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TECHNICAL NOTE

Determination of Coefficient of Consolidation from Index Properties of Soil

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Introduction

One of the areas in soil mechanics where predictions are of profound importance is the time required for a specified amount of consolidation and the probable consolidation settlement at a given time. A realistic estimation of the rate of consolidation settlement or the rate of dissipation of pore water pressure depends on the selection of the most appropriate value for the coefficient of consolidation (C_v). However, the determination of C_v from laboratory consolidation tests requires a great amount of time and effort. Often the C_v observed in the field is significantly different from the values determined from laboratory experiments.

Terzaghi and Peck (1967) had found that the laboratory consolidation tests could not be made on more than 10 to 15 samples without causing undue delay in the project, while the physical properties of clay were likely to be significantly different from one point to another even in the case of a relatively homogenous clay strata. They worked towards a statistical relationship between compressibility and the index properties, especially the liquid limit.

Several such studies were taken up by other researchers also. Narasimha Raju et al (1995) determined the coefficient of consolidation from void ratio at liquid limit, e_L, using the stress state - permeability relationship. The equations proposed by them for normally consolidated soils is as follows.

$$C_V = \frac{1 + e_L(1.23 - 0.276 \log p)}{e_L} \times \frac{1}{p^{0.353}} \times 10^{-3} \tag{1}$$

where p is the overburden pressure in kPa and C_v is in cm^2/sec. Narasimha Raju et al (1997) later proposed equation (2) for normally consolidated soils and equation (3) for over-consolidated soils.

$$C_V = \frac{1 + e_L(1.229 - 0.119 \ln \sigma_v)}{e_L} \times \frac{5.64 \times 10^{-4}}{\sigma_v^{0.212}} \tag{2}$$

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$$C_v = \frac{1 + e_L(1.229 - 0.102\ \ln\sigma_{v0} - 0.017\ \ln\sigma_v)}{e_L} \times \frac{(3.964 \times 10^{-3}) \times (\sigma_v)^{0.827}}{(\sigma_{v0})^{1.04}} \qquad (3)$$

where σ_v and σ_{v0} are the consolidation pressure and the pre-consolidation pressure respectively in kPa and C_v is in cm^2/ sec. The authors proposed that more tests needed be carried out on a variety of soils and the coefficients refined so that the equations become applicable to a wider spectrum of soils.

Theoretical Considerations

The normalization of void ratio with its value at liquid limit was derived from the fact that there were experimental evidences that at liquid limit the fine grained soils acquire a unique state. Mitchell (1992) mentioned that at liquid limit the fine grained soils have a shear strength of about 1.7 – 2.5 kPa and pore water suction of about 6 kPa (Russel and Mickle, 1970; Wroth and Wood, 1978; Whyte,1982). Observing that the coefficients of permeability of different clays are very nearly the same (1.28×10^{-7}cm/sec to 2.83×10^{-7} cm/sec) at liquid limit although the water contents and void ratios vary over a wide range, Mitchell (1992) concluded that the effective pore sizes controlling fluid flow must be about the same for all clays. Pandian et al (1993a) and Nagaraj et al (1991,1993,1994) presented that the coefficients of permeability values of various clays at liquid limit ranged from 1.28×10^{-7}cm/sec to 3.4×10^{-7} cm/sec. According to Nagaraj et al (1994) the physico chemical potential per unit volume at liquid limit is constant for all soils and this logically explains the macro level constant magnitudes of soil suction or consolidation pressure, constant shear strength and the same order of permeability. These unique conditions at liquid limit represent a reference state in relation to which all other states can be normalized.

In this study the coefficients of permeability at liquid limit were obtained by compressing soils from a slurry state. Thirteen soil samples were used in the present study. The soils were reconstituted at water contents slightly greater than their liquid limit water contents and kept for a minimum period of 24 hours in the form of slurry for uniform distribution of moisture. The slurry was then transferred to the oedometer rings. Consolidation tests were carried out using a loading sequence of 5 kN/m^2 , 10 kN/m^2, 20 kN/m^2, 40 kN/m^2, 80 kN/m^2, 160 kN/m^2, 320 kN/m^2 and 640 kN/m^2. At each pressure after equilibrium was achieved, falling head permeability test was performed to determine the coefficient of permeability. A thin layer of kerosene was placed over the water to prevent evaporation from the burette. The coefficients of permeability at liquid limit void ratio (e_L) were obtained from the plots of void ratio (e) versus log k for each soil. Table 1 summarizes the Atterberg limits, the specific gravity values and the coefficient of permeability at liquid limit state. The liquid limits of the soils were determined using the 60 gm 60^0 cone and plastic limits were determined using the 400 gm 30^0 cone (Sharma and Bora, 2003, 2004). It is observed from the table that even though the liquid limit varies from 33.8% to 78%, the coefficient of permeability at liquid limit is of the same order ranging from 1.28×10^{-7} cm/sec to 3.2×10^{-7} cm/sec. This range is consistent with the range given by Nagaraj et al, (1991, 1993,1994) and by Mitchell (1992) also.

Based on the above observations, further experiments in the present work were undertaken to develop a correlation between the coefficient of consolidation c_v

and the liquid limit. In this work, refined correlations for the coefficient of permeability (k) and the coefficient of volume compressibility (m_v) using the liquid limit and the consolidation pressure have been developed using the stress state permeability relationship (Nagaraj et al. 1994). These values are then substituted in the equation $C_V = k/m_v \gamma_w$ as defined by Terzaghi.

Table1 Values of Permeability at Liquid Limit Water Content

SL.No.	Liquid limit w_L(%)	Plastic limit w_P (%)	Specific Gravity G_S	Permeability at Liquid limit (cm/sec)
1	77	28	2.68	2.5×10^{-7}
2	38.5	17	2.63	1.42×10^{-7}
3	42	20	2.65	1.28×10^{-7}
4	60	24.2	2.71	1.42×10^{-7}
5	52.5	21	2.71	2.6×10^{-7}
6	33.8	16	2.68	1.95×10^{-7}
7	76	29.5	2.74	3.2×10^{-7}
8	45.8	16.2	2.65	1.62×10^{-7}
9	44	16	2.7	2.2×10^{-7}
10	61	22.5	2.68	2.6×10^{-7}
11	78	29.5	2.71	3.1×10^{-7}
12	69	24	2.72	2.38×10^{-7}
13	56	24.2	2.694	1.62×10^{-7}

Generalised Permeability Behaviour

Using the void ratio at liquid limit (e_L) for generalization, Nagaraj et. al (1993) presented a method for predicting the coefficient of permeability k of fine grained soils at different void ratios as follows.

$$\frac{e}{e_L} = 2.38 + 0.233 \log k \qquad (4)$$

where k is in cm/sec with a correlation coefficient of 0.93.

Nagaraj, Pandian and Narasimha Raju (1994), later correlated the permeability k of over-consolidated soils with the generalized state parameter e/e_L as follows

$$\frac{e}{e_L} = 2.162 + 0.195 \log k \qquad (5)$$

where k is in cm/sec with a correlation coefficient of 0.98. In this paper the authors showed that the e - log k paths did not reflect the effect of stress history.

Pandian, et. al. (1993b) studied the changes in permeability due to drying of soil. The normalized plot for natural, partially air dried and dried conditions of Parur clay was given by the following expression.

$$\frac{e}{e_L} = 2.25 + 0.21 \log k \tag{6}$$

where k is in cm/sec with a correlation coefficient of 0.989.

Similarly the authors expressed the normalized relationship for the air dried conditions of red, brown and black cotton soils as follows.

$$\frac{e}{e_L} = 2.375 + 0.223 \log k \tag{7}$$

with a correlation coefficient of 0.934.

Narasimha Raju, et al (1995) found the following correlation for red, black cotton and marine soils.

$$\frac{e}{e_L} = 2.23 + 0.204 \log k \tag{8}$$

This was later modified by the same authors (1997) as

$$\frac{e}{e_L} = 2.398 + 0.098 \ln k \tag{9}$$

At liquid limit, since e/e_L equals 1, the value of coefficient of permeability (k) works out to be 1.19×10^{-6} cm/sec from equation (4). Similarly the value of k is 1.099×10^{-6} cm/sec, 1.116×10^{-6} cm/sec and 6.82×10^{-6} cm/sec respectively from equations (5), (6) and (7) respectively at liquid limit water content. Equations (8) and (9) give the permeability coefficients as 9.3×10^{-7} cm/sec and 6.38×10^{-7} cm/sec respectively. These values do not agree with the range of values of permeability at liquid limit mentioned by Nagaraj et al (1991,1993,1994), Pandian et al (1993a) and Mitchell (1992) implying some inconsistencies in the results from equations (4), (5), (6), (7), (8) and (9).

In the present study the void ratio (e) versus log k plots of the 13 soils tested came out to be linear. All the e - log k plots were then normalized with the respective void ratios at liquid limit water content. This is presented in Figure 1. It is observed from the plot that all the points fall within a narrow band which can be fitted with a linear equation of the form

$$\frac{e}{e_L} = 3.606 + 0.392 \log k \tag{10}$$

with a correlation coefficient of 0.9745. From equation (10) at $e/e_L = 1$, i.e, at liquid limit water content, k comes out to be 2.22×10^{-7} cm/sec which is well within the range of values stated by Mitchell (1992) and Nagaraj et al(1991,1993,1994). Hence equation (10) can be considered as a refined version of equations (4), (5), (6), (7), (8) and (9). Equation (10) was generated from experimental results and can be treated as the generalized permeability equation for reconstituted soils. The equation can be rewritten as,

$$k = 10^{\left(\frac{e/e_L - 3.606}{0.392}\right)} \text{ cm/sec} \qquad (11)$$

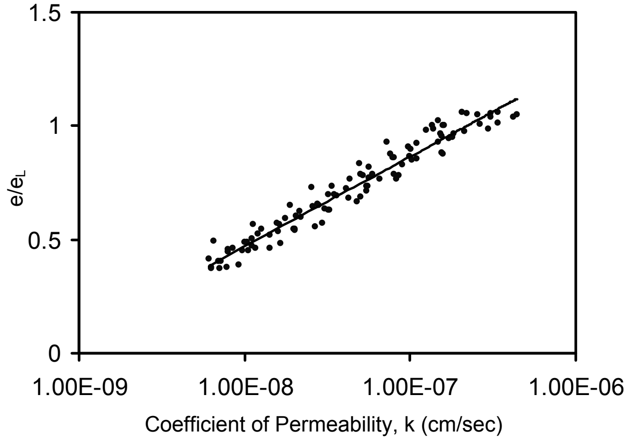

Fig. 1 e/e_L Versus Coefficient of Permeability

Generalised Compressibility Behaviour

Nagaraj, Srinivasa Murthy and Vatsala (1991) used the void ratio (e_L) at liquid limit to generalize compressibility behaviour of eleven, normally consolidated uncemented soils as follows

$$\frac{e}{e_L} = 1.122 - 0.234 \log P \qquad (12)$$

where P is the consolidation pressure in kN/m^2 of the soils. In the present work compressibility characteristics of reconstituted soils were obtained by performing slurry consolidation tests on 17 soil samples with liquid limit ranging from 33.8% to 82%. The soils were reconstituted at water contents slightly greater than the liquid limit water contents and kept in the form of a slurry for a minimum period of 24 hours for uniform distribution of moisture. The void ratio versus log of consolidation pressure (P) plots were normalized with the void ratio at liquid limit water content (e_L). All the seventeen e versus log P plots got clustered into a narrow band as presented in Figure 2 that can be fitted with a linear equation of the form

$$\frac{e}{e_L} = 1.2315 - 0.2933 \log P \qquad (13)$$

with a correlation coefficient of 0.985. In the equation P is the consolidation pressure in kN/m^2. Equation (13) was generated from experimental results and can be treated as the generalized compressibility equation for reconstituted soils.

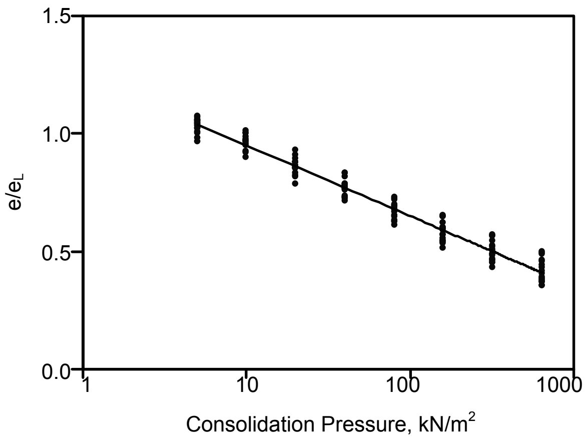

Fig. 2 e/e_L Versus Log P

The coefficient of volume change (m_v) which is volume change per unit initial volume due to a unit increase in pressure can be written as

$$m_V = \frac{\Delta e}{1+e_o} \times \frac{1}{\Delta \sigma} = \frac{e_1 - e_2}{1+e_1} \times \frac{1}{P_1 - P_2} \qquad (14)$$

where e_1 and e_2 are the void ratios corresponding to consolidation pressures P_1 and P_2 respectively.

From equation (13), one can write

$$e_1 = 1.2315\, e_L - 0.2933\, e_L \log_{10} P_1 \qquad (15)$$

$$e_2 = 1.2315 e_L - 0.2933\, e_L \log_{10} P_2 \qquad (16)$$

$$e_1 - e_2 = 0.2933\, e_L \log_{10} \frac{P_2}{P_1} \quad \text{and}$$

$$m_V = \frac{0.2933\, e_L \log_{10} \frac{P_2}{P_1}}{1+e_1} \times \frac{1}{P_2 - P_1}$$

or

$$m_V = \frac{0.2933 \log_{10} \frac{P_2}{P_1}}{(P_2 - P_1)\left(\frac{e_1}{e_L} + \frac{1}{e_L}\right)}\ m^2/kN \qquad (17)$$

where e_1/e_L is the generalized state parameter corresponding to a consolidation pressure P_1 which can be computed from equation (13). Equation (17) can therefore be used for predicting the coefficient of volume compressibility using the knowledge of only the liquid limit and specific gravity of the soil. Using equation (11) and (17) for k and m_V respectively, the coefficient of consolidation (C_V) can be obtained as $C_V = \frac{k}{m_V \gamma_w}$ where the numerator is the coefficient of permeability in cm/sec, denominator is the coefficient of volume compressibility in m^2/kN and γ_w is the unit weight of water in kN/m^3.

$$C_V = \frac{10^{\left(\frac{e/e_L - 3.606}{0.392}\right)}}{\frac{0.2933 \log_{10}\frac{P_2}{P_1}}{(P_2 - P_1)\left(\frac{e_1}{e_L} + \frac{1}{e_L}\right)}} \times \frac{1}{\gamma_w} \times 100 \left(cm^2/sec\right) \tag{18}$$

Equation (18) shows that coefficient of consolidation is a function of stress and liquid limit of the soil and that it is possible to determine coefficient of consolidation for a given pressure increment by knowing only the liquid limit and specific gravity of the soil.

In equation (18) the generalized state parameter (e/e_L) that can be computed from equation (13) corresponding to any pressure P exists in both the numerator and the denominator. In the denominator, the generalized state parameter corresponds to the stress level before the pressure is applied. In the numerator e/e_L is the generalized state parameter to determine permeability of the soil. Since permeability gradually decreases as the void ratio decreases, it is reasonable to take the e/e_L values which correspond to the mid point of the pressure increment considered. For example, for a pressure range of 160 kN/m^2 to 320 kN/m^2, e/e_L could be determined from equation (13) corresponding to 240 kN/m^2. Hence the generalized state parameter in the numerator of equation (18) may be rewritten as $(e/e_L)_m$ ('m' corresponding to mid point of the pressure increment considered.)

$$C_V = \frac{10^{\left(\frac{(e/e_L)_m - 3.606}{0.392}\right)}}{\frac{0.2933 \log_{10}\frac{P_2}{P_1}}{(P_2 - P_1)\left(\frac{e_1}{e_L} + \frac{1}{e_L}\right)}} \times \frac{1}{\gamma_w} \times 100 \left(cm^2/sec\right) \tag{19}$$

Test Results and Discussion

Nineteen soil samples with liquid limits ranging from 33.8% to 78% were tested to determine their consolidation, permeability and other relevant physical properties. The coefficients of consolidation based on consolidation tests were determined by the Taylor's square root of time fitting method, Casagrande's logarithm of time fitting method, the method proposed in this study and by Pandian's bilinear method (Pandian et al., 1992). By examination of Terzaghi's theoretical degree of consolidation (U) and theoretical time factor (T) curve, Pandian et al (1992), showed that log U/T versus log T plot exhibits a bilinear behaviour. The intersection point corresponds to a theoretical time factor of 0.793 and a degree of consolidation of 88.5%. Using this property, the following equation for the coefficient of consolidation was proposed

$$C_v = 0.793 d^2 / t_{88.5} \quad (20)$$

where, time t corresponds to 88.5% consolidation which can be obtained from the log δ/t versus log t plot. Figures. 3, 4, 5, 6 and 7 show a comparison between the C_V values obtained from the four methods used in this study.

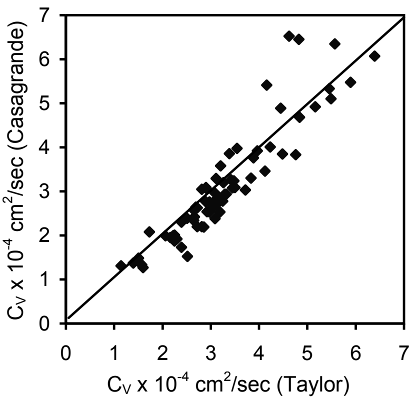

Fig. 3 Comparison of C_V Values between Taylor's Method and Casagrande's Method

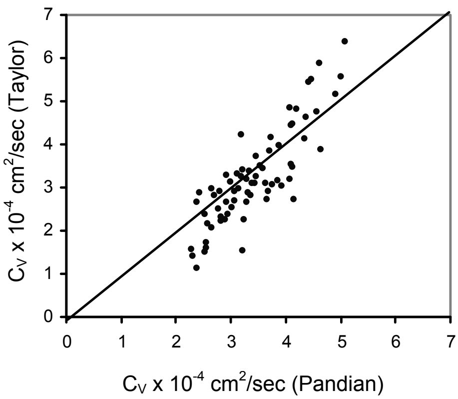

Fig. 4 Comparison of C_V Values between Taylor's Method and Pandian's Method

Figure 8 shows the test results of Figure 6 in a log-log plot. These C_V values which happen to be within the range of C_V values of remoulded soils recorded in NAVFAC- DM 7.1 – 1985, show their characteristic inconsistency usually observed. (Terzaghi and Peck, 1967; Lambe and Whitman,1969).

In general, the Taylor's and Pandian's methods give higher values of C_V compared to the Casagrande method and the proposed new method yields C_V values that are mostly higher than those of Taylor's and Pandian's methods. It has been observed (Duncan,1993; Lerouseil, 1988) that the actual rates of settlement are faster – often two to four times faster (Lambe and Whitman, 1969) than the rates predicted on the basis of laboratory consolidation tests. In view of this fact, the rates of settlement generally predicted by the new method which is based on reconstituted soil behaviour are likely to serve as an upper bound for prediction for a wide pressure range and thus be close to the actual observations of settlement. Therefore, the proposed method with its inherent simplicity and convenience is likely to be of help for the practicing engineer in the initial predictions of the rate of consolidation in the field. However, the method cannot be recommended in situations where the rate of consolidation is critical to the design of the structure. Actual field measurements of excess pore pressure generated and its dissipation become necessary in such cases.

The proposed method is limited to one dimensional consolidation of inorganic clays.

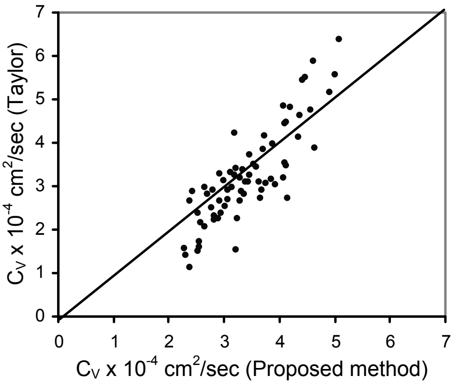

Fig. 5 Comparison of C_V Values between Proposed Method and Taylor's Method

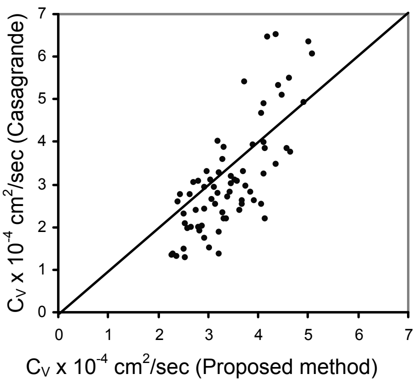

Fig. 6 Comparison of C_V Values between Proposed Method and Casagrande's

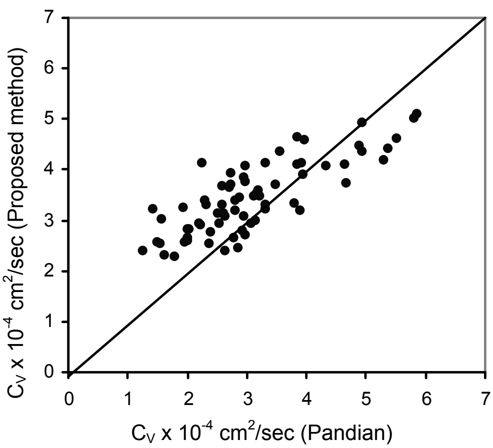

Fig. 7 Comparison of C_V Values between the Proposed Method and Pandian's Method

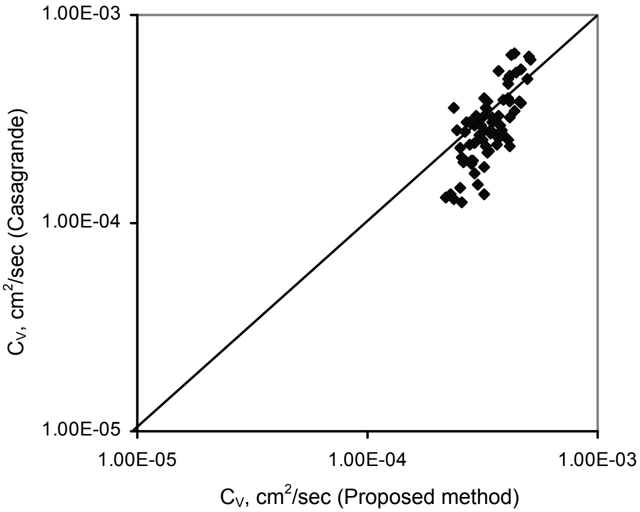

Fig. 8 Comparison of C_V Values between Casagrande's Method and Proposed Method in Log- Log Scale

Conclusions

The coefficient of volume compressibility, permeability and hence the coefficient of consolidation can be determined corresponding to any pressure for reconstituted soils by using a simple generalized state parameter. The parameter consists of void ratio of the soil at any required pressure normalized with respect to its void ratio at liquid limit.

Results obtained from the new method for determination of C_V are in broad agreement with those of the conventional methods with indications that the new method is likely to give better estimates of the actual settlements over a wide pressure range. In view of the time and effort needed in the conventional methods, the new method would be of help for the practicing engineer in the general situation where an approximate rate of settlement is all that is required.

Notations

C_v = coefficient of consolidation of soil;

δ = compression

γ_w= unit weight of water;

e = void ratio;

e_L = void ratio at liquid limit;

e_1 = void ratio at consolidation pressure P_1;

e_2 = void ratio at consolidation pressure P_2;

d = Length of drainage path

k = coefficient of permeability of soil

m_v = coefficient of volume compressibility

P_1 = consolidation pressure P_1

P_2 = consolidation pressure P_2

t = time

References

Duncan, J.M.(1993): 'Limitations of conventional analysis of consolidation settlement', *Journal of Geotechnical Engineering, American Society of Civil Engineers,* 119(a): pp 1331 – 1359.

Lambe , T.W., and Whitman, R.V. (1969): *Soil Mechanics,* John Wiley and Sons, New York.

Lerouseil, S. (1988): 'Recent developments in consolidation of natural clays'. *Tenth Canadian Geotechnical Colloquium. Canadian Geotechnical Journal*, 25. pp 85 – 107.

Mitchell, J. K. (1992): *Fundamentals of Soil Behaviour*, John Wiley, New York.

Nagaraj,T.S., Pandian,N.S., and Narasimha Raju, P.S.R. (1991): 'An approach for prediction of compressibility and permeability behaviour of sand – bentonite mixes', *Indian Geotechnical Journal*, 21(3), pp 271- 282.

Nagaraj,T.S., Pandian,N.S. and Narasimha Raju, P.S.R. (1993): 'Stress – state – permeability relationship for fined grained soils', *Geotechnique*, 43(2), pp 333 – 336.

Nagaraj,T.S., Pandian,N.S., and Narasimha Raju, P.S.R. (1994): 'Stress – state – permeability relationship for overconsolidated soil', *Geotechnique*, 44(2), pp 349 – 352.

Nagaraj, T.S., Srinivasa Murthy, B.R., and Vatsala A. (1991): 'Prediction of soil behaviour. Part 1–Saturated uncemented soils', *Indian Geotechnical Journal,* 21(1), pp 137 – 160.

Narasimha Raju, P.S.R., Pandian, N.S. and Nagaraj, T.S. (1995): 'Analysis and estimation of coefficient of consolidation', *Geotechnical testing journal,* ASTM, 18(2), pp 252- 258.

Narasimha Raju, P.S.R., Pandian, N.S. and Nagaraj, T.S. (1997): Determination of coefficient of consolidation from independent measurement of permeability and compressibility. *Geotechnical Engineering* (London), 125, pp 224- 229.

NAVFAC. (1985): *Soil Mechanics*, DM 7.1, Naval Facilities Engineering Command, Alexandria, VA, page 144.

Pandian, N.S., Sridharan, A. and Kumar, K.S. (1992): 'A new method for the determination of coefficient of consolidation', *Geotechnical Testing Journal, GTGODJ*, 15(1), pp 74 – 79.

Pandian, N.S., Nagaraj, T.S., and Sivakumar Babu, G.L. (1993a): 'Behaviour of tropical clays. Part 1 – Index properties and microstructural considerations', *Journal of Geotechnical Engineering, American Society of Civil Engineers* 19 (5): pp 826-839.

Pandian, N.S., Nagaraj, T.S., and Sivakumar Babu, G.L. (1993b): 'Behaviour of tropical clays. Part 2 – Engineering Behaviour', *Journal of Geotechnical Engineering, American Society of Civil Engineers.* 119(5) : pp 840-861

Russel, E.R., and Mickle, J.L. (1970): 'Liquid limit values by soil moisture tension', *Journal of Soil Mechanics and Foundation Engineering Division, American Society of Civil Engineers* 96: pp 967 – 989.

Sharma, B., and Bora, P.K. (2003): 'Plastic limit, liquid limit and undrained shear strength of soils – Reappraisal', *Journal of Geotechnical and Geoenvironmental Engineering. ASCE,* 129(8), pp 774 – 777.

Sharma, B., and Bora, P.K. (2004): 'Determination of plastic limit of soils by cone penetration method', *Indian Geotechnical Journal.* 34(4). pp 297 – 312.

Terzaghi, K., and Peck, R.B. (1967): *Soil Mechanics in Engineering Practice.* John Wiley and Sons, Inc, New York. 2nd Edition.

Wroth, C.P., and Wood, D.M. (1978): 'The correlation of index properties with some basic engineering properties of soil', *Canadian Geotechnical Journal*,15(2), pp 137 - 145

Whyte, I.L. (1982): 'Soil plasticity and strength. A new approach using extrusion', *Ground Engineering*. 15(1), pp 16 – 24.