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TECHNICAL NOTE

Analysis of Rigid Piles in Clays

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Introduction

Several theoretical methods are available for predicting the behavior of short rigid piles under lateral loads in cohesive soils. (Hansen, 1961; Broms 1964a, 1964b; Poulos 1971;, Briaud et al, 1983a, 1983b; Budhu and Davis,1988 and Rao and Rao, 1995). In this paper, a simple method is proposed to estimate the ultimate lateral load of rigid piles in cohesive soils. The results of the proposed method are compared with published field and model test results and with some of the available methods. Statistical analysis is carried out to analyze the performance of available methods; based on which, the proposed method is found to be more efficient.

Theories

Under a lateral load pile deflects in the direction of load application and the soil in front of the pile offers resistance to this movement. The key for computation of ultimate lateral capacity is the estimation of soil pressure distribution along the length and across the width of the pile and the deflection profile under the applied lateral load. There are guidelines suggested to arrive at the ultimate lateral capacity. Here, an attempt is made to estimate the ultimate lateral capacity, knowing the deflected profile and distribution of various forces acting on the pile. The ultimate lateral load is estimated from the consideration of static equilibrium of the pile.

The analyses are usually based on simplified theoretical assumptions of soil pressure distribution along the pile length. In most of the analyses (Hansen, 1961; Matlock, 1970; Reese et al., 1975), the variation in soil reaction with depth is assumed to vary from $2c_uD$ at ground level to 8 c_uD to $12c_uD$ at a depth of 3D beyond which, it remains constant. Broms (1964a) proposed an earth pressure diagram in which, the soil resistance from the ground level to a depth of 1.5D is ignored and beyond this depth, a uniform pressure of $9.0c_uD$ is considered. As pointed out by Smith (1987), the earth pressure distribution across the pile width is not uniform, as shown in Figure 1.

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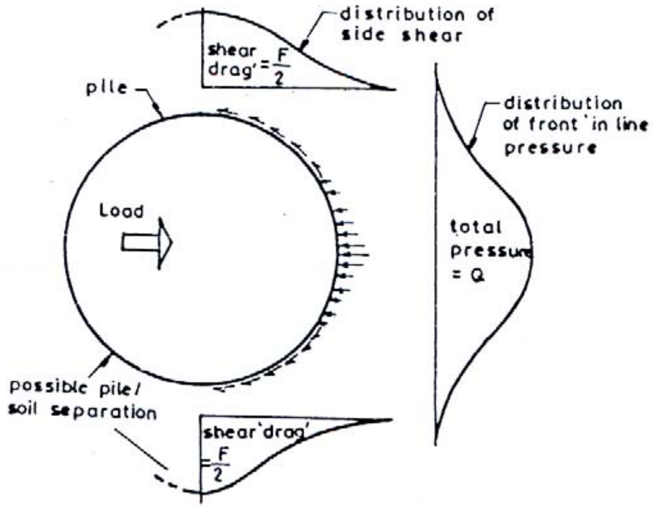

Fig. 1 Distribution of Earth Pressure and Lateral Side Shear Around the Pile (after Smith, 1987)

Following Smith (1987) and Briaud et al (1983a, 1983b), Narasimha Rao et al. (1996) proposed an earth pressure diagram as shown in Figure 2. In this diagram, the soil resistance in the top zone up-to depth 1.5 pile diameter is neglected (Broms 1964). Below this zone, the soil resistance remains constant equal to $9c_u\eta D$. (η is the shape factor to account for the non-uniform distribution of the soil resistance in front of the pile). According to Briaud et al. (1983a), η is taken as

η =1.0 for square piles and (1a)

η = 0.75 for circular piles (1b)

In addition to the soil resistance, Briaud et al (1983a) and Smith (1987) have pointed out side shear component (Figure1). The skin friction developed over a depth equal to 1.5times pile diameter is neglected and below this depth a constant value of $\alpha c_u \beta D$ is assumed. The factor α is introduced to account for the pile soil adhesion and β is the shape factor to account for non- uniform distribution in lateral shear drag.

According to Das and Seely (1982), the average values of α for concrete piles is given approximately by

$\alpha = 0.9 - 0.00625\ c_u$ for $c_u < 80kPa$ (2a)

$\alpha = 0.4$ for $c_u > 80kPa$ (2b)

For aluminium and mild steel piles, the average values of α is given by

$\alpha = 0.715 - 0.0191\ c_u$ for $c_u < 27kPa$ (2c)

α = 0.20 for c_u **>** 27*kPa* (2d)

Smith (1983) reported that, the shape factor β for lateral shear drag could be taken as

β = 0.79 for circular piles and (3a)

β = 1.76 for square piles (3b)

When a lateral load acts on a rigid pile, it can be assumed that, pile rotates about point O at a depth ξL as shown in Figure 2. Broms (1964a) suggested that, point of rotation of rigid short piles occurred between 0.70 to 0.75 times the embedded lengths; where the larger value coincides with the largest lateral loadings. Another possible way to verify this is the finite element method. Varghese (2004) studied the behavior of a single rigid pile in cohesive soils using finite element method. In his analysis he developed a program code in which, pile was modeled as a three-dimensional linear-elastic material and soil was modeled as an elastic-perfectly-plastic material obeying Von Mises yield condition. In addition, soil was assumed incapable of sustaining any tensile stresses.

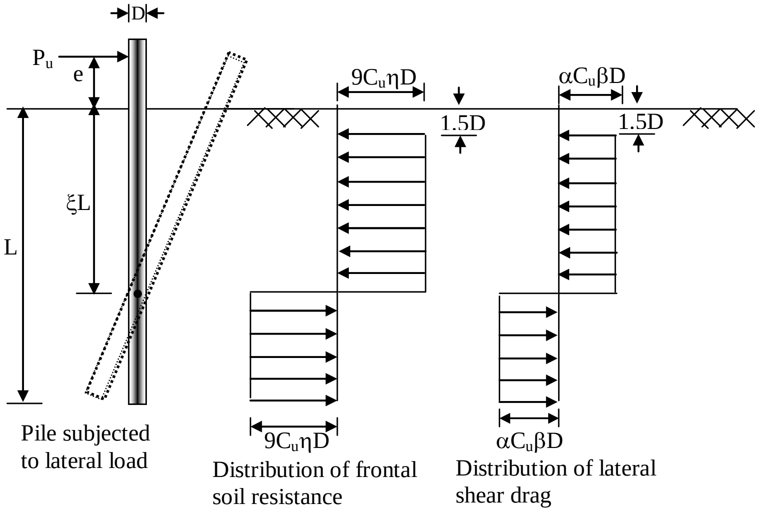

Fig. 2 Distribution of Frontal Soil Resistance and Lateral Shear Drag

Varghese's investigations showed that, free headed short rigid piles of lengths 10m and 20m and flexibility factors ($K_R = (E_pI_p)/(E_sL^4)$) equal to 0.35 and 0.02 respectively behaved like rigid beams, rotating about a point located at 0.75L below the ground line.

Prediction of Ultimate Lateral Capacity

In this investigation, the scope is limited to study the reliability of four methods namely, Broms (1964 a & b), Budhu and Davis (1988) and Rao and Rao (1995) and the proposed method.

Method A (Proposed method)

The earth pressure diagram is taken as shown in Figure 2 (Narasimha Rao et al., 1996) and the location of point of rotation is taken as $\xi L = 0.75L$ (Varghese 2004) below the ground line. Now considering the equilibrium of moments about the bottom of the pile, the ultimate lateral capacity, P_u is calculated as

$$\begin{aligned} P_u\left(L+e\right) = {} & (9c_u\eta D)(\xi L - 1.5D)[L - \xi L + \tfrac{1}{2}(\xi L - 1.5D)] - \\ & (9c_u\eta D)(L - \xi L)\tfrac{1}{2}(L - \xi L) + \\ & (\alpha c_u\beta D)(\xi L - 1.5D)[L - \xi L + \tfrac{1}{2}(\xi L - 1.5D)] - \\ & (\alpha c_u\beta D)(L - \xi L)\tfrac{1}{2}(L - \xi L) \end{aligned} \tag{4}$$

The only unknown parameter in the above equation is P_u. Substituting $\xi L = 0.75L$ and simplifying,

$$P_u = \frac{\left(9c_u\eta D + \alpha c_u\beta D\right)}{L+e}\left[\left(1.125\right)\left(\tfrac{L}{2} - D\right)\left(\tfrac{L}{1.2} - D\right) - L^2/32\right] \tag{5}$$

where,

c_u = un-drained soil shear strength

D = pile diameter

e = load eccentricity

L = pile embedment length

α = soil-pile adhesion factor as per Eqs. 2-a to 2-d

β = shape factor for lateral shear drag as per Eqs.3-a and 3-b

η = shape factor to account for the non-uniform distribution of the soil resistance in front of the pile, as per Eqs. 1-a and 1-b.

Method B (Broms, 1964a)

Ultimate lateral load capacity is obtained using the curves developed by him, which relate the pile embedment ratio L/D to the ultimate lateral soil resistance for various e/D ratios.

Method C (Budhu and Davies, 1988)

The following expression is proposed for lateral capacity calculation..

$$\frac{P_u}{c_u D L^2} = \frac{1.2}{g + 0.88} \tag{6}$$

where,

P_u = ultimate lateral capacity

c_u = un-drained soil shear strength

D = pile diameter

e = load eccentricity and

g = eccentricity ratio = e/L (∞>g>2/3)

L = pile embedment length

Method D (Rao and Rao, 1995)

They proposed the following relationship between P_u/c_uDL and e/L based on a non-linear regression analysis to estimate the lateral capacity of the rigid pile in cohesive soil.

$$P_u = 2.44(0.32)^{e/L} c_u DL \tag{7}$$

where,

P_u = ultimate lateral capacity

c_u = un-drained soil shear strength

D = pile diameter

L = pile embedment length and

e = load eccentricity

Comparison of Measured and Predicted Lateral Capacities

The reliability of an empirical equation or formulation can be established by comparing the estimated results with the test results or the results that are not used in developing the empirical equation. In this investigation, an attempt is made to verify the existing methods for prediction of ultimate lateral resistance by comparing their results with laboratory and field test data. The data of 69 pile load tests collected from the literature is used to evaluate the four methods reported above. The accuracy and precision of each method is examined statistically. The details of pile dimensions, soil properties and observed and predicted lateral capacities are presented in Tables 1 for some of the cases.

In order to predict the performance of a method in a more reliable way, three statistical techniques are used. Before passing judgment on a method, one must always inspect the plot of predicted versus measured ultimate loads. The predicted ultimate loads are plotted against the measured ultimate loads in Figures 3 to 6.

For all the four methods, the mean and standard deviation of the ratio of predicted ultimate load P_{up} to the measured ultimate load P_{um} are calculated. The mean values of (P_{up} / P_{um}) as calculated by Methods A, B, C and D are 1.069, 1.129, 1.166 and 1.063 respectively. In general, all the methods are over predicting the lateral capacity. Methods A and D are found to be in close agreement as compared to Methods B and C. Three statistical techniques are selected to quantify the performance of each method (Goon et al 2000). They are (1) Root square mean method (2) χ^2 method and (3) Statistical hypothesis test (F-test).

Table 1: Comparison of predicted and observed lateral capacities

S. No.	D (mm)	L (mm)	e (mm)	Cu (kPa)	α (deg)	β (deg)	η	Observed capacity, P_{ub}	Proposed method	Predicted capacity, P_{up} Broms Method (1964)	Budhu and Davies (1988)	Narasimha Rao et al. (1995)	Reference of the test data	L/ R
1	6.35	139.7	25.40	38.8	0.2	0.79	0.75	69.5N	74.70N	90.49N	77.79N	114.9N	Druery and Ferguson (1969)	1.4654
2	13.0	260.0	0.0	24.0	0.2566	0.79	0.75	225.0N	206.0N	251.27N	221.23N	197.9N	Meyerhof (1981)	1.434
3	750.0	4500.0	0.0	95.8	0.4	0.79	0.75	454.3kN	499.76 KN	678.98kN	881.79kN	595.2kN	Briaud et al. (1983)	1.654
4	18.0	300.0	50.0	10.0	0.5240	0.79	0.75	116.5N	116.57 N	137.70 N	123.82 N	108.97 N	Rao et al. (1993)	1.30
5	13.5	300.0	50.0	5.5	0.6099	0.79	0.75	50.0 N	51.40 N	53.13 N	51.07 N	44.95 N	Rao et al. (1993)	1.61
6	18.4	300.0	50.0	10.0	0.5240	0.79	0.75	114.0 N	118.54 N	126.62 N	126.57 N	111.39 N	Rao et al. (1993)	1.275
7	25.4	300.0	50.0	5.5	0.6099	0.79	0.75	75.0 N	82.76 N	83.38 N	96.10 N	84.57 N	Rao et al. (1993)	1.02
8	21.5	300.0	150.0	7.2	0.5775	0.79	0.75	72.5 N	74.91 N	83.21 N	80.76 N	100.0 N	Rao et al. (1995)	1.135
9	13.5	300.0	100.0	7.2	0.5775	0.79	0.75	52.5 N	58.67 N	53.14 N	57.68 N	48.67 N	Rao et al. (1996)	1.609
10	12.5	190	0.0	24.0	0.2566	0.79	0.75	128.0N	136.2N	174.38N	155.45kN	127.75 KN	Meyerhof and Yalcin (1984)	1.079

Note: R is the stiffness factor (IS 2911 Part I, Sec.3). For short rigid piles, L/R is less than two. All the piles are free-headed.

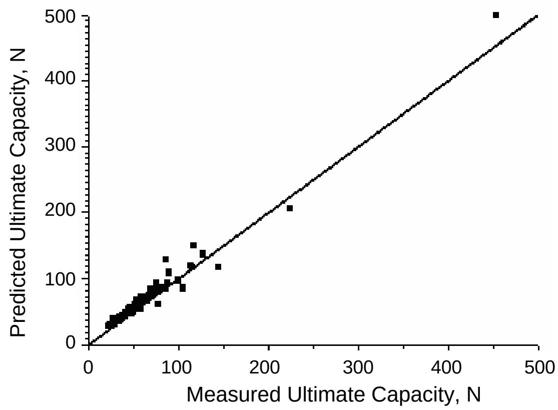

Fig. 3 Predicted Vs Measured Lateral Capacity (Proposed Method)

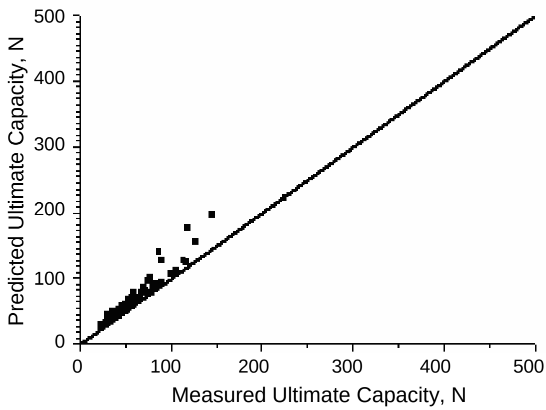

Fig. 4 Predicted Vs Measured Lateral Capacity (Brom's Method)

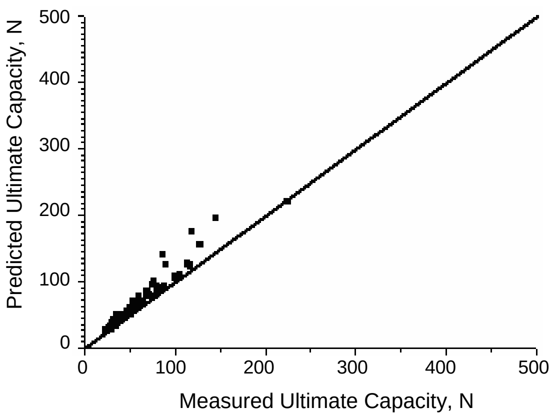

Fig. 5 Predicted Vs Measured Lateral Capacity (Budhu and Davies Method)

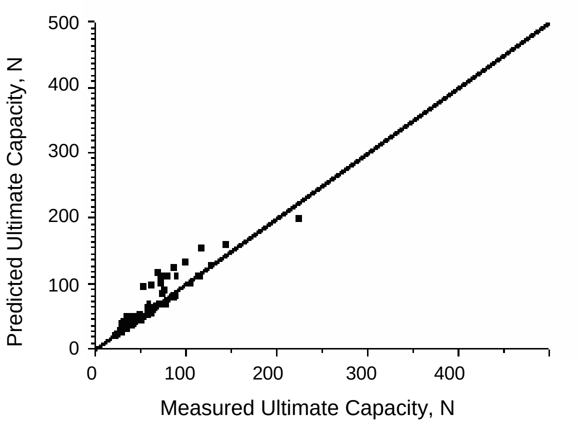

Fig. 6 Predicted Vs Measured Lateral Capacity (Rao and Rao Method)

Root Mean Square Deviation Method

This method measures a total deviation of the response values from the measured values. In order to give a proper idea about the overall nature of the given values of a set of variables, it is necessary besides mentioning the average value, to state how scattered the given values are about the estimated values.

Here root mean square deviation is calculated using the following expression.

$$\sqrt{\frac{1}{n}\sum_{i}\left(P_{um}-P_{up}\right)^{2}} \tag{8}$$

where,

n = number of pile loading test data collected to test the performance of each method

P_{um} = measured ultimate load capacity value of laboratory and field test results reported in the literature

P_{up} = predicted ultimate load capacity value by selected method

Root mean square deviations of Methods A, B, C and D are 28.4, 43.14, 102.63 and 37.64 respectively, from which it can be judged that, proposed method predicts the ultimate load capacity more reliably.

χ^2 Method

Chi square is used most frequently to test the statistical significance of reported results. Maximum and minimum numbers of data points that can be tested using chi-square method are 99 and 2. Here chi-square is computed using the following expression.

$$\chi^2 = \sum_{i=1}^{n} \frac{\left(P_{um} - P_{up}\right)^2}{P_{up}} \tag{9}$$

where,

n = number of pile loading test data collected to test the performance of each method

P_{um} = measured ultimate load capacity value of laboratory and field test results reported in the literature

P_{up} = predicted ultimate load capacity value by selected method.

Chi-squares obtained from Methods A, B, C and D are 160.43, 274.66, 750.31 and 267.57 respectively. The value of χ^2 with a degree of freedom as (n-1) =68 is 90, which is smaller than the values obtained from all the four methods. It is seen that, results of all the methods are significantly varying from the test data. As χ^2 value of Method A is noticeably smaller than other methods, it can be concluded that, the proposed method is performing better than the other methods.

Statistical Hypothesis Test (F-Test)

F- Test also known as variance ratio test given by Volk (1969) is used here for testing the significance of the difference of the difference between two variances. For carrying out the test of significance, the ratio F is calculated using the following formula.

$$F = \frac{\overline{X_1} - \overline{X_2}}{\sqrt{\dfrac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2}}} \tag{10}$$

where,

$\overline{X_1}$ and $\overline{X_2}$ are means of the two samples

σ_1 and σ_2 are the group variances of the two samples and

n_1 and n_2 are sample sizes equal to 69

Since we have four groups of samples, three sets, i) Methods A and B ii) Methods A and C and iii) Methods A and D were considered to calculate F value. The mean and standard deviation for the ratio of predicted ultimate load P_{up} to the measured ultimate load P_{um} are calculated for all the four methods and given in Table 2.

F values of Methods A and B, A and C and A and D are 2.3, 3.74 and 0.105 respectively. These values are compared with the table values of F for ν_1 and ν_2 (ν_1 = degrees of freedom for sample with larger variance and ν_2 = degrees of freedom for sample with small variance) degrees of freedom at 5% level of significance. If the calculated value of F is less than the table value, the null hypothesis is accepted with the inference that, both the samples have come from the population having same variance. If the calculated value of F is greater than the table value, then the F-ratio is considered significant.

Table 2 Calculation of mean and standard deviations for various methods

SR. No.	*Method*	*Mean* $\overline{X}$	*Standard deviation* σ^2
1	A	1.069	0.126
2	B	1.129	0.172
3	C	1.166	0.175
4	D	1.063	0.225

In the present analysis, table F-value is 1.53, which is smaller than F-values of Methods A and B and Methods A and C. Since, the calculated values of F are greater than the table value of F at 5% level of significance, the F of ratios (P_{up} / P_{um}) is considered significant and null hypothesis is rejected. By observing the mean values of P_{up} / P_{um} calculated by Methods A, B and C it can be concluded that, ultimate lateral capacity predicted by the proposed method gives values safer than Brom's and Budhu and Davies methods.

Now considering Methods A and D, since F-value is smaller than table F-value, null hypothesis is accepted (there is no difference between means of the ratios (P_{up} / P_{um}) obtained from the methods). By observing the standard deviations of Methods A and D, it can be concluded that the proposed method gives values safer than Rao and Rao method.

The accuracy of a method is nothing but its ability to predict the measured ultimate load. By observing, the results of all these statistical analysis it can be concluded that, the proposed method gives values that are safer than the other methods considered in this investigation.

Conclusions

Based on the statistical analysis, the proposed method shows a better agreement with the field and model tests data vis-à-vis other methods. The location of point of rotation is considered on the basis of finite element analysis and non-uniform distribution of lateral shear drag as well as pile soil adhesion effect are incorporated in the proposed method which is simple to operate.

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