Indian Geotechnical Journal, 37(3), 2007, 159-173

Estimation of Maximum Shear Modulus of Sand from Shear Wave Velocity Measurements by Bender Elements

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Introduction

The designer of earthquake-resistant structures needs some understanding of how soils respond during an earthquake; not only is this important for the foundation design itself, but the nature of soil overlaying bedrock may have a crucial modifying influence on the overall seismic response of the site. The response of soils to earthquake excitation is highly complex and depends on a large range of factors, many of which cannot be established with any certainty. The information on dynamic soil properties, such as dynamic shear modulus and damping ratio, is more essential for accurate computations of seismic site-soil response, design of foundations subjected to dynamic loads and evaluation of liquefaction potential. Characterization of free-field soil modulus and its spatial variation is an essential component in the simulations of dynamic soil-foundation-structure interaction (SFSI) problems. The initial tangent shear modulus of soil, also termed as maximum shear modulus (G_{max}) is a fundamental parameter in static and dynamic analyses involving deformation calculations. At very small strains ($\approx$ 0.0001%) the shear moduli are independent of shear strain amplitude and they reach a constant limiting value, G_{max}. In the case of a homogeneous deposit of sands with uniform density, the gravity-induced stress variation alone is known to create a depth-dependent shear modulus and shear-wave velocity profiles. The G_{max} is a key parameter for predicting the dynamic response and behaviour of soils both in total and effective stress analyses and acts as a basis for evaluation of secant shear modulus (G) corresponding to the dynamic shear strain level at the existing confinement. As the strain level increases shear modulus starts decreasing from its maximum value. In the literature, a degradation of soil stiffness is usually represented by modulus reduction curve and the G_{max} is an important parameter required in the development of modulus reduction curves. The estimation of G_{max} can be made either by carrying out field tests or by laboratory tests, in which the deformation properties of soil are related to elastic shear wave velocities. Low-strain field tests typically induce seismic waves in the soil and

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seek to measure the velocities at which these waves propagate. Because the strain amplitudes associated with these waves are quite low, the measured velocities of shear waves can be used, along with the soil density, to compute the corresponding low-strain shear modulus, as

$$G_{max} = \rho V_s^2 \quad (1)$$

where ρ is the mass density of the soil and V_s is the shear wave velocity.

The bender element test, the resonant column test and the ultrasonic pulse test are the common laboratory tests for the estimation of G_{max}. The piezoelectric bender element tests offer a simple and reliable method for determining shear wave velocity and hence shear modulus corresponding to very small strains. As the deformation levels due to piezoelectric transducers is very small (0.0001%), the shear modulus can be considered as equal to G_{max}. This paper describes the method of estimating G_{max} for saturated sand by using the piezoelectric bender element test set up mounted on a cyclic triaxial apparatus. The precautions are taken to minimize the distortion of output waves based on the experiences of previous researchers available in the literature. The interpretation of the travel time from the bender element test is improved by adopting two performance criteria such as signal-to-noise ratio of at least 4 dB for the receiver signal and a wave path length to wavelength ratio of at least 3.33. The estimated G_{max} values are compared with the empirical equations developed using the results of resonant column tests and it is noted that the bender element tests can be used with confidence for estimating G_{max} values for all practical purposes.

Background and Literature Review

The value of small-strain shear modulus can be obtained from shear wave velocity measurements using piezoelectric transducers. Bender elements are piezoelectric cantilever beam shaped transducers, used to generate and detect shear motion. The measurement is based on transmission of a mechanical signal through a soil sample. The bender elements are first developed by Shirley and Hampton (1977) as part of the program to measure in situ acoustic parameters of sediments. The bender element consists of two sheets of piezoelectric ceramic materials such as lead zirconate titanate, barium titanate or lead titanate sandwiching a center shim of brass, stainless steel or other ferrous nickel alloys to add strength to it. The bender element is an electromechanical transducer capable of converting mechanical energy into electrical energy and vice versa. When a driving voltage is applied to the bender element, the polarization will cause a bending displacement and, thus, the bender element acts as a signal generator. When the element is forced to bend, a voltage is generated and, thus, the bender element can act as a signal receiver. In the bender element test for measurement of shear wave velocity, a pair of bender elements is used whereby one of the bender elements acts as the shear wave transmitter and the other bender element acts as the receiver. A schematic diagram of the bender elements is shown in Figure 1. The bender elements have to be inserted into the top and bottom of a soil specimen. The measurement of shear wave velocity is based on transmission of a mechanical signal through a soil sample. A sharp voltage pulse applied to one element causes a shear wave that travels through the soil specimen. When that shear wave reaches the other bender element, the deflection of the receiving bender element produces a voltage that can be measured. By measuring the time required for the wave to travel from the source to the receiver, and knowing the distance between them, the shear wave velocity of the specimen can be measured nondestructively.

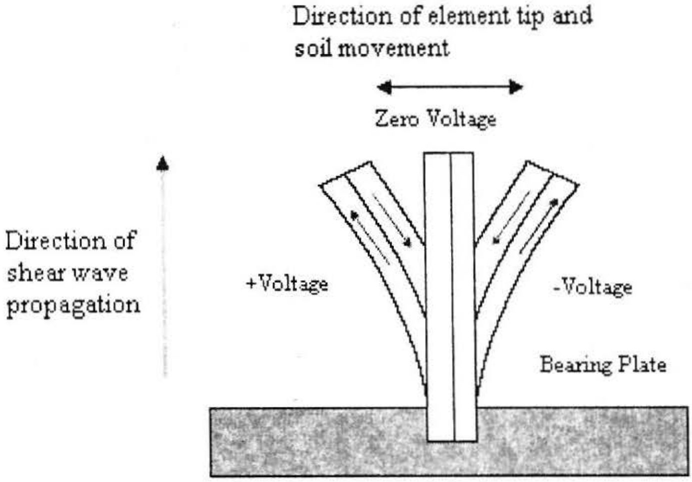

Fig. 1 Piezoelectric Bender Element (after Kramer 1996)

By measuring the travel time of the wave, shear wave velocity V_s is determined as

$$V_s = \frac{L}{t} \tag{2}$$

where L is the tip to tip distance between the transmitter and the receiver bender elements and t is the travel time of the wave form – the transmitter to the receiver and the G_{max} is calculated using Equation (1). Travel length L can be taken as the distance between the tips of the generator and receiving bender elements (Viggiani and Atkinson 1995a). Since L can be measured with relative ease and accuracy, the error in the shear wave velocity would mainly be caused by the errors in the travel time measurement. Figures 2 and 3 show the transmitter and receiver bender elements, and the bender elements mounted on top and bottom platens of the cyclic triaxial apparatus, respectively. The experimental technique and interpretative methodology to estimate G_{max} using bender elements are described in the subsequent sections of the paper.

The bender elements can be utilized in most laboratory equipments, but are particularly versatile when used in triaxial tests as described by Dyvik and Madshus (1985). The principal advantage of bender elements is that they can be incorporated into a variety of testing devices and anisotropy of soil stiffness can be investigated by locating bender elements on two vertical and opposite sides of the sample. Since this is a nondestructive test, the samples are not disturbed and can be subsequently tested for other soil characteristics.

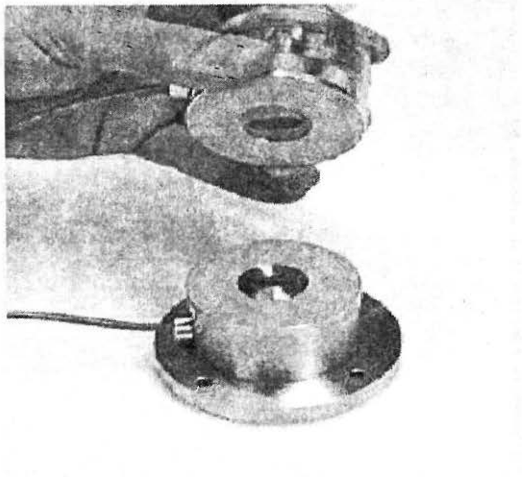

Fig. 2 Transmitter and Receiver Bender Elements

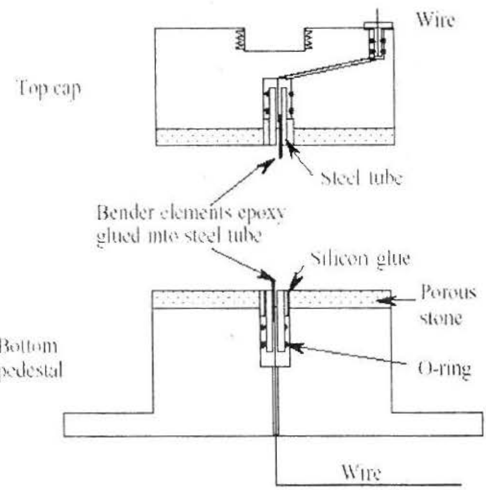

Fig. 3 Bender Elements Setup in Cyclic Triaxial Platens

Near Field Effect and Interpretation of Travel Time from Output Waveforms

The bender element test uses soil specimens of a finite size, hence there is reflection and refraction of the wave at the specimen's boundary as it travels from the transmitter bender element to the receiver bender element. The interference of incident and reflected waves at rigid boundaries can affect the interpretation of travel time. The travel time of the shear wave can be estimated as the time between the start of the voltage pulse input to the transmitting bender element and the first arrival of the output signal from the receiving bender element. The output signals are distorted by near field effects, which can lead to erroneous shear wave velocity calculations. Various methods have been used to determine the travel time and the methods can be divided into two categories. They are time of flight technique and phase sensitive detection technique. The time of flight technique uses a single pulse, while the phase sensitive detection technique uses continuous waves. A typical bender element wave trace is shown in Figure 4. In the time of flight method, difficulty arises in identifying an unambiguous first arrival in the output signal. Several options of interpretations for the travel time have been suggested in the literature (Dyvik and Madshus 1985; Arulnathan et al. 1998; Viggiani and Atkinson 1995a; 1995b; Lee and Santamarina 2005). As shown in Figure 5, they include first arrival of the receiver signal (A-D), first reversal point of the receiver signal (A-E), first point of deflection (A-F), based on corresponding peaks (B-G) or troughs (C-H). The problem in correct interpretation of travel time arises due to a phenomenon called the near field effect and is characterized by an initial deflection of the output signal before the significant motion of the receiving bender element, as depicted in the output wave between points D and F (Figure 5). This has the effect of masking the true arrival of the first shear wave.

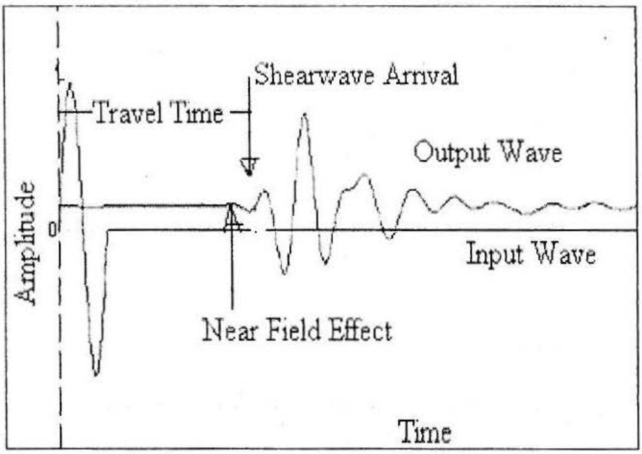

Fig. 4 Typical Bender Element Signal

The determination of travel time by cross correlation of input and output waves is one of the methods described in the literature. In cross –correlation between transmitter and receiver signals, the travel time is taken as the time shift that produces the peak correlation between the input and output signals. In the phase sensitive detention technique, a signal is generated at a particular frequency, and the change in phase angle between the transmitting and receiving waves is determined. The relationship between the change in phase angle and the travel time is given by

$$t = \Phi_{angle} / 2\pi f \qquad (3)$$

where t is the travel time in seconds, Φ_{angle} is the change in phase angle in degrees and f is the frequency of input wave in Hz. The following paragraphs will provide a brief review on the earlier studies in the area of small strain stiffness measurement of soils.

Dyvik and Madshus (1985) showed the agreement between G_{max} measured with bender element tests and resonant column tests. Brignoli et al. (1996) determined shear wave velocity of Ticino sand with bender element test by performing velocity measurement at a frequency for which a clear wave form is obtained and found that shear wave velocity values averaged about 8% higher than those determined with the resonant column test. Kramer (2000) compared G_{max} values evaluated from bender element tests with that of resonant column and seismic cone tests on Mercer slough peat and observed that the G_{max} values from the bender element tests and the seismic cone tests appear to lie below the values obtained from the resonant column tests.

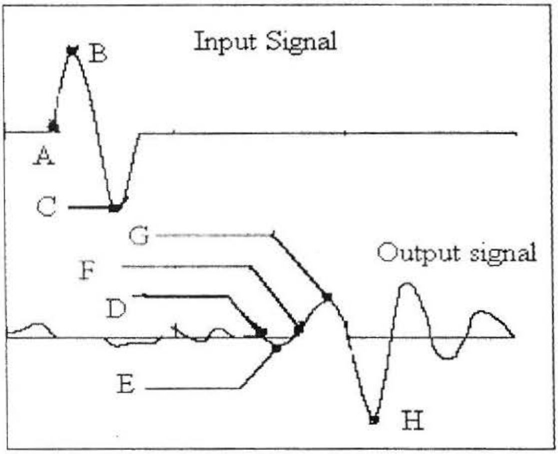

Fig. 5 Different Methods of Interpretation of Travel Time

Drnevich and Richart (1970), Seed and Idriss (1971) and Kokusho (1980) showed that the G_{max} was basically related to the mean principal stress σ_0' and the void ratio e of the soil, as expressed by the well known Hardin and Richart (1963) equation taking the general form:

$$G_{max} = A\ F(e)(\sigma_0')^n \tag{4}$$

where G_{max} and σ_0' are in terms of kPa and A is an empirical constant reflecting soil fabric formed through various stress and strain histories, n is an empirically determined exponent, approximately equal to 0.5, e is the void ratio and F(e) is the void ratio function, which is usually given by

$$F(e) = (2.17 - e)^2 / (1 + e) \tag{5}$$

Hardin and Richart's equation had undergone several adjustments by the various authors. Iwasaki et al. (1978) proposed the following empirical equation for G_{max} based on resonant column tests:

$$G_{max} = 9000\left[(2.17 - e)^2/(1 + e)\right](\sigma_0')^{0.38} \tag{6}$$

Yu and Richart (1984) proposed the following empirical equation for G_{max} based on resonant column tests:

$$G_{max} = 7000\left[(2.17 - e)^2/(1 + e)\right](\sigma_0')^{0.5} \tag{7}$$

Viggani and Atkinson (1995b) and Jovicic et al. (1996) proposed the use of single shot sinusoidal wave to aid in identifying points of similarity between the input and the output waveforms and lead to an improvement over square pulse excitation. They showed that the use of high frequency signals helps to erase the near field effects and the near field components attenuates with distance and frequency.

Arulnathan et al. (1998) suggested four methods for the determination of travel time. They are: first arrival time, travel time between the characteristic points, cross-correlation of input and output signals, and cross-power of transmitter and receiver signals. Authors found that the travel time based on characteristic peaks or cross-correlation between transmitter and receiver signals is incorrect because of wave interference at the boundaries, phase lag or signal distortion, and near-field effects. The authors further suggested that the travel time should be determined from the second wave arrival, which is less affected by wave interference at the boundaries or the transfer functions relating electrical signals to physical waves. However, they admitted that the second wave arrival still suffers from near-field effects.

Santamarina and Fam (1997) pointed out that the determination of travel time using cross correlation is only valid if both the input and output signals are of same nature. Gajo et al. (1997) noted that the cross correlation method underestimates the arrival time based on their bender element tests.

Blewett et al. (1999) developed phase sensitive detection technique in which a continuously cycled sinusoidal wave is transmitted through the source element and the output and received signals are displayed simultaneously in a digital oscilloscope. The frequency of the source signal is gradually increased until the source and received signals come in to phase. The travel time can be calculated from the slope of a frequency versus number of wavelengths line. The results obtained from their tests showed that there is no significant difference in shear wave velocity obtained from the phase sensitive detection and conventional time of flight techniques.

Kawaguchi et al. (2001) stated that the bender elements response is enhanced when the frequency of the input sine wave signal approaches the resonant frequency of the bender element-soil system. In this case, the first arrival is not affected by the selected input frequency, but the ability to detect the arrival time can change dramatically. Authors concluded that the use of characteristic points to determine the travel time is only acceptable at high frequencies.

Lee and Santamarina (2005) found that the multiple reflections detected with bender element provide a simple method to overcome travel length and travel time uncertainties. Leong et al. (2005) proposed that by adopting a signal to noise ratio of at least 4dB for the received signal and a wave path length to wavelength ratio of at least 3.33 make more accurate interpretation of shear wave velocity possible from the bender element tests. This approach is followed in the present experimental investigations.

Experimental Investigations

Test Apparatus

The bender element test setup procured from Wykeham Farrance, UK is used in the present study, which is available in the Department of Civil Engineering, IIT Madras, Chennai. The bender elements are incorporated in a servo controlled cyclic triaxial system. The bender element test setup consists of a transmitter element, which is energized to produce the shear waves through the soil sample, and when waves reach the receiver it generates the electrical signal (Figure 6). The insertion of bender elements into the soil sample is shown in Figure 7. The function generator generates a signal with specified amplitude and frequency as input signal and PC based oscilloscope records data from transmitter bender element. The travel time of the shear wave from the transmitter to the receiver is determined via specific software that allows the user to quickly and easily calculate the shear wave velocity.

Soil Sample and Preparation

The soil sample obtained from Kalpakkam, India is used in the present study. Geotechnical characterization has been done and soil is identified as uniformly graded sand (SP) as per the Indian Soil Classification System (IS: 1498-1970). The index properties of the sand are summarized in Table 1. A grain size distribution curve for the sand is shown in Figure 8. The reconstituted specimens are cylindrical and have a diameter of 50 mm and length of 100 mm. Samples are prepared by air pluviation technique at relative densities of 20%, 50%, 65% and 85%. The travel distance and density are greatly influence the shear modulus, therefore, accurate determination of density and volume change measurements have been made during saturation and consolidation. The initial specimen dimensions are measured after the mould is removed. After assembling and filling the triaxial chamber with water, de-aired water is allowed slowly to seep through the specimen from the bottom. Back pressure is applied in the specimen in steps and evaluation of degree of saturation is done at appropriate intervals by measuring Skempton's porewater pressure parameter B and a value of more than 0.95 is ensured. Then the saturated soil sample is isotropically consolidated to the required effective confining stress. The consolidated height and volume of the sample are noted by using the data acquisition system incorporated with the cyclic triaxial apparatus.

Parameters of Applied Voltage to the Transmitter Element

The receiver bender element signal is dependent on the voltage applied to the transmitter bender element and the soil type. The applied voltage has three parameters: wave form, magnitude and frequency. The main problem associated with the use of square wave excitation is that group velocity dispersion in the sample leads to the separation of different frequency components within the voltage edge waveform. These propagate at different speeds, arriving at different times, producing a severely distorted received signal. The simplest way to obtain a bender element trace that may be interpreted objectively is to use a sinusoidal wave rather than a more usual square wave. For better interpretation of arrival time as proposed by Viggiani and Atkinson (1995b) and Jovicic et al. (1996) sine wave pulse was adopted in this study. For most interpretable wave forms, Brignoli et al. (1996) suggested frequency range of 3 to 10 kHz for shear wave measurements on 10 to 14 cm high specimens. In the present study a frequency of 10 kHz is used. The use of relatively high frequency is desirable since the near field effect is reduced and error in the interpretation methods is also minimized. Jovicic et al. (1996) stated the use of higher frequencies for attenuating the near field effects with distance. The input to output voltage relationship between the elements depends on the type of element, type of connection, geometry of sample and material being tested. The typical input voltages are 10 to 20 V_{pp} (peak to peak voltage) with outputs ranging between 1 and 10 mV. The voltage is a direct effect of the displacement of the tip of the element. This in turn represents the shear strain induced in the source element. Based on the collected information related to the input voltage parameters available in the literature, for better interpretation of travel time measurement, a sine wave form with 10 kHz frequency and input voltage amplitude of 20 V_{pp} are used in the study.

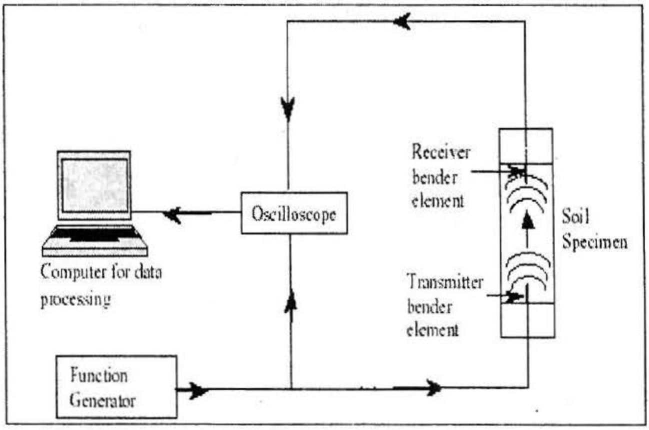

Fig. 6 Layout of Bender Element Test Setup

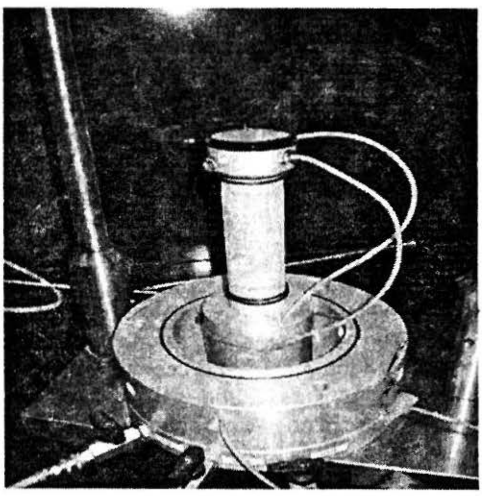

Fig. 7 Cyclic Triaxial Cell and Soil Sample Equipped with Bender Elements

TABLE 1: Properties of Sand

Parameter	Value
Specific gravity, G_s	2.68
Maximum dry density (kg/m^3)	1720
Minimum dry density (kg/m^3)	1510
D_{10} (mm)	0.17
D_{60} (mm)	0.40
Coefficient of uniformity (C_U)	2.35

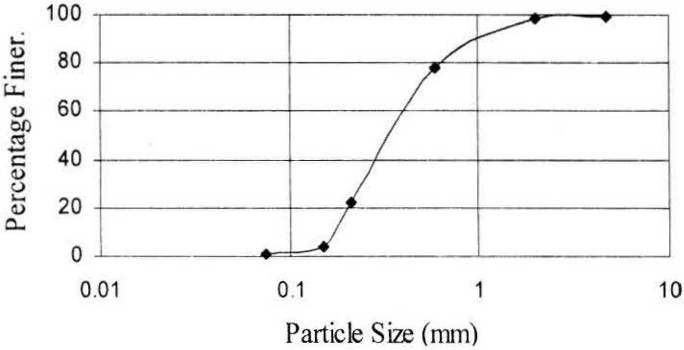

Fig. 8 Grain Size Distribution of Sand

Calibration of Bender Element System

Calibration of the bender element system is necessary so that any time delay introduced in the measurements by the electronics, ceramics and coating materials can be properly accounted. The calibration is carried out according to the procedure suggested by Brignoli et al. (1996). The two bender elements are placed in direct contact and measurement is made for the time interval between the initiations of the electrical impulse sent to the transmitter and the initial arrival of the waveform recorded at the receiver. Initial polarity of the output signal has to be in phase with the alignment and polarization directions of the input signal. It is found that the time delay in the system is only 5700ns. During calibration process, the check for the absence of wave transmission paths other than the soil specimen is necessary and accordingly it is ensured that the wave path is only through the soil specimen.

Measurement of Travel Time from the Wave Trace

The travel time of the shear wave is estimated as the time between the start of the voltage pulse input to the transmitting bender element and the first deflection in the output signal from the receiving bender element. In this approach, difficulty arises in identifying an unambiguous first arrival of shear wave in the output signal. Leong et al. (2005) compared the travel time for both the bender element and ultrasonic tests and realized that they are similar only if the first deflection of receiver signal was used for the bender element test. Travel times based on characteristic peak and characteristic trough and cross correlation methods are found unreliable. So in this study the travel time is estimated as the first deflection in the output signal. A typical wave trace obtained and the interpretation of travel time by the software is shown in Figure 9.

Results and Discussion

The evaluated G_{max} value with effective confining stress is shown in Figure 10. To eliminate the influence of void ratio nonuniformity, G_{max} has been further divided by F(e) and the results are plotted in Figure 11, where e is the void ratio corresponding to the G_{max} at a given loading stage. As shown in Figure 11, if divided by F(e), the data points of the test series are almost identical, and the maximum shear modulus (i.e. G_{max}) is well correlated with the effective stress regardless of the void ratio. To account for the influence of varying stress conditions, the maximum shear modulus is normalized [$(G_{max})_n$] by dividing it with $(\sigma_0')^{0.5}$ and is shown in Figure 12. From Figure 12 it is clear that as G_{max} is normalized against effective confining stress, the data points collapse into a single relationship. These results show that a definite relationship exists between G_{max} and void ratio under a certain confining stress. Based on the experimentally evaluated G_{max} values, it has been found that this relationship can be expressed as

$$G_{max} = 6340\ F(e)\ (\sigma_0')^{0.5} \tag{8}$$

where $F(e) = (2.17 - e)^2/(1 + e)$, G_{max} and σ_0' are in kPa.

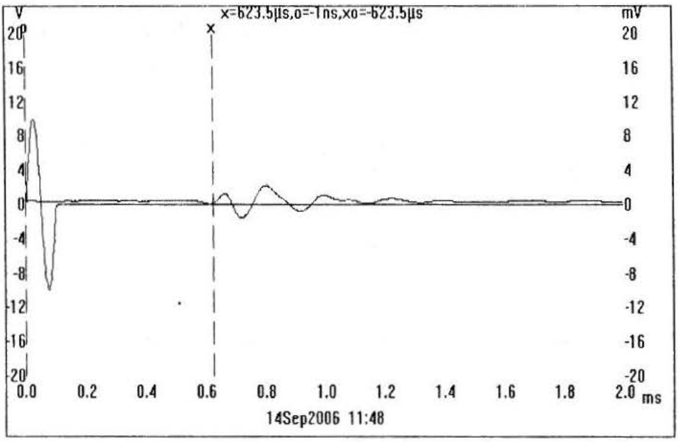

Fig. 9 Typical Wave Trace from Bender Element Test

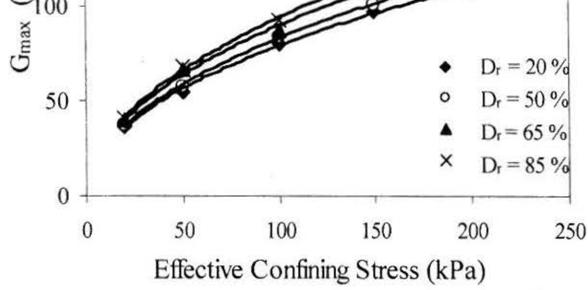

Fig. 10 Variation of G_{max} with Effective Confining Stress

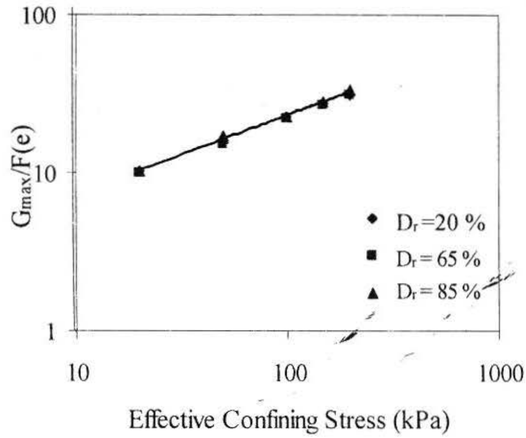

Fig. 11 Variation of $G_{max}/F(e)$ with Effective Confining Stress

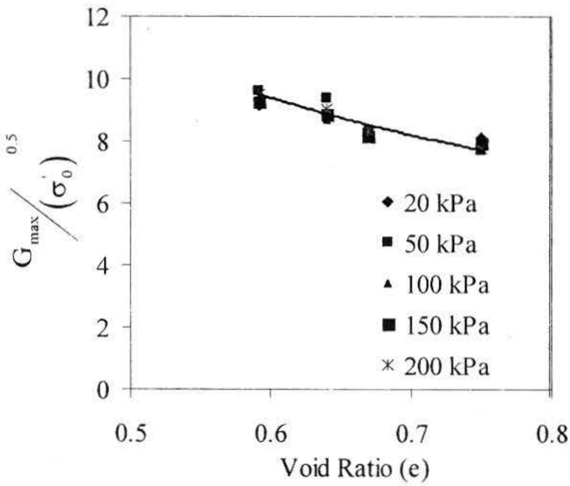

Fig. 12 Variation of $G_{max}/(\sigma_0')^{0.5}$ with Void Ratio for Different Confining Stress

The bender element G_{max} values compare reasonably well with the empirical equation of Yu and Richart (1984) if the multiplier (A) involved is varied from 10 to 20% (Figure 13). The value of constant (A) generally depends on grain size composition and hence it will vary over a wide range reflecting variability of the modulus for individual sands. The G_{max} values evaluated using the above empirical equation are compared with the results of empirical equations of G_{max} derived from the resonant column tests (Iwasaki et al. 1978; Yu and Richart 1984). The comparison of G_{max} values estimated from bender element tests with the two empirical equations proposed based on resonant column tests suggests that the bender element results match well with the general equation of G_{max} for sand proposed by Yu and Richart (1984), as depicted in Figure 14. It has also been noted that Yu and Richart's equation overestimates the G_{max} values by 10 to 20% for uniform sand in saturated conditions and Iwasaki and co-worker's equation underestimates the effect of confining stress on G_{max}.

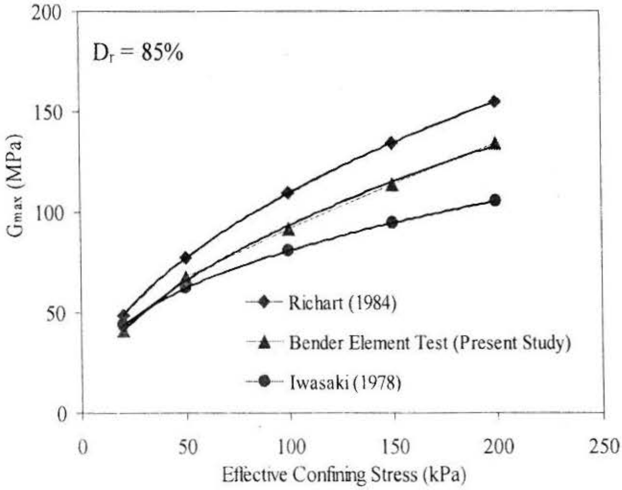

Fig. 13 Comparison of G_{max} from Bender Element Test with Empirical Equations

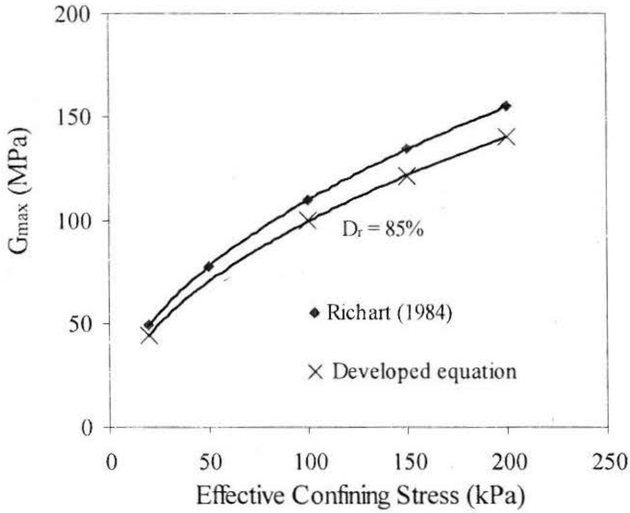

Fig. 14 Comparison of G_{max}: Present and Existing Empirical Equation

Conclusions

The following conclusions are arrived based on the study presented in the paper. The reliable estimates of G_{max} are obtained from bender element tests by using the sinusoidal input voltage and travel time based on first deflection in the output signal. Based on the estimated values of G_{max} from bender element tests, easy to use predictive relationship is developed for the uniform saturated sand. A developed empirical equation for estimating G_{max} for the uniform saturated sand is, $G_{max} = 6340\ F(e)\ (\sigma_0')^{0.5}$. The comparison of developed equation with empirical equations reported in the literature based on the resonant column tests shows that the developed empirical equation will provide practically acceptable values of G_{max} for all possible confining stresses if the multiplier involved in the equation is varied from 10 to 20% for the uniform saturated sand. There are some uncertainties involved in the interpretation of bender element test results, which arise mainly from the quality of the receiver signal. Moreover, measurements obtained from bender elements always need to be interpreted carefully and also a closer attention to certain experimental details of the bender element tests are needed to put axial bender element testing measurements on a more solid basis and a further work on these lines is currently under investigation.

Acknowledgements

The authors are grateful to the financial support provided by IGCAR, Kalpakkam in the form of sponsored research project titled "Effect of Embedment on Seismic Response Analysis of Nuclear Structures". The content of this paper is a part of this research work. The first author would like to thank the College of Engineering, Trivandrum for the sponsorship to pursue the Doctoral studies in IIT Madras, Chennai.

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