Indian Geotechnical Journal, 40(1), 2010, 67-77

Vertical Vibration of Embedded Rectangular Footing in Layered Soils

Shambhu P Dasgupta[1] and Gopalakrishna Kamireddi[2]

ABSTRACT: A finite element solution has been presented for the vertical mode of vibration of a rectangular footing resting on a semi infinite isotropic and layered elastic medium. A combination of Ralyeigh wave and standard viscous boundary has been used for finite dimensional idealization of the semi infinite soil continuum. Software was developed for the solution of these problems and displacement profile of the elastic half space has been determined. Numerical results were obtained for the vertical mode of vibration for rectangular, square and strip footings for a layered soil deposit for different Poisson's ratios under surface and embedment conditions. The results have been compared with the existing solutions and are presented graphically.

KEYWORDS: Dynamic response, elasticity, finite element method, layered soils, embedment, rectangular footing and wave propagation.

Introduction

The vibration, whether originating as an impulse from a forge hammer or vibrations from reciprocating engines are transmitted as waves through the ground. These waves in turn excite other structures distant from the source as well as the foundation from which vibration energy is emanating.

The key to the solution of such problems is to compute the matrix of dynamic impedance functions relating the steady-state force and displacements at the base of an associated foundation-soil system. Such a system is identical with the actual one, except that the mass of superstructure and the foundation set equal to zero. The dynamic response of soil is normally represented by the compliance functions produced by the loading of the structure and depends on the mechanical properties of the soil, the shape of loading area, and frequency of excitation.

Though not considered, the elastic constants of soil depend on normal stresses and the elastic deformations may affect the initial internal stress which always exists in soil. It should also be noted that the solution of problems related to the propagation of waves may be greatly influenced by dissipative properties of soil which govern the absorption of wave energy. Once the harmonic response of such a mass-less (but rigid) foundation is determined, the response of massive foundation or any supported structure can be evaluated.

Various methods have been developed over the years for computing the dynamic response of soil-foundation systems. Anandakrishnan and Krishnaswamy(1973) experimentally studied the dynamic response of embedded footings to the vertical vibrations. Extensive reviews of these developments are presented by, among others, Lysmer (1978), Roesset (1980), Luco (1982), Gazetas (1982) and Lin and Tassoulas (1986).

Nogami and his colleagues used another form of simplified soil model for both static and dynamic response analyses of embedded strip foundations (Nogami and Lam 1987; Nogami and Leung 1990). In their approach they proposed a numerical assessment of dynamic soil stiffness at the side of embedded structures with rectangular footing.

The effect of layer thickness and embedment depth on the stiffness for various modes has been studied by Johnson et al. (1975) using the finite element method. Their studies showed that, the effect of both decreasing layer thickness and increasing the depth of embedment was to increase the stiffness, resulting in an increase in the resonant frequency.

Kagawa and Kraft (1981) carried out a parametric study to investigate the effect of layering on the vertical vibration response. They considered two-layer soil system, the bottom layer being a half-space and stiffer than the top layer. It was concluded that the layering has a large effect on the resonant frequency for small mass ratio but has negligible effect at large mass ratios.

Sridharan et al. (1990) experimentally studied the dynamic response of footing on layered soil system under a wide range of static and dynamic loads. They showed the effect of position and thickness of the layer on resonant frequency and amplitude for both two and three layered soil systems.

Meek and Wolf (1992a,b) presented a simplified methodology to evaluate the dynamic response of a base mat on the surface of a homogeneous half-space. The cone model concept was extended to a layered cone to compute the dynamic response of a footing or a base mat on a soil layer resting on a rigid rock and on flexible rock (Wolf and Meek, 1993). Meek and Wolf (1994) performed dynamic analysis of embedded footings by idealizing the soil as a translated cone instead of elastic half-space. They have found out the dynamic stiffness coefficients of foundations resting on or embedded in a horizontally layered soil using cone frustums.

1 Professor; Department of Civil Engineering, Indian Institute of Technology, Kharagpur 721 302, West Bengal, India. E-mail: dasgupta@civil.iitkgp.ernet.in

2 Graduate Student, Department of Civil Engineering, Indian Institute of Technology, Kharagpur 721 302, West Bengal, India.

Baidya and Sridharan (1994), Baidya and Muralikrishna (2001) studied the dynamic response of foundation response on a stratum underlain by a rigid layer using lumped parameter model and also suggested a method to estimate the equivalent stiffness and the equivalent damping for such systems.

By combining the advantages of the finite element and the boundary element method with unique properties of their own, Deeks and Wolf (2002) introduced the scaled-boundary finite element method. It is a novel semi-analytical technique. A new virtual work formulation and modal interpretation of the method for elastostatics was developed. The formulation follows the similar procedure of traditional virtual work derivation of the standard finite element method. This approach leads to a new technique for the treatment of body loads, side-face loads and axisymmetric conditions that can be simply implemented.

Nogami and Chen (2003) proposed a method to obtain the soil behavior at the side of an embedded structure with rectangular base area. The soil medium is idealized as a stack of horizontal sheets interconnected by distributed vertical springs. Each sheet is made of column-spring system.

Deeks and Augarde (2004) had shown that the scaled boundary method is an excellent way to model an unbounded domain. However, it is limited to linear problems. Many soft-ground geotechnical problems require both non-linear constitutive behavior for the soil to capture pre-failure deformations and the presence of an unbounded domain. They introduced mesh-free methods. It is ideally suited to such problems. They coupled a mesh-less local Petrov-Galerkin method for the near field with a mesh-less scaled boundary method of similar type for the far field.

Baidya, Krishna and Pradhan (2006) presented an investigation of the dynamic response of foundations resting on a layered soil underlain by a rigid layer. A simple method to estimate the equivalent stiffness of the foundations resting on any multilayered soil system is presented.

The present work is related to the evaluation of the dynamic force displacement relationship, thereby finding vertical compliance functions of embedded rectangle footing subjected to harmonic type of loadings using a three dimensional finite element model. Displacement profile of the elastic half space has been determined considering the footing embedment, layering effect with varying length to width ratio of the footing. For a two layered half-space influence of the overlaying layer on the distribution of surface waves for different Poisson's ratio is determined and compared with the available theoretical results. Dynamic soil stiffness at the side of embedded footing with rectangular base in plan view is determined with the elastic half space theory and compared with the previous results. Computer codes have been developed for the solution of these problems.

Analysis

The soil strata below the footing is sub-divided into a finite number of simply connected elements and the original region is then considered as an assemblage of these elements connected at nodes. The appropriate functional value of the solution is assumed over the element. The equations of equilibrium for the entire region are then obtained by the combination of equilibrium equation of each element such that the continuity of displacement is ensured at each node where elements are connected. In discrediting the finite region the following criteria were followed: (1) The distance from the edge of the footing to the vertical boundary should be of the order of the Rayleigh wave length and, (2) The largest dimension of any element in the finite element mesh does not exceed one tenth of the Rayleigh wave length. The reason for this is that the waves propagating from the foundation would reflect back from such boundary and would induce spurious modes in the vibration resulting in erroneous results.

Finite Dimensional Idealization of Infinite Media

The present study treats the footing soil system (Figure 1) as a wave propagation problem.

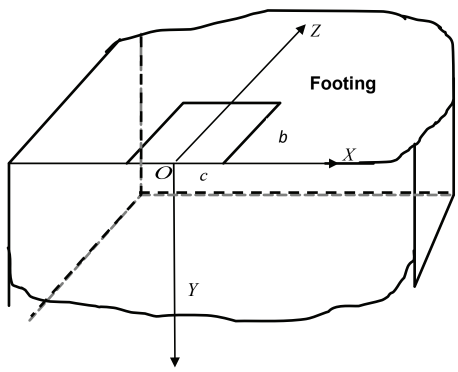

Fig. 1 Problem Description of a Rectangular Footing on an Elastic Half-space

Viscous boundary is defined as the waves arriving at this boundary get lost to infinite region immediately beyond it and do not reflect back to the vibrating footing. Considering the reflection of plane waves on the boundary, it is found that the stress boundary (standard viscous boundary) condition given below takes care of elastic wave propagation for P and S-waves;

$$\sigma = \rho V_p \dot{w} \;\; : \; \tau = \rho V_s \dot{u} \qquad (1)$$

in which, $v_s^{\,2} = G / \rho$; $V_p = V_s / s$; $s = \frac{(1-2v)}{2(1-v)}$;

where, $\sigma =$ normal stresses; $\tau =$ shear stress, u, v and w = displacements along x, y and z directions at its own time derivatives respectively; $\rho =$ mass density of soil, V_p = P-wave velocity; V_s = S-wave velocity; G = shear modulus; $\nu =$ Poisson's ratio of the soil.

The Rayleigh surface waves account for a major part of the total energy propagation through an elastic half space. A stress boundary condition mentioned below takes care of the major portion of the energy in elastic wave propagation.

$$\sigma = \frac{G}{V_R}\frac{[\frac{1-s_1^2}{2}\{2r_1^2-\eta(1+r_1^2)\}e^{ir_1k_y}-2s_1r_1e^{ir_1k_y}]}{[\frac{1-s_1^2}{2}e^{ir_1k_y}+s_1r_1e^{ir_1k_y}]}\dot{u} \qquad (2)$$

$$\tau = \frac{G}{V_R}\frac{[(1-s_1^2)e^{ir_1k_y}-e^{is_1k_y}]}{[\frac{1-s_1^2}{2}e^{ir_1k_y}+e^{is_1k_y}]}\dot{w} \qquad (3)$$

in which, $s_1^2 = (V_R / V_s)^2 - 1$; $r_1^2 = (V_R / V_p)^2 - 1$;

$\eta = V_p / V_s$; $k = \omega / V_R$ = wave number, $i^2 = -1$; ω = angular velocity of vibration; V_R= Rayleigh wave velocity of the medium.

For any value of ν, σ and τ can be obtained, knowing the roots of Rayleigh equation,

$$R^6 - 8R^4 + R^2(24 - 16Q_1^{*2}) - 16(1 - Q_1^{*2}) = 0 \qquad (4)$$

where;

$R = V_R / V_s$; $Q_1^* = V_S / V_P$;

and subjected to condition, $0 < V_R < V_S < V_P$ (5)

These descriptions are valid for steady state vibration problems.

A combination of these two boundaries has been used in the analysis. Along vertical faces, Rayleigh wave absorbing boundaries are used as stress boundary conditions where as along the base, the stresses arising from SVB have been used.

Finite Element Model

For discretizing into a finite number of elements, the semi infinite media is idealized as bounded region by adopting appropriate boundary conditions as mentioned earlier. A combination of SVB and Rayleigh wave absorbing boundaries has been used for present analysis.

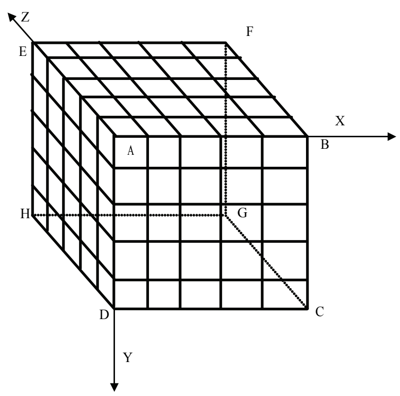

Fig. 2 Description of the Half-space

For the description of the halfspace shown in Figure 2, the normal (N) and Shearing (S) stresses along the boundaries are:

At each node along BCGF

$$N_x = a^*\dot{u},\quad S_y = b^*\nu,\quad S_z = \rho V_s\dot{w} \qquad (6)$$

At each node along DCGH

$$N_y = \rho V_p\nu,\quad S_x = \rho V_s\dot{u},\quad S_z = \rho V_s\dot{w} \qquad (7)$$

At each node along EFGH

$$N_z = a^*\dot{w},\quad S_x = \rho V_s\dot{u},\quad S_y = a^*\dot{v} \qquad (8)$$

Here suffixes indicate the direction along which the stress acts, a^* and b^* are coefficients which depend on Poisson's ratio.

In discretizing the finite region, following conditions are satisfied__

- Ratio of the largest element dimension and the minimum of the elastic wave length (least of P, S and R wave length) should be less than 1/10, and
- Distance of viscous boundary should be the order of Rayleigh wave length.

A simple hexahedral isoparametric element with eight nodes has been chosen as the basic element. The best element, in general, is the one with the lowest trace of the element stiffness matrix. The element chosen here follows this criterion.

The equation of motion of finite element model of the system is

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{Q\} \qquad (9)$$

in which

[M] = element mass matrix = $\iiint_\Omega [N]^T \frac{\rho}{g}[N]d\Omega$

[C] = element damping matrix = $\iiint_\Omega [N]^T \mu^* [N]d\Omega$ (10)

[K] =element stiffness matrix = $\iiint_\Omega [B]^T[D][B]d\Omega$

{Q} = load vector = $\iiint_\Omega [N]^T\{F\}d\Omega + \iiint_\Omega [N]^T\{S\}d(d\Omega)$

where *[N]* = shape functions, $d\Omega$ = boundary; Ω = volume, *S* = surface tractions; *F* = force per unit volume, μ^* = linear proportionality coefficient in viscous damping (e.g. damping force = $-\mu^*\dot{u}$).

For study state vibration case, as is used in present analysis, if

$$\{Q\} = \{Q_0\}e^{i\omega t} \quad \Rightarrow \{u\} = \{u_0\}e^{i\omega t} \qquad (11)$$

where ω = frequency of vibration in radians per unit time, *t* = time period.

The element stiffness matrix can be expressed as

$$[K] = \iiint [B]^T[D][B]|J|drdsdt \qquad (12)$$

In which, |*J*| is the determinant of the Jocobian.

Equation (12) is integrated numerically using Gaussian quadrature. Numerically integrated elements yield convergence towards the correct value as the mesh size is refined and if the numerical integration is adequate to evaluate the element volume exactly. It is found that the jocobian determinant yield the number of Gaussian points needed to obtain the volume of particular element. For the present problem, a (2x2x2) Gaussian quadrature formula is found to be sufficient for the integration of the expression in equation (11) however; at least a (3x3x3) quadrature rule is required for skewed elements. The total number of elements used for the space shown in Figure 2 is 10x10x10.

Element Mass Matrix

Element mass matrix [m] for a lumped mass system can be written as;

$$[m] = \frac{1}{8}\rho I \int_{-1}^{+1}\int_{-1}^{+1}\int_{-1}^{+1} |J|\, drdsdt \qquad (13)$$

where I is an identity matrix; and J is the Jacobian matrix.

An equivalent form of the consistent mass matrix can be obtained from equation (13). The result obtained by these two types of mass formulations is not significantly different. So, the lumped mass formulation has been chosen for the present analysis. The approximation inherent in the lumped mass model may results in small reduction in accuracy as compared to a consistent mass model, but of greater importance is the fact that the diagonal character of the mass matrix leads to a considerable saving in computer storage and execution time.

Element Damping Matrix

Element damping matrix *[C]* in equation (10) is diagonal one and can be obtained by considering the element along the energy absorbing boundaries (BCFG, EFGH and CDHG in Figure 2). The coefficients along the diagonal of *[C]* are non zeros corresponding to the degree of freedom on the viscous boundary. In the present work, only radiation damping is considered to compare the theoretical results. For material damping experimental work is required. For the i^{th} degree of freedom on the viscous boundary, the contribution of one element to *[C]* can be written as;

$c_u = 0.25A^*a^*$, along BCFG

$= 0.25A^*\rho V_p$, along CDHG

$= 0.25A^*a^*$, along EFGH (14)

Here, A^* is the area of an element along the prescribed boundaries; a^* and b^* are coefficients which depend on Poisson's ratio.

Surface Loads

Considering the nature of soil structure interaction, three probable contact pressures namely, uniform, parabolic, and the one corresponding to rigid base conditions may be assumed between the footing and soil medium. These pressures are considered as surface tractions for the finite element model considered. In static cases the nature of contact pressure, depends upon the relative rigidity of the soil and the footing but in dynamic cases the nature of contact pressure, for the same loading, may vary from one to another depending upon the magnitude of frequency of vibration. In this paper only uniform contact pressure is considered and is given by;

$$Q(x,z,t) = \frac{We^{i\omega t}}{4cb} \text{ for } |x| \le c, |z| \le b \qquad (15)$$

$$= 0, \text{ otherwise.}$$

in which, *Q(x, z, t)* =dynamic contact pressure distributions at any time t; 2c = width of footing; *2b* = length of footing; *W* = total load on the footing, ω = frequency of vibration and *z, x* = space variables.

The load vector *{Q}* in equation (10) is computed considering the contact pressures as surface traction over the surface of the element under consideration. Equivalent nodal forces at the element level are computed, using equation (15) neglecting body forces, by numerical techniques

Once all the terms in equation are computed, the equation of motion for the entire system can be obtained by 'Direct Stiffness Method'. Response vector $\{u_0\}$ of equation can be evaluated by solving a set of linear algebraic using Gaussian Elimination Technique.

Footing Embedment

There are three possible effects of embedment on the static stiffness of a rigid foundation. First, a foundation mat placed at a depth bellow free ground surface transmit the load to a soil at deeper and therefore, different (usually stiffer) than the soil affected by a similar surface foundation. The other two effects that modify the behavior of embedded foundations are the 'trench effect' and the 'side wall contact effect'. The trench effect stems from the fact that, even a perfectly homogeneous half-space, the displacement of foundation placed at the bottom of an open trench are smaller than those of same foundation on the ground surface, because normal and shear stresses from the overlying soil restrict its movement. The side wall contact effect arises from the fact that when the vertical side wall of an embedded foundation are in contact with the surrounding soil, part of the applied load is transmitted to the ground through the shear and normal tractions acting on the vertical sides, depending on whether a particular side is parallel or perpendicular to the direction of loading. Inclined sides apply both the normal and shear tractions. These additional transmission paths lead to a further increase in the stiffness of an embedded foundation.

The radiation damping coefficient C_x and C_y represents the vibration energy transmitted into the soil and carried away by outward and downward spreading waves. These are generated at every point of the soil foundation interface so that, in general, the damping coefficient increases with the increase in the area of contact. For swaying embedded foundation, in addition to shearing waves originate at the basement, shearing and compression-extension waves are emitted from vertical side. For vertical loading embedded foundation, in addition to compression-extension waves originate at the basement, shearing and compression-extension waves are emitted from vertical side, depending on whether a particular side is parallel or perpendicular to the direction of loading. The shear waves emitting from the sides that are parallel to the direction of motion of the foundation propagate predominantly in the horizontal direction with an apparent velocity equal to S-wave velocity. It is assumed that the travel of these waves is restricted in the horizontal direction and they are not affected by the presence of the soil bellow the base. The propagation velocity of the compression-extension wave energy is not the P-wave velocity; it is taken as Lysmer's analog velocity,

$$V_p = [3.4 / \{\pi(1 - v)\}]V_s \qquad (16)$$

in which ν = Poisson's ratio; V_s = shear wave velocity.

Propagation of Elastic Waves in Layered Soil

Natural soils are formed in layers characterized by different mechanical properties. The layered structure of soil is responsible many peculiarities in wave propagation which can not be explained if the soil is considered to be a homogeneous body.

In the process of surface wave propagation in layered medium, there is a dispersion of waves; thus the velocity of wave propagation is not constant and is determined not only by the elastic and internal properties of the medium, but also by the lengths of propagating waves and their frequencies.

It was established that the presence of layer overlaying a homogeneous soil mass shown in Figure 3 and having properties different from this layer does not produce the monotonous change of amplitude with depth which takes place when surface waves are propagated in the homogeneous semi infinite mass.

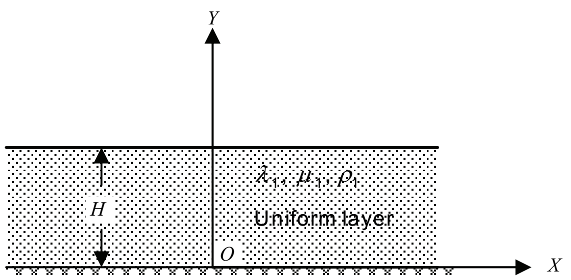

Fig. 3 Two layer system with coordinates

To simplify calculations, the situation is confined to establishing the influence of the top layer on the distribution of free plane surface waves. The material properties considered therein may be written as

$$\lambda = \frac{v}{(1+v)(1-2v)}E : \mu = \frac{1}{2(1+v)}E \qquad (17)$$

in which

λ and μ =Lame's parameters of the lower layer; $\lambda_1$and μ_1 =Lame's parameters of the upper layer; ρ and ρ_1 = density of the bottom and top layer, respectively (assumed to be constant for the present case).

Compliance Function and Displacement Rectangular Footing

Complex displacement in the vertical direction at the centre of the rectangular footing under the action of Dynamic loads is obtained from equation (15). This can be written in the form;

$$w(0,0,0,t) = \frac{W}{Gr_0}(f_1 + i f_2)e^{i\omega t} \qquad (18)$$

In which; $w(0,0,0,t)$ = vertical complex displacement at $x = y = z = 0$, at time t,

W = total soil reaction, G = shear modulus of soil,

ω = frequency of vibration in rad/sec.

r_0 = equivalent radius of footing = $\sqrt{\frac{4bc}{\pi}}$

$2c$, $2b$ = length and width of the rectangular footing, and

f_1, f_2 = compliance functions in the vertical direction.

Dynamic displacement amplitude at the centre of the rectangular footing of mass, m_0 for contact dynamic load is given by

$$w_0 = \frac{W}{Gr_0}\sqrt{\frac{f_1^2 + f_2^2}{(1 - b_1 a_0^2 f_1)^2 + (b_1 a_0^2 f_2)^2}} = \frac{W}{Gr_0} = M \qquad (19)$$

where; M = response factor; W = amplitude of the exciting force.

b_1 = mass ratio ($m_0 / \rho r_0^3$); a_0 = frequency ratio ($\omega r_0 / V_s$).

Results and Discussion

Plane Problems

Homogeneous Soil

Before the vibration of a real footing is taken up a plane problem of elastodynamics for vibration amplitude on the surface near the source of vibration is considered. In Figure 1, if c =0 and b extends to a large value i.e. a harmonic line load along the z direction the problem in Fig 1 reduces to a plane problem. For finite element idealization, the half of the halfspace (not the quarter space as shown in Figure 2) as shown in Figure 4 is considered for discretisation.

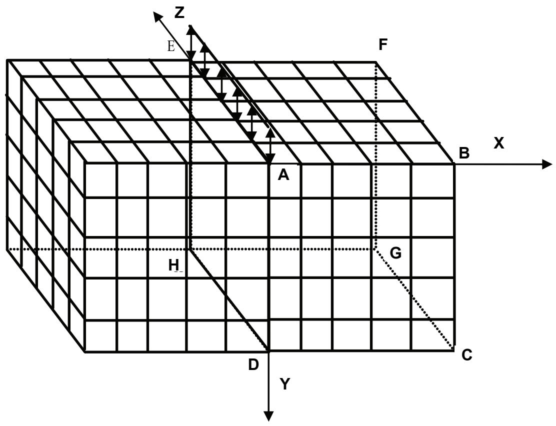

Fig. 4 Description of Plane Elastodynamic Problem

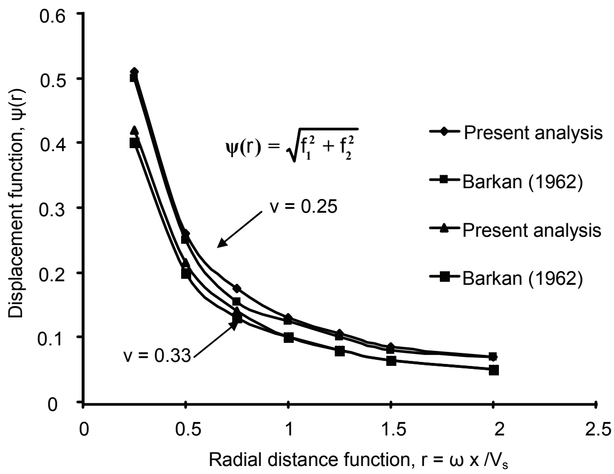

Fig. 5 Displacement function Ψ(r) versus distance function r

The vertical displacement on the soil surface at a small distance x from the loading source is defined as

$$w_0 = -\frac{Q_0 \omega}{V_s G} e^{i\omega t}\left(f_1 + i f_2\right) \quad (20)$$

Considering the real part, one can obtain

$$w = A_0 \psi(r) \sin(\omega t - j) \quad (21)$$

In which

$$A_0 = \frac{Q_0 \omega}{G V_s};\ \psi(r) = \sqrt{f_1^2 + f_2^2};\ \tan j = f_1 / f_2 \text{ where } r = \frac{\omega x}{V_s}$$

The influence of a layered soil structure on the distribution of surface waves proposed by Barkan (1962), is the only available theoretical solution.

For a non layer case, with the discretisation shown in Figure 4, the results are compared with the theoretical results of Barkan and presented in the Figure 5.

The function, ψ plotted for different values of poison's ratios with the distance ratio (r) and the results are compared and are presented. The results are nearly matching.

Layered Soil

The influence of a layered soil structure on the distribution of surface waves is plotted in Figure 6 for different cases like the effect of the surface layer thickness H to the wave length L on the Velocity ratio C_1/C for different Lame coefficients (μ / μ_l). The effect of the ratio of the wave length L to the upper layer thickness H on the ratio of horizontal u to vertical v displacements on the surface of the upper layer for different Lame coefficients (μ / μ_l).

Figures 6 indicates that for all shear moduli values the influence of top layer is confined mostly to the Raleigh wave length distance.

The following parameters were used for describing the problem:

H = top layer thickness; *L* = shear wave wave length for the surface layer; C_1 = surface wave velocity of the top layer; *C* = surface shear wave velocity without overlying layer, bottom layer shear wave velocity; u_1 = horizontal displacement; v_1 = vertical displacement.

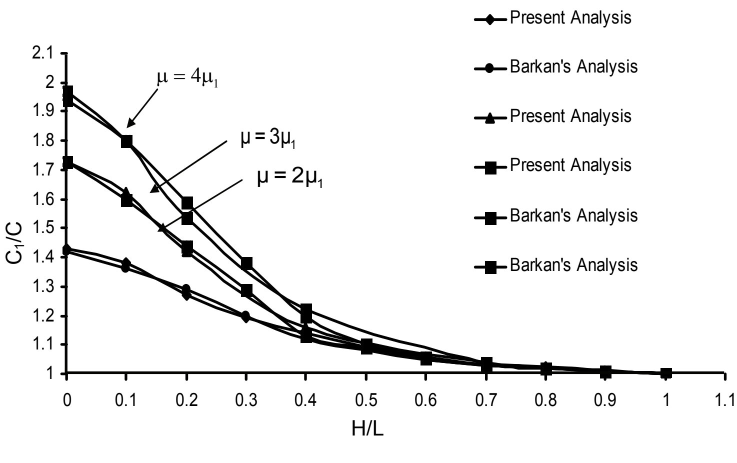

Fig. 6 Effect of the Surface Layer Thickness H to the Wave Length L on the Velocity Ratio C_1/C for Different Lame's Parameters

Figure 7 shows that for all Poisson ratios, horizontal to vertical distance ratio is same within the wave length *L*

It is clear from the Figures 6 and 7 that

> For *H/L* ratio less than 0.5, velocity of wave increases in proportion to the wave length, i.e. inversely proportional to the frequency. For larger *H/L* ratios waves propagate mostly through the upper layer and layering does not have any significance (Figure 6).

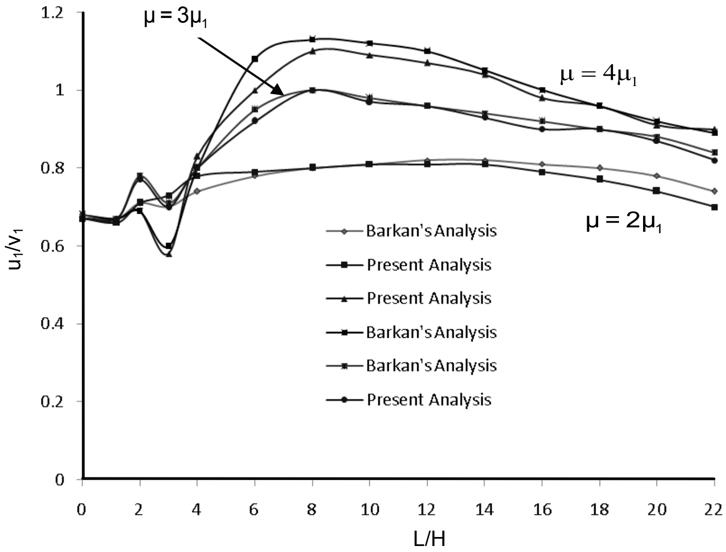

Fig. 7 Effect of the Ratio of the Wave Length to the Upper Layer Thickness on the Ratio of Horizontal to Vertical Displacements on the Surface of the Upper Layer for Different Lame's Parameters

> For $L/H < 1$, the ratio u_1/v_1 does not differ much from the homogeneous half space value of 0.6811. However for values between 2 to 4, the ratio decreases and after 4 to 6 it increases (Figure 7).

Three-Dimensional Problems

Material Properties and Footing Dimensions

Material properties used for present analysis are as follows:

Young's modulus of soil, $E = 4.80 \times 10^7 N / m^2$ to $4.80 \times 10^8 N / m^2$

Young's modulus of pile, E = $4.80 \times 10^8 N / m^2$; Poison's ratio, ν = 1/4, 1/3

Mass density of soil medium, $\rho = 2040 N - sec^2 / m^4$

Mass density of pile, $\rho = 2400 N - sec^2 / m^4$

Amplitude of the harmonic force intensity = $525 N / m^2$

Footing dimensions: Minimum length and width of footing, *2c, 2b* = 3.0m.

Numerical results have been obtained for the vertical compliance functions for rectangular, square and strip footings for uniform layer and different layered soil deposit by considering different Poisson's ratios, and also for surface and embedment conditions.

The problem under study is symmetric about XY and ZY plane, hence, only a quarter of half space bounded by OA, OE and OC has been considered for discretization. The half space is divided into finite number of 8-point hexahedral elements (ZIB-8) as shown in Figure 2 although for this particular problem only rectangular prisms are considered for discretization, any six-sided solid can be incorporated in the analysis.

Effect of Layered Soils

The compliance functions are obtained for different frequency ratios keeping the poison's ratio and length to width ratio of the footing constant for homogeneous soil and layered soil are presented in the Figure 8.

From Figure 8, it can be seen that the real part of the compliance function, f_1 decreases with the increase in the frequency ratio, while the other parameters constant. The imaginary part of the compliance function f_2 increases with the increase in the frequency ratio, while the other parameters constant.

For layered soil when young's modulus is increasing with depth the real part of the compliance function and the imaginary part of the compliance function doesn't vary that much.

The variation of compliance functions with varying top layer and middle layer thickness with constant length to width ratio and Poisson's ratio are compared and plotted in the Figure 9. The dynamic response factor decreases with the increase in the layer thickness and frequency ratio that means the effect of the layer is only up to the layer thickness of *B/2*. The stiffness is more if the top layer thickness is small compared to the middle layer.

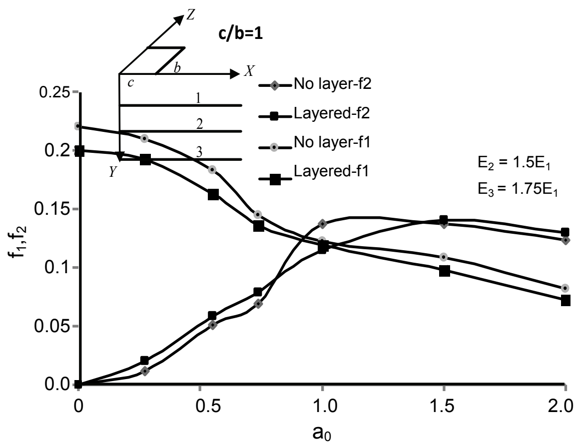

Fig. 8 Compliance Functions, f_1, f_2 versus a_0

The dynamic response of the footing in three layered soil for different mass ratios are presented in Figures10 to 11. It can be observed that with the increase of mass ratio the response factor increases at lower frequency ratio. The influence of mass ratio on the response factor is more for the layer thickness of *B/4*.

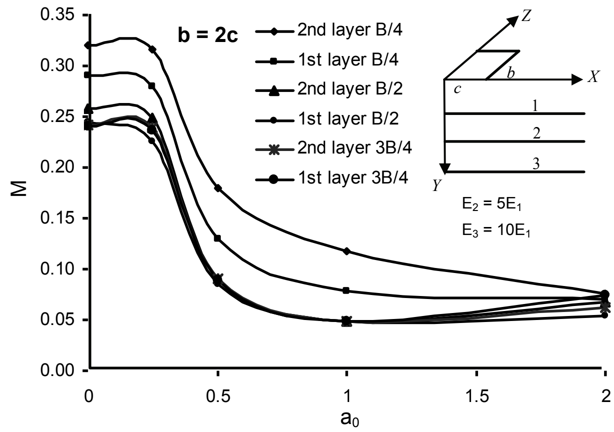

Fig. 9 Response factor M Vs a_0 for Different Top Layer and Middle LayerThickness

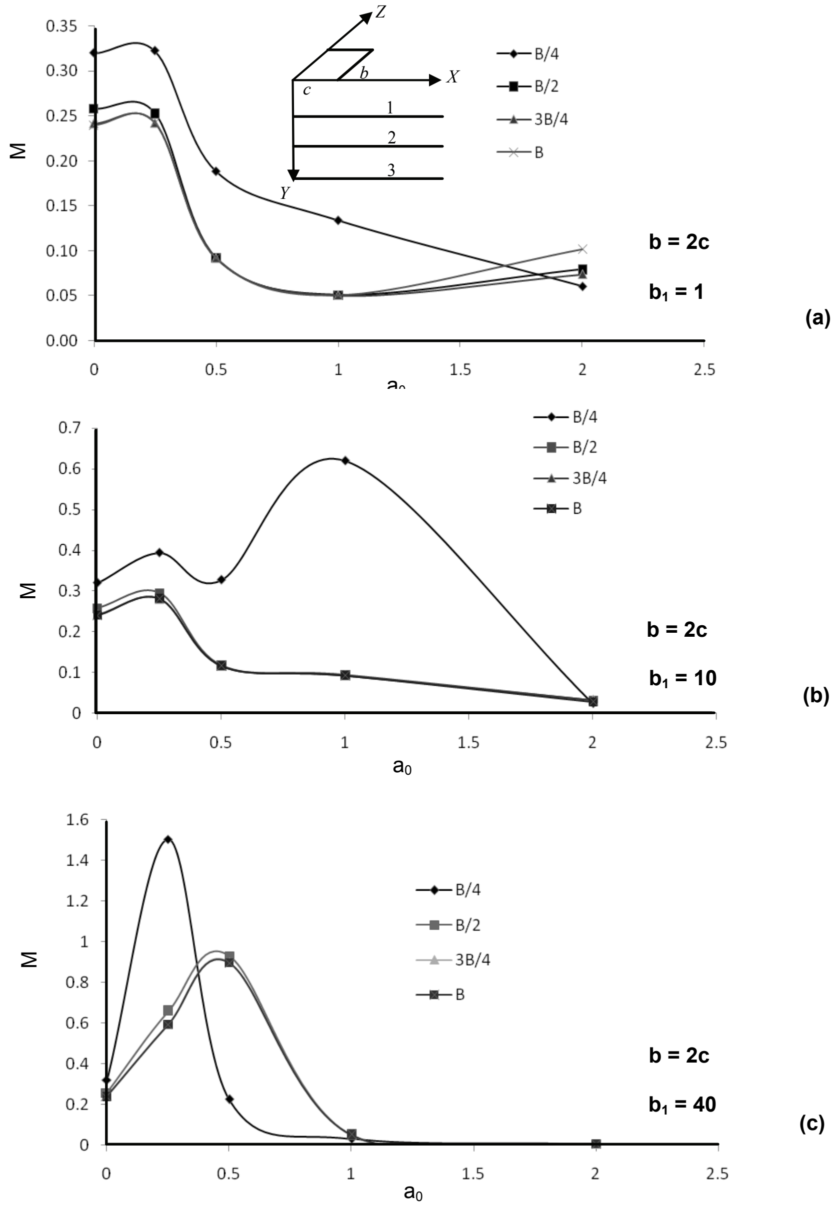

Fig. 10 Response Factor M Vs a_0 for Different Top Layer Thickness

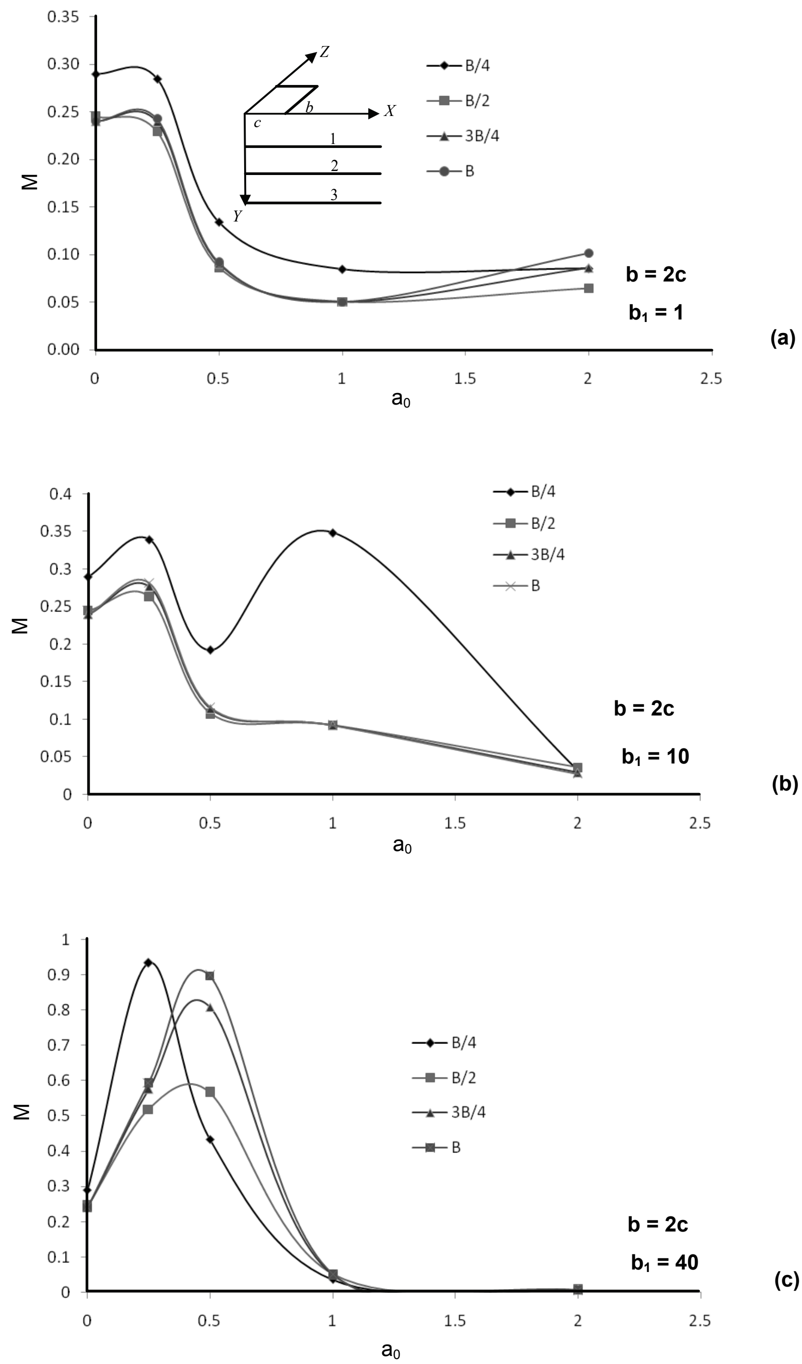

Fig. 11 Response Factor M Vs a_0 for Different Middle Layer Thickness

Length to width Ratio Effect

The compliance functions are obtained for different frequency ratios and different length to width ratios keeping the poison's ratio of the footing constant for homogeneous soil are presented in the Figure 12.

From Figure12 it can be observed that with the increase in the length to width ratio at lower frequency ratios keeping the Poisson's ratio constant both f_1 and f_2 are increasing and f_2 is increasing at higher frequency ratio.

Embedment Effect

The compliance functions are obtained for different frequency ratios and different embedment depths keeping the poison's ratio and the length to width ratio of the footing constant for homogeneous soil. From the Figure 13 we can observe that with the increase in the embedment depth f_1 reduces and f_2 also reduces at higher a frequency ratio that means the stiffness increases with the embedment depth.

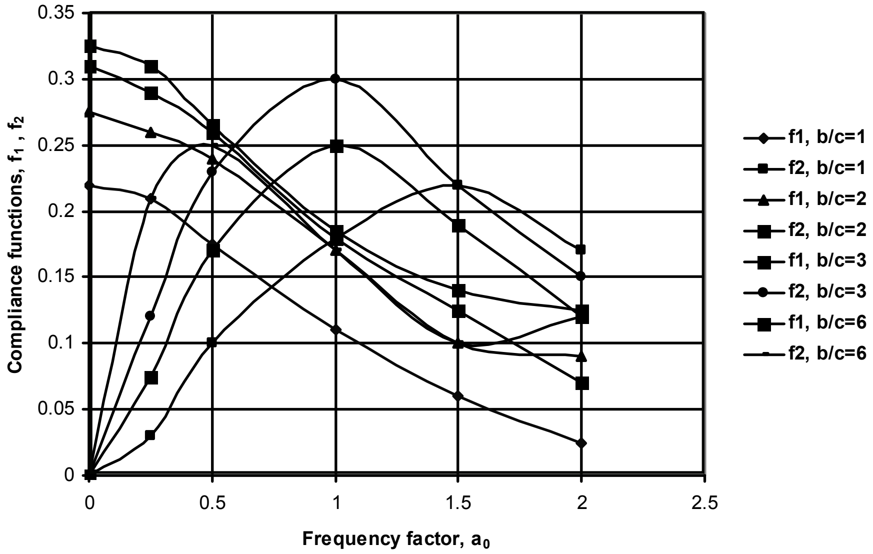

Fig. 12 Compliance function f_1,f_2 Vs a_0 for different b/c ratios, ν = 0.33

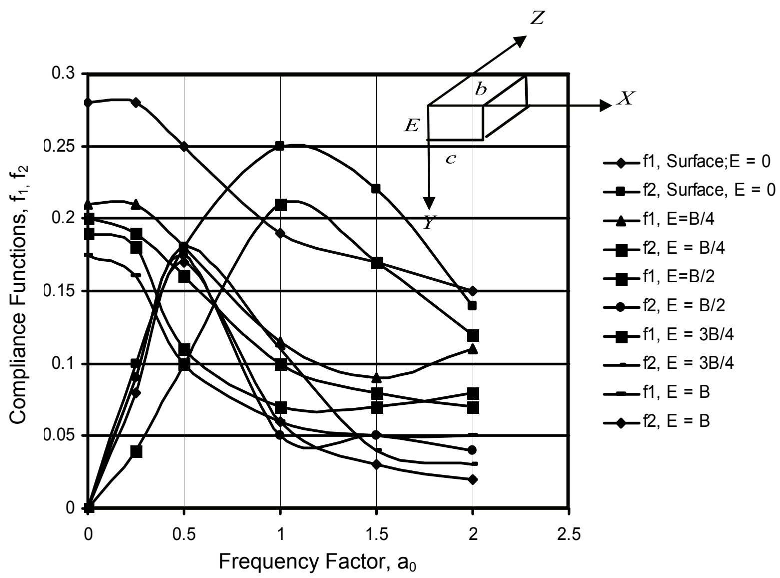

Fig. 13 Compliance Functions f1, f2 Vs a_0 for Different Embedment Depths, b/c = 2

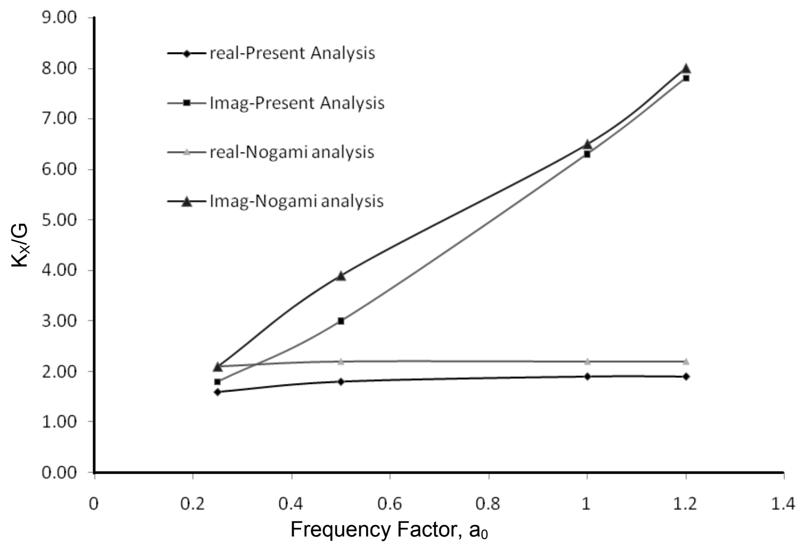

Fig. 14 Side Soil Stiffness for Structures with Square Cross Sections

Soil Stiffness at the Side of Embedded Structure

The side soil stiffness per depth are computed for the structure with square cross section and the computed values are compared with those previously computed (Nogami and Chen 2003) in Figure 14, they are nearly matching with the results.

Conclusions

The following conclusions are drawn from the present analysis:

> For layered soil when the lower layers stiffer than the upper layer the real part of the compliance function is almost same at the frequency ratio 1 and the imaginary part of the compliance function varying much at the frequency ratio 1.

> The compliance functions are increasing with the increase in length to width to ratio and frequency ratio but the imaginary part of the compliance function is decreasing with the increase in length to width ratio at the higher frequency ratio.

> The compliance functions are decreasing with the increase in the embedment depth and frequency ratio, it is indicated that the stiffness is increasing with the embedment depth. There is no effect of embedment if the embedment is more than the depth of *2B*.

> The influence of top layer and middle layer thickness with the increase in frequency ratio are obtained in the form of dynamic response factor ψ, these results indicating that the layering effect is only up to the layer thickness of *B/2* and the stiffness is more for the lower top layer thickness compared to middle layer thickness.

References

Anandakrishnan, M., and Krishnaswamy, N. R. (1973). 'Response of embedded footings to the vertical vibrations', *Journal of Soil Mechanics and Foundation Division*, ASCE, 99(10), pp 863-883.

Barkan, D. D. (1962). *Dynamics of bases and foundations*, McGraw Hill Book Co. New York.

Baidya, D. K, Muralikrishna, G, and Pradhan, P. K. (2006). 'Investigation of foundation vibrations resting on layered soil system', *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, 132(1), 116-123.

Baidya, D.K. and Sridharan, A. (1994): 'Stiffness of the foundations embedded into elastic stratum', *Indian Geotechnical Journal*, 24(4), pp 353-367.

Baidya,D.K. and Muralikrishna, G. (2001): 'Investigation of resonant frequency and amplitude of vibrating footing resting on a layered soil system', *Geotech. Testing J.*, ASTM, 24(4), pp 409-417.

Deeks, A.J. and Augarde, C.E. (2004): 'Stresses around a square tunnel using a meshless scaled boundary method', *ACME conference*, Cardiff.

Deeks, A. J. and Wolf, J. P. (2002): 'A virtual work derivation of the scaled boundary finite-element method for elastostatics', *Comput. Mech.* 28, pp 489-504.

Dobry R and Gazetas G (1986): 'Dynamic response of arbitrary shaped foundations', *Journal of Geotechnical Engineering*, ASCE, 112(2), pp109-135.

Dobry R., Gazetas G and Tassoulas J. L. (1987). 'Horizontal stiffness of arbitrarily shaped embedded foundations', *Journal of Geotechnical Engineering*, ASCE, 113(5), pp 450-457.

Gazetas G. (1982): 'Analysis of machine foundation vibrations: State-of-the-Art", *International Journal of Soil Dynamics and Earthquake Engineering*, Vol. 2(1), 2-43.

Kagawa, T. and Kraft, L. (1981): 'Lateral pile response during earthquakes', *J. Geotech. Eng.* ,ASCE, 107(12), pp.1713-1731.

Lin, H.T. and Tassoulas, J.L. (1986): 'A hybrid method for three-dimensional problems of dynamics of foundations', *Earthquake Engineering and Structural Dynamics*, 4, pp61-74.

Luco, J.E., (1982): 'Soil-structure interaction: a review', *Winter Annual Meeting*, 53, ASCE, New York.

Lysmer, J. (1978): 'Analytical procedures in soil dynamics', *Report No. UCB/EERC-78/29*, University of. California at Berkeley, Earthquake Engineering Research Centre, Richmond, CA.

Meek, J.W. and Wolf, J.P (1992a): 'Cone models for homogeneous soil', *J. Geotech. Engng*, ASCE,118(5), pp 667-685.

Meek, J.W. and Wolf, J.P (1992b): "Cone model for soil layer on rigid block", *J. Geotech. Engng*, ASCE, 118(5), pp 686-703.

Meek, J.W. and Wolf, J.P. (1994): 'Cone models for an embedded foundation', *J. Geotech. Engng*, ASCE, 120(1), pp 60-80.

Nogami, T. and Lam Y. (1987): 'A two-parameter layer model for analysis of slab on elastic foundation', *J. Engng. Mech*, ASCE, 113(9), pp1279–1291.

Nogami, T. and Leung, M.B. (1990): 'A simplified mechanical subgrade model for dynamic response', *Int. J. Earthquake Engng. Struct Dyn,* 19, pp 1041–1055.

Nogami, T. and Chen, H. S. (2003): 'Dynamic soil stiffness at side of embedded structures with rectangular base', *Journal of Engineering Machanics*, ASCE, 129(8), pp 963-975.

Pradhan, P. K, Baidya D. K, and Ghosh D. P. (2003):. "Impedance functions of circular foundation resting on layered soil using cone model", *The Electronic journal of Geotechnical Engineering*,Vol-8.

Roesset, J. (1980): 'Stiffness and damping coefficients of foundations, Dynamic Response of Pile Foundations: Analytical Aspects', Ed. M.O'Neill and R. Dobry, ASCE, 1-30.

Sridharan, A., Gandhi, N.S.V.V.S.J. and Suresh, S. (1990): 'Stiffness coefficients of layered soil system', *J. Geotech. Engng*., ASCE,116(4), pp 604-624.

Wolf, J.P. and Meek, J.W. (1993): 'Cone models for soil layer on flexible rock half-space', *Earthquake Engng. Struct Dyn*, 22, pp185-193.