

# **Effect of Particle Size and Gradation on the Behaviour of Granular Materials Simulated using DEM**

**G. Madhavi Latha<sup>\*</sup> and T.G. Sitharam<sup>\*\*</sup>**

## **Introduction**

**T**he effect of particle size and gradation of granular soils on their strength and volume change behaviour is not clearly understood. Literature gives conflicting statements about the individual effect of each of these parameters. These statements were deduced based on triaxial compression and direct shear tests conducted on different sizes of granular soil samples. Some of the experimental studies indicated that the particle size has no effect on the shear strength of granular soils viz. Bishop (1948), Holtz and Gibbs (1956), Vallergera et al. (1956) and Selig and Roner (1987). The studies reported by Lewis (1956), Dunn and Bora (1972) and Chattopadhyaya and Saha (1981) indicated increase in shear strength with increase in particle size, whereas Rowe (1962), Kirkpatrick (1965), Marsal (1965), Koerner (1970) and Marachi et al. (1972) observed decrease in shear strength with increase in particle size. Considering the studies on the effects of gradation, Rico et al. (1977), Marsal (1965) and Marachi et al. (1972) reported increase in strength with broader gradation in contrast to Leslie (1963) and Susan (1999), who have reported decrease in strength with broader gradation.

Laboratory tests appear to provide the best means of investigation into the behaviour of granular materials but it would be worthwhile to supplement the experimental studies with theoretical research. Theoretical research is particularly advantageous in case on rock fill materials, where experiments on prototype materials become impossible due to size limitations in laboratory equipments. Constitutive modeling, which treats soil as a continuum, has successfully addressed many problems of design and modeling techniques in geomechanics analysis. However, the effects of size and shape of individual particles and the gradation of the system comprising of particles of many sizes on the overall behaviour of the physical system can not be understood using continuum models because the grain level properties can not be modeled. An alternative to the continuum description for particulate mechanics problems is the particle modeling approach, also referred to as discrete element modeling (DEM) that models the material as a collection of individual particles that interact only at inter-particle contact points. A discrete element model consists of a set of

---

\* Assistant Professor, Department of Civil Engineering, Indian Institute of Science, Bangalore, India, E-mail: madhavi@civil.iisc.ernet.in, Tel:+91-80-22933123 Fax:+91-80-23600404

\*\* Professor, Department of Civil Engineering, Indian Institute of Science, Bangalore, India, E-mail: sitharam@civil.iisc.ernet.in, Tel: +91-80- 22932919

particles in which each has an individual collection of attributes (e.g., mass, particle position, velocity) and some constitutive relationships describing the interaction among particles. The particle attributes evolve according to the equations of motion.

This paper studies the effect of particle size and gradation on the strength and volume change behaviour of granular soils through series of numerical experiments on granular assemblies using discrete element modeling. Discrete models have successfully been applied to wide class of problems in Geomechanics. Cundall (1971) is the first one to use particle modeling techniques for evaluating soil and rock mechanics problems. Several researchers have used DEM to model granular assemblies, e.g. Cundall and Strack (1979), Bathurst (1985), Thornton and Branes (1986), Rothenburg and Bathurst (1989), Jenkins et al. (1989), Ng and Dobry (1992), Ting et al. (1993), Chantawarangul (1993), Washington (1996), Sitharam and Nimbkar (1996), Sitharam (1999), Sitharam and Dinesh (2003) and Jensen et al. (1999, 2001)

In this study, the modified version of the computer program TRUBAL [1989] is used for modeling granular soils. The program was originally developed by Cundall and Strack (1979) for modeling the mechanical behaviour of assemblies of spheres in three dimensions using the DEM numerical technique using an explicit finite difference formulation. This program is most widely used to simulate dry granular material.

## Discrete Element Method and Program Trubal

The Discrete Element Method keeps track of the motion of individual particles and updates any contact with neighbouring elements by using a constitutive contact law. The DEM runs according to an explicit time difference scheme. Each calculation cycle includes two stages: the application of a simple interaction law at all particle/particle or particle/wall contacts involving contact force and relative displacement; and the application of Newton's Second Law to determine the particle motion resulting from any unbalanced force. The integration of the law of motion provides the new sphere positions and, therefore, the contact displacements (and velocities). The contact force displacement law is then used to obtain the new contact forces, which are to be applied to the spheres in the next time step.

The program TRUBAL based on discrete element method is a FORTRAN computer code to simulate mechanical (micro and macro) behaviour of three-dimensional assemblies of spheres. The translational motion of a sphere is given by

$$\ddot{u}_i + \alpha \dot{u}_i = \frac{f_i}{m} + g_i \quad (1)$$

where,  $\ddot{u}_i$  = acceleration of the sphere,  $\dot{u}_i$  = translational velocity of sphere,  $\alpha$  = viscous damping constant,  $f_i$  = resultant force acting on the sphere,  $m$  = mass of the sphere and  $g_i$  = acceleration due to gravity.

The equation for the rotation of the sphere is given by

$$\ddot{\theta}_i + \alpha \dot{\theta}_i = \frac{M_i}{I} \quad (2)$$

where  $\ddot{\theta}_i$  = angular acceleration,  $\dot{\theta}_i$  = angular velocity of the sphere,  $M_i$  = moment acting on the sphere and  $I$  = mass moment of inertia.

The equations of motion are integrated by centered finite-difference technique. The loading of the assembly is by distorting the periodic cell. This results in volume change corresponding to strain controlled condition. Therefore the strain of the assembly is the strain of the periodic cell.

The average stress tensor  $\sigma_{ij}$  is obtained from:

$$\sigma_{ij} = \frac{1}{V} \sum_{c \in V} f_i^c l_j^c \quad (3)$$

where  $l_j^c$  = contact vector length,  $f_i^c$  = contact force and  $V$  = volume of the periodic cell.

## Numerical Experiments

The numerical experiments reported in this paper were conducted on three-dimensional test assemblies, each consisting of 1000 frictional spheres. The test assemblies can be broadly divided into two categories: assemblies with uniform size particles and graded assemblies. Tests were carried out on 4 assemblies of uniform size particles, referred as Uni1, Uni2, Uni3 and Uni4 with diameter of spheres 30, 50, 75 and 100 mm respectively and on 7 graded assemblies referred as Grad1 to Grad7 with diameter of spheres varying from 20 mm to 100mm. All of the graded assemblies contain spheres of 21 different radii. The number of spheres of each size was chosen to represent the lognormal particle size distribution curve, which is typical for many natural granular soils. The distribution of the number of particles of different sizes for the test assembly grad1 is shown in Figure 1.

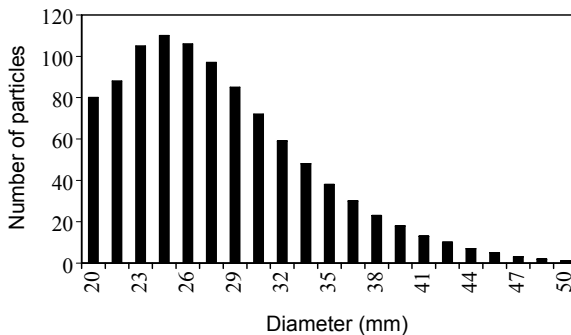
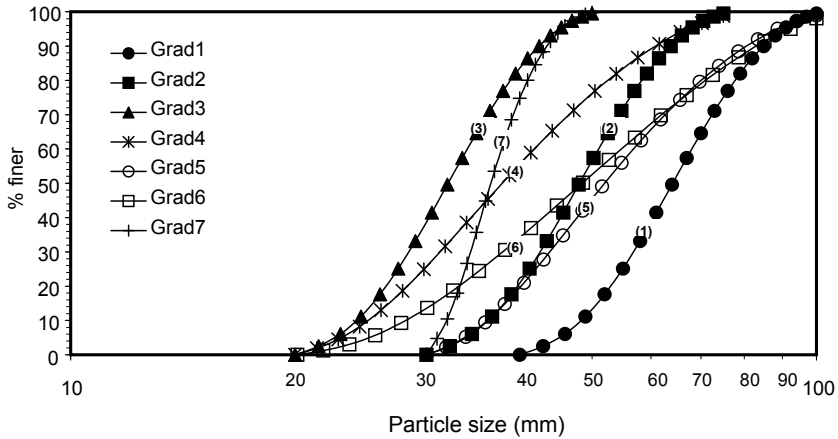
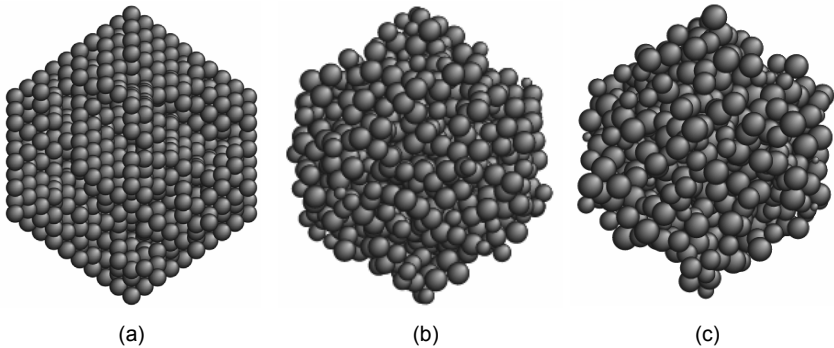


Fig. 1 Distribution of the Number of Particles for the Test Assembly Grad1

The range of particle diameters the program TRUBAL can allow is 20-100 mm. All the gradations were chosen within this range with proper care to utilize the maximum possible range for each gradation. The grain size distribution curves for all the 7 graded assemblies are given in Figure 2. For the assemblies with uniform size particles, the distributions are straight lines parallel to the y-axis (representing % finer) at the respective particle size represented on x-axis. Particle size reported in this study correspond to the diameter of the sphere. The visual representation of some graded particle assemblies is shown in Figure 3.



**Fig. 2 Grain Size Distribution Curves for Graded Test Assemblies**



**Fig. 3 Visual Representation of Graded Spherical Granular Assemblies**  
 (a) Uni 1 (b) Grad 3 (c) Grad 6

Mean Particle size corresponding to 50% finer, maximum particle size, minimum particle size and the sizes of particles corresponding to 10, 30 and 60% finer for different test assemblies are presented in Table 1. Table also gives the coefficient of curvature ( $C_c$ ) and the coefficient of uniformity ( $C_u$ ) for different assemblies. All these samples are poorly graded as per Unified Soil Classification System.

**Table 1 Gradation Properties of Test Samples**

<i>Sample</i>	<i>Mean particle size (mm)</i>	<i>Min Particle size (mm)</i>	<i>Max Particle size (mm)</i>	<i>D<sub>10</sub> (mm)</i>	<i>D<sub>30</sub> (mm)</i>	<i>D<sub>60</sub> (mm)</i>	<i>C<sub>c</sub></i>	<i>C<sub>u</sub></i>
Grad1	64	40	100	48.82	57.06	68.4	1.68	1.17
Grad2	48	30	75	36.18	42.56	57.8	2.17	1.18
Grad3	32	20	50	24.26	28.54	34	1.67	1.18
Grad4	38	20	75	25.18	31.34	42	2.23	1.24
Grad5	52	30	100	36.18	43.9	57.06	2.05	1.21
Grad6	48.6	20	100	28.18	38	51.32	2.46	1.35
Grad7	36.8	30	50	32	34.5	37.8	1.29	1.08
Uni1	30	30	30	30	30	30	1	1
Uni2	50	50	50	50	50	50	1	1
Uni3	75	75	75	75	75	75	1	1
Uni4	100	100	100	100	100	100	1	1

From Figure 2 and Table 1, we can divide the test samples into 4 different groups for comparison of numerical results: samples with perfectly parallel gradations, samples with minimum particle size same, samples with maximum particle size same and samples with uniform size particles, which also have perfectly parallel grain size distributions. These groups are summarized in Table 2. The objective of selecting these groups is to have maximum possible range of gradations within the sizes allowed by the numerical program to clearly bring out the effects of both particle size and gradation on the behaviour of granular soils.

**Table 2 Groups of Sample Gradations**

<i>Name of the group</i>	<i>Set</i>	<i>Samples under the group</i>
Perfectly parallel	1	Grad1, Grad2, Grad3
	2	Grad4, Grad5
Maximum particle size same	1	Grad1, Grad5, Grad6
	2	Grad3, Grad7
	3	Grad2, Grad4
Minimum particle size same	1	Grad3, Grad4, Grad6
	2	Grad2, Grad5, Grad7
Uniform size particles	1	Uni1, Uni2, Uni3, Uni4

Input parameters for numerical simulations are given in Table 3. These parameters were chosen by trial basis to avoid overlapping of spheres during compression and to get stable and accurate results.

**Table 3 Input Parameters Selected for Numerical Simulation**

<i>Property</i>	<i>Value</i>
Normal stiffness ( $K_n$ )	100 kN/m
Shear stiffness ( $K_s$ )	100 kN/m
Density of spheres ( $\gamma_d$ )	2000 kg/m <sup>3</sup>
Cohesion at particle contacts	0.0
Coefficient of interparticle friction ( $\mu$ )	0.0 and 0.5

Mindlin and Deresiewicz (1953) have shown that for elastic spheres, the ratio of the shear stiffness to the normal stiffness ranges from 1 to 1.5 depending on the value of Poisson's ratio. The value of the shear contact stiffness  $k_s$  is taken as equal to the normal contact stiffness  $k_n$  in this study as used by several other researchers in similar studies on granular materials using DEM. Global mass-proportional Rayleigh type damping, which is effective in reducing low frequency motion where the particles of the whole assembly are on the move, is used in the analysis. Samples were subjected to isotropic compression in the first stage to simulate the application of cell pressure in a triaxial test. All samples were brought to equilibrium under a mean isotropic compressive stress of 25 kPa in the first stage. The sample Grad1 is tested at higher confining pressures (50, 100 kPa) also to study the effect of confining pressure on the results. In the second stage, the sample is subjected to deviatoric stress by setting the strain rate of the periodic space. Volume change is allowed in the test to simulate drained condition. The coefficient of interparticle friction ( $\mu$ ) was set zero during isotropic compression to generate dense samples and to a value of 0.5 to generate loose samples. During shearing, invariably the value was set to 0.5. For generating denser samples, interparticle friction is taken as zero in the first stage to allow for smooth sliding of particles, which ensures the densification and homogeneity of the generated test assembly as far as possible. Before shearing, the friction coefficient is set to 0.5 and the system is equilibrated for the new friction coefficient so that friction forces are generated on the dense assembly before it is sheared. Shearing is done as in case of real triaxial experiment.

## Model Sensitivity Analysis

The sensitivity of the model to various input properties is systematically studied in this section to verify the applicability of the model to simulate the behaviour of granular soils.

### Effect of Confining Pressure

To study the effect of confining pressure on model behaviour, the test sample Grad1 was compacted to equilibrium under confining pressures ( $\sigma_3$ ) of 25, 50 and 100 kPa separately and subjected to shearing. The stress paths for these three different tests are shown in Figure 4. By joining the peaks of each line in the figure and the origin, the failure envelope for the sample can be plotted as shown in figure. Thus the sensitivity of model to the confining pressure is established from this study.

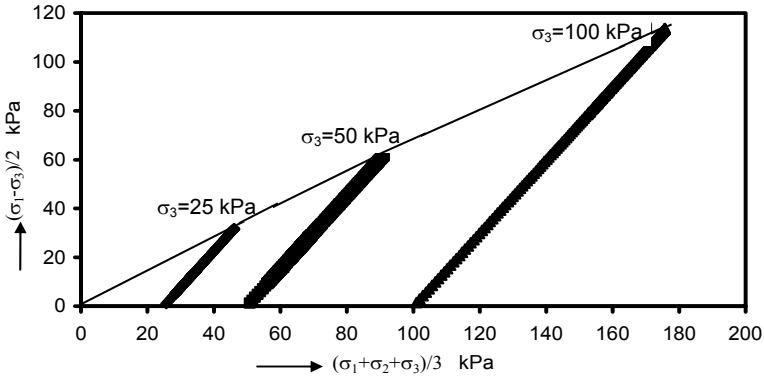


Fig. 4 Stress Paths for Sample Grad1 at Different Confining Pressures

### Effect of Initial Density

To test the model sensitivity for initial density of the sample, the test sample Grad4 was generated to loose and dense conditions under same confining pressure of 25 kPa and results of triaxial shear tests with two different initial densities were compared. For generating loose sample, an interparticle friction ( $\mu$ ) of 0.5 was used during isotropic compression, whereas  $\mu$  was set zero during isotropic compression for generating dense sample. The initial void ratio was 0.473 for dense sample and 1.48 for loose sample and the initial coordination number, which represents the average number of interparticle contacts per particle in the system, was 5 for loose sample and 7.24 for dense sample. The coordination number is arrived from the total number of contacts and the number of particles in the generated assembly. Clearly each physical contact contributes two contacts to the assembly. The average co-ordination number,  $\gamma (= M / N)$  of the assembly is defined as the ratio of the total number of contact points ( $M$ ) within the assembly volume ( $V$ ) to the total number of particles ( $N$ ) in the assembly. The comparison of stress-strain curves and volumetric strain response for loose and dense samples are presented in Figures 5 and 6 respectively.

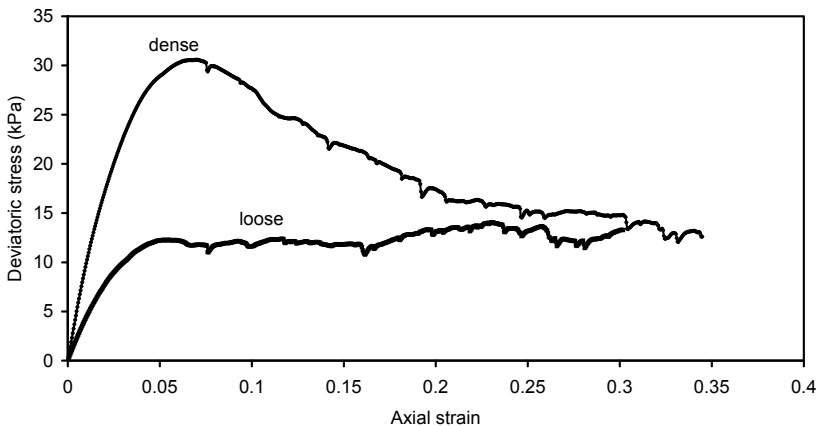
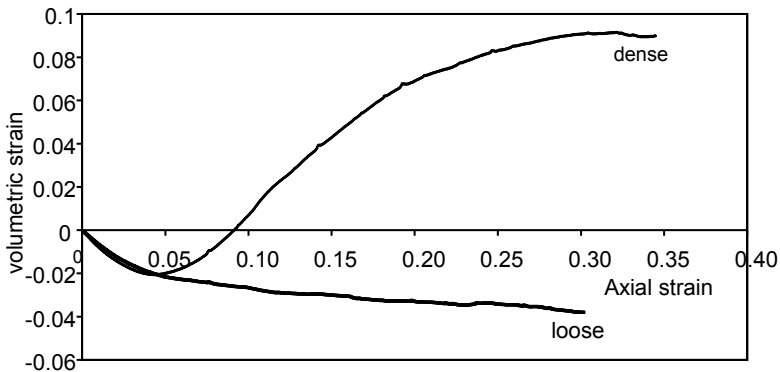


Fig. 5 Stress-strain Curves for Sample Grad4 in Dense and Loose Condition



**Fig. 6 Volumetric Strain Response of Sample Grad4 in Dense and Loose Conditions**

### Effect of Interparticle Friction

The sensitivity of the model to the interparticle friction ( $\mu$ ) is studied by using two different  $\mu$  values for isotropic compaction for samples Grad3, Grad4 and Grad6. Initial porosity of both the samples is 0.727. Coordination number is defined as the number of contacts per particle. It was observed that increasing interparticle friction results in decrease in average coordination number attained by the sample at the end of isotropic compression at higher confining pressures. At very low confining pressures ( $<10$  kPa), the trend is reverse, because the number of sliding contacts will be more when there is no interparticle friction. However, with increase in confining pressure, there will be reduction in number of sliding contacts and hence if the  $\mu$  value is less, particles move close easily making more number of contacts and average coordination number increase rapidly. These results are in conformation with the observations of Thornton (1996) regarding the effect of interparticle friction on the number of sliding contacts, demonstrating that the effect of  $\mu$  is well represented by the numerical model.

### Results and Discussion

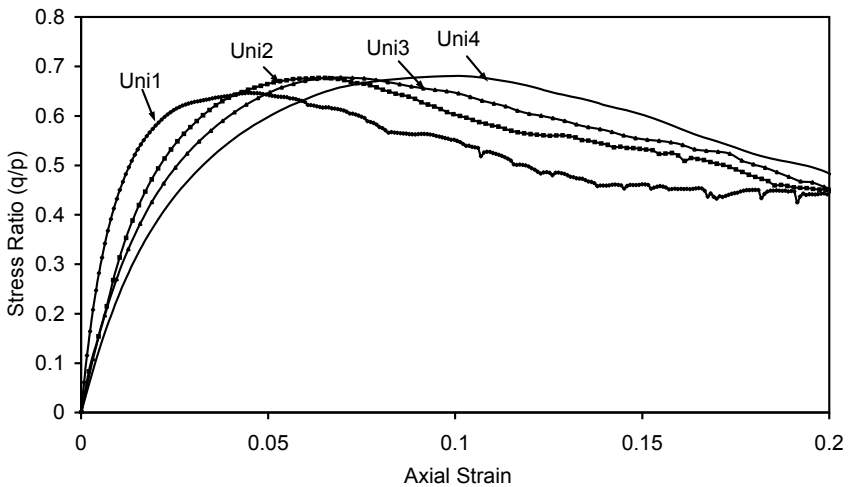
The objective of present study is to understand the effect of particle size and gradation on the shear strength of granular soils measured in terms of angle of internal friction ( $\phi$ ). The influence of particle size is studied from the numerical experiments on different groups of samples having perfectly parallel gradation with particles of different sizes and also on samples with uniform size particles within a sample and particle size varying for different samples. The effect of gradation is studied through experiments on samples with different grain size distributions, considering the groups with minimum particle sizes same and groups with maximum particle size same in order to test different combinations of wide and narrow gradations. Results from this study are presented and discussed in the following subsections.



## Effect of Particle Size

### Stress-strain behaviour

Figure 7 gives the stress ratio ( $q/p$ ) vs. axial strain curves for different samples with uniform size particles (particle size ranging from 30 to 100 mm) tested at a confining pressure of 25 kPa. In this figure,  $q$  is  $(\sigma_1 - \sigma_3)/2$  and  $p$  is  $(\sigma_1 + \sigma_2 + \sigma_3)/3$ , where  $\sigma_1$  is the vertical normal stress and  $\sigma_2$  and  $\sigma_3$  are the horizontal normal stresses. It can be observed from the figure that the peak stress ratio is almost same for all these samples. The only difference in the behaviour is samples with smaller size particles reached peak stress at very low deviator strains, whereas the failure strain increased for samples with bigger size particles. The failure strains for samples with particle sizes 30, 50, 75 and 100 mm, as observed from the figure are 4.5%, 6%, 7% and 10% respectively.

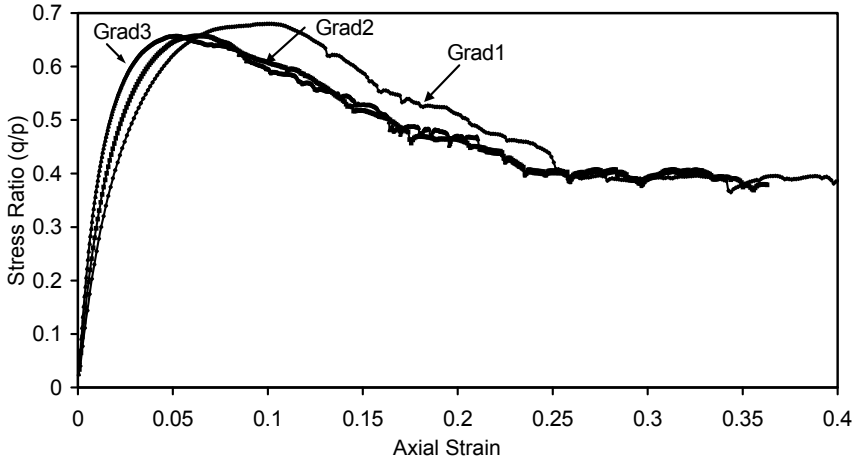


**Fig. 7 Stress Ratio Vs. Axial Strain for Samples with Uniform Size Particles**

The axial strain vs. strain ratio curves for parallel graded samples Grad1, Grad2 and Grad3 are given in Figure 8. There is no significant change in the behaviour of these three samples except that the failure strain increased with particle size in this case also. The range of particle size in test samples Grad1, Grad2 and Grad3 is 40-100, 30-75 and 20-50 respectively and the failure strain for these samples is 11%, 6% and 5% respectively. Thus the failure strain increased with increase in particle size. Similar observations were made for the other parallel graded group of samples 4 and 5.

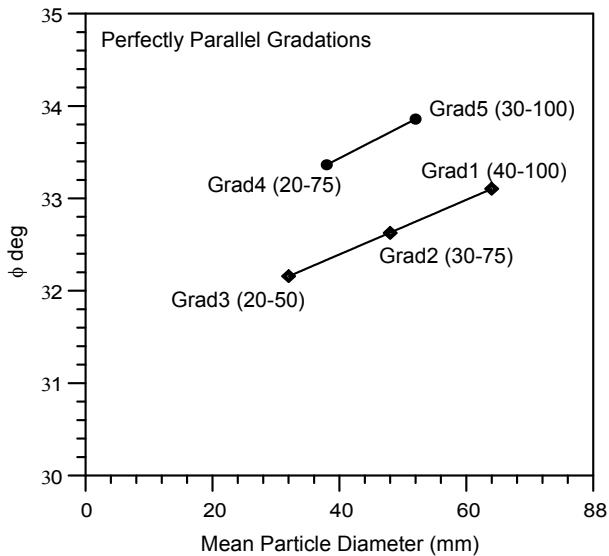
Careful examination of Figures 7 and 8 reveals that all samples reached a state of constant volume, called critical state at very large strains (about 40%). The value of  $\phi$  at residual strength corresponding to constant volume state for all the samples tested in the confining stress range of 25 to 100 kPa irrespective of the size and gradation of particles is found out to be about 20.5. The residual friction angles obtained in this study are less than the interparticle friction angle used in the analyses ( $\phi_{it} = 26.5^\circ$ ). Thornton (2000) indicated that such small

values of residual friction angles are normal with models that ignore the possibility of particle rolling resistance at contacts.



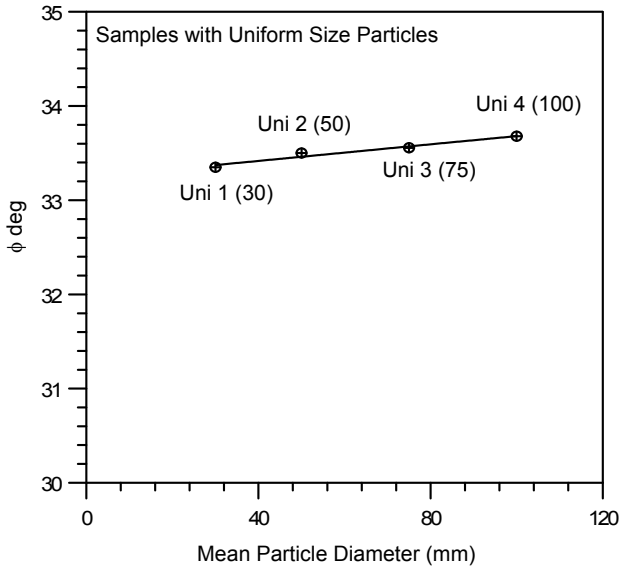
**Fig. 8 Stress Ratio Vs. Axial Strain for Samples with Parallel Gradation**

The variation of peak angle of internal friction ( $\phi$ ) with mean particle size for the two groups of parallel graded samples, Grad1, Grad2 & Grad3 and Grad4 & Grad5 is presented in Figure 9. The minimum and maximum particle sizes for each gradation in mm are indicated in brackets. From the figure it is observed that the shear strength measured in terms of peak angle of internal friction ( $\phi$ ) slightly increased with increase in particle size for parallel graded samples. All the samples shown in Figure 9 are graded samples.



**Fig. 9 Variation of  $\phi$  with the Mean Particle Size for Parallel Graded Samples**

To separate the effects of size and gradation and to bring out the individual influence of size of the particles on strength, the results from tests on groups of samples with uniform particle size are shown in Figure 10. Figure shows that the angle of internal friction remained almost constant with the variation in particle size among the group of samples with uniform size particles. The value of  $\phi$  is 33.35 for the sample Uni1 in which the size of the particles was 30 mm and it increased slightly with increase in particle size, the increase being insignificant. The values of  $\phi$  observed are 33.5, 33.55 and 33.68 for particle sizes 50, 75 and 100 mm respectively. This slight increase in the peak angle of internal friction with increase in particle size could be attributed to the increase in contact area with increase in particle size. But this effect is insignificant and for all practical purposes, we can consider that the size of the particles has no influence on the shear strength of granular soils.

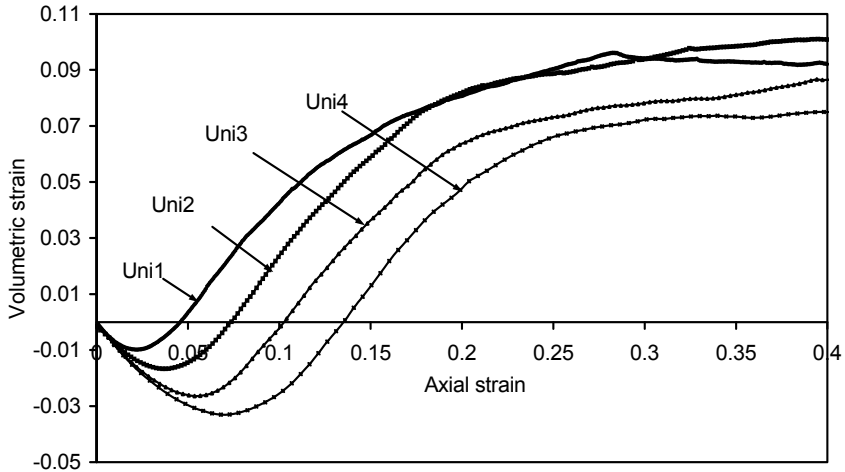


**Fig. 10 Variation of  $\phi$  with the Mean Particle Size for Samples with Uniform Particle Size**

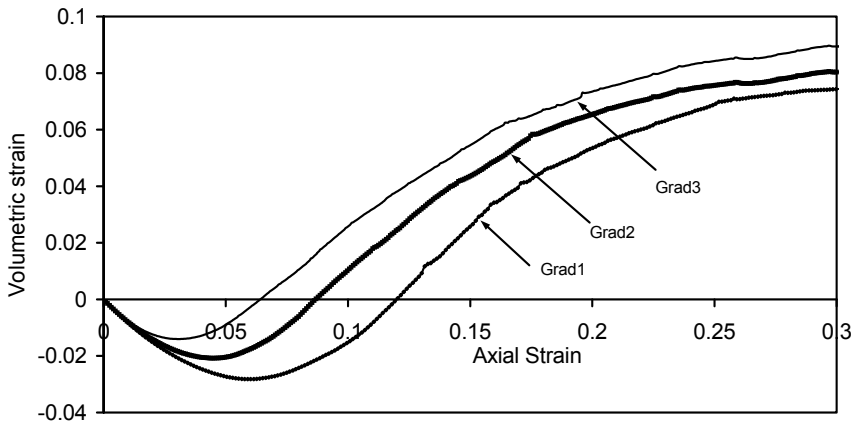
### ***Volume change behaviour***

The volume change behaviour of samples with uniform size particles during shearing is shown in Figure 11. As all the samples are dense, the samples contracted initially and then started to dilate. This is because of the mean stress level during the tests that suppresses the dilative behaviour of the assembly at the beginning. It can be observed from the figure that among the samples Uni1, Uni2, Uni3 and Uni4, the sample Uni4 having bigger particles (size 100 mm) is exhibiting higher compressibility and thus expected to give higher load carrying capacity compared to other samples at any particular deviatoric strain. It has exhibited a maximum volume compression of 3.3% before undergoing dilation. The compressibility of the sample has decreased with decrease in particle size and the dilatancy increased. The samples Uni4 having particles of size 100 mm, Uni3 having particles of size 75 mm, Uni2 containing particles of size 50 mm and Uni1 containing particles of size 30 mm

have exhibited volume compression of 3.3%, 2.6%, 1.7% and 1% respectively. Also the deviatoric strain at which the deformation of the sample changed from compression to dilation is increased with increase in particle size: 2.3, 4, 6 and 7% respectively for samples Uni1, Uni2, Uni3 and Uni4. This variation in behaviour is due to the difference in arrangement of grains due to slight variation in the initial void ratio of different samples.



**Fig. 11 Volume Change Behaviour of Samples with Uniform Size Particles during Shear in Triaxial Test**



**Fig. 12 Volume Change Behaviour of Parallel Graded Samples during Shear in Triaxial Test**

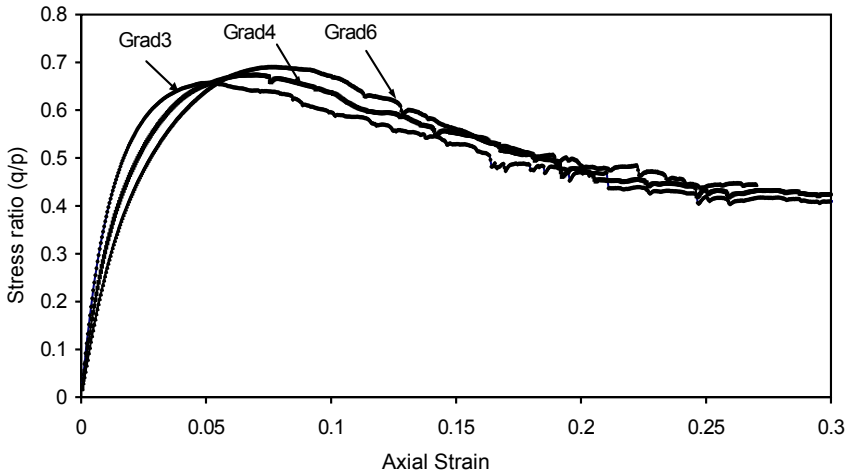
The test samples with bigger size particles have lower initial void ratio due to overlapping of particles resulting from increase in contact area between any two particles. Note that the systems are in equilibrium and the percentage overlap is less than 1% of the diameter of the sample in any case. As the initial void ratio for samples is decreasing with the increase in particle size, the samples with bigger size particles are exhibiting slightly higher compressibility and strength compared to those with smaller size particles.

The variation of volumetric strain with deviatoric strain for the parallel graded samples is shown in Figure 12. These samples also exhibited similar behaviour of slight increase in compressibility and increase in the deviatoric strain at which the samples started undergoing dilation with the increase in particle size.

### Effect of Particle Size Distribution

#### *Stress-strain behaviour*

The effect of particle gradation on the stress-strain behaviour of granular soils is studied by conducting a series of numerical triaxial tests on samples with different gradations, keeping the minimum or maximum particle size same to facilitate the comparison of the influence of wider or narrower gradations on the behaviour of these soils. The groups of soil samples identified for this study with same maximum or minimum particle size are given in Table 2. The variation of stress ratio ( $q/p$ ) with axial strain for samples Grad3, Grad4 and Grad6 are given in Figure 13. The size of the smallest particle in the sample is same (20 mm) for these samples and the gradation is narrower for Grad3 with maximum particle size of 50 mm, slightly wider for Grad4 with maximum particle size of 75 mm and wider for Grad6 with maximum particle size of 100 mm (ref. Figure 2).



**Fig. 13 Stress Ratio vs. Axial Strain for Samples with Different Gradations**

From the figure it can be observed that the peak stress ratio is high for samples with wider gradation. The axial strain at which the sample attained the peak stress has decreased for samples with narrow gradation. Similar observations were made in case of other group of samples (Grad2, Grad5 and Grad7) with same minimum particle size. The shear strength of samples in these groups measured in terms of  $\phi$  is plotted with respect to the mean particle size and shown in Figure 14. The values given in brackets for each sample represent the minimum and maximum particle sizes in mm.

By examining the figure, it is understood that the shear strength of granular materials with wider gradation is more when compared to the narrower gradation. This aspect can be more clearly understood by analyzing the results

from tests on samples with same maximum particle size and different gradation also. The variation of  $\phi$  with mean particle diameter for this group of samples is given in Figure 15. These results also clearly indicate the effect of gradation on the strength of granular soils. All the three groups of samples exhibited identical behaviour of increase in shear strength with the widening of particle size distribution curve.

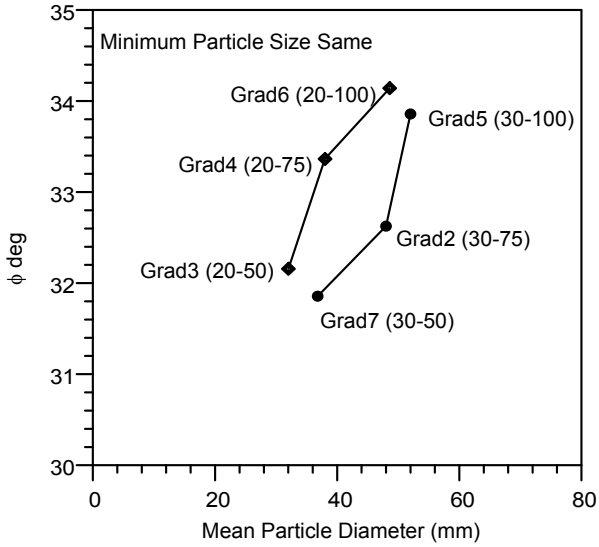


Fig. 14 Variation of  $\phi$  for Samples with Same Minimum Particle Size

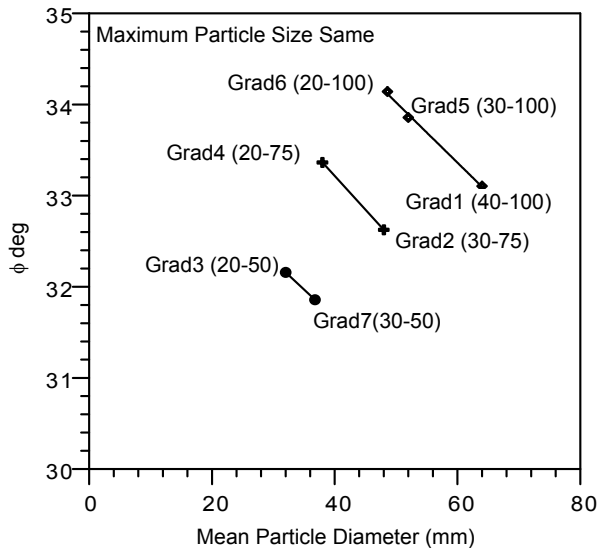


Fig. 15 Variation of  $\phi$  for Samples with Same Maximum Particle Size

Another important observation made from Figure 15 is that the apparent increase in strength with increase in particle size as observed from the results of tests on parallel graded samples and on samples with same particle size is masked by the effect of gradation in this set of tests. A steep decrease in strength with increase in particle size is observed for all the cases. For example, among the group of samples Grad1, grad5 and Grad6, the size of particles ranged from 40-100 mm for Grad1, 30-100 mm for Grad5 and 20-100 mm for Grad6. The sample Grad6 has exhibited higher strength irrespective of the smaller particles. As the Grain size distribution curve moved rightwards from Grad6 to Grad1, the size of particles increased but the angle of internal friction decreased because the gradation curve became narrow (ref. Figure 2). Thus by comparing the results of different tests on samples with different gradations from Figures. 9, 10, 14 and 15, it can be concluded that the size of particles has insignificant effect on the strength of the granular soils. It is the gradation that is important in causing the variation in strength. Samples with wider gradation exhibit higher strength as a result of decrease in void ratio due to compact packing possible with wider gradation.

All the earlier studies reported in this section are carried out on dense samples generated without any interparticle friction ( $\mu$ ) during isotropic compression. To check the validity of the results for loose samples, similar tests were carried out on loose samples of Grad3, Grad4 and Grad6 generated with  $\mu$  value of 0.5 during isotropic compression. The variation of peak angle of internal friction with mean particle size for these samples is presented in Figure 16. Results from this series of tests also shown increase in strength with widening of gradation.

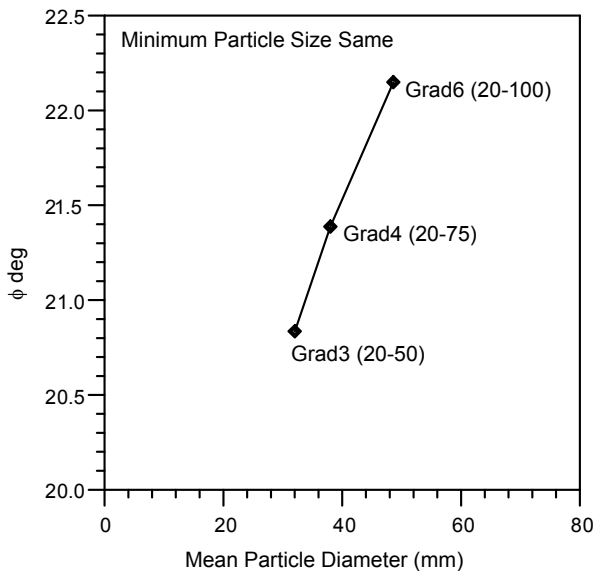
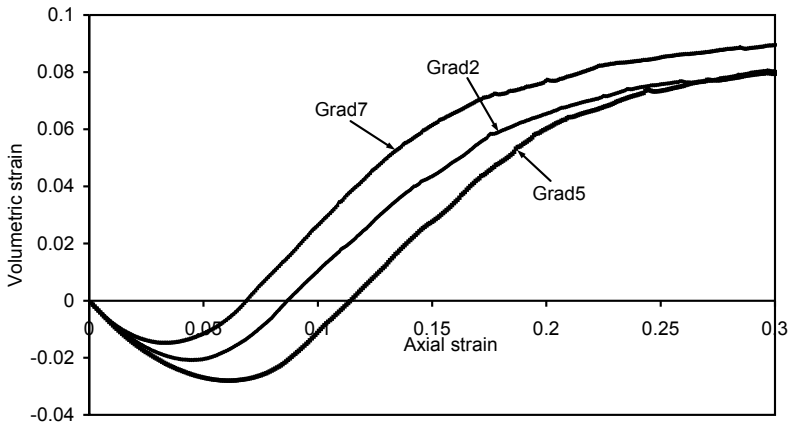


Fig. 16 Variation of  $\phi$  for Loose Samples with Same Minimum Particle Size

### Volume change behaviour

The volume change response of a group of dense samples with different gradations during shearing is shown in Figure 17. Among the three samples Grad2, Grad5 and Grad7, sample Grad5 with wider gradation has exhibited highest compressibility compared to the other two samples and the deviatoric strain at which the sample starts dilating increased for wider gradations. This is because of the smaller void ratio of samples with wider gradation due to compact packing possible as the range of particles available is more. Similar behaviour was observed for other set of samples (Grad3, Grad4 and Grad6) under this group in dense as well as in loose conditions.



**Fig. 17 Volume Change Behaviour of Dense Samples with Different Gradations During Shear in Triaxial Test**

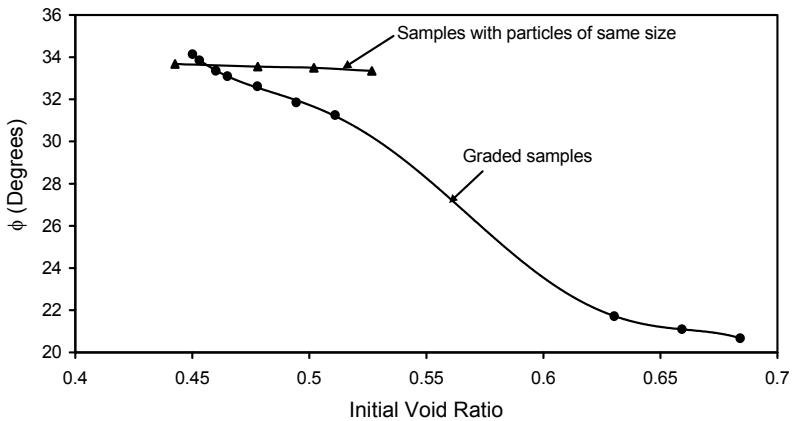
### Effect of Initial Void Ratio

Studying the individual effects of size and gradation of particles often becomes difficult because change in size or gradation of particles also involve simultaneous change in void ratio. In this section, the effect of initial void ratio of different test samples irrespective of particle size and gradation on their shear strength is discussed. The initial void ratios and the peak angles of internal friction ( $\phi$ ) for all the test samples are presented in Table 4. The variation of  $\phi$  with respect to the initial void ratio of samples is shown in Figure 18. It is interesting to note that the samples with particles of same size exhibit almost same shear strength irrespective of the initial void ratio of the sample. This observation further reinforces the conclusion that the particle size has no effect on the shear strength of granular materials. However, for graded samples, the strength decreased with increase in the initial void ratio. The curve representing the initial void ratio vs.  $\phi$  value followed a nonlinear path as shown in figure. As all the graded samples fall in this path irrespective of the particle size and gradation, it is very clear that the major parameter that influences the strength of granular materials is the initial void ratio rather than the size and distribution of particles.



**Table 4 Initial Void Ratio and Peak Angles of Internal Friction for all Test Samples**

Sample	Initial void ratio	Peak angle of internal friction ( $\phi$ )
Uni1	0.527	33.35
Uni2	0.502	33.50
Uni3	0.478	33.55
Uni4	0.443	33.68
Grad1	0.465	33.10
Grad2	0.478	32.62
Grad3	0.494	31.85
Grad4	0.460	33.36
Grad5	0.453	33.86
Grad6	0.450	34.14
Grad7	0.511	31.26
Grad3 (loose)	0.684	20.66
Grad4 (loose)	0.659	21.10
Grad6 (loose)	0.630	21.72

**Fig. 18 Variation of  $\phi$  with initial void ratio of test samples**

## Comparison with Experimental Results

From the analysis of results from the present study, the major observations that can be made are: The shear strength of granular soils is not affected by the size of the particles but the volume change behaviour is slightly affected with the variation in size of particles. Particle gradation significantly influences both the strength and volume change behaviour of granular soils.

As literature gives contradictory statements regarding the influence of particle size and gradation on the behaviour of granular soils, the selection of experimental studies for comparing with numerical studies should be done with caution. The experimental studies reported by Bishop (1948), Holtz and Gibbs (1956), Vallerga et al. (1956) and Selig and Roner (1987) indicated that the effect of particle size on the shear strength of granular soils is insignificant. The results from present study are in agreement with this statement. However, Rowe (1962), Kirkpatrick (1965), Marsal (1965), Koerner (1970) and Marachi et al. (1972) observed decrease in shear strength with increase in particle size. The decrease in strength with increase in particle size can be due to the higher confining pressures used in experiments and also due to the microfissures that are dominant in larger size particles, because of which particle crushing occurs during triaxial test. In the present study, the fissures on the surface of the particles are not modeled and also the particle crushing is not taking place. Also, these tests are done on graded samples. Hence effect of size cannot be completely separated from the effect of gradation. On the other hand, Lewis (1956) conducted series of triaxial tests on crushed granite samples of different particle sizes, each sample containing particles of same size without grading, exactly similar to the samples Uni1 to Uni4 in the present study. Results from this study indicated slight increase in friction angle with increase in particle size, as observed from the present study.

Most of the researchers e.g. Rico et al. (1977), Marsal (1965) and Marachi et al. (1972) reported increase in strength with broader gradation as observed from the present study. Leslie (1963) reported contradictory results about the effect of gradation for different series of tests and concluded that further tests with different gradation and samples are needed for better understanding. However, widening the gradation by increasing the range of particle sizes should increase shear strength as a result of decreased void ratio due to compact packing.

## Conclusions

The effect of particle size and gradation on the stress-strain and volume change behaviour of granular soils is studied through systematic series of numerical experiments. Results from numerical triaxial tests on samples with uniform size particles and also on parallel graded samples indicated that the particle size has no effect on the strength of granular materials. Irrespective of the initial void ratio, samples with particles of same size exhibit same shear strength. The volume change response of samples during isotropic compression is independent of the particle size. However, Samples with bigger size particles exhibit higher compressibility resulting in slightly higher load carrying capacity at any particular deviatoric strain during triaxial tests and also the failure strain increase slightly with increase in particle size.

Gradation of the particles significantly influences the strength and volume change behaviour of granular soils. Widening the gradation curve by keeping same minimum or maximum particle size increases the shear strength irrespective of the initial density of the sample. The volume change behaviour during isotropic compression is also different for different gradations, the volumetric compression for any value of mean isotropic compressive stress being small for wider gradations due to low initial void ratio resulting from compact packing because of large range of particle sizes available. Also the compressibility of samples and the failure deviatoric strain increased for wider gradations.

Initial void ratio of the samples is the major factor that influences the shear strength of granular soils. The  $\phi$  values of samples with the initial void ratio of the samples follow a unique nonlinear trendline, suggesting that initial void ratio is the single parameter that can effectively replace the individual effects of size and gradation of the granular soils.

## References

Bathurst, R.J. (1985). *A study of stress and anisotropy in idealized granular assemblies*. Ph.D. Thesis, Queen's University, Kingston, Ontario.

Bishop, A.W. (1948). "A large shear box for testing sand and gravels." *Proc., 2nd Int. Conf. on Soil Mech. and Found. Engg.*, Vol.1, pp. 207-211.

Chantawarangul, K. (1993). *Numerical simulations of three-dimensional granular assemblies*. Ph.D. Thesis, University of Waterloo, Ontario, Canada.

Chattopadhyaya, B.C. and Saha, S. (1981). "Effect of grain size of cohesionless soil on its shear strength characteristics." *Proc. Geomech-81, Symp. on Engrg. behaviour of coarse grained soils*, Boulders and Rocks, Hyderabad, India, pp. 41-46.

Cundall, P. A. and Strack, O. D. L. (1979). "Discrete numerical model for granular assemblies." *Géotechnique*, 29, pp. 47-65.

Cundall, P.A. (1971). "A computer model for simulating progressive, large-scale movements in blocky rock systems." *Proc. Symp. of Soc. Rock. Mech.*, Nancy, France, Vol. 2, pp. 129-136.

Dunn, S. and Bora, P.K. (1972). "Shear strength of untreated road base aggregates measured by variable lateral pressure triaxial cell." *J. of Mechanics*, 7(2), pp. 131-142.

Feda, J. (1982). *Mechanics of particulate materials: The principles*, Elsevier Scientific Publishing Company, New York.

Holtz, W. G. and Gibbs, H. J. (1956). "Triaxial shear tests on previously gravelly soils." *J. Soil Mech. and Found. Engng.*, ASCE, 82, SM1, pp. 1-22.

Jenkins, J.T., Cundall, P.A. and Ishibashi, I. (1989). "Micromechanical modeling of granular materials with the assistance of experiments and numerical simulations." *Powders and Grains*, J. Biarez, and Gourvès, eds., Balkema, Rotterdam, pp. 257-264.

- Jensen, R.P., Bosscher, P. J., Plesha, M. E. and Edil, T. B. (1999). "DEM simulation of granular media-structure interface: effect of surface roughness and particle shape." *Int. J. Numer. and Analytical Methods in Geomech.*, 23, pp. 531-547.
- Jensen, R.P., Edil, T. B, Bosscher, P. J., Plesha, M. E. and Kahla, N. B. (2001). "Effect of particle shape on interface behavior of DEM-simulated granular soils." *Int. J. Geomech.*, 1(1), pp. 1-19.
- Kirkpatrick, W. M. (1965). "Effects of grain size and grading on the shearing behaviour of granular materials." *Proc., Sixth Int. Conf. on Soil Mech. and Found. Engg.*, Montreal, Quebec, Canada, Vol. 1, pp. 273-276.
- Koerner, R. M. (1970). "Effects of particle characteristics on soil strength." *J. Soil Mech. and Found. Div.*, ASCE, 96(4), pp. 1221-1234.
- Leslie, D. D. (1963). "Large scale triaxial tests on gravelly soils." *Proc., Second Pan American Conf. on Soil Mech. and Found. Engrg.*, Brazil, pp. 181-202.
- Lewis, J. G. (1956). "Shear strength of rockfill." *Proc., 2nd Australia-New Zealand Conf. on Soil Mech. and Found. Engg.*, Christchurch, pp. 50-52.
- Marachi, N. D., Chan, C. K. and Seed, H. B. (1972). "Evaluation of properties of rockfill materials." *J. Soil Mech. and Found. Div.*, ASCE, 98(1), pp. 95-113.
- Marsal, R. J. (1965). "Large scale testing of rockfill materials." *J. Soil Mech. and Found. Div.*, ASCE, 93(2), pp. 27-43.
- Mindlin, R. D., and Deresiewicz, H. (1953). "Elastic spheres in contact under varying oblique forces". *J. Appl. Mech.*, 20, pp. 327-344.
- Ng, T. T. and Dobry, R. (1992). "A non-linear numerical model for soil mechanics." *Int. J. Numer. and Analytical Methods in Geomech.*, 16, pp. 247-263.
- Rico, A., Orozco, J. M. and Aztegui, T. T. (1977). "Crushed stone behaviour as related to grading." *Proc., Ninth Int. Conf. on Soil Mech. and Found. Engrg.*, Tokyo, Japan, Vol. 1, pp. 263-265.
- Rothenburg, L. and Bathurst, R. J. (1989). "Analytical study of induced anisotropy in idealized granular materials." *Géotechnique*, 39(4), pp. 601-614.
- Rowe (1962). "The stress-dilatancy relation for static equilibrium of an assembly of particles in contact." *Proc. of Royal Society of London*, 269, pp. 500-527.
- Schofield, A. and Wroth, P. (1968). *Critical state soil mechanics*, McGraw-Hill, New York.
- Selig, E. T and Roner, C. J. (1987). "Effect of particle characteristics on behaviour of granular material." *Transportation Research record*, 1131, pp. 1-6.
- Sitharam, T. G. (1999). "Micromechanical modeling of granular media: The power of discrete element modeling." *Distinct Element Modeling*, V. M. Sharma, K. R. Saxena, and D. M. Woods, eds., OXFORD-IBH Publishing company, New Delhi, pp. 47-88.

Sitharam, T. G. and Nimbkar, M. S. (2000). "Micromechanical modelling of granular materials: Effect of particle size and gradation." *Geotech. and Geolog. Engg.*, 18, pp. 91-117.

Sitharam, T.G. and Dinesh S.V. (2003). "Numerical Simulation of Liquefaction Behaviour of Granular Materials Using Discrete Element Method", Proc. Indian Academy of Sciences. (Earth and planetary sciences), 112 (3), pp. 479-484

Susan, K. (1999). *Shear strength and volume change characteristics of granular materials and aggregate mixtures*. M.Sc. Thesis, Indian Institute of Science, Bangalore, India.

Thornton C. (2000). Numerical simulation of deviatoric shear deformation of granular media. *Geotechnique*, 50(1), pp. 43–53.

Thornton (1996). "From contact mechanics to particulate mechanics." *Solid-Solid interactions*, H. Adams, Briscoe and Biswas, eds., Imperial College Press, pp. 250-264.

Thornton, C. and Branes, D. J. (1986). "Computer aided deformation of compact granular assemblies." *Acta Mech.*, 64(1), pp. 45-61.

Ting, J.M., Khwaja, M., Meachum, L.R. and Rowell, J.D. (1993). "An Ellipse\_based Discrete Element Model for Granular Materials." *Int. J. Numer. and Analytical Methods in Geomech.*, 17(9), pp. 603-623.

Trubal (1989) User's manual for program Trubal Version 1.5.1.

Vallerga, B. A., Seed, H.B., Monismith, C. L. and Cooper, R. S. (1956). "Effect of shape, size and surface roughness of aggregate particles on the strength of granular materials." *Road and paving materials*, Special Tech. Publication 212, ASTM, pp. 63-76.

Washington, D. W. (1996). *Discrete element modeling of dry granular material using a massively parallel supercomputer*. Ph.D. Thesis, New Jersey Institute of Technology, Newark, N.J.