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Effect of Non-Linear Consolidation for Radial Flow on Pore Pressure Dissipation

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ABSTRACT: Installation of vertical drains with surcharge loads is one of the widely used methods of soft ground improvement. The drains accelerate the rate of consolidation by shortening the drainage path considerably. The conventional radial consolidation theory used often to predict the behavior of the vertical drains in soft clays is based on linear void ratio-effective stress relationship. A theory for consolidation with radial flow is developed considering the linear $e - \log\sigma'$ relationship but assuming the coefficient of consolidation to be constant (Davis and Raymond 1965) and the results compared with the conventional radial consolidation theory proposed by Barron. The degrees of settlement predicted by both linear and non-linear theories are identical. However the rate of dissipation of pore pressure is shown to be dependent on the stress increment ratio.

KEYWORDS: Ground improvement, Consolidation, Non-linear theory, Vertical drain, Finite difference method, Pore pressure

Introduction

Preloading with a system of vertical drains, is one of the widely used methods of soft ground improvement. The vertical drains accelerate the rate of consolidation by shortening the drainage path considerably. This ground improvement technique increases the shear strength and reduces compressibility of soft ground due to acceleration of consolidation of the soil. The vertical drains were initially composed of sand columns. With the advent of prefabricated vertical drains (PVDs) and their relative ease in installation into the ground, ground improvement with PVDs has gained popularity world wide.

Barron (1948) presented a comprehensive analytical solution to the problem of radial consolidation by drain wells. In this classical theory, the formulation is based on small strains and constant coefficients of volume compressibility (m_v) and horizontal permeability (k_h). However, for a relatively large applied stress both the coefficients of volume compressibility and permeability of the soil would decrease during the consolidation process. To overcome the limitations involved in the conventional consolidation theory, several attempts have been made by researchers to obtain a more realistic theory of consolidation.

Richart (1957) reviewed the theories for consolidation due to vertical flow and radial flow of water to a drain well. He found that including void ratio as a variable did not significantly change the consolidation – time characteristics of consolidation by vertical flow. Davis & Raymond (1965) developed a theory of non-linear consolidation for vertical flow considering $e - \log\sigma'$ relationship but assuming coefficient of consolidation, c_v, to be constant during consolidation. Basak and Madhav (1978) analyzed the problem of sand drain consolidation incorporating the variation of compressibility and permeability. Hansbo (1979, 1981) presented a simple and equally accurate solution for radial consolidation with band shaped vertical drains by transforming the drain into an equivalent circular one. Vaid (1985) presented a solution for vertical non-linear consolidation under constant rate of loading and introduced a new dimensionless parameter to influence the magnitude of difference between linear and non-linear results. Lekha et al. (1998) presented a non-linear theory for sand drain consolidation under time dependent loading for equal strain case. Almeida et al. (2000) analyzed the behavior of a very soft organic clay with PVDs and concluded that the effect of secondary consolidation is negligible compared to that of primary consolidation. Full-scale test conducted by Bergado et al (2002) on soft Bangkok clay with PVDs revealed that the degree of consolidation obtained from pore pressure measurements is lower than the corresponding values obtained from settlement measurements. Teh and Nie (2002) investigated the influence of non-Darcian flow on radial consolidation and found that it doesn't significantly change the pattern of deformation and excess pore pressure distribution. The various modeling aspects of PVDs are comprehensively discussed by Indraratna et al. (2003) along with the evaluation of their effectiveness in practice. Indraratna et al. (2005 a) developed a theory for consolidation with radial flow using $e - \log\sigma'$ (C_c and C_r) & e – log k_h (C_k) relationships and the loading increment ratio $(\Delta\sigma' / \sigma_i')$. Zhuang et al. (2005) presented a semi-analytical solution for one dimensional consolidation of clays with variable compressibility and permeability. The ratio C_c/C_k (the slopes of $e - \log\sigma'$ and $e\text{-}\log k$ respectively) determine the need to consider the effect of nonlinear property. Indraratna et al. (2005 b) proposed a modified consolidation theory for vertical drains incorporating vacuum preloading for both axi-symmetric and plane

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strain conditions. Rujikiatkamjorn and Indraratna (2006) developed 3D FEM analysis of soft soil consolidation improved by PVDs by considering the variation of horizontal permeability along the radial direction and the actual cross sectional shape of band drains. Sathananthan and Indraratna (2006) formulated equivalent plane-strain consolidation equations for radial consolidation in soft soils with PVDs and found good agreement between equivalent plane-strain and axi-symmetric solutions. A critical review of analytical solutions and numerical analysis of soft clay stabilization with PVDs beneath road and railway embankments is made by Indraratna et al. (2007). Inflection point method for predicting settlement of PVD improved soft clay is presented by Sinha et al. (2007). Two- and three-dimensional multi-drain finite-element analyses of a case study of a combined vacuum and surcharge preloading with vertical drains is presented by Rujikiatkamjorn et al. (2008). The numerical predictions compare well with field observed data. A new technique is developed by Indraratna et al. (2008) to model consolidation by vertical drains beneath a circular loaded area by transforming the system of vertical drains into a series of concentric cylindrical drain wall. A simple analytical solution for radial consolidation under time dependent loading is given by Conte and Troncone (2009). A spectral method is presented for analysis of vertical and radial consolidation in multilayered soil with PVDs assuming constant soil properties within each layer by Walker et al. (2009).

In the present paper, a theory of non-linear consolidation for radial flow around a vertical drain is developed considering the coefficient of consolidation to be constant but based on the non- linear theory of consolidation presented by Davis & Raymond (1965).

A typical triangular pattern of drain installation is shown in Figure1. The zone of influence is approximated by circle of equivalent diameter, d_e (=C.S where C=1.05 and 1.13 for triangular and square patterns of arrangements respectively and S - the spacing of the drains). For strip drains, the equivalent diameter, d_w is equal to $2(a+b)/\pi$ where a and b are the width and the thickness of the PVD. Thus, the unit cell constitutes the drain and the surrounding zone of influence and consolidates due to radial flow of water. A theory for consolidation with vertical flow based on non-linear theory had been presented by Davis and Raymond (1965). Herein, the corresponding equation for consolidation with radial flow is derived incorporating the linear e - log σ′ relationship.

Formulation

For normally consolidated soils, the void ratio, e, is related to effective vertical stress, σ′, as

$$e = e_o - C_c \log_{10} \frac{\sigma'}{\sigma'_0} \tag{1}$$

where e_o is the initial void ratio corresponding to stress, σ_o', and C_c is compression index of the soil. Differentiating void ratio, e, with respect to effective pressure (σ′), the coefficient of volume change, m_v, (Lambe and Whitman 1969) is

$$m_v = \frac{0.434.\ C_c}{(1+e)\ \sigma'} \tag{2}$$

During consolidation process (1+e) varies with time far less than the effective pressure σ′. So (1+e) may be considered constant for any load increment. With this assumption equation (2) becomes

$$m_v = \frac{A}{\sigma'} \tag{3}$$

$$\text{where A is a constant} = \frac{0.434\ C_c}{(1+e)} \tag{4}$$

For normally consolidated soils, the coefficient of consolidation, $c_r = k_h / (m_v.\gamma_w)$, due to radial flow varies much less than the m_v and may be taken relatively constant, where k_h is permeability of the soil in the horizontal direction and γ_w the unit weight of water. This is equivalent to assuming that the rate of decrease in permeability is proportional to the rate of decrease of compressibility.

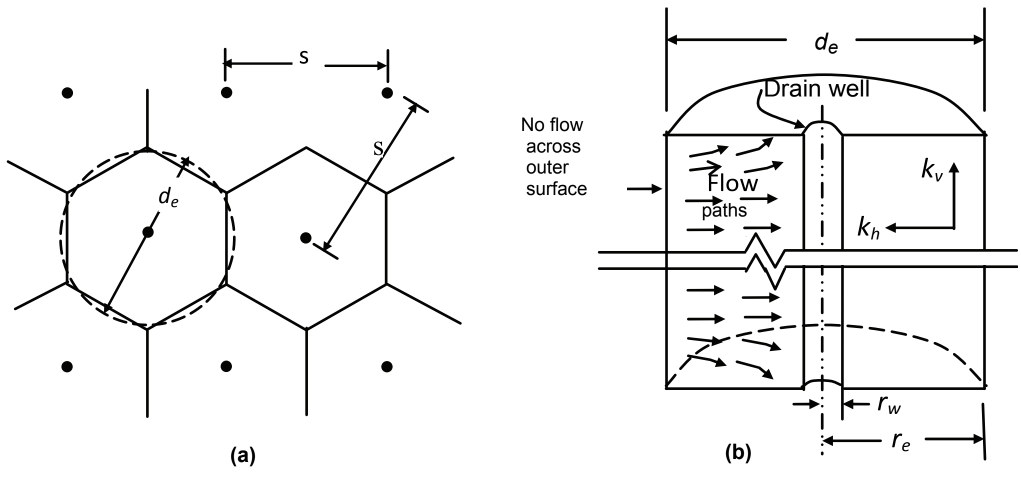

Fig. 1 (a) Triangular Arrangement of PVDs and (b) Flow in Unit Cell

Considering an element of soil, (Figure 2) and using Darcy's law, the expression for continuity of flow in radial direction is

$$\frac{1}{V.}\frac{\partial q_r}{\partial r}.dr = -\left[\frac{\partial^2 u}{\partial r^2}\frac{1}{\sigma'} + \left[\frac{\partial u}{\partial r}\right]^2 \frac{1}{(\sigma')^2} + \frac{1}{\sigma'}\frac{1}{r}\frac{\partial u}{\partial r}\right] c_r.A \quad (5)$$

where V is the volume of the element, q_r the flow rate in radial direction, r the radial distance and u the excess pore water pressure. Neglecting secondary compression or creep the volumetric strain developed within the soil element is

$$f = \frac{e_0 - e}{1+ e_0}$$

$$\text{or } f = \frac{C_c}{(1+e_0)}.\log_{10}\frac{\sigma'}{\sigma'_0} \quad (6)$$

Differentiating f with respect to time, t,

$$\frac{\partial f}{\partial t} = \frac{C_c}{(1+e_0)}\frac{0.434}{\sigma'}\frac{\partial \sigma'}{\partial t} \quad (7)$$

Assuming (1 + e_0) is equal to (1 + e)

$$\frac{\partial f}{\partial t} = \frac{A}{\sigma'}\frac{\partial \sigma'}{\partial t} \quad (8)$$

Equating the rate of water lost per unit volume to the rate of volume decrease per unit volume both within a small element of soil

$$\frac{1}{V}.\frac{\partial q_r}{\partial r}dr = \frac{\partial f}{\partial t} \quad (9)$$

Substituting equations (5) and (8) gives:

$$-\left[\frac{\partial^2 u}{\partial r^2}\frac{1}{\sigma'} + \frac{1}{(\sigma')^2}\left[\frac{\partial u}{\partial r}\right]^2 + \frac{1}{\sigma'}\frac{1}{r}\frac{\partial u}{\partial r}\right] c_r = \frac{1}{\sigma'}\frac{\partial \sigma'}{\partial t} \quad (10)$$

Equation (10) is the general equation of non-linear consolidation with radial flow.

$$\text{Let} \quad w = \log_{10}\frac{\sigma'}{\sigma'_f} = \log_{10}\frac{(\sigma'_f - u)}{\sigma'_f} \quad (11)$$

where σ'_f is the final effective stress.

Differentiating equation (11) with respect to radial distance, r,

$$\frac{\partial w}{\partial r} = -\frac{0.434}{\sigma'}.\frac{\partial u}{\partial r} \quad (12)$$

$$\frac{\partial^2 w}{\partial r^2} = -0.434\left[\frac{1}{\sigma'}\frac{\partial^2 u}{\partial r^2} + \frac{1}{(\sigma')^2}\left(\frac{\partial u}{\partial r}\right)^2\right] \quad (13)$$

Differentiating equation (11) with respect to t,

$$\frac{\partial w}{\partial t} = \frac{0.434}{\sigma'}.\frac{\partial \sigma'}{\partial t} \quad (14)$$

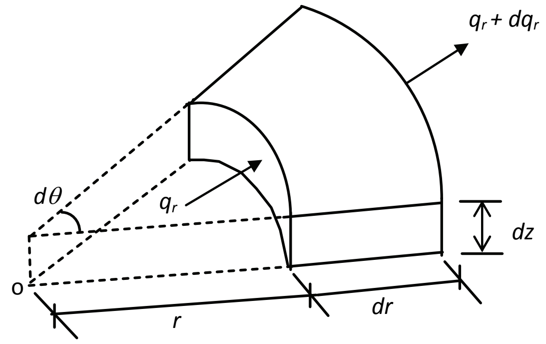

Fig. 2 Flow through Soil Element

Substituting Eqs. (12), (13) & (14) into Eqn. (10) leads to a simple differential equation in terms of the function w as

$$\frac{\partial w}{\partial t} = c_r\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) \quad (15)$$

This equation is identical in form to that of the conventional Barron's radial consolidation theory and can be solved in the same way with the boundary conditions that are the same in terms of u and w.

Initial and Boundary Conditions

For $t=0$ and $r_w \le r \le r_e$; $u=(\sigma'_f - \sigma'_i)$ or $w=\log_{10}\frac{\sigma'_i}{\sigma'_f}$

For $t>0$ and $r=r_w$; $u=0$ or $w=0$

For $t>0$ and $r=r_e$; $\frac{\partial u}{\partial r}=0$ or $\frac{\partial w}{\partial r}=0$

where σ_i' is the initial effective stress and $\sigma'_f = (\sigma'_i + \Delta\sigma')$, where $\Delta\sigma'$ is the increment in applied stress.

Equation (15) is re-written in non-dimensional form as:

$$\frac{\partial w}{\partial \bar{t}} = \left(\frac{\partial^2 w}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial w}{\partial \bar{r}}\right) \quad (16)$$

where $\bar{r}$ and $\bar{t}$ are the normalized parameters $\bar{r} = \frac{r}{r_R}$ and $\bar{t} = \frac{t}{t_R}$

where r_R is the reference distance and t_R reference time.

The reference parameters are taken as

$r_R = r_w$ and $t_R = \frac{r_R^2}{c_r}$.

Eqn. (16) is solved numerically using the finite difference approach, discretizing the unit cell radially into

100 elements of equal thickness, for the corresponding boundary and initial conditions for various r_e/r_w (=n) and σ'_f/σ'_i values. A trial analysis discretizing the unit cell into 50, 80,100 numbers of units for n of 40 gave identical results. However, in the present numerical analysis, the unit cell is discretized into 100 elements for all the n values and results obtained.

Results and Discussion

Results, in the form of degree of settlements, U_s, normalized excess pore water pressures, U_p (= u/u_i), and normalized maximum excess pore water pressures, U_{max} (= u_{max}/u_i) which occurs at a radial distance, r_e, are obtained for σ'_f/σ'_i of 1.5, 2, 4, 8 and 16 and for n values of 5, 10, 15, 25 and 40 where, u_i and u_{max} are the initial and maximum excess pore water pressures respectively. The present results are compared with those of Barron's free strain theory (Linear Theory).

The variation of degree of settlement, U_s, with respect to time factor, $T_h (= c_r.t/d_e^2)$ for different n values for σ'_f/σ'_i of 2 (conventional oedometer test) is presented in Figure 3 along with the results of linear theory. The degree of settlement computed by the present non-linear theory agrees exactly with the degree of settlements predicted by Barron's free strain theory since the governing equations for settlement for both linear and non-linear theories are the same. An identical result is obtained for consolidation with vertical flow (Davis and Raymond 1965).

A new term, the degree of dissipation of maximum excess pore pressure is defined as equal to (1-U_{max}) 100. The variation of the normalized maximum pore pressure, U_{max} i.e. at $r=r_e$, with T_h is presented in Figure 4 for various n values and for σ'_f/σ'_i of 2. The maximum pore pressure is minimum (or the degree of dissipation is maximum) for the closest spacing of drains (n=5) and the dissipation decreases, as expected, with the increase of n. Points farther from the centre naturally need larger times for dissipation of excess pore pressure. U_{max} decreases to 10% at times of 0.342 and 1, about three-fold increase, for n value increasing from 5 to 40.

The variations of degree of settlement and normalized maximum excess pore pressure, with time factor for various σ'_f/σ'_i and for n=10 are shown in Figure 5 along with the results of linear theory.

While the degree of settlement is independent of the ratio, σ'_f/σ'_i and is equal to the degree of settlement predicted by Barron's free strain theory, the rate of dissipation of maximum pore pressure, U_{max}, is sensitive to the ratio σ'_f/σ'_i. The rate of dissipation decreases with increase of σ'_f/σ'_i. For example, at 90% degree of settlement, i.e. at T_h equal to 0.46, the degree of dissipation of maximum pore pressure decreases from about 85% for stress increment ratio of 0.5 (σ'_f/σ'_i =1.5) to about 70% for stress increment ratio of 15 (σ'_f/σ'_i =16). The most significant aspect of

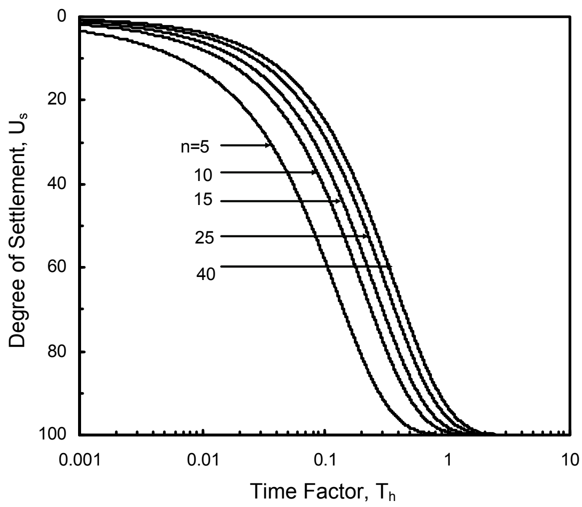

Fig. 3 Variation of Degree of Settlement with Time Factor for both Linear and Non Linear Theories

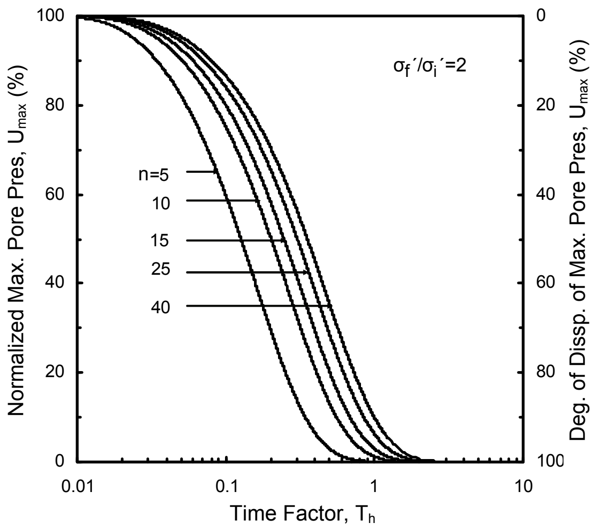

Fig. 4 U_{max} - T_h Relations -- Effect of n

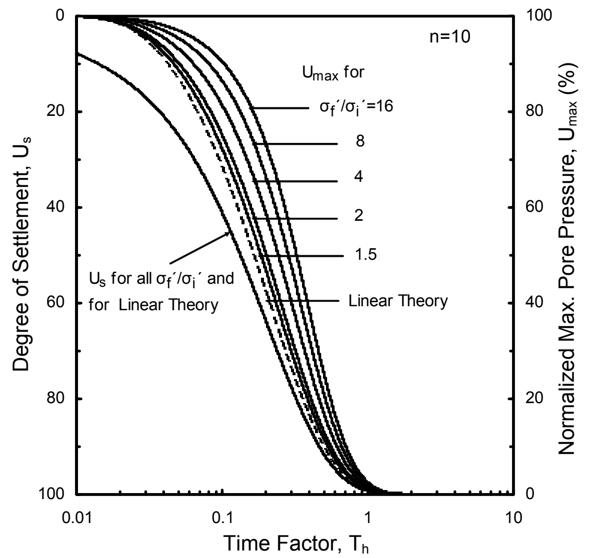

Fig. 5 U_s and U_{max} - T_h Relations – Effect of σ'_f/σ'_i

this result is that for large stress increments with respect to the initial effective stress, the residual pore pressure at the farthest end of the unit cell would be large. The degree of settlement may suggest a significant value, but the pore pressure dissipations could be at variance from this value especially for large stress increments. The results are similar for all values of *n.*

The variations of pore pressures at various radial distances from the centre of the drain are shown in Figure 6 and Figure 7 along with the results of linear theory for n =15 and $\sigma'_f / \sigma'_i = 2$. The pore pressures predicted by non-linear theory at different radial distances are higher compared to those predicted by linear theory. The pore pressure closer to the drain gets dissipated rather quickly and consequently effective stresses at these points increase sharply compared to pore pressures at points farther from the drain. The coefficient of permeability decreases with increase of effective stress and this variation of permeability with effective stress is considered in the present non-linear analysis (The rate of decrease in permeability is assumed to be proportional to the rate of decrease of compressibility which in turn depends on effective stress). Thus, due to the decreased permeability nearer to the drain, dissipation of pore pressure gets retarded in the non-linear theory and hence the difference in the pore pressures from linear and non-linear theories is more at points closer to the drain compared to those at farther points during the initial stages ($T_h \leq 0.10$). With increasing time, points farther from the drain consolidate with a consequent reduction in permeability. Hence the effect of non- linear consolidation moves farther from the drain at larger times.

The implication of the above theory is that preloading of normally consolidated soils treated with PVDs is monitored by measuring both the settlements and the pore pressures. While the settlements would indicate a value which is independent of the stress increment ratio, the rate of dissipation of pore pressure would be at variance with this value and could be significantly smaller for higher stress increment ratios. The residual pore pressures could be much larger for larger stress increment ratios, i.e. at shallow depths where the initial effective stress is very low indeed. This may explain some of the anomalous failures of embankments built on soft clay.

Design Example

A soft clay layer of 8 m thickness (Figure 8) is to be preloaded using PVDs and surcharge loading. The water table at the site is at ground level. The characteristics of the clay layer are: saturated unit weight, γ_{sat} =16 kN/m^3, coefficient of radial consolidation, c_r = 3 m^2/year. PVDs, 100 mm wide and 4 mm thick, are proposed to be used in triangular pattern at a spacing of 1.20 m. If a surcharge intensity of 50 kPa is applied on the clay, the variations of residual excess pore pressures along the radial distance from the centre of the drain

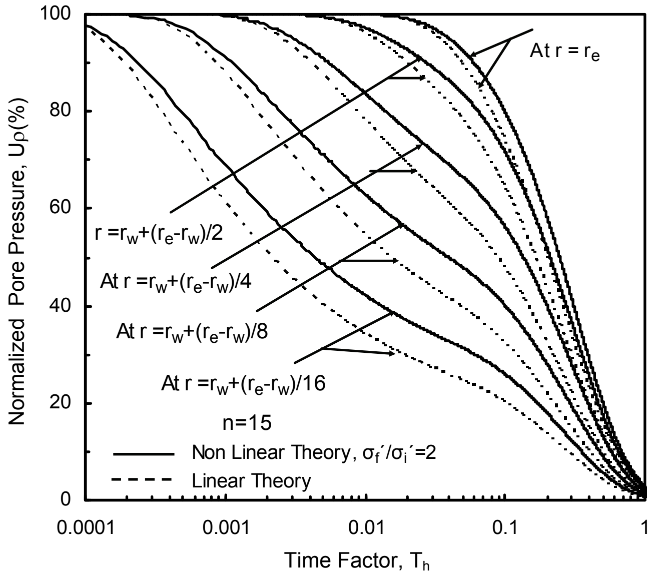

Fig. 6 U_p - T_h Relations --Effect of Radial Distance

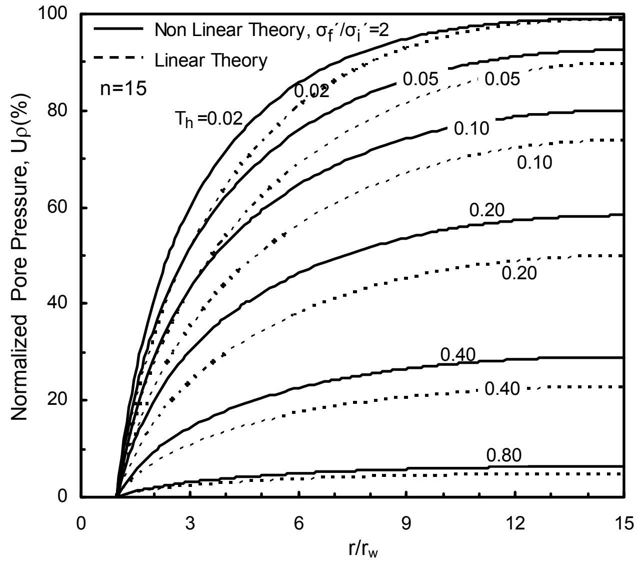

Fig. 7 Variation of Pore Pressures with Radial Distance

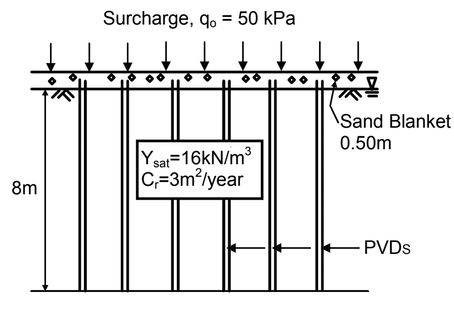

Fig. 8 Definition Sketch for Design Example

to the boundary of influence zone at times of 1, 2, 3 and 6 months are predicted based on the proposed theory and compared with those based on linear theory.

Solution

The equivalent diameter of band drain, d_w =2(a+b)/π=66.21 mm, effective drain spacing or the equivalent diameter of the influence zone, d_e =1.05x 1.20=1.26 m.

The drain spacing ratio, n= d_e/d_w=19. The residual excess pore pressure depends on the stress ratio, σ'_f / σ'_i in the proposed non-linear theory. The initial effective stress, σ'_i at the centre of the clay deposit = 0.5H.γ′ = 24.80 kPa. For an applied load intensity, q_o = 50 kPa the final effective stress, σ'_f = (σ'_i +Δσ′) = (σ'_i +q_o) = 74.8 kPa and the stress ratio, σ'_f / σ'_i =3. The initial input parameter, $w = \log_{10}(\sigma'_i / \sigma'_f) = -0.48$, to be used in the non-linear theory for the given data.

The time factors are calculated for different durations (t) based on the equation, $T_h = (c_r.t)/d_e^2$. Using the finite difference method the residual excess pore pressures are obtained at different radial distances from the centre of the drain to the boundary of influence zone at times of 1, 2, 3 and 6 months, based on the proposed non-linear theory and the conventional linear theory (Barron's free strain theory) and presented in Figure 9. After 1 month (T_h =0.1575), the maximum residual excess pore pressures, u_{max} which occurs at the boundary of influence zone, are 37.2 kPa and 31.1 kPa respectively as per non-linear and linear theories, for a degree of settlement of 44.2% in both the theories. Thus, the residual excess pore pressures are relatively high or the degree of dissipation of excess pore pressures small as per the proposed non-linear theory compared to those based on conventional linear theory.

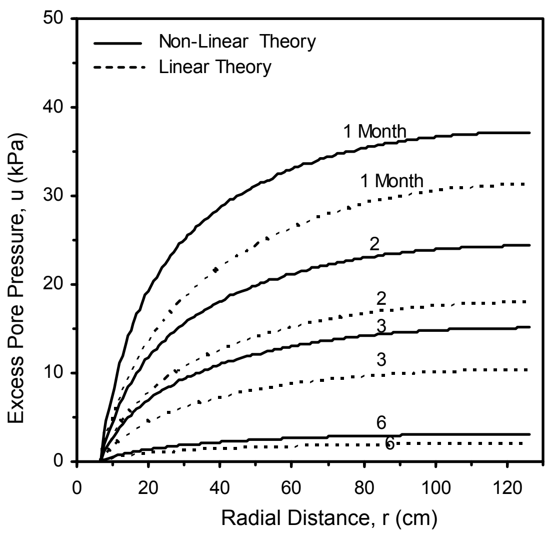

Fig. 9 Comparison of Excess Pore Pressures Predicted by Linear and Non-linear Theories

Summary and Conclusions

A non-linear theory of consolidation for radial flow is developed for a normally consolidated soil considering $e - \log\sigma'$ relation but assuming c_r to be constant. The theory is based on the non-linear theory of vertical consolidation developed by Davis and Raymond (1965). The governing differential equation of consolidation is similar in form to Barron's equation of radial consolidation of linear theory. The governing equation is solved by the finite difference approach and results obtained in the form of variations of degrees of consolidation and maximum pore pressure with time factor. The results are compared with the solutions presented by Barron for free strain case.

The degree of settlement is independent of the ratio of final to initial effective stresses, σ'_f / σ'_i , and is identical to that from the linear theory. However, the rate of dissipation of excess pore pressures is strongly dependent on the stress increment ratio. The rate of dissipation of excess pore pressures is overestimated by Barron's linear theory. For large ratios of final to initial effective pressures, the pore pressures are considerably larger than those predicted by Barron's theory at all times.

Notation

A	$\frac{0.434\ C_c}{(1+e)}$
a	width of the prefabricated vertical drain
b	thickness of the prefabricated vertical drain
C_c	compression index
C_k	permeability index
C_r	recompression index
c_r, c_v	coefficients of consolidation for radial flow and vertical flow
d_e	equivalent diameter of zone of influence of vertical drain
d_w	equivalent diameter of prefabricated vertical drain
e	void ratio
e_o	initial void ratio for effective pressure σ'_0
f	volumetric strain
H	thickness of clay deposit
k_h	coefficient of permeability in horizontal direction
m_v	coefficient of volume change
n	ratio of r_e to r_w
q_o	surcharge load intensity
q_r	flow rate in radial direction
r	radial distance from centre of the drain
r_R	reference radial distance

r_e equivalent radius of zone of influence of vertical drain

r_w equivalent radius of prefabricated vertical drain

$\bar{r}$ normalized radial distance, $\frac{r}{r_R}$

S spacing of vertical drains

T_h time factor, $\frac{c_r.t}{d_e^2}$

t time of consolidation

t_R reference time

$\bar{t}$ normalized time, $\frac{t}{t_R}$

U_p normalized excess pore water pressure, u/u_i

U_s degree of settlement

U_{max} normalized maximum excess pore water pressure, u_{max}/u_i

u excess pore water pressure

u_i initial excess pore water pressure

u_{max} maximum excess pore water pressure

V volume of soil element

w $\log_{10}\frac{\sigma'}{\sigma'_f}$

γ' submerged unit weight of clay deposit

γ_w unit weight of water

σ' effective stress

σ'_i initial effective stress

σ'_f final effective stress

$\Delta\sigma'$ increment in applied stress

References

Almeida, M.S.S., Santa Maria, P.E.L., Martins, I.S.M., Spotti, A.P. and Coelho, L.B.M. (2000): 'Consolidation of very soft clay with vertical drains', *Geotechnique,* 50 (6), pp. 633–643.

Barron, R.A. (1948): 'Consolidation of fine grained soils by drain wells', *Transactions of* ASCE, 113, pp. 718 – 754.

Basak, P. and Madhav, M.R. (1978): 'Analytical solution of sand drained problems', *Journal of Geotechnical Engineering*, ASCE, 104, pp. 129 – 135.

Bergado, D.T., Balasubramaniam, A.S., Fannin, R.J. and Holtz, R.D. (2002): 'Prefabricated vertical drains (PVDs) in soft Bangkok clay: a case study of the new Bangkok International Airport project', *Can.Geotech*, ***J*.,** 39, pp. 304-315.

Conte, E. and Troncone, A. (2009): 'Radial consolidation with vertical drains and general time-dependent loading', *Can. Geotech. J.*, 46, pp. 25-36.

Das, B.M. (1983): *Advanced Soil Mechanics*, Hemisphere Publishing Corporation, pp. 253 – 331.

Davis, E.H. and Raymond, G. P. (1965): 'A non – linear theory of consolidation', *Geotechnique*, 15, pp161–173.

Hansbo, S. (1979): 'Consolidation of clay by band shaped prefabricated drains', *Ground Engineering*, 12 (5), pp.16-25.

Hansbo, S. (1981): 'Consolidation of fine grained soils by prefabricated drains', *Proceedings of the 10th International Conference on Soil Mechanics and Foundation Engineering,* Stockholm, Sweden, June, 3, pp. 667-682.

Indraratna, B., Aljorany, A. and Rujikiatkamjorn, C. (2008): 'Analytical and numerical modeling of consolidation by vertical drain beneath a circular embankment', *International Journal of Geomechanics,* 8 (3), pp. 119–206.

Indraratna, B., Bamunawita, C., Redana, I.W and McIntosh, G. (2003): 'Modeling of prefabricated vertical drains in soft clay and evaluation of their effectiveness in practice', *Ground Improvement,* 7 (3), pp. 127–137.

Indraratna, B., Rujikiatkamjorn, C. and Chu, J. (2007): 'Soft clay stabilization with geosynthetic vertical drains beneath road and railway embankments: A critical review of analytical solutions and numerical analysis', *Advances in Measurement and Modeling of Soil Behavior (GSP173),* 236, 7.

Indraratna, B., Rujikiatkamjorn, C. and Sathananthan, I. (2005, a): 'Radial consolidation of clay using compressibility indices and varying horizontal permeability', *Can. Geotech. J.*, 42 (5), pp. 1330–1341.

Indraratna, B., Sathananthan, I., Rujikiatkamjorn, C. and Balasubramaniam, A.S. (2005, b): 'Analytical and numerical modeling of soft soil stabilized by prefabricated vertical drains incorporating vacuum preloading', *International Journal of Geomechanics,* 5 (2), pp. 114–124.

Lambe, T.W. and Whitman, R.V. (1969): *Soil Mechanics*, John Wiley & Sons, Inc, pp. 157

Lekha, K. R., Krishnaswamy, N. R. and Basak, P. (1998): 'Consolidation of Clay by Sand drain under time – dependent loading', *Journal of Geotechnical and Geoenvironmental Engineering*, 124 (1), pp. 91 – 94.

Richart, F. E. (1959): 'Review of the theories for sand drains', *Transactions of* ASCE, 124, pp. 709 – 736.

Rujikiatkamjorn, C. and Indraratna, B. (2006): 'Three-dimensional numerical modeling of soft soil consolidation improved by prefabricated vertical drains', *Ground Modification and Seismic Mitigation (GSP 152),* 196 (22), pp. 161–168.

Rujikiatkamjorn, C., Indraratna, B. and Chu, J. (2008): '2D and 3D numerical modeling of combined surcharge and vacuum preloading with vertical drains', *International Journal of Geomechanics,* 8 (2), pp. 114–156.

Sathananthan, I. and Indraratna, B. (2006): 'Plane-strain lateral consolidation with non-Darcian flow', *Can.Geotech*, ***J*.,** 43, pp. 19-133.

Sinha, A.K., Havanagi, V.G. and Mathur, S. (2007): 'Inflection point method for predicting settlement of PVD improved soft clay under embankments', *Geotextiles and Geomembranes, 2*5 (6), pp. 336–345.

Teh, C.I. and Nie, X. (2002): 'Coupled consolidation theory with non-Darcian flow', *Computers and Geotechnics*, 29 (3), pp. 169-209.

Vaid, Y. P. (1985): 'Constant rate of loading nonlinear consolidation', *Japanese Society of Soil Mechanics and Foundation Engineering, Soils and Foundations*, 25 (1), pp. 105 – 108.

Walker, R., Indraratna, B. and Sivakugan, N. (2009): 'Vertical and radial consolidation analysis of multilayered soil using the spectral method', *J. Geotech. Geoenviron. Eng., ASCE,* 135 (5), pp. 657–663.

Zhuang, Y. C., Xie, K. H. and Li, X.B. (2005): 'Nonlinear analysis of consolidation with variable compressibility and permeability', *Journal of Zhejiang University (Engineering Science)*, 6A (3), pp. 181 – 187.