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Two-dimensional Modeling of Contaminant Transport using Meshfree Method

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Introduction

Contamination of soil and groundwater involves degradation due to the presence of physical, biological, chemical, or radiological materials in undesirable concentrations or concentrations considered abnormal. Concentrations considered acceptable for one type of use may be unacceptable for another. The presence of contaminants can be the result of natural occurrences, but is commonly considered to be a condition associated with human activities. The bulk of subsurface contaminant migration takes place in the presence of moving groundwater – both the flowing category and the more or less stationary groundwater condition – due to the process of advection and hydrodynamic dispersion. The configuration of a contaminant plume is influenced by an array of factors, such as the physical properties of the soil stratum being polluted, the presence of groundwater, the properties of the contaminant, including the concentration, solubility and density and flow characteristics and time (Freeze and Cherry 1979; Charbeneau 2000). The development of numerical methods for simulating groundwater contamination is of major importance in view of the numerous applications in the contaminant hydrogeology and geoenvironmental engineering. With the increased concern about the contamination of groundwater around the world, more and more emphasis is being placed on the ability of the numerical models to predict the fate of contaminants in the subsurface. The numerical simulation of contaminant transport is an important tool to quantify the potential danger of pollution for uncontaminated aquifers and also to design the remedial actions to be taken up for contaminated aquifers (Sun 1996; Zheng and Bennett 2002).

Mathematical models in groundwater have been greatly expanded and improved in recent years. One great advantage of a computer model is its flexibility – it can be used to model a large number of different problems by simply varying input data. There are many numerical models employing a wider variety of solution techniques such as finite difference, finite element, finite layer and boundary element (Bear 1979; Rowe and Booker 1985; Frind 1988; Craig and Rabideau 2006) and idealisations have been developed. However, it is often difficult to solve

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contaminant transport problems numerically because of the mixed character of the governing equation, with particular difficulties associated with the hyperbolic part of the equation. The traditional finite difference and finite element methods are poorly suited for these problems, and often leave the user with a choice between nonphysical oscillations and excessive numerical diffusion (Binning and Celia 2002). As the entire formulations of finite difference and finite element methods are based on the grid/mesh, a time-consuming and costly process of generating a quality grid/mesh is necessary.

A fairly recent class of computational methods — Meshfree methods such as Smooth Particle Hydrodynamics (Monaghan 1988; 1992), Reproducing Kernel Particle Method (Liu et al. 1995), Element Free Galerkin Method (Belytschko et al. 1994b), Meshless Local Petrov–Galerkin Method (Atluri and Zhu 1998), Radial Point Interpolation Method (Wang and Liu 2002) and Natural Neighbour Galerkin Method (Sukumar et al. 2001), have been developed in recent years. The key idea of the meshfree methods is to provide accurate and stable numerical solutions for integral equations or partial difference equations with all kinds of possible boundary conditions with a set of arbitrarily distributed nodes (or particles) without using any mesh that provides the connectivity of these nodes or particles (Liu and Liu 2003). The meshfree methods are successfully applied to numerous boundary-value problems related to various fields of study (Belytschko et al. 1994a; Lu et al. 1995; Belytschko and Tabbara 1996; Rao and Rahman 2000; Praveen Kumar et al. 2006, 2007).

A limited amount of research is undertaken regarding the potential usage of the meshfree methods for modelling the contaminant transport. The success of the Element Free Galerkin Method (EFGM) in solving various types initial and boundary value problems provided a strong impetus for the utilisation of the method in the modelling of geoenvironmental engineering problems. The present study is the extension of the earlier work carried out by Praveen Kumar et al. (2006). In this paper, the EFGM is used for modelling two-dimensional (2D) advection-dispersion-sorption processes through saturated porous media. Student's t distribution function is used as a weight function in the meshfree analysis. Lagrange multiplier method is used for enforcing the essential boundary conditions. MATLAB program is developed to code the procedure of the EFGM for 2D contaminant migration. The results of the EFGM are compared with the analytical and finite element results.

Element Free Galerkin Method

The Element Free Galerkin Method (EFGM) is a meshfree method developed by Belytschko et al. (1994b) based on the diffuse element method originated by Nayrole et al. (1992). The EFGM employs moving least-square (MLS) approximants formulated by Lancaster and Salkauskas (1981) to approximate the function $C(x, y)$ with $C^h(x, y)$ in which $C(x, y)$ is the contaminant concentration at (x, y), here (x, y) is a position coordinates.

These approximants are constructed from three components: a weight function of compact support associated with each node; a basis usually consisting of a polynomial and a set of coefficients that depend on position. For solving integrals in the weak form formed due to Galerkin approximation procedure, a background mesh is used. Connectivity between the nodes and the approximation functions are completely constructed by the method.

Moving Least-Square Approximations

Consider a continuous function $C(x, y)$ defined on a domain $\Omega \subseteq \Re^K$, where K = 1 and 2. Let $\Omega_x \subseteq \Omega$ denote a sub-domain describing the neighbourhood of a point, $x \in \Re^K$ located in Ω. According to the moving least-squares (MLS) proposed by Lancaster and Salkauskas (1981), the approximation, $C^h(x, y)$ of $C(x, y)$ is

$$C(x,y) \cong C^h(x,y) = \sum_{i=1}^{m} p_i(x,y)\, a_i(x,y) \qquad (1)$$
$$= \mathbf{p}^T(\mathbf{x})\, \mathbf{a}(\mathbf{x}) \qquad \forall (x,y) \in \Omega$$

where

$$\boldsymbol{p}^T(\mathbf{x}) = [1, x, y] \qquad (2)$$

$$\boldsymbol{a}^T(\mathbf{x}) = [\, a_0(\mathbf{x}), a_1(\mathbf{x}), a_2(\mathbf{x}), \ldots, a_m(\mathbf{x}) \,] \qquad (3)$$

$$\mathbf{x} = [\, x, y \,] \qquad (4)$$

where $p(x)$ is a monomial basis function and $a(x)$ is a vector of undetermined coefficients, whose values can vary according to the position (x, y) in Ω and m is the order of the basis. The basis function should satisfy the following properties:

- $p_1(\mathbf{x}) = 1$,
- $p_i(\mathbf{x}) \in C^S(\Omega)$, i = 1, 2, . . . , m where $C^S(\Omega)$ is a set of functions that have continuous derivatives up to order S on Ω, and
- $p_i(\mathbf{x})$, i = 1, 2, . . . , m constitute a linearly independent set.

The coefficient vector $a(\mathbf{x})$ at any point is determined by minimising L_2 error norm of the weighted least-squares. The L_2 error norm is defined as

$$J = \sum_{I=1}^{n} w(\mathbf{x}-\mathbf{x}_I)\,[\, C_L^h(\mathbf{x}_I,\mathbf{x}) - C_I \,]^2 = \sum_{I=1}^{n} w(\mathbf{x}-\mathbf{x}_I)\,[\, \boldsymbol{p}^T(\mathbf{x}_I)\, \boldsymbol{a}(\mathbf{x}) - C_I \,]^2 \qquad (5)$$

where $w(\mathbf{x}-\mathbf{x}_I)$ is the weight function associated with the nodal point $\mathbf{x}_I$ in the domain Ω and n is a local node number whose support includes $\mathbf{x}$ as shown in Figure 1.

By minimising J with respect to $\boldsymbol{a}(\mathbf{x})$ gives

$$\boldsymbol{A}(\mathbf{x})\, \boldsymbol{a}(\mathbf{x}) = \boldsymbol{B}(\mathbf{x})\, \boldsymbol{C} \quad \Rightarrow \quad \boldsymbol{a}(\mathbf{x}) = \boldsymbol{A}^{-1}(\mathbf{x})\, \boldsymbol{B}(\mathbf{x})\, \boldsymbol{C} \qquad (6)$$

where

$$\mathbf{A}(\mathrm{x}) = \sum_{I=1}^{n} w(\mathrm{x}-\mathrm{x}_I)\,\mathbf{p}(\mathrm{x}_I)\,\mathbf{p}^T(\mathrm{x}_I)$$
$$= w(\mathrm{x}-\mathrm{x}_1)\begin{bmatrix} 1 & x_1 & y_1 \\ x_1 & x_1^2 & x_1y_1 \\ y_1 & x_1y_1 & y_1^2 \end{bmatrix} + \ldots + w(\mathrm{x}-\mathrm{x}_n)\begin{bmatrix} 1 & x_n & y_n \\ x_n & x_n^2 & x_ny_n \\ y_n & x_ny_n & y_n^2 \end{bmatrix} \quad (7)$$

$$\mathbf{B}(\mathrm{x}) = [\,w(\mathrm{x}-\mathrm{x}_1)\,\mathbf{p}(\mathrm{x}_1), \ldots, w(\mathrm{x}-\mathrm{x}_n)\,\mathbf{p}(\mathrm{x}_n)\,]$$
$$= \left[\, w(\mathrm{x}-\mathrm{x}_1)\begin{bmatrix} 1 \\ x_1 \\ y_1 \end{bmatrix}, w(\mathrm{x}-\mathrm{x}_2)\begin{bmatrix} 1 \\ x_2 \\ y_2 \end{bmatrix}, \ldots, w(\mathrm{x}-\mathrm{x}_n)\begin{bmatrix} 1 \\ x_n \\ y_n \end{bmatrix} \right] \quad (8)$$

$$\boldsymbol{C}^{\mathrm{T}} = [\,C_1, C_2, \ldots, C_n\,] \quad (9)$$

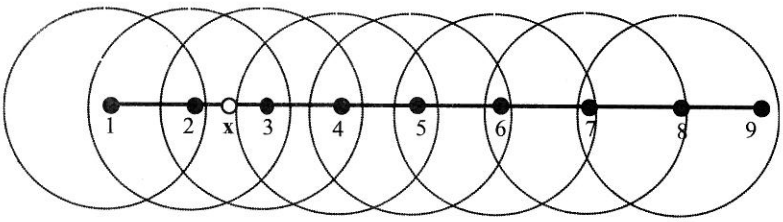

Fig. 1 Domains of Influence for Node x

By substituting Eq. (6) in Eq. (1), the MLS approximate for $C(\mathbf{x})$ is

$$C(\mathbf{x}) \cong C^h(\mathbf{x}) = \sum_{I=1}^{n} \boldsymbol{p}^T \boldsymbol{A}^{-1} \boldsymbol{B}_I\, C_I = \sum_{I=1}^{n} \Phi_I(\mathbf{x})\, C_I = \boldsymbol{\Phi}\, \boldsymbol{C} \quad (10)$$

where

$$\Phi_I(\mathbf{x}) = \boldsymbol{p}^T \boldsymbol{A}^{-1} \boldsymbol{B}_I \text{ and } \boldsymbol{\Phi} = \{\Phi_1, \Phi_2, \ldots, \Phi_n\}. \quad (11)$$

The spatial derivative of the MLS shape function of node I is expressed as

$$\Phi_{I,\mathbf{x}} = (\boldsymbol{p}^T \boldsymbol{A}^{-1} \boldsymbol{B}_I)_{,\mathbf{x}} = \boldsymbol{p}^T{}_{,\mathbf{x}}\, \boldsymbol{A}^{-1} \boldsymbol{B}_I + \boldsymbol{p}^T (\boldsymbol{A}^{-1})_{,\mathbf{x}}\, \boldsymbol{B}_I + \boldsymbol{p}^T \boldsymbol{A}^{-1} \boldsymbol{B}_{I,\mathbf{x}} \quad (12)$$

where

$$\boldsymbol{B}_{I,\mathbf{x}} = \frac{dw}{d\,\mathbf{x}}(\mathbf{x}-\mathbf{x}_I)\,\boldsymbol{p}(\mathbf{x}_I) \quad (13)$$

$$A_{,\mathbf{x}}^{-1} = -A^{-1} A_{,\mathbf{x}} A^{-1} \tag{14}$$

in which

$$\begin{aligned} \boldsymbol{A}_{,\mathbf{x}}(\mathbf{x}) &= \sum_{I=1}^{n} w_{,\mathbf{x}}(\mathbf{x}-\mathbf{x}_I)\,\boldsymbol{p}(\mathbf{x}_I)\,\boldsymbol{p}^T(\mathbf{x}_I) \\ &= \frac{dw}{d\mathbf{x}}(\mathbf{x}-\mathbf{x}_1)\begin{bmatrix} 1 & x_1 & y_1 \\ x_1 & x_1^2 & x_1y_1 \\ y_1 & x_1y_1 & y_1^2 \end{bmatrix} + \ldots + \frac{dw}{d\mathbf{x}}(\mathbf{x}-\mathbf{x}_n)\begin{bmatrix} 1 & x_n & y_n \\ x_n & x_n^2 & x_ny_n \\ y_n & x_ny_n & y_n^2 \end{bmatrix} \end{aligned} \tag{15}$$

Description of Weight Function

A weight function is an important ingredient of the EFGM. The weight function is non-zero only over a small neighbourhood of $\mathbf{x}_I$, called the domain of influence of node I. In order to generate a set of sparse discrete equations, the weight functions should have a compact support. The support of the weight function defines a node's domain of influence, which is the sub-domain over which a particular node contributes to the approximation. The overlap of the nodal domains of influence defines the nodal connectivity as depicted in Figure 1.

A choice of the weight function affects the resulting approximation $C^h(x)$. In this paper, a Student's t distribution weight function (Rao and Rahman, 2000) is adopted. The weight function is written in terms of normalised radius r as

$$w(r_x) = \left\{ \begin{array}{ll} \dfrac{(1+\beta^2 r_x^2)^{-\left(\frac{(1+\beta)}{2}\right)} - (1+\beta^2)^{-\left(\frac{(1+\beta)}{2}\right)}}{1-(1+\beta^2)^{-\left(\frac{(1+\beta)}{2}\right)}} & r \le 1 \\ 0 & r > 1 \end{array} \right\};$$

$$w(r_y) = \left\{ \begin{array}{ll} \dfrac{(1+\beta^2 r_y^2)^{-\left(\frac{(1+\beta)}{2}\right)} - (1+\beta^2)^{-\left(\frac{(1+\beta)}{2}\right)}}{1-(1+\beta^2)^{-\left(\frac{(1+\beta)}{2}\right)}} & r \le 1 \\ 0 & r > 1 \end{array} \right\}; \tag{16}$$

$$w(\mathbf{x}-\mathbf{x}_I) = w(r_x)\,w(r_y)$$

where

β = parameter controlling the shape of the weight function,

$$r_x = \frac{\|x-x_I\|}{d_{mxI}}\ ,\ r_y = \frac{\|y-y_I\|}{d_{myI}}\ ,\ d_{mxI} = d_{\max} z_{xI}\ ,\ d_{myI} = d_{\max} z_{yI} \tag{17}$$

in which $d_{\max}$ is a scaling factor and z_{xI} and z_{yI} are the distances to the nearest node in the neighbourhood and are determined by searching enough neighbouring nodes for **A** to be non-singular.

The smoothness of the shape functions $\Phi_I(x)$ is governed by the smoothness of the weight function and basis functions. If $p_I(x) \in C^S(\Omega)$ and $w_I(x) \in C^r(\Omega)$, then it can be shown that $\Phi_I(x) \in C^{\min(r,s)}(\Omega)$. To avoid poorly formed shaped functions, $w_I(x)$ should be unity at the centre and zero along the boundary of Ω_x. Also, appropriate values of β depending on the basis function should be selected.

Enforcement of Essential Boundary Conditions

The shape functions of the EFGM do not satisfy the Kronecker delta criterion: $\Phi_i(\mathbf{x}_j) \neq \delta_{IJ}$. Lack of this property poses some difficulty in the imposition of essential boundary conditions. Several techniques have been developed and reported in the literature to enforce essential boundary conditions. Lagrange multiplier technique (Belytschko et al., 1994b; Dolbow and Belytschko, 1998), modified variational principle approach (Lu et al., 1994), penalty approach (Zhu and Alturi, 1998), coupling with FEM (Krongauz and Belytschko, 1996) and transformation technique (Rao and Rahman, 2000) are some of the techniques that are used for the enforcement of essential boundary conditions. In this paper, Lagrange multiplier technique is used to enforce the essential (Dirichlet) boundary conditions.

Discretisation of Governing Equation

The transport of contaminants is described by advection, mechanical dispersion and chemical reaction between the contaminant and surrounding soil particles. In this study, the steady state flow of an incompressible fluid contaminated by a solute through the saturated homogenous porous media is considered. The fluid is assumed to flow in the longitudinal axis (positive *x*-axis) direction. Under these assumptions, the governing equation for contaminant transport can be written as

$$\left(1+\frac{\rho_d}{n}K_d\right)\frac{\partial C}{\partial t} = D_L\frac{\partial^2 C}{\partial x^2} + D_T\frac{\partial^2 C}{\partial y^2} - \left(\frac{u}{n}\right)\frac{\partial C}{\partial x} \tag{18}$$

Initial condition

$$\text{at t} = 0, C(x,y,0) = C_i \text{ in } \Omega \tag{19a}$$

Boundary conditions

$$C(0,y,t) = C_0 \quad \text{on } \Gamma_s \text{ (Dirichlet boundary conditions)} \tag{19b}$$

$$\nabla(C)\cdot n_S = g \quad \text{on } \Gamma_E \text{ (Neumann boundary conditions)} \tag{19c}$$

where (x, y) are the spatial coordinates, n is the porosity of the soil, ρ_d is the bulk density of the soil, K_d is the distribution coefficient, C is the concentration of contaminant, D_L and D_T are the longitudinal and transverse dispersion coefficients respectively, u is the discharge velocity, C_0 and g are the concentration of contaminant at the source and concentration gradient at the exit boundary respectively, n_S is a unit normal to the domain Ω and, Γ_S and Γ_E are the portions of boundary Γ where source concentration and concentration gradient are prescribed.

The weak form of Eq. (18) with boundary conditions is expressed as

$$\int_\Omega \delta C^T D_L \frac{\partial^2 C}{\partial x^2}\, d\Omega + \int_\Omega \delta C^T D_T \frac{\partial^2 C}{\partial y^2}\, d\Omega - \int_0^L \delta C^T \left(\frac{u}{n}\right) \frac{\partial C}{\partial x}\, dx$$
$$- \int_\Omega \delta C^T \left(1 + \frac{\rho_d}{n} K_d\right) \frac{\partial C}{\partial t}\, d\Omega - \int_{\Gamma_S} \delta\lambda^T (C - C_0)\, d\Gamma - \int_{\Gamma_S} \lambda^T \delta C\, d\Gamma = 0 \tag{20}$$

where L is the length of the domain and λ is a Lagrange multiplier, expressed as

$$\lambda(\mathbf{x}) = N_K(\kappa)\, \lambda_I, \quad \mathbf{x} \in \Gamma_S \tag{21a}$$

$$\delta\lambda(\mathbf{x}) = N_K(\kappa)\, \delta\lambda_I, \quad \mathbf{x} \in \Gamma_S \tag{21b}$$

in which $N_K(\kappa)$ is a Lagrange interpolant, κ is the arclength along the boundary and repeated indices designate summations.

By using divergence theorem, Eq. (20) is written as

$$\int_{\Gamma_E} \delta C^T D_L \frac{\partial C}{\partial x} n_S\, ds + \int_{\Gamma_E} \delta C^T D_T \frac{\partial C}{\partial y} n_S\, ds - \int_\Omega \delta\left(\frac{\partial C^T}{\partial x}\right) D_L \frac{\partial C}{\partial x}\, d\Omega$$
$$- \int_\Omega \delta\left(\frac{\partial C^T}{\partial y}\right) D_T \frac{\partial C}{\partial y}\, d\Omega - \int_0^L \delta C^T \left(\frac{u}{n}\right) \frac{\partial C}{\partial x}\, dx \tag{22}$$
$$- \int_\Omega \delta C^T \left(1 + \frac{\rho_d}{n} K_d\right) \frac{\partial C}{\partial t}\, d\Omega - \int_{\Gamma_S} \delta\lambda^T (C - C_0)\, d\Gamma - \int_{\Gamma_S} \lambda^T \delta C\, d\Gamma = 0$$

Eq. (22) can be split into two parts:

$$\int_{\Omega} \delta C_{,x}^{T} D_L C_{,x} \, d\Omega + \int_{\Omega} \delta C_{,y}^{T} D_T C_{,y} \, d\Omega + \int_{0}^{L} \delta C^{T} \left(\frac{u}{n} \right) \frac{\partial C}{\partial x} dx$$

$$+ \int_{\Omega} \left(\delta C^{T} \left(1 + \frac{\rho_d}{n} K_d \right) \frac{\partial C}{\partial t} \right) d\Omega \qquad (23a)$$

$$+ \int_{\Gamma_S} \lambda^{T} \delta C \, d\Gamma = \int_{\Gamma_E} \delta C^{T} D_L g_x \, d\Gamma + \int_{\Gamma_E} \delta C^{T} D_T g_y \, d\Gamma$$

$$\int_{\Gamma_S} \delta \lambda^{T} \left(C - C_0 \right) d\Gamma = 0 \qquad (23b)$$

As δC and $\delta \lambda$ are arbitrary values and by using Eqs. (10) and (11) in the discretisation of Eq. (23), the following relationship can be obtained:

$$\left[\boldsymbol{K}^{(1)} \right] \{\boldsymbol{C}\} + \left[\boldsymbol{K}^{(2)} \right] \{\boldsymbol{C}\}_{,t} + [\boldsymbol{G}] \{\boldsymbol{\lambda}\} = \{\boldsymbol{Q}\} \text{ and } \left[\boldsymbol{G}^{T} \right] \{\boldsymbol{C}\} = \{\boldsymbol{q}\} \qquad (24)$$

where,

$$\boldsymbol{K}_{IJ}^{(1)} = \int_{\Omega} \left[\begin{bmatrix} \Phi_{I,x} \\ \Phi_{J,y} \end{bmatrix}^{T} \begin{bmatrix} D_L & 0 \\ 0 & D_T \end{bmatrix} \begin{bmatrix} \Phi_{I,x} \\ \Phi_{J,y} \end{bmatrix} + \begin{bmatrix} \Phi_I \\ \Phi_J \end{bmatrix}^{T} \left(\frac{u}{n} \right) \begin{bmatrix} \Phi_{I,x} \\ \Phi_{J,x} \end{bmatrix} \right] d\Omega \qquad (25a)$$

$$\boldsymbol{K}_{IJ}^{(2)} = \int_{\Omega} \left[\begin{bmatrix} \Phi_I \\ \Phi_J \end{bmatrix}^{T} \left(1 + \frac{\rho_d}{n} K_d \right) \begin{bmatrix} \Phi_I \\ \Phi_J \end{bmatrix} \right] d\Omega \qquad (25b)$$

$$\boldsymbol{G}_{IK} = \int_{\Gamma_S} \Phi_I N_K \, d\Gamma \qquad (25c)$$

$$\boldsymbol{Q}_I = \int_{\Gamma_E} \Phi_I \, \boldsymbol{D} \, g \, d\Gamma \qquad (25d)$$

$$\boldsymbol{q}_K = \int_{\Gamma_S} N_K \, C_0 \, d\Gamma \qquad (25e)$$

Using Crank-Nicolson method for time approximation, Eq. (24) can be written as

$$\begin{bmatrix} \boldsymbol{K}^{(1)*} + \boldsymbol{K}^{(2)} & \boldsymbol{G} \\ \boldsymbol{G}^{T} & \boldsymbol{0} \end{bmatrix} \begin{Bmatrix} \boldsymbol{C}_n \\ \boldsymbol{\lambda} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{R}_n \\ \boldsymbol{q} \end{Bmatrix} \qquad (26)$$

where

$$\boldsymbol{R}_n = \left(\left[\boldsymbol{K}^{(2)} \right] - \frac{\Delta t}{2} \left[\boldsymbol{K}^{(1)} \right] \right) \{ C \}_{n-1} + \frac{\Delta t}{2} \left(\{ \boldsymbol{Q} \}_n + \{ \boldsymbol{Q} \}_{n-1} \right) \qquad (27a)$$

$$\boldsymbol{K}^{(1)*} = \frac{\Delta t}{2}\left[\boldsymbol{K}^{(1)}\right] \tag{27b}$$

in which $\boldsymbol{C}_n$ and $\mathbf{C}_{n-1}$ are the nodal concentrations at start and end of the time increment and, $\boldsymbol{Q}_n$ and $\boldsymbol{Q}_{n-1}$ are the nodal mass fluxes at start and end of the time increment.

Numerical Examples and Results

MATLAB program that implements the procedure of EFGM for modelling contaminant transport through the saturated homogenous porous media, is used to analyse three numerical examples. The source of contaminant is assumed as continuous. It is assumed that the governing equation of contaminant transport is governed by the advection-dispersion-sorption processes.

In the EFGM, a linear basis function is used for constructing the shape functions. As the shape functions are linear one, it is required to take the shape parameter β = 2 in the weight function. Based on the parametric study, it has been found that $d_{\max}$ = 2.5 for the present analysis and the same value is used for all the numerical examples.

Example 1: Advection-Dispersion (Source along Bottom Half-width)

The parameters considered for this example are given in Table 1.

Table 1 Data Used for Advection-Dispersion: Example 1

Parameter	Value
Seepage velocity (m/day)	0.1
Length of the reach (m)	200
Width of the reach (m)	100
Longitudinal dispersivity (m)	1.0
Transverse dispersivity (m)	0.1
Total duration of simulation (days)	730
Time step (Δt) (days)	0.4
Number of divisions in length direction	40
Number of divisions in width direction	20
Initial concentration (g/m^3)	0.0
Concentration at source boundary (g/m^3) ($x = 0; 0 \le y \le 50$)	1.0

The EFGM model is applied to the problem considered by Rao and Medina (2006). The problem domain is shown in Figure 2. The problem domain has been divided into 41 × 21 uniformly spaced nodes and 800 cells, solely for numerical integration purpose. Nodes of the background mesh are chosen such that they coincide with the meshfree nodes. The simulation has been carried out for 730 days.

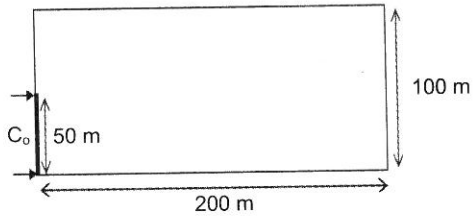

Fig. 2 Sketch of the Problem Domain

A comparison between the concentration profiles obtained from the EFGM and analytical method for advection–dispersion case at different cross-sections for four time periods varying from 0.5 to 2 years is made. Figures 3 and 4 depict comparative values of the normalised concentrations at two cross-sections with 0.5 and 1 year and with 1.5 and 2 years as time periods, respectively. From the figures, it is noted that both the EFGM and analytical results are comparing well with one another. This advocates the applicability of the proposed EFGM and shown to perform well for contaminant transport modelling.

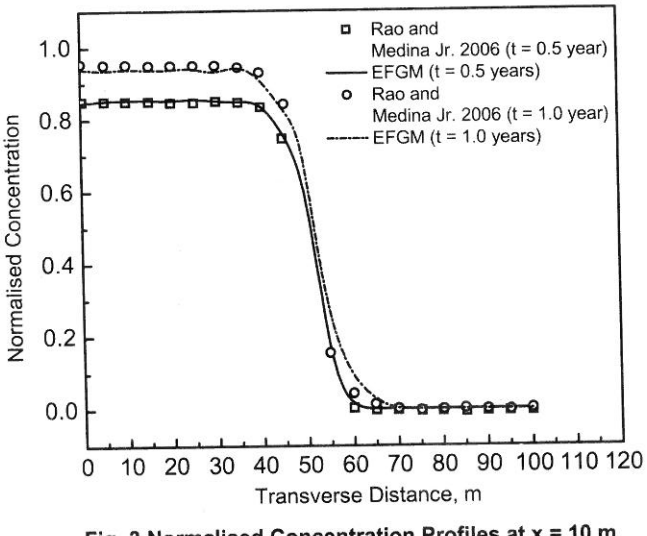

Fig. 3 Normalised Concentration Profiles at x = 10 m

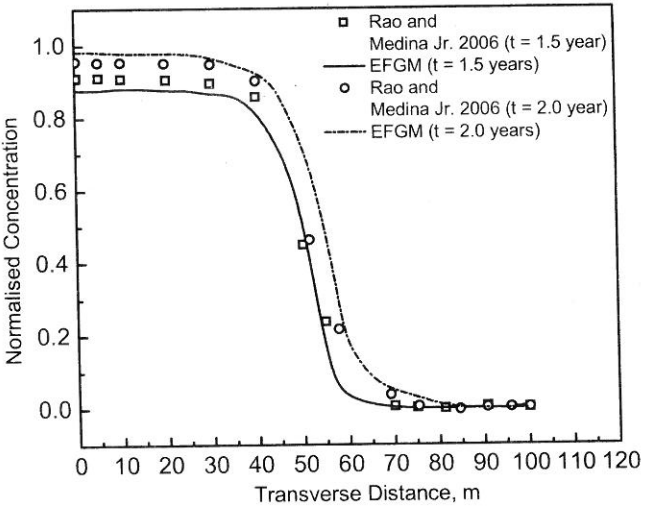

Fig. 4 Normalised Concentration Profiles at x = 25 m

Example 2: Advection-Dispersion (Source along Entire Width)

Table 2 gives the parameters used in this example. In this case, the contaminant source spreads along the entire width of the problem domain. The EFGM model is divided into 21 × 11 uniformly spaced nodes and 200 cells. For this example, the numerical simulation has been carried out for 1500 days with a time step of 20 days.

Table 2 Data Used for Advection-Dispersion: Example 2

Parameter	Value
Seepage velocity (m/day)	0.029
Length of the reach (m)	975
Width of the reach (m)	335
Longitudinal dispersivity (m)	79.248
Transverse dispersivity (m)	0.79248
Total duration of simulation (days)	1500
Time step (Δt) (days)	20
Number of divisions in length direction	51
Number of divisions in width direction	9
Initial concentration (mg/l)	0.0
Concentration at source boundary (mg/l)	30

A finite element package, CTRAN/W (GeoStudio 2007), has been used for solving this example problem and the results are compared with that of the EFGM results. In finite element analysis, the domain is discretised into 4-noded 459 elements with 520 nodes as shown in Figure 5. The concentration profiles obtained

from both the finite element method (FEM) and EFGM are shown in Figures 6 and 7 respectively. It is seen from the figures that the results are agreeing reasonably well.

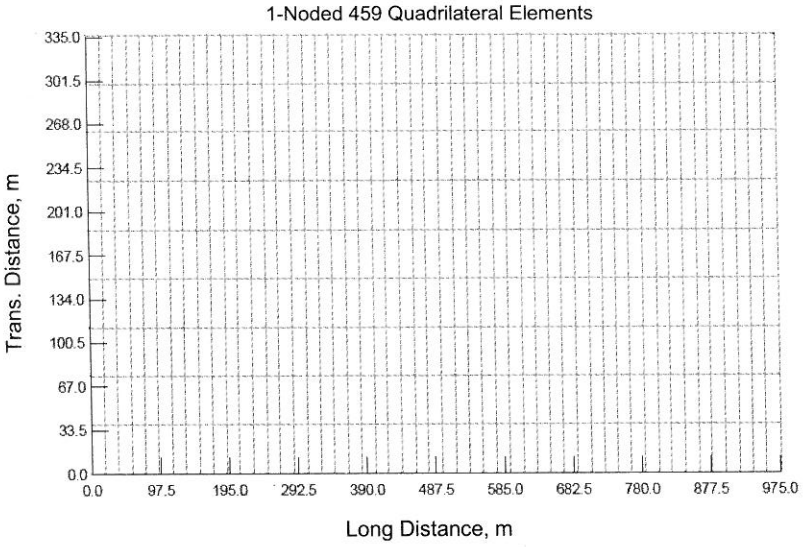

Fig. 5 Finite Element Mesh – Example 2

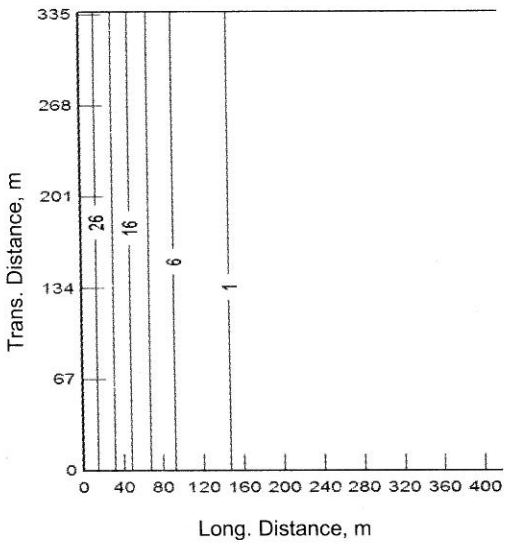

Fig. 6 Contours of Concentration Profile for Advection-Dispersion (FEM)

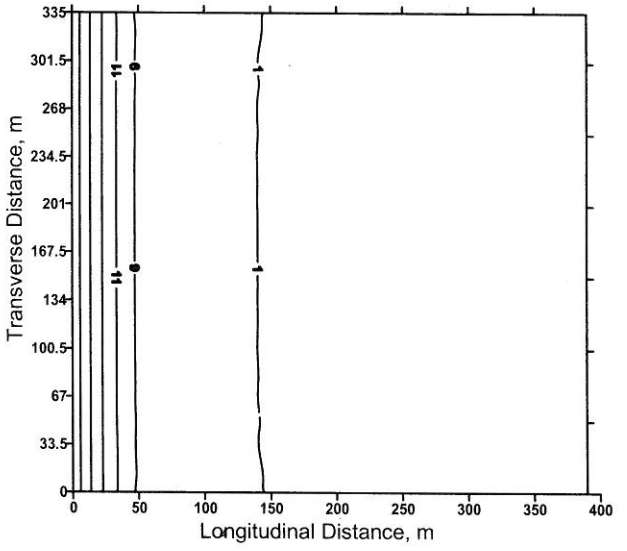

Fig. 7 Contours of Concentration Profile for Advection-Dispersion (EFGM)

Example 3: Advection-Dispersion-Sorption

Table 3 gives the parameters that are considered in the analysis of Example 3. The EFGM model has been divided into 21 × 21 uniformly spaced nodes and 400 cells, solely for numerical integration purpose. In this case, the simulation has been carried out for 840 days with a time step of 12 days.

Table 3 Data Used for Advection-Dispersion-Sorption: Example 3

Parameter	Value
Seepage velocity (m/day)	0.864
Length of the reach (m)	80
Width of the reach (m)	80
Longitudinal dispersivity (m)	1.75
Transverse dispersivity (m)	1.75
Retardation factor (R_d)	7.2674
Total duration of simulation (days)	840
Time step (Δt) (days)	12
Number of divisions in length direction	41
Number of divisions in width direction	11
Initial concentration (mg/l)	0.0
Concentration at source boundary (mg/l)	1.0

In the finite element analysis, the domain is discretised into 4-noded 400 elements with 451 nodes as shown in Figure 8. A comparison between the concentration profiles obtained from the EFGM and FEM for advection–dispersion-sorption case for the total duration of 840 days is made. Figures 9 and 10 depict the concentration contours obtained from the FEM and EFGM respectively. From the figures, it is noted that both the EFGM and finite element results are matching well.

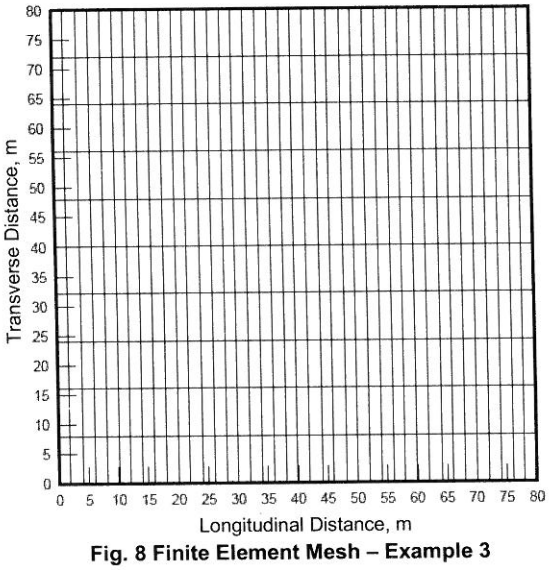

Fig. 8 Finite Element Mesh – Example 3

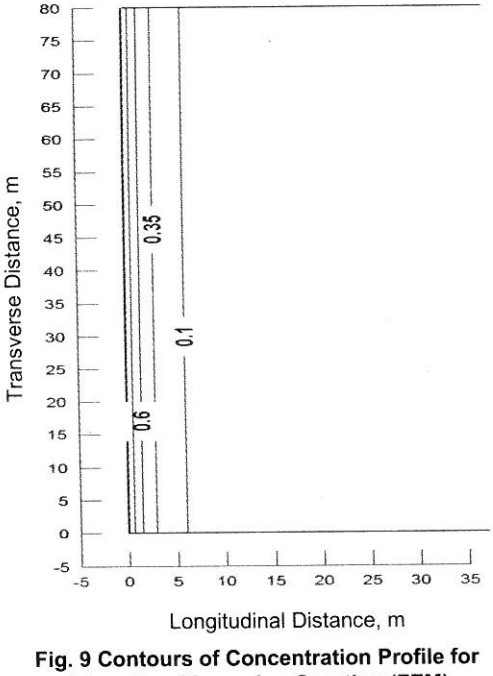

Fig. 9 Contours of Concentration Profile for Advection-Dispersion-Sorption (FEM)

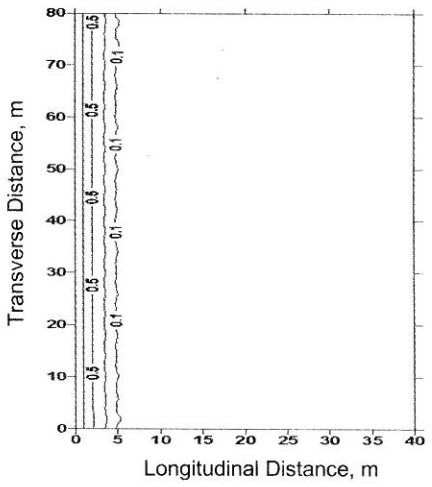

Fig. 10 Contours of Concentration Profile for Advection-Dispersion-Sorption (EFGM)

Summary and Conclusions

The numerical model of solute transport, expressed in terms of governing equations, initial and boundary conditions, together with the applicable flow and transport parameters, and information on sources and sinks must be solved to obtain the concentration distribution in the region and time span of interest. The numerical simulation of contaminant transport offers means that can integrate, in some approximate way at least, the effects of several controlling processes; and it is the only method of calculation which can accommodate complex geometries and parameter distributions. In groundwater systems having irregular geometry, non-uniform aquifer properties and complicated boundary conditions, numerical techniques are used to determine approximate solutions to the contaminant transport equation.

Details of the Element Free Galerkin Method (EFGM) and its numerical implementation for modelling two-dimensional contaminant transport through the saturated homogeneous porous media are presented in the paper. In the EFGM, a structured mesh is not required; only a scattered set of nodal points is needed in the domain of interest. The EFGM employs moving least squares approximants to approximate the function and uses the Lagrange multiplier method for imposing the essential boundary conditions. The implementation of the EFGM is very simple and straight forward, irrespective of the dimension of the problem and shape of the domain under consideration. The MLS approximants and Student's t distribution weight function are used for the shape function construction. Numerical results obtained from the MATLAB program are compared with the analytical and FEM results for the case of advection-dispersion-sorption contaminant transport processes. A good agreement is obtained between the results of the EFGM and FEM. Further investigations are being pursued for three-dimensional contaminant transport modelling through the saturated porous media.

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