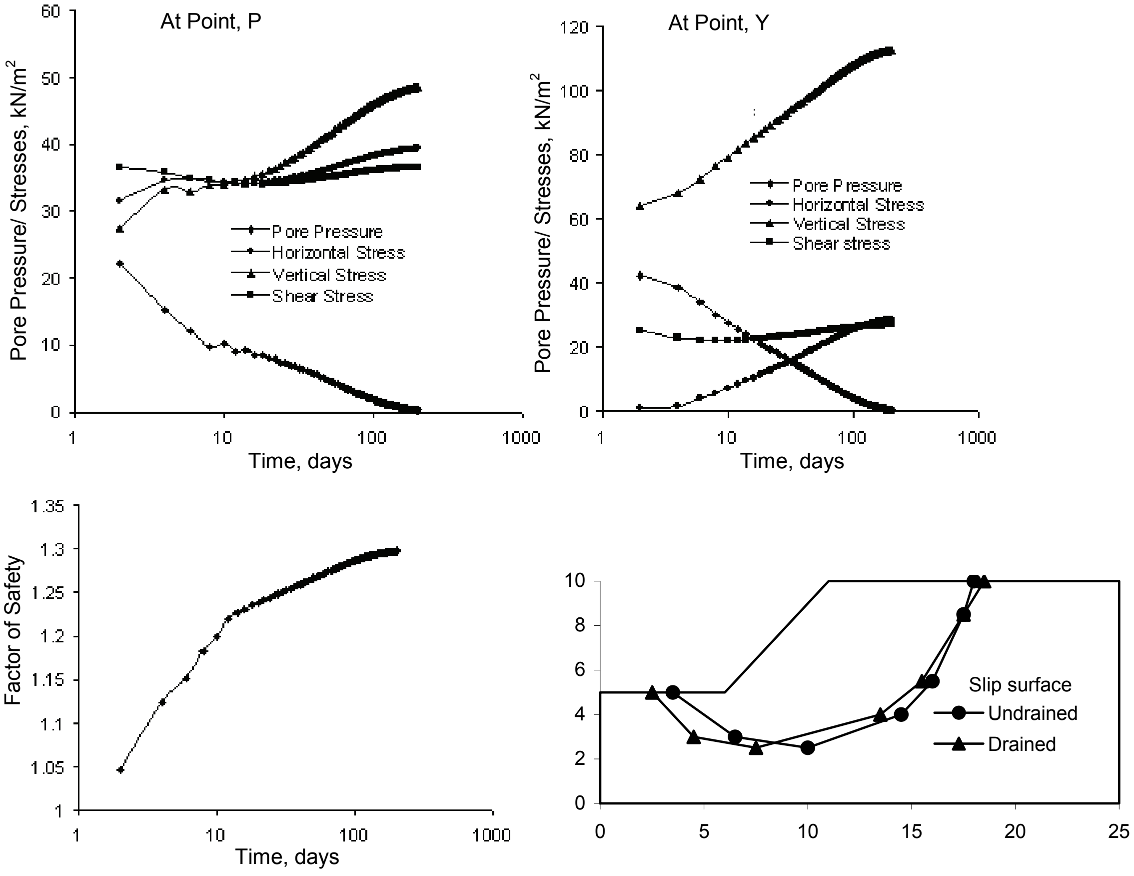

Fig. 9 Results Obtained from the Analysis of Slope due to Surcharge

stresses are 21.0 kN/m^2, 35.0 kN/m^2 and 19.0 kN/m^2 at point P and 17.0 kN/m^2, 55.0 kN/m^2 and 4.0 kN/m^2 at point Y. From Figure 9, it can be observed that, due to the application of the surcharge, the factor of safety of the slope decreases suddenly from 2.37 to 1.04 and then increases to 1.30 after 202 days. Pore pressure immediately after the application of surcharge is 22.2 kN/m^2 at point P and 42.2 kN/m^2 at point Y. Excess pore pressure dissipates completely after 202 days. Due to dissipation of pore pressure, effective horizontal stress increases from 31.0 kN/m^2 to 39.0 kN/m^2 at point P and 1.2 kN/m^2 to 28.6 kN/m^2 at point Y. Effective vertical stress increases from 27.4 kN/m^2 to 48.5 kN/m^2 at point P and 64.2 kN/m^2 to 112.0 kN/m^2 at point Y. The shear stress will not change much at point P as well as at point Y. As in case 1, the slip surface corresponding to both drained and undrained conditions, passes completely through the slope.

Summary and Conclusions

A procedure to obtain the factor of safety of a consolidating slope at various time intervals is developed. The effective stresses and pore pressure in soil are obtained using finite element method and the critical slip surface is obtained using a Monte Carlo technique. The analysis is used to obtain the factor of safety, pore pressure and stresses for consolidating slope at various time intervals for various cases. Based on the analysis, it is concluded that

- The method of locating critical slip surface is simple since it requires few trial slip surfaces.
- The proposed method can be used to obtain the factor of safety, pore pressure and effective stresses for consolidating slope at various time intervals.
- Effective horizontal and vertical stresses will increase with time where as the effective shear stress will not change much with time during consolidation.
- Effective horizontal and vertical stresses in a dry or fully consolidated slope may change with time if the slope is constructed on a soft consolidating foundation.

References

Baker, R. (1980): 'Determination of the critical slip surface in slope stability computations', *International Journal for Numerical and Analytical Methods in Geomechanics*, (4), pp. 333-359.

Bardet, J.P. and Kapuskar, M.M. (1989): 'A simplex analysis of slope stability', *Computers and Geotechniques*, (8), pp. 328-348.

Biot M.A. (1941): 'General theory of three dimensional consolidation', *Journal of Applied Physics*, (12), pp. 155-164.

Bishop, A. W. (1955): 'The use of the slip circle in the stability analysis of slopes', *Geotechnique*, 5(1), pp. 7 – 17.

Delwyn, G. F., Raymond, E.G. and Noshin, Z. (1999): 'Using a finite element stress analysis to compute the factor of safety', *Proc., 52nd Canadian Geotechnical Conf.,* Saskatchewan, pp. 73 – 80.

Donald, I.B. and Giam, S.K. (1988): 'Application of the nodal displacement method to slope stability analysis', *Proc., 5th Australia – New Zealand Conf. on Geomechanics,* Sydney, pp. 456 – 460.

Duncan, J.M. and Dunlop, P. (1969): 'Slopes in stiff – fissured clay and shales', *Journal of Soil Mechanics and Foundation Division , ASCE*, 95(2), pp. 467 – 492.

Greco, V.R. (1996): 'Efficient Monte Carlo technique for locating critical slip surface', *Journal of Geotechnical Engineering, ASCE*, (122), pp. 517-525

Janbu, N. (1957): 'Earth pressures and bearing capacity calculations by generalized procedure of slices', *Proc., 4th International conference on Soil Mechanics and Foundation Engg.,* pp. 207 – 212.

Kim, J.Y. and Lee, S.R. (1997): 'An improved search strategy for the critical slip surface using finite element stress fields', *Computers and Geotechnics*, 21 (4). pp. 295-313.

Lewis, R.W., Roberts, G.K., Roberts and Zienkiewicz, O.C. (1976): 'Nonlinear flow and deformation analysis of consolidated problems', *Proc. 2nd Int. Conf. on Numerical Methods in Geomechanics,* pp. 1106-1118

Manoharan N. and Dasgupta, S.P. (1995): 'Consolidation analysis of elasto-plastic soil', *Computers and Structures*, 54(6), pp. 1005-1021.

Morgenstern, N.R. and Price. V.E. (1965): 'The analysis of the stability of general slip surfaces', *Geotechnique*, 15(1), pp. 79 – 93.

Sandhu R.S. and Wilson E.L. (1969): 'Finite element analysis of seepage in elastic media', *Journal of Engineering Mechanics, ASCE*, 95(3), pp. 641-652.

Sarma, S.K. (1979): 'Stability analysis of embankments and slopes', *Journal of the Geotechnical Engineering Division, ASCE,* 105(12), pp. 1511 – 1524.

Scott, L., Huang. and Yamasaki, K. (1993): 'Slope failure analysis using local minimum factor of safety approach', *Journal of Geotechnical Engineering*, 119(12), pp. 1974 – 1987.

Spencer, E. (1967): 'A method of analysis of the stability of embankments assuming parallel interslice forces', *Geotechnique*, 17(1), pp. 11 – 26.

Venanzio, R.G. (1996): 'Efficient Monte Carlo technique for locating critical slip surface', *Journal of Geotechnical Engineering*, 122(7), pp. 517 – 525.

Zienkiewicz, O.C. (1997): *The Finite element method,* McGraw - Hill Book Company, UK.