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Modeling of Degradation of Clayey Soils under Repeated Loading

K. V. Sridhanya[1], K. Rajagopal[2] and C. Lakshmana Rao[3]

Introduction

Many structures resting on soft soils are subjected to cyclic loading, such as, wave loading in marine structures or traffic loading in pavement structures, with frequencies in the range of 3 cycles per minute to 60 cycles per minute (i.e. 0.05 Hz to 1 Hz). In order to analyze the stability of the structures resting on soft clay soils, the behaviour of such soils under cyclic loads has to be assessed properly. Consequently, a wide variety of experimental investigations on the engineering behaviour of soils under such loading conditions has been carried out in the past by many researchers (Seed and Chan 1966, Castro and Christian 1976, Vucetic and Dobry 1988, Yusuhara and Hyde 1997, Moses 2003). These researchers have reported the development of excess pore pressures, a resultant increase in shear strain and degradation of strength and stiffness under cyclic loading. Based on the work done by these researchers it is clear that the soil either fails during cyclic loading itself or reaches a non-failure, stable state depending upon the cyclic stress level.

A number of constitutive models based on plasticity principles have been proposed for modeling the cyclic response of soils (e.g. Dafalias and Herrmann 1982, Pastor et al. 1990 on generalized plasticity model). Though these models have been proven to predict the soil behaviour adequately, the main limitation of these models is that they are formulated in terms of a large number of parameters. These parameters, which may not have much physical significance, need to be determined through specialized test apparatus which may not be possible in many laboratories. However a few simpler models were developed based on Cam Clay and Modified Cam Clay (MCC) models for describing the cyclic response of soft clay soils. These models are easier to apply because of the involvement of a fewer parameters that may be determined from conventional oedometer test and triaxial compression tests (Roscoe et al. 1958, Roscoe and Burland 1968 etc). The model developed by Carter et al. (1982) is one such model. This model for cyclic loading is similar to the classical Modified Cam Clay model (Roscoe and Burland 1968), developed to describe monotonic loading behaviour of soil, except for the presence

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1 Former Research Scholar, Dept. of Civil Engineering, Indian Institute of Technology Madras,Chennai - 600 036. Email: sridhanyakv@gmail.com

2 Professor, Dept. of Civil Engineering, Indian Institute of Technology Madras, Chennai - 600 036. Email: gopalkr@iitm.ac.in

3 Professor, Dept. of Applied Mechanics, Indian Institute of Technology Madras, Chennai - 600 036. Email: lakshman@iitm.ac.in

of a degradation parameter that was introduced to describe the degradation under cyclic loading. Also the model requires only six parameters, the values of which can be obtained from standard oedometer or triaxial compression tests. However, this model has a few limitations such as its inability to describe non failure equilibrium under stress controlled loading for low cyclic stress levels. Though the soil does not fail at low cyclic stress levels (non-failure equilibrium), there would be a reduction in the strength and stiffness of the soil and a consequent effect on the serviceability of the structure. Hence, it is important to model this response also.

An attempt has been made in this paper to address some of these issues that were not considered in the earlier model, and to improve the model such that it predicts experimentally observed phenomenon (of pore pressure development, strain development etc.). The concept of degradation in the hardening parameter with every unloading, that was proposed by Carter et al. (1982) is also adopted for the development of the present model. However, in the present study, the degradation equation for the hardening parameter is proposed from the undrained cyclic triaxial test data on soft clays, rather than consolidation test data (drained test) that was adopted by Carter et al. (1982). The motivation for such a proposal is because, clay being fine grained soil with low permeability experiences undrained response under repeated loading conditions. The improvement in the predictive capabilities of the proposed modified model is verified with the published experimental data on natural marine clays. The discussions dealt with in this paper emphasize on predicting the non failure behaviour of soil, which the Carter et al. model cannot predict, as it predicts failure for all cases.

Theoretical Development

Features of Cyclic Loading Model by Carter et al. (1982)

In order to clarify the presentation, some of the essential features of critical state soil mechanics are summarized here. One of the earliest models developed based on the critical state soil mechanics theory is the Cam Clay model. Roscoe et al. (1958) developed the Cam Clay model at the Cambridge University. Later, Roscoe and Burland (1968) modified the model.

The basic assumption made in the Modified Cam Clay model is that the soil is homogeneous, isotropic and saturated. Carter et al. (1982) model is an improved version of the Modified Cam Clay (MCC) model that can predict cyclic behaviour. The model defines three state variables and five material parameters. The variables are the effective mean stress (p′), the deviator stress (q) and the void ratio (e). The state of effective stress of a soil specimen is expressed in terms of the stress invariants p' and q defined by,

$$p' = \frac{1}{3}(\sigma_1' + \sigma_2' + \sigma_3') \text{ and}$$
$$q = \sqrt{\{\frac{1}{2}[(\sigma_1' - \sigma_2')^2 + (\sigma_2' - \sigma_3')^2 + (\sigma_3' - \sigma_1')^2]\}} \quad (1)$$

where, σ'_1, σ'_2 and σ'_3 are the principal effective stresses.

The measures for strain used are the volume strain (ν) and a measure of octahedral shear stain (ε), given by

$$\nu = \varepsilon_1 + 2\varepsilon_3 \text{ and } \varepsilon = \frac{2}{3}\left(\varepsilon_1 - \varepsilon_3\right) \quad (2)$$

where, ε_1 and ε_3 are the major and minor principal strains respectively. Only saturated cohesive soils are dealt with here. The model requires the specification of five parameters, values of which can be obtained from standard oedometer or triaxial compression tests (Roscoe et al. 1958, Roscoe and Burland 1968, Carter et al. 1982). These parameters are:

> λ, the gradient of the normal consolidation line in e-ln (p′) space

> k, the gradient of the swelling and recompression lines in e-ln(p′) space

> e_{cs}, the critical void ratio, this is taken as the void ratio at unit p′ on the critical state line. It is the void ratio of the soil at failure.

> M, the value of the stress ratio q/p′ at critical state. This is related to the angle of friction through the relationship given below.

$$M = \frac{6\sin\phi'}{3-\sin\phi'}$$

> G, the elastic shear modulus

The values of λ and κ parameters can be obtained from conventional oedometer tests, in which the test must include unloading–reloading cycles. The slope of the loading path in the e-ln p′ gives the value of λ, while the slope of the unloading path gives the value of κ. Figure 1 shows the conventional oedometer test result to obtain the values of parameters λ and κ. Higher the value of λ and κ, higher is the compressibility (plasticity) of the soil. Similarly, if the current voids ratio (e) or current stress ratio (η) is closer to the critical state voids ratio e_{cs} or the critical state parameter M respectively, higher would be the susceptibility of the soil to failure.

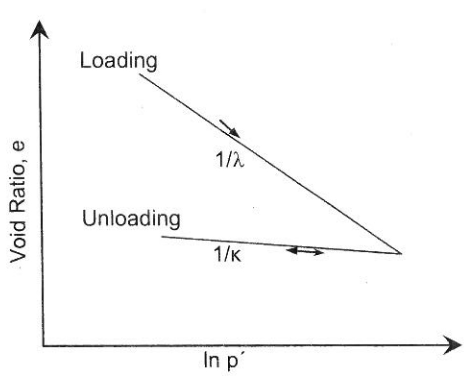


Fig. 1 Determination of Model Parameter from Conventional Oedometer Tests

Being plasticity based model, the theory defines yield surface, flow rule and hardening law which are the essential features of an elasto-plastic model. The model uses the state variables of the Critical State Soil Mechanics (CSSM) theory to derive the Flow rule, Hardening law and the yield function.

Like the MCC model, this model also assumes that the dissipation of strain energy is a function of the irrecoverable volumetric strains (v^p) and shear strains (ε^p). The work done per unit volume and the dissipation of energy are related as,

$$\delta W = p'\delta v^p + q\delta\varepsilon^p = p'\sqrt{(\delta v^p)^2 + (M\delta\varepsilon^p)^2} \tag{3}$$

Based on this equation for dissipation, the flow rule and the yield functions are derived as:

Yield function,

$$q^2 + M^2 p'^2 = M^2 p' p'_c \tag{4a}$$

Flow rule,

$$\frac{\delta v^p}{\delta\varepsilon^p} = \frac{M^2 - \eta^2}{2\eta} \tag{4b}$$

For a state of stress that falls within the current yield surface, the soil responds elastically and the incremental effective stress strain law is the same as in the classical theory of elasticity. When a stress state lies on the yield surface, plastic deformation takes place and the behaviour is governed by an elastic plastic constitutive matrix. Incremental stress strain law during yielding is defined by Eq. 5.

$$\begin{pmatrix} dv \\ de \end{pmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{pmatrix} dp' \\ dq \end{pmatrix} \tag{5}$$

dv and *de* are the incremental volumetric and shear strains

where,

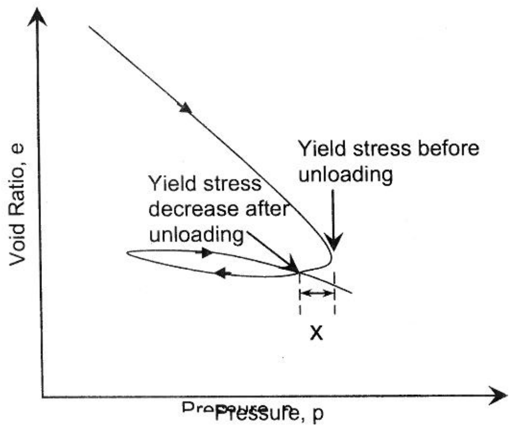
$$C_{11} = \frac{(\lambda-\kappa)a}{(1+e)p'} + \left(\frac{\kappa}{(1+e)}\right)\frac{1}{p'}; \qquad C_{12} = C_{21} = \frac{(\lambda-\kappa)}{(1+e)}\left(\frac{1-a}{p'}\right);$$

$$C_{22} = \frac{(\lambda-\kappa)}{(1+e)}\left(\frac{b}{p'}\right) + \left(\frac{1}{3G}\right)$$

where $a = \dfrac{M^2-\eta^2}{M^4-\eta^4}$ and $b = \dfrac{4\eta^4}{M^4-\eta^4}$

And η is the stress ratio q/p'. For further details on this, the reader is referred to the paper by Carter et al. (1982).

Repeated loading usually causes permanent strain to occur earlier than what is observed in previous loading cycles (Carter et al. 1982). That is, when saturated clay is unloaded and then reloaded it is found that permanent strains occur earlier than predicted by the Cam Clay model. This implies that the yield stress limit continuously decreases during every unloading. This is illustrated in Figure 2.



X=Decrease in Yield Stress During Unloading

Fig. 2 Decrease in Yield Stress during Unloading

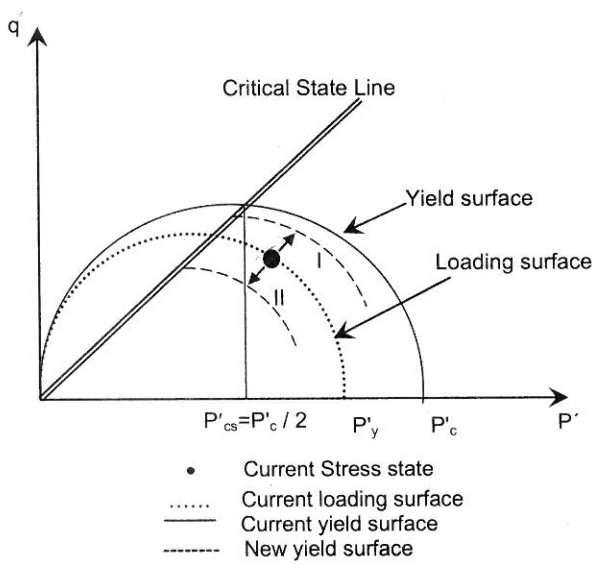


Fig. 3 Cyclic Loading Model (Carter et al. 1982)

One way of interpreting this real behaviour, as proposed by Carter et al. (1982) is to assume that the position and perhaps the shape of the yield surface are in some way affected by the elastic unloading. In the conventional Modified Cam Clay model this cannot be reproduced as the yield surface is unaffected during the elastic unloading of the material (feature of an elasto-plastic model). But, in the modified model by Carter et al. (1982), it is assumed that the size of the yield surface reduces gradually with elastic unloading as shown by the loading path II in Figure 3. This is achieved by reducing the hardening parameter P'_c during the process of unloading. It is this modification to the MCC, given by Carter et al. (1982), which enabled it to predict the cyclic response. The loading path *I* shows the expansion of the yield surface during loading. In comparison with the other models, this kind of modification by Carter et al. does not define any additional surfaces to predict cyclic response and hence, requires relatively lesser number of parameters.

Limitations of the Model for Cyclic Loading

The predictions made by the model developed by Carter et al. (1982) are compared with the experimental data reported by Moses (2003) to illustrate the limitations of this model. Moses (2003) reported the results of cyclic triaxial tests in which stress-controlled cycles were applied to laboratory prepared saturated clay samples. The value of λ, κ, e_{cs} and M values in this model were obtained by using the data from oedometer and triaxial compression tests as 0.395, 0.196, 3.26 and 0.852 respectively using data reported by Moses (2003).

The model prediction for a confining pressure of 200 kPa, frequency of 0.05 Hz and cyclic stress ratio of 0.45 is shown in Figure 4.

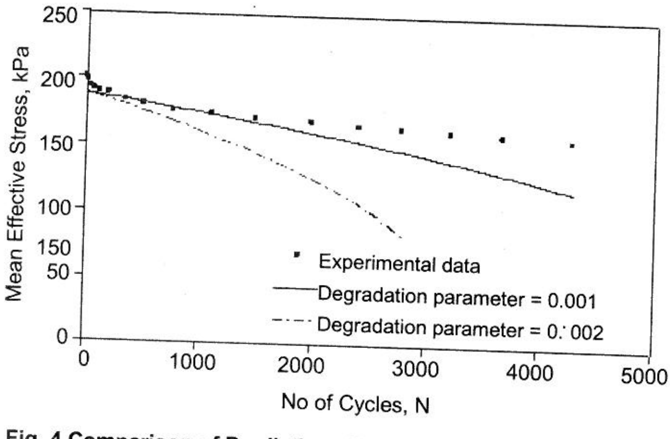


Fig. 4 Comparison of Predictions Made by Carter et al. (1982) Model with the Experimental Data of Moses (2003)

Evidently, the predictions seem to deviate from the experimental data for higher number of cycles. The value of degradation parameter adopted in this case is as low as 0.001, which is the smallest among the values in the range proposed by Carter et al. (1982) for the parameter. For higher values of the degradation parameter, say 0.002, the model predicts quicker degradation response with the number of cycles as shown in Figure 4. The predictions are shown with two different degradation parameters of 0.001 and 0.002 in the figure for illustrative purposes. For both the values of degradation parameters, the model gives a non-stabilizing trend for pore pressure development. In the original paper, the author suggests the use of the number of cycles to failure or axial strain or excess pore pressures values to pick the value for degradation parameter. However, there is visible arbitrariness in selecting the value of the degradation parameter in this model. Hence, the lowest value for the degradation parameter in the range specified by Carter et al. (one that gives predictions closest to the experimental results) was used for further predictions.

An attempt has been made in this paper to address some of these short comings of this model, that is, the inability to predict non failure equilibrium for smaller cyclic stress ratios under stress controlled loading. This is done by modifying the equation for the degradation of pre-consolidation pressure with the number of cycles. Further, the modifications are based on the critical state concept within the frame work of Cam Clay models.

Proposed Model

The model proposed by Carter et al. (1982) derives the pre-consolidation degradation equation from the observations in a consolidation test, which is a drained phenomenon. However, the clay being fine grained material with low permeability, experiences undrained response when loaded, especially under repeated loading, where there is no time allowed for the excess pore pressure to dissipate. In a consolidation test, during unloading, the mean effective stress on the soil reduces causing the plastically stressed soil grains to relax, thus becoming less dense. Hence, the dense soil (densified during loading) softens during the unloading process, and yielding occurs earlier than expected during the reloading phase. However, in an undrained test (as in repeated loading conditions) the phenomenon is entirely different. Unlike drained test, the undrained test is a constant volume phenomenon, which results in the development of pore pressure with every loading. Though this pore pressure is relaxed during unloading, not all the pore pressure developed during the loading phase would be relaxed during the unloading phase.

There would be some amount of residual pore pressure left within the soil after every unloading. Evidently, the softening response of the soil would be due to the increase in this residual pore pressures with successive cycles, that is, the degradation in pre-consolidation pressure would depend predominantly on the increase in residual pore pressure or decrease in the mean effective stress with every successive cycle.

Figure 5 illustrates the variation in pore pressure with loading and unloading during drained as well as undrained tests. When a sample is subjected to a stress pulse of magnitude P_1, as shown in (a), the pore pressure begins to develop within the sample. Let us say the pore pressure developed within the sample is U_1. During drained conditions, if the load is allowed to stand for a period t_1, the pore pressure developed would begin to dissipate. If this t_1 is greater than the time required for complete consolidation to take place, then the excess pore pressure left within the sample would reduce to zero, as shown in (b). During the unloading, there would be no change in the pore pressure. On the other hand, if the pore pressures are not allowed to dissipate, as in an undrained phenomenon, there will be residual pore pressures left within the sample even after complete unloading of the sample, as shown in (c).

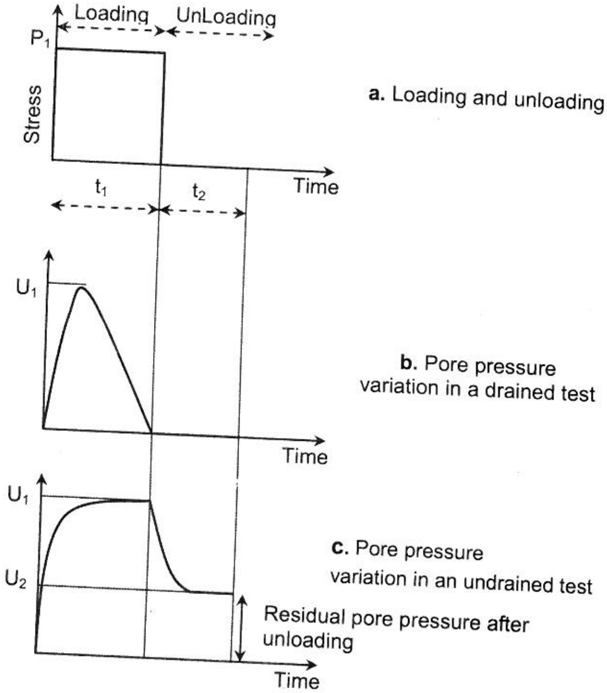


Fig. 5 Pore Pressure Variation with Loading and Unloading in a Drained and Undrained Test

It is the accumulation of this residual pore pressure that causes the yield stress (pre-consolidation pressure) to degrade with the number of cycles. In the proposed model, the degradation of pre-consolidation pressure, due to the increase in residual pore pressure, is assumed to be affected by

> Initial confining pressure

> Level of loading and unloading

The general degradation of strength and stiffness of the soils depend on these factors (Koutsoftas 1978, Lee 1979), hence, it is assumed that the degradation in pre-consolidation pressure would also depend on these factors.

The degradation in the mean effective stress with the number of cycles was studied using the data published by Moses (2003). The mean effective stress degradation plot is obtained by normalizing the current pressure with the initial mean effective stress and plotting its variation with number of cycles in the logarithmic scale. The advantage of the exponential variation in mean effective stress with the number of cycles is used to approximate the curve to a straight line in a log-log plot. A typical mean effective stress degradation plot for marine clay under a confining pressure of 50 kPa and frequency of 0.17 Hz for four cyclic stress ratios is shown in Figure 6 (Moses 2003).

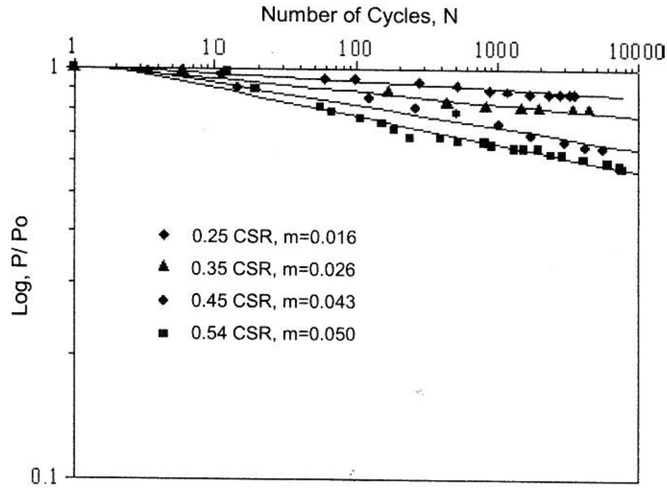


Fig. 6 Plot of Mean Effective Stress Degradation

The slope 'm' of the mean effective stress degradation line can be considered as decay constant. Thus from the mean effective stress degradation plot, it is possible to formulate an equation to estimate the degraded value of the mean effective stress P' at the end of each cycle as,

$$\frac{p'}{p'_i} = N^{-m} \tag{6}$$

From Eqn. 4a, the Yield locus equation for the MCC model can be rewritten as

$$\frac{p'}{p'_c} = \frac{M^2}{M^2 + \eta^2} \tag{7}$$

Here η is the stress ratio given by q/p'. Substituting the equation for the degradation of mean effective stress into the yield function equation for MCC, we arrive at the modified equation for the hardening parameter P'_c. That is, substituting Eq. 6 in the Eq. 7, we get,

$$p'_c = \frac{p'(M^2 + \eta^2)}{M^2} = \frac{p'_i(N^{-m})(M^2 + \eta^2)}{M^2} \tag{8}$$

This gives the equation for the degraded value of pre-consolidation pressure after every successive cycle.

Here, the value of the degradation constant 'm' is obtained from standard cyclic triaxial tests. As discussed previously, the mean effective stress degradation plot is drawn by normalizing the current pressure with the initial mean effective stress and plotting its variation with the number of cycles in the logarithmic scale. The slope of the mean effective stress degradation line is considered as decay constant 'm' (Figure 6). From the observations of the data from cyclic triaxial tests, the decay constant was found to follow a definite trend as a function of the CSR values, the confining pressure and frequency. This trend can be approximated to a set of straight lines parallel to each other and making intercepts on the CSR axis as shown in Figure 7. This chart can be used for picking the value of degradation parameter for similar soils.

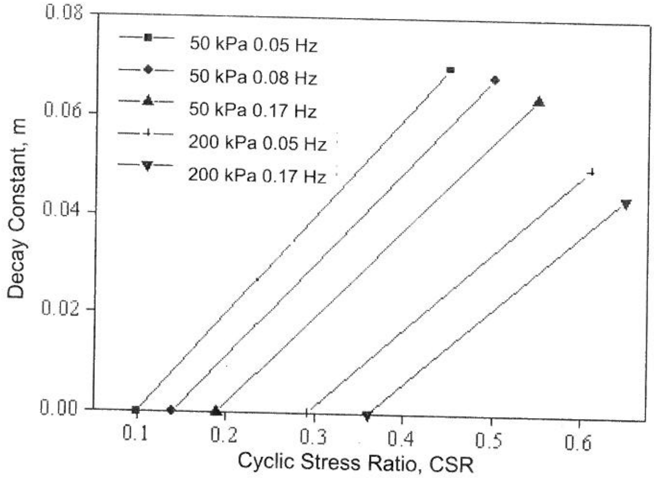


Fig. 7 Approximated Chart Showing the Variation of Degradation Parameter 'm'

On comparing the proposed Eq. 8 with the equation proposed by Carter et al. (1982), it can be observed that the proposed equation takes in to account the loading and unloading amplitude and the number of cycles to which the sample is loaded. Unlike the cyclic Cam Clay model proposed by Carter et al. (1982), this formulation takes into account the effect of the material property and the stress to which the material is loaded and unloaded through the parameters M and η respectively. The proposed equation is used with the conventional MCC model for predicting the nonlinear behaviour of soil under cyclic loading. This model does not define any additional surfaces to simulate the cyclic response; hence it uses only a minimum number of parameters.

Evaluation of Material Parameters

The physical significance of the five basic critical state parameters along with the corresponding test procedure that was used to determine them has been discussed in the section on theoretical developments. The values adopted for the same were mentioned while explaining the limitations of the model of Carter et al. (1982). The value of the additional parameter, the decay constant, can be determined as described before.

Comparison of Model Predictions with Experimental Results

The improvement in the predictions made by the proposed model over the Carter et al. (1982) model can be observed in the comparison shown in Figure 8. The figure shows the predictions for mean effective stress degradation, for a confining pressure of 200 kPa, frequency of 0.05 Hz and cyclic stress ratio of 0.45. A value of 0.001 is taken as the degradation parameter for the model of Carter et al. (1982). The value of the decay constant used for proposed model is 0.0275. This value is obtained from the chart given in Figure 7. The values for the other model parameters and the source for each are as given earlier in Section 3.1. As seen from the figure, the proposed model seems to make good predictions of the experimental data in comparison with the Carter et al. (1982) model. Some predictions on the pore pressure development and strain development are also made using the new model.

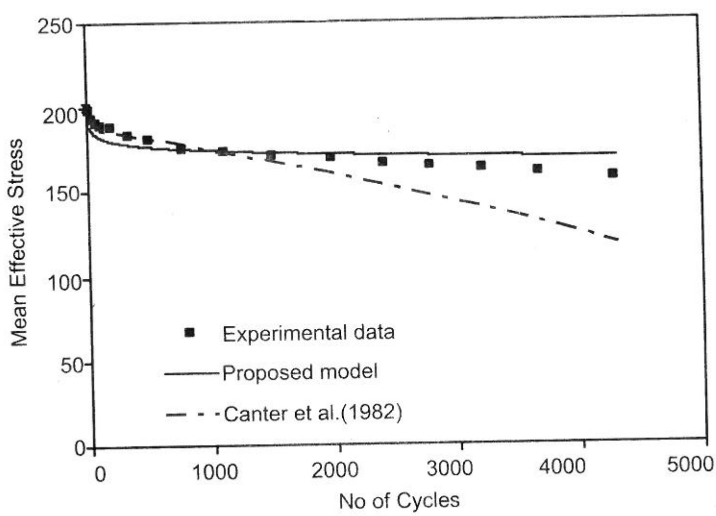


Fig. 8 Comparison of the Proposed Model with Carter et al. (1982) Model

The results for a confining pressure of 200 kPa and a frequency of 0.05 Hz are plotted in Figure 9 and 10 respectively. The decay constants used in this case for the proposed model are 0.0275 (for stress ratio of 0.45) and 0.0348 (for stress ratio of 0.52). These values have been obtained from the chart in Figure 7. The pore pressure and strain development during the cyclic loading of the clay sample tends to stabilize after some cycles. This is evident from the experimental results of Moses (2003), shown in Figure 9 and 10. The new model seems to predict satisfactorily these responses of soil under cyclic loading.

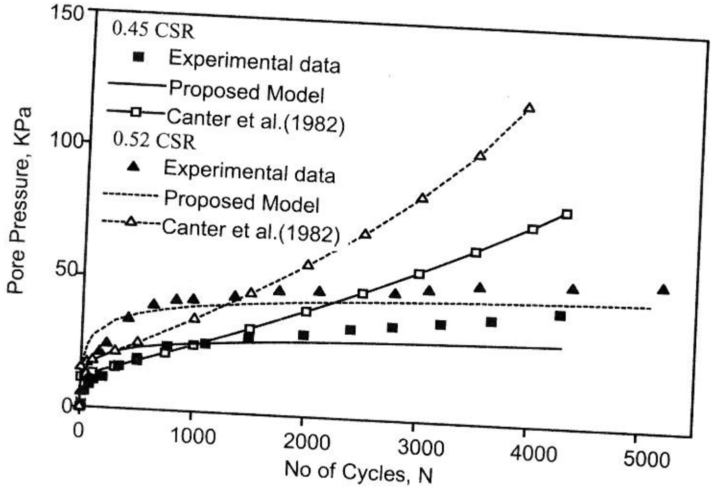


Fig. 9 Prediction of the Variation of Pore Pressure

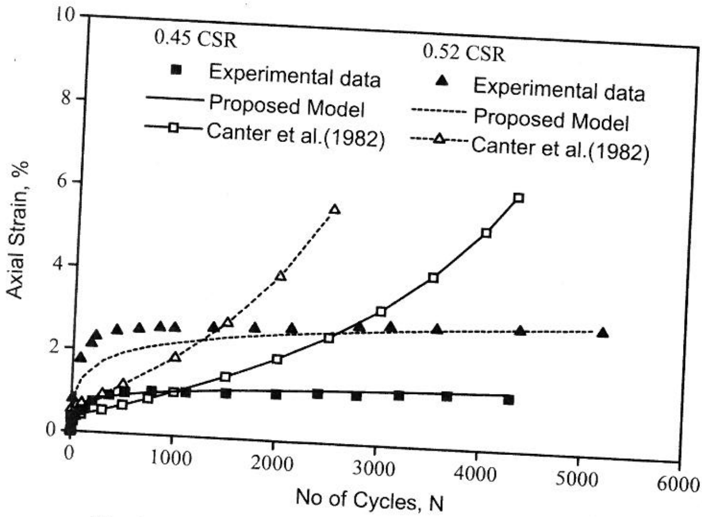


Fig. 10 Prediction of Development of Axial Strain

Conclusions

A model is developed based on the critical state concept, within the mathematical framework of critical state soil mechanics. The model is observed to be capable of predicting many of the observed features of the behaviour of clay when subjected to repeated loading. The various factors affecting the behaviour of clayey soils under cyclic loading, such as the initial confining pressure, the unloading-reloading amplitude and material property are taken into account in developing the model. The model possesses most of the characteristics of the former critical state models but with a simple modification. With the introduction of this modification an additional parameter is required which can easily be determined from the cyclic triaxial tests. This parameter is dependent on confinement, frequency and CSR and hence is capable of reflecting the cyclic loading responses under varied confinement, frequency and CSR.

Calculations have been made using this model and the results have been presented. The proposed model is found to accurately predict the mean effective stress degradation in clayey soils under cyclic loading. Some aspects of the soil behaviour under cyclic loading (such as factors influencing the degradation in pre-consolidation pressure) that was not considered in the Carter et al. model are also addressed herein. The improvement in the predictive capabilities of the model in comparison with the cyclic Cam Clay model published by Carter et al. (1982) is also illustrated.

Notation

G	Shear modulus of soil (kPa)
M	slope of the Critical state line in the q-p plot
P'	Mean effective stress (kPa)
P'_c	Pre-consolidation pressure (kPa)
dW	Work done per unit volume
dv^p	Plastic volumetric strain
$d\varepsilon^p$	Plastic shear strain
e	Void ratio
e_{cs}	Critical state void ratio
m	Degradation parameter
q	Deviatoric stress (kPa)
$\sigma'_1, \sigma'_2, \sigma'_3$	Effective principle stresses (kPa)
v	Volumetric strain
ε	Shear strain
$\varepsilon_1, \varepsilon_2$	Principle strains
η	Stress ratio
λ	Slope of the loading line in the e- ln (p') plot
κ	The slope of the unloading reloading line in the e- ln (p') plot
ϕ'	Effective Angle of Internal Friction (°)

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