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Dynamic Earth Pressure on Rigid Unyielding Walls under Earthquake Forces

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Introduction

Since the development of Mononobe and Okabe's (M-O) method for evaluation of the dynamic earth pressure under earthquake forces in 1920's (Mononobe and Matsuo 1929), a number of research works have been carried out till date to ameliorate the same and further evaluate its adequacy. These include the work of Wood (1973), Seed and Whitman (1970), Whitman and Christian (1990), Whitman (1990; 1991), Richard and Elms (1979), Matsuzawa et al. (1984), only to name a few. All these methods are extensions of M-O method and are pseudo-static in nature and calculate the forces based on the maximum ground acceleration without taking into cognizance the time period of the system. Strictly speaking M-O method is valid for gravity type of retaining walls, where the wall itself has a very high structural stiffness and is expected to undergo large movements under seismic forces when the active failure wedge behind the wall is mobilized. The first pseudo-dynamic method of analysis was proposed by Steedman and Zeng (1990), where the pressure was computed based on the natural frequency of the soil medium.

Chowdhury and Dasgupta (2003; 2004) proposed a method, where dynamic moments and shears for cantilever and counterfort retaining walls may be calculated adapting an improved Rayleigh Ritz technique. The solutions however will be valid only when the wall is flexible enough to mobilize the failure wedge behind the wall and shall not be applicable for the problems where the wall is unyielding. However, it has been shown that when the wall is flexible, the time period of the system plays a significant role and the results can have significant variation with the results obtained by using the M-O method.

Some recent observations on field data reported by Ostadan and White (1997) and back checking the same based on Finite Element Analysis (Lysmer et al. 2000), it has been found that the pressures induced on such rigid walls are significantly different in comparison to M-O method. This has prompted United States Nuclear Regulatory Committee (NRC) to abandon M-O and M-O based methods for designing of their nuclear facilities.

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While SASSI (Lysmer et al. 2000), based on FEM, are available to correctly predict the dynamic pressure on rigid walls but these are too elaborate and exhaustive to be used for day-to-day practice in design offices. Based on his exhaustive experimental observations and analysis, Ostadan has proposed a pressure equation based on least square fitted curve (Ostadan 2004), which may be used for the evaluation of dynamic pressures and is being adapted by NEHRP (NEHRP 2001) in design offices. However, the use of this curve would still require the use of softwares like SHAKE (Schnabel et al. 1972) or SASSI and is valid for soils having shear modulus invariant with depth.

In the present paper, a method has been proposed where users not having the access to softwares like SHAKE, SASSI can still arrive at a dynamic pressure, which is in close agreement to what has been proposed by NEHRP and takes into cognizance the linear and parabolic distribution of shear modulus with depth.

THE PROPOSED METHOD

Shown in Figure 1 is a basement of a building where the basement can be considered as rigid. Since in many cases the building basements consists of different levels with floor slabs catering to sundry building services like car parks, HVAC floors, pump rooms etc. it is quite justified to assume such basement walls to be quite rigid and unyielding. In such cases as the wall does not yield, one can design the wall for static loading under earth pressure at-rest condition. Under dynamic loading such unyielding walls, as the active failure wedge does not get mobilized, the soil may be assumed to continue in its state of plane strain condition under both static and earthquake forces.

The objective is to find out the dynamic pressure on the wall of the basement due to earthquake waves propagating through the soil medium. The soil medium and the basement are assumed to be resting on stiff soil considered as the bedrock, the level from which the ground acceleration propagates.

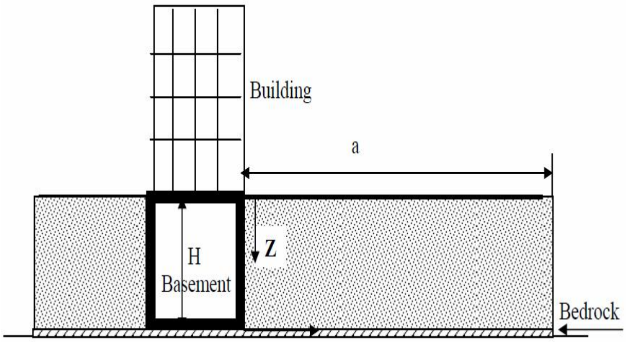

Fig. 1 Sketch of the Building with Basement Wall

Let the depth of the basement be H and the soil medium in the horizontal direction has been considered having a finite dimension a, where in reality lim a $\rightarrow \infty$. Having defined the problem with basic conditions one can argue that for shear waves propagating through the soil medium, the wave propagation equation can be represented by

$$\frac{\partial^2 u(x,z,t)}{\partial x^2} + \frac{\partial^2 u(x,z,t)}{\partial z^2} = V_S^2 \frac{\partial^2 u(x,z,t)}{\partial t^2} \tag{1}$$

where V_s = shear wave velocity of the soil medium; u(x, z, t) = the displacement function and can be considered as u = H(x)Q(z)P(t) [H, Q, P are the three independent functions of x, z and t respectively].

Without getting into the details it can be shown (Kreyszig 1969) that Equation (1) may be reduced to three independent ordinary differential equations of second order given by

$$\frac{d^2P}{dt^2} + \lambda^2 P = 0 \text{ where } \lambda = \ell V_S \text{, where } \ell \text{ is a constant} \tag{2}$$

$$\frac{d^2H(x)}{dx^2} + k^2 H(x) = 0 \text{, where k is another constant, and} \tag{3}$$

$$\frac{d^2Q(z)}{dz^2} + p^2 Q(z) = 0 \tag{4}$$

where p, ℓ and k are related through $p^2 = \ell^2 - k^2$.

The solutions of Equations (3) and (4) are given by

$$H(x) = A\cos kx + B\sin kx \tag{5}$$

$$Q(z) = C\cos pz + D\sin pz \tag{6}$$

Imposing the boundary conditions

At x = 0, u = 0 → H (x) = 0, which implies A = 0.

At x = a (where a may be very large), u = 0, → H (a) = 0, which implies H (a) = $B\sin ka = 0$ and

$$k = \frac{m\pi}{a} \text{, and hence } H_m(x) = \sin\frac{m\pi x}{a} \tag{7}$$

At the free surface i.e. the superstructure-base interface, the boundary conditions are

At z = 0, shear strain, $\frac{\partial u}{\partial z} = 0$ or $\frac{dQ(z)}{dz} = 0$ which implies D = 0.

At z = H, displacement $u = 0$, i.e. $Q(H) = 0$.

It implies that

$$p = \frac{(2n-1)\pi}{2H} \quad (8)$$

and hence

$$Q(z) = \cos\frac{(2n-1)\pi z}{2H} \quad (9)$$

Thus, the eigenvectors, Φ of the problem can be established as

$$\Phi(x,z) = H(x)Q(z) = \sin\frac{m\pi x}{a}\cos\frac{(2n-1)\pi z}{2H},$$

where, m, n =1,2,3… (10)

Again, from the description of Equations (2) and (3)

$$\lambda = V_S\sqrt{p^2 + k^2}$$

Substituting the value of p and k from Equations (7) and (8), one can have

$$\lambda = V_S\pi\sqrt{\frac{m^2}{a^2} + \frac{(2n-1)^2}{4H^2}} \quad (11)$$

For the fundamental mode [m, n = 1] and lim a $\rightarrow \infty$, the value of λ reduces to

$$\lambda = \omega = \pi V_S\sqrt{0 + \frac{1}{4H^2}}\text{, implying } \omega = \frac{\pi Vs}{2H} \quad (12)$$

The period, T can be derived from Equation (12) as $T = 4H/V_s$ which is basically the free field time period in one dimension for the site.

For limit a $\rightarrow \infty$, the first term of eigen function (in the x direction) can be dropped from Equation (11) and to determine the displacement and pressure at the wall face and hence the eigen function φ can be computed as

$$\varphi(z) = \cos\frac{(2n-1)\pi z}{2H} \quad (13)$$

Based on the modal response technique (Clough and Penzien 1975), the maximum amplitude function can be defined as

$$S_d = S_a/\omega^2 \quad (14)$$

where S_d = maximum displacement; S_a = acceleration which is a function of the time period, $4H/V_s$, and can be read off from the normalized response given in Codes.

Considering $\beta = ZI/(2R)$ = the code factor, u(z) may be written as $u(z) = \kappa_i \beta \dfrac{Sa}{\omega^2} \phi(z)$ in which, Z = zone coefficient; I = importance factor; R = response reduction or ductility factor.

Now, the modal mass participation factor may be written as

$$\kappa_i = \sum m_i \phi_i \Big/ \sum m_i \phi_i^2$$

Substituting the value of ω from Equation (12), the displacement u(z) can be expressed as

$$u(z) = \frac{4}{\pi^2} \kappa_i \beta \frac{S_a \gamma_s H^2}{Gg} \cos \frac{\pi z}{2H} \qquad (15)$$

in which G = ρVs^2 ; γ_S = bulk density of soil ; g = acceleration due to gravity.

The modal participation factor can be computed as

$$\kappa_i = \sum m_i \phi_i \Big/ \sum m_i \phi_i^2 = \int_0^H \gamma z \cos \frac{\pi z}{2H} \Bigg/ \int_0^H \gamma z \cos^2 \frac{\pi z}{2H} \qquad (16a)$$

Using integration by parts, κ_i is expressed as

$$\kappa_i = 8/(\pi + 2) \qquad (16b)$$

The strain within the soil body is given by

$\varepsilon_{xx} = \dfrac{\partial u}{\partial x} = 0 ; \varepsilon_{zz} = \dfrac{\partial u}{\partial z}$ which gives $\varepsilon_{zz} = -\dfrac{2}{\pi} \kappa_i \beta \dfrac{S_a \gamma_s H}{Gg} \sin \dfrac{\pi z}{2H}$ and hence

$$\varepsilon_{zz} = -\frac{16}{\pi(\pi + 2)} \beta \frac{S_a \gamma_s H}{Gg} \sin \frac{\pi z}{2H} \qquad (17)$$

The constitutive stress-strain relationship under plane strain condition is given by

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{zz} \\ \tau_{xz} \end{bmatrix} = \frac{2G}{(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \dfrac{1-2\nu}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{zz} \\ \gamma_{xz} \end{bmatrix} \qquad (18)$$

resulting in $\sigma_{xx} = \dfrac{2G(1-\nu)}{1-2\nu}\varepsilon_{xx} + \dfrac{2G\nu}{1-2\nu}\varepsilon_{zz}$

For the present case as $\varepsilon_{xx} = 0$, one can have $\sigma_{xx} = \dfrac{2G\nu}{1-2\nu}\varepsilon_{zz}$. The dynamic pressure on the wall may be obtained from

$$p_{dyn} = -\frac{16}{\pi(\pi+2)}\frac{2\nu}{1-2\nu}\beta\frac{S_a\gamma_s H}{g}\sin\frac{\pi z}{2H} \tag{19}$$

The negative sign above indicates that the pressure is acting towards the wall. Equation (19) can be rewritten as

$$p_{dyn} = -\frac{32}{\pi(\pi+2)}\psi_\nu\beta\frac{S_a\gamma_s H}{g}\sin\frac{\pi z}{2H} \tag{20}$$

where $\psi_\nu = \nu/(1-2\nu)$.

The pressure coefficients as obtained in Equation (20) may be compared with the pressure equation proposed in the literature. The dynamic pressure (Ostadan 2004), as adapted by NEHRP is given by

$$p(z) = -0.0015 + 5.05z - 15.84z^2 + 28.25z^3 - 24.59z^4 + 8.14z^5$$

The results obtained using Equation (20) and the one proposed by Ostadan (2004) have been compared and presented in Figure 2.

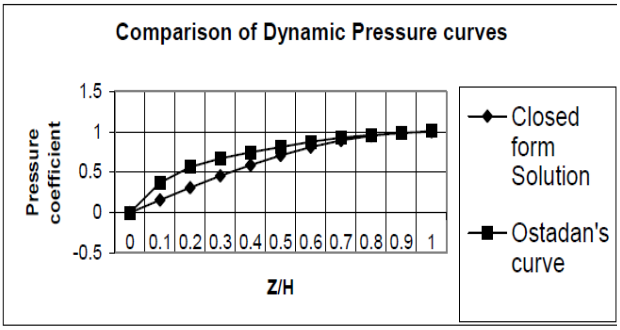

Fig. 2 Comparison of Normalized Pressure on the Wall

It can be observed that the pressure coefficient as proposed in Equation (20) is in close agreement with Ostadan's solution. Equation (20) can be further simplified to

$$p_{dyn}(z) = -Coeff_\nu\beta\frac{S_a\gamma_s H}{g} \tag{21}$$

where the coefficients may be read off for different values of Poisson's ratios (0.25, 0.3 and 0.4) as shown in Figure 3.

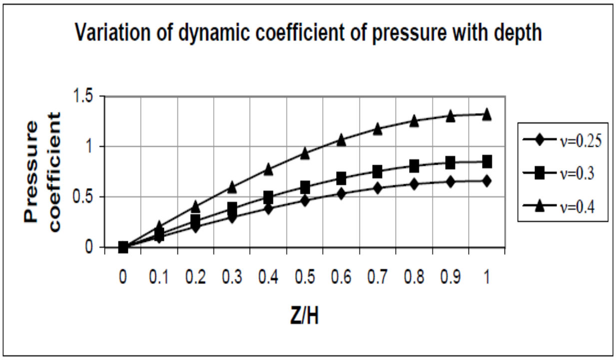

Fig. 3 Variation of Dynamic Pressure Coefficient with Depth for Different Poisson's Ratios

Dynamic Pressure on Wall When Shear Modulus Varies With Depth

Equation (20) is valid when the dynamic shear modulus is invariant with depth; however there could be cases when shear modulus varies with depth based on the expression:

$$G' = G\left(\frac{z}{H}\right)^{\alpha} \tag{22}$$

where α = 0 when G is invariant with depth,
= 1 when G varies linearly with depth, and
= 2 when G varies as parabolic with depth.

The use of Equation (22) for solving the partial differential equation expressed in Equation (1), makes the solution complicated and accordingly the following approach will be followed.

The strain energy equation of a soil body, in general, is given by

$$V = \frac{\lambda e^2}{2} + G\left(\varepsilon_x{}^2 + \varepsilon_y{}^2 + \varepsilon_z{}^2\right) + \frac{G}{2}\left(\gamma_{xy}{}^2 + \gamma_{yz}{}^2 + \gamma_{xz}{}^2\right) \tag{23}$$

where V= strain energy density of the soil body;. $\lambda = 2G\nu/(1-2\nu)$; G = dynamic shear modulus of the soil medium and ν its Poisson's ratio; e = $\varepsilon_x + \varepsilon_y + \varepsilon_z$; ε_x, ε_y and ε_z are strains, respectively in the x, y and z directions and γ_{xy}, γ_{yz} and γ_{zx} are shear strains in the xy, yz and zx planes, respectively.

With reference to Figure 1 and assuming the condition of plane strain, Equation (23) can be rewritten as

$$V = \frac{G\nu}{1-2\nu}\left(\varepsilon_x + \varepsilon_z\right)^2 + G\left(\varepsilon_x{}^2 + \varepsilon_z{}^2\right) + \frac{G}{2}\left(\gamma_{xz}{}^2\right) \tag{24}$$

For impulsive seismic excitation, ε_z = 0 which reduces Equation (24) further to

$$V = \frac{G(1-\nu)}{1-2\nu}\varepsilon_x^2 + \frac{G}{2}\gamma_{xy}^2 \tag{25}$$

Considering u(x,z) = N(x,z), q(t) one can have

$$\frac{\partial V}{\partial q_r} = \frac{2G(1-\nu)}{1-2\nu}\frac{\partial u}{\partial x}\frac{\partial}{\partial q_r}\left(\frac{\partial u}{\partial x}\right) + G\frac{\partial u}{\partial z}\frac{\partial}{\partial q_r}\left(\frac{\partial u}{\partial z}\right)$$

where

$N(x,z) =$ Generalized shape function with respect to x and z co-ordinate, and q(t) = Displacement function with respect to time in generalized co-ordinate.

That is

$$\frac{\partial V}{\partial q_r} = \frac{2G(1-\nu)}{1-2\nu}\frac{\partial N_i}{\partial x}\frac{\partial N_r}{\partial x}q_i q_r + G\frac{\partial N_i}{\partial z}\frac{\partial N_r}{\partial z}q_i q_r \tag{26}$$

From which it can be proved (Hurty and Rubenstein 1967) that the stiffness and mass matrix can be expressed as

$$K_{ir} = \int_0^H\int_0^a \left[\frac{2G(1-\nu)}{1-2\nu}\frac{\partial N_i}{\partial x}\frac{N_r}{\partial x} + G\frac{\partial N_i}{\partial z}\frac{\partial N_r}{\partial z}\right]dx\,dz \tag{27}$$

$$M_{ir} = \frac{\gamma_s}{g}\int_0^H\int_0^a N_i N_r dx\,dz \tag{28}$$

in which K = stiffness matrix of the soil medium; M = mass matrix of the soil medium; i and r subscripts indicate different modes 1, 2, 3, ...

The K and M for the fundamental mode (r = 1, i = 1) are given by

$$K_{11} = \int_0^H\int_0^a \left[\frac{2G(1-\nu)}{1-2\nu}\left(\frac{\partial N}{\partial x}\right)^2 + G\left(\frac{\partial N}{\partial z}\right)^2\right]dx\,dz \tag{29}$$

and $M_{11} = \frac{\gamma_s}{g}\int_0^H\int_0^a (N)^2\,dx\,dz$ (30)

It was shown earlier that when limit $a \to \infty$, the first term can be dropped and Equations (29) and (30) reduce to

$$K_{11} = \int_0^H\left[G\left(\frac{\partial N}{\partial z}\right)^2\right]dz \tag{31}$$

$$M_{11} = \frac{\gamma_s}{g}\int_0^H (N)^2\, dz \tag{32}$$

Considering the shape function as given in Equation (13) as $\phi(z) = \cos\frac{(2n-1)\pi z}{2H}$ and substituting it in Equations (31) and (32) for a constant G value (i.e. when α= 0) and by integrating one can have

$$K_{11} = \frac{\pi^2 G}{8H} \tag{33}$$

and $M_{11} = \frac{\gamma_s H}{2g}$ (34)

Considering $T = 2\pi\sqrt{M/K}$ substituting Equations (33) and (34) one can arrive at $T = 4H/V_S$ the same expression derived earlier. This shows that the stiffness and mass matrix formulation as represented here is correct. When G varies linearly with depth, the stiffness matrix equation in Equation (31) it gets modified to

$$K_{11} = \int_0^H \left[G'\left(\frac{\partial\phi}{\partial z}\right)^2 \right] dz \tag{35}$$

or $K_{11} = \int_0^H \left[G\left(\frac{z}{H}\right)^\alpha \left(\frac{\partial\phi}{\partial z}\right)^2 \right] dz$ [using α=1] (36)

Substituting Equation (13) in Equation (36) and considering $\xi = z/H$, Equation (36) can be rewritten as

$$K_{11} = \frac{\pi^2 G}{4H}\int_0^1 \xi\left[\frac{1}{2} - \frac{1}{2}\cos\pi\xi\right] d\xi \tag{37}$$

which on integration by parts gives

$$K_{11} = \frac{\pi^2 G}{8H}\left[1 - \frac{2}{\pi^2}\right] \tag{38}$$

The mass expression remaining the same as Equation (34) and the period reduces to

$$T = \frac{4H}{V_s}\sqrt{1 \Big/ \left(1 - \frac{2}{\pi^2}\right)} \tag{39}$$

which is the expression for the time period using a linearly varying dynamic shear modulus.

Obtaining the value S_a/g from the above time period and proceeding in the same manner as in Equations (14) through (19), one can obtain the dynamic pressure on the wall for linearly varying soil modulus G as

$$p_{dyn} = -\frac{32\pi}{(\pi+2)\left(\pi^2-2\right)}\psi_v \beta \frac{S_a \gamma_s H}{g}\sin\frac{\pi z}{2H}$$
$$\text{where, } \psi_v = \frac{v}{1-2v} \tag{40}$$

When the dynamic shear modulus varies parabolically with depth, the stiffness equation becomes

$$K_{11} = \int_0^H \left[G\left(\frac{z}{H}\right)^{\alpha} \left(\frac{\partial \phi}{\partial z}\right)^2 \right].dz \quad \text{[using } \alpha = 2\text{]} \tag{41}$$

which gives

$$K_{11} = \frac{\pi^2 G}{4H}\int_0^1 \xi^2 \left[\frac{1}{2} - \frac{1}{2}\cos \pi\xi\right].d\xi \tag{42}$$

Equation (42) on integration by parts gives

$$K_{11} = G\left(\pi^2 - 3\right) \Big/ (12H) \tag{43}$$

The mass expression remains the same as Equation (34) and the period may be computed as

$$T = \frac{2H}{V_s}\sqrt{6 \Big/ \left(1 - \frac{3}{\pi^2}\right)} \tag{44}$$

Again, obtaining the value of S_a/g based on Equation (44) and proceeding through Equations (14) to (19), the expression for p_{dyn} using parabolic shear modulus variation can be obtained as

$$p_{dyn} = -\frac{48\pi}{(\pi+2)\left(\pi^2-3\right)}\psi_v \beta \frac{S_a \gamma_s H}{g}\sin\frac{\pi z}{2H} \tag{45}$$

where $\psi_v = \dfrac{\nu}{1-2\nu}$

Figure 4 shows the variation of dynamic pressure on the wall for varying G.

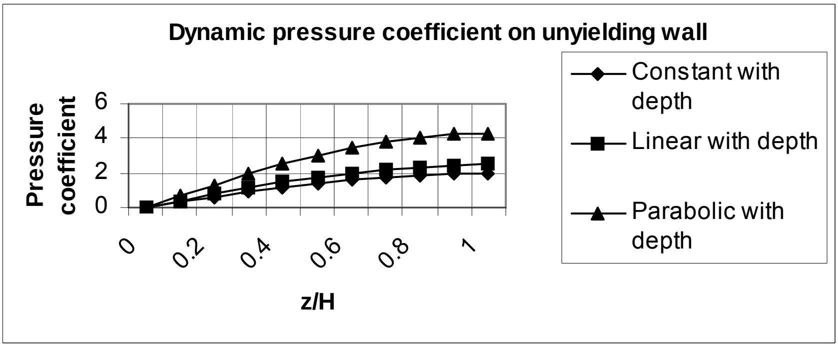

Fig. 4 Variation of Dynamic Pressure Coefficient for Varying Shear Modulus

Solution of the Problem

To evaluate how the computed dynamic pressures on the wall compare for a real life problem, the results of a 30 ft (9.14 m) deep rigid wall located in earthquake zone IV in India, are obtained. The shear wave velocity of soil is taken as 1000 ft/sec (305 m/s) (assumed constant over the depth) and unit weight of soil is 125 pcf (20 kN/m^3). The angle of friction of soil is ϕ = 15^o. Poisson's ratio of soil is ν = 0.3. The zone factor considered is Z = 0.24 as per IS 1893 - 2002, I = 1.2 and R = 2.0. The values of dynamic pressure generated on the wall are as shown in Figure 5 [the results have been presented using the conventional fps units, as was used there].

It can be observed in Figure 5 that while M-O method gives significantly lower value for this particular case, Ostadan (2004) and the proposed analytical method give results where the variation is not significant and well within an acceptable tolerance limit.

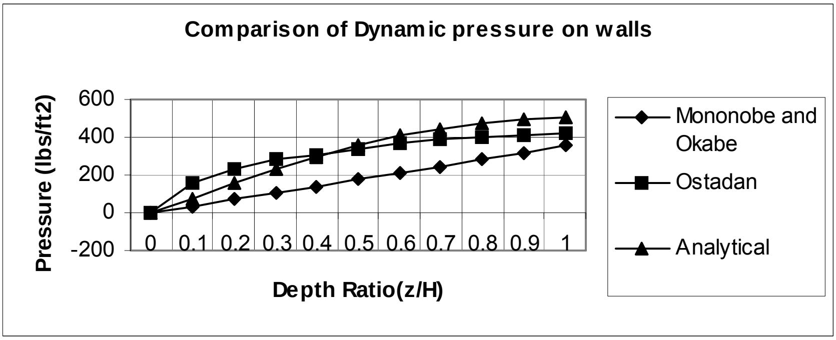

Fig. 5 Comparison of DynamicPressure on wall by M-O and other Methods

Conclusions

An analytical model for the dynamic pressure on rigid wall under the earthquake forces has been presented and is in close agreement with the results proposed in NEHRP. The NEHRP expression advocates the use of computer programs like SHAKE or SASSI which may not be accessible to all the engineers across the industry around the world. The present analysis gives comparable results as those proposed in NEHRP. The analysis also addresses the case of varying property of soil –which NEHRP method does not cater to.

Acknowledgement

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