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Effect of Vertical Reinforcement on Bearing Capacity of Footing on Sand

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Introduction

The concept of reinforced earth for increasing the bearing capacity of subgrade soil or to increase the resistance for retaining structures by means of thin metal strips placed horizontally in layers held together by internal friction between the reinforcing strips and the material has been confirmed both theoretically and experimentally by many investigators. It has been used frequently in USA and other countries to make reinforced earth retaining wall, highway embankments, shallow foundation etc. The biggest disadvantage with horizontal alignment of reinforcement is that it cannot be used in 'In-Situ' construction. Such system requires large scale excavation below footing which destroys the strength of soil developed with age. Further compaction becomes essential after placing the reinforcing elements. In recent years major research effort has been applied on use of geotextile and geogrid soil improvements. Hence it appears possible to use semi-flexible nonhorizontal reinforcement in soil to improve its load bearing capacity for supporting shallow foundation. Basset and Last (1978) investigated the possibility of using non-horizontal reinforcing elements and suggested further work in this direction. Installation of root piles for improving foundation had been advocated by Lizzi (1979). If inclined or vertical reinforcements are found effective in improving the subgrade, it can be installed more easily in new construction and can also be used for strengthening of existing foundation as well. Tatsuoka and Miki (1982) studied the relative performance of horizontal and vertical form of reinforcement in their two dimensional model tests. The authors reported that in case of vertical form of reinforcement (named as tensile reinforcement), restraining of sand zone under the footing was more indirect, therefore the quantity of reinforcement required is more than the horizontal form of system. Verma and Char (1986) evaluated the efficiency of vertical reinforcing elements in improving the sand subgrade. The reinforcements were provided throughout the soil even below the base of footing. The authors concluded that for a given type of reinforcement the bearing capacity increases with the increasing density of reinforcement. Puri et al. (2005) reported significant improvements in the ultimate bearing capacity of lose and medium sand by reinforcing it with flexible vertical reinforcements. Joshi et al. (1994) in their investigation used vertical and horizontal reinforcements in silty fine sand and reported that there is no additional advantage of placing the reinforcement simultaneously below the footing and along the sides except at very high settlement. It is probably due to the fact that reinforcement which is below the footing inhibits dilation and soil

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mobilizes less friction with side reinforcement. During the construction of a subway in Japan in 1927 arrow plates were placed around the foundation to improve resistance and prevent lateral displacements. Katoda (1987) reported that Nigata city Administration Building (1957) and Nigata Electrical Building (1960) were made strong by binding the loose sand subsoil with arrow plates. This prevented their collapse during 1964 Nigata earthquake. Juran et al. (1992) also justified the engineering assumption that in reinforced soil system the vertical force transferred to the foundation soil is relatively small as compared to the lateral earth pressure retained by the reinforcement. Mahmoud and Abdrabbo (1989) reported that vertical reinforcing elements installed along each side of a strip footing were found to be a good method of increasing the bearing capacity of the footing soil system.

The restraining effects in the subgrade under the footing can be provided by different methods and using different materials. One such method of strengthening existing foundation is to place the vertical reinforcing bars beyond the footing base quite easily without disturbing the subgrade just below the footing base. Placement of vertical reinforcement in the subgrade laterally around the footing or beyond the footing base if found effective may prove to be quite beneficial for existing footing where improvement is necessary due to the apprehension of danger to the footing. Keeping this in view Verma and Jha (1992) and Jha et al. (1990) reported results of three and two dimensional model footing tests and they were found to be encouraging.

In the present study two and three dimensional model footing tests were conducted where the vertical reinforcements were installed either beyond the footing edge or surrounding the footing thus without disturbing the subgrade directly below the footing base. Working formula derived by Janbu (1957) for factor of safety of slip surfaces by the method of slices has been extended in the present investigation for the calculation of bearing capacity of reinforced sand subgrade. The theoretical value of bearing capacity thus obtained has been compared with experimental results.

Test Programme

Two types of tests were conducted:

1. Two dimensional model footing test
2. Three dimensional model footing test

Two-dimensional model-footing tests were carried out using a 955 mm x 480 mm x 100 mm box. A 8 mm thick perspex sheet was used in the frontage for observing the failure surface. Special care was taken to make the box as rigid as possible. Four pair of teak wood pieces of 480 mm x 51 mm x 22 mm was used as vertical stiffeners on both side of the box. These four pair of vertical stiffeners was again tightened by two pair of horizontal stiffeners of mild steel of section 50 mm x 50 mm x 5 mm. Each pair was connected tightly together with nuts and bolts at two ends. After each test, stiffeners were removed for observing failure surface. The inside wall of the box was polished to reduce the side friction. The cohesionless test beds were prepared by pouring local sand in layers of 20 mm through a funnel held at constant height 300mm above the surface. After pouring each layer of sand, coloured sand was spread by the side of the perspex sheet to obtain a colour band of 1mm thickness. Thus the compacted test bed showed alternate layers of coloured and ordinary sand.

This arrangement facilitated the observation of developed failure surface through the side perspex wall.

Three- dimensional model- footing tests were performed in a well stiffened square wooden box 1000 mm x 1000mm x 1000mm. The sides of the box were braced with stiffeners to avoid lateral yielding during soil placement and loading of the model foundation. Test beds were prepared using local sand by rainfall technique. The sand used was uniformly graded passing through 250 micron sieve and retained on 75 micron sieve. The height of fall was kept constant as 250mm from the top surface.

The accuracy of sand placement and consistency of placement density during the raining process was checked by placing small cans of known volume at three different locations in the box. The global density of the sand was also calculated by weighing the total sand used for preparing the test bed in the box. The accepted soil beds satisfied the following two conditions: the difference in densities at three measured location was less than 1% and the difference between global density of the soil bed and the average density of the three measured values was less than 1.5 %.

In most of the experiments reported earlier, strip footings were used, despite the fact that rectangular and square footings are far more in practice. Therefore a 40 mm thick square footing of teak wood of desired size was selected for the investigation. To avoid the boundary effects in three dimensional tests, the ratio of the size of box and size of footing was kept as 10. The ratio of depth of soil and size of footing was also kept as 10, which is in conformity with the earlier reports like Omar et al (1993), Akinmusuru and Akinbolade (1981). Square footing will also minimize the dimensional effects. Base of the footing was made rough by gluing sand grains to the base. After completion of the backfilling operation, the sand on the top surface was levelled. The footing was placed on a predefined alignment such that the load from the jack and loading frame would be transferred concentrically to the footing. On levelled sand layer, markings were made upto required extend and at required grid spacing on both sides of the central line of the footing.

Reinforcement should be oriented in the direction of principal tensile strain in order (Bassett and Last 1978) to mobilize as much tensile resistance in the reinforcement as possible. An analysis of principal strains in a direct shear test by Jewell (1980) showed that the principal tensile strain in dense sand is oriented approximately 60 degrees to the shear surface. Grey and Ohashi (1983) suggested that the simpler, perpendicular fiber reinforcement model is a satisfactory mean approximation for predicting shear strength increase along a shear surface crossed by randomly oriented fibers, e.g., a shear surface in a root permeated soils. Mahmoud and Abdrabbo (1989) reported the effect of reinforcing element inclination on bearing capacity ratio. Tests were carried out with reinforcing element inclination angle of 5, 10, 20 and 30 degrees to the vertical. A significant decrease in BCR was observed with the increasing inclination of reinforcing element. This is because the vertical component of the frictional force developed along the reinforcing element (which in turn affects the bearing resistance underneath the footing) decreases with the increase of inclination of element. Again the confined region of soil beneath the footing decreases as the inclination angle of reinforcing element increases, which leads to a decrease in bearing capacity. Therefore, it was decided that only plain galvanised iron wire of 1.7 mm diameter (generally used for binding reinforcements for R.C.C. works) and required length be pushed vertically into the sand bed. While pushing the wire utmost care was taken to maintain its verticality. After placing the reinforcements, the sand was again leveled and checked with spirit level before placing the model footing. No attempt was made in the present study to create an ideally rough surface by cementing a layer of sand grains to the reinforcement. The procedure results in full scale mobilization of the internal friction angle of sand along the surface of the reinforcement. The maximum length of reinforcement used in this study was kept equal to 150 mm, because beyond this length its installation poses problems. The loading arrangement and reinforcement pattern used in the tests are shown in Figures 1 and 2.

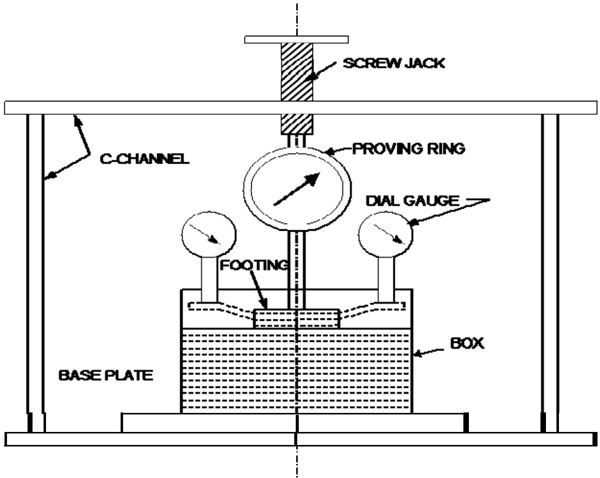

Fig. 1(a) Loading Arrangement (Two Dimensional)

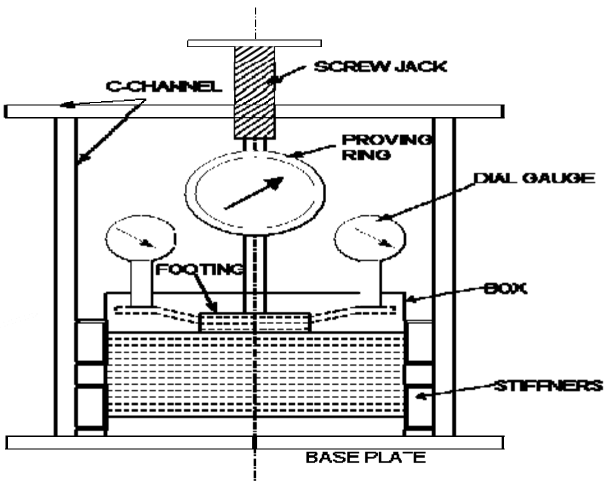

Fig. 1(b): Loading Arrangement (Three Dimensional)

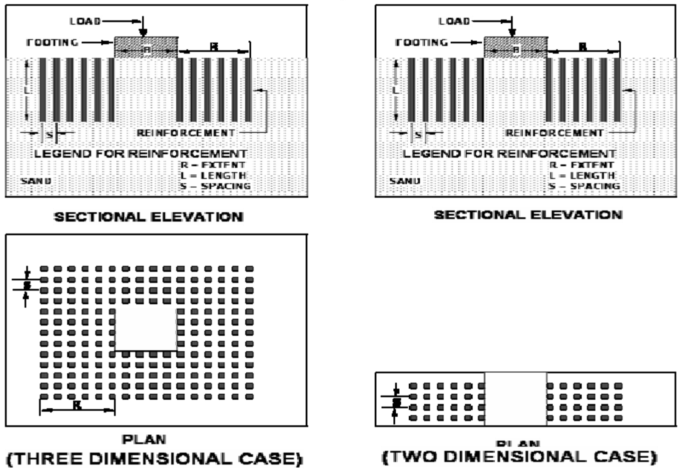

Fig 2: Reinforcement Pattern

Load was applied to the footing through hydraulic jack. Loads were applied carefully and recorded by a proving ring installed between the jack and the test footing. The footing was loaded at constant rate of 1 mm/min until an ultimate state was reached. The ultimate state was defined as that state at which settlement continued to increase without any further increase in load or where there was an abrupt change in load – settlement relationship. The settlement of the footing was recorded by dial gauge fixed adapter and resting on two extension plates fixed on either side of the footing. The settlement reported were the average of the two dial gauge readings, which were nearly identical until the ultimate state was reached. After completion of each test (two dimensional case) the position of the footing and deformed coloured bands were traced. The failure surface was obtained by joining the kinks of the coloured bands. Position of reinforcing rods after failure was also traced out after gradual removal of sand from outside the reinforced zone on either side of the footing. Tracing of failure surface was not possible in case of three dimensional test, but deflection of reinforcements from initial position after failure was observed by careful removal of sand around the outer periphery of the reinforced zone. The properties of sand and reinforcement are given in Table 1 and the variables used in the study are provided in Table 2.

TABLE 1: Properties of Sand and Reinforcement

Effective Size D10 (mm)	Uniformity Coefficient (Cc)	Specific Gravity	Relative Density (%)	Placement Density (kN/m3)	Angle of Internal Friction (ϕ)	Tensile Strength (MPa)	Modulus of Elasticity (MPa)
0.48	1.729	2.74	74	17.6	34^0	3.6	2.29×10^5

TABLE 2: Variables of Study

(B = Width of footing, R = Extent of Reinforcement, L = Length of Reinforcement, S = Spacing of Reinforcement)

Type of test	Two dimensional	Three dimensional	
B mm	100	100	75
R	0.25B, 0.5B, 1.0B and 2.0B	0.25B, 0.5B, 0.75B, 1.0B, 1.5B and 2.0B	0.25B, 0.5B, 0.75B, 1.0B, 1.5B and 2.0B
L	B, 1.5B	B, 1.5B	B, 1.5B and 2B
S	0.1B, 0.13B, 0.17B, 0.25B	0.06B, 0.1B, 0.13B, 0.17B	0.09B,0.13B, 0.18B, 0.22B

Stability Analysis for Bearing Capacity Calculation

Janbu (1957) derived a working formula for the factor of safety of slip surfaces by the method of slices. This method is extended in the present investigation for the calculation of bearing capacity of reinforced sand subgrade. For the sake of completeness the salient working formula is reproduced here. The elementary condition of equilibrium for a slice is shown in Figure 3.

The shearing resistance τ_f along an element of the slip surface is given in terms of effective normal stress, σ by Coulomb's equation

$$\tau_f = c + \sigma \tan \phi \quad (1)$$

Along this element the shear stress τ necessary for equilibrium can be expressed as a certain proportion of τ_f, say

$$\tau = \tau_f / F = c_e + \tan \phi_e \quad (2)$$

where $c_e = c/F$, $\tan \phi_e = \tan \phi_e/F$, and F is a factor of safety with respect to shear strength.

For dry sand,

$$\tau = \sigma \tan\phi_e \quad (3)$$

$$\sigma = dN/dl \quad (4)$$

Three equations of equilibrium for a slice are

Vertical: $dW + dP + dT = dS \sin\alpha + dN \cos\alpha$ (5)

Horizontal: $dE = - dS \cos\alpha + dN \sin\alpha$ (6)

Moment about M: $Tdx + E\, dy_t - dE\, h_t = 0$ (7)

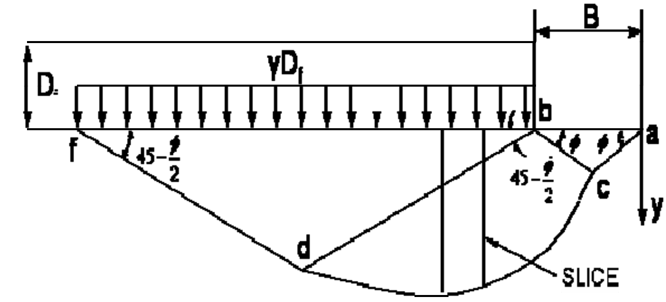

Fig. 3(a) Slip Surface Showing the Elementary Slice

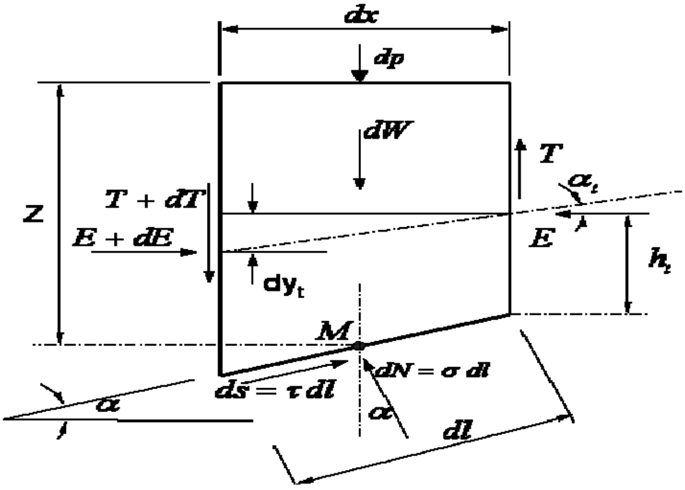

Fig. 3(b) Forces on the Elementary Slice for Equilibrium

If P = dW/dx + dP/dx and t = dT/dx and substituting the value of P and t in Equation (5)

$dN = (P + t)dx/\cos\alpha - ds \tan\alpha$ (8)

Combining Equations (4) and (8) and putting $ds = \tau\, dl$

$\sigma = (P+t) - \tau \tan\alpha$ (9)

Substituting this value of σ in Equation (3) and rearranging

$\tau = [(P+t) \tan\phi_e]/ (1+ \tan\alpha \tan\phi_e)$ (10)

Substituting the value of dN from Equation (8) into Equation (6)

$dE = (P+t) \tan\alpha\, dx - \tau \cos^{-2}\alpha\, dx$ (11)

The horizontal thrust, E at any vertical section is obtained by integrating Equation (11) from 0 to x and the shear force, T is expressed in terms of E on dividing Equation (7) by dx

$$T = - E \tan\alpha_t + h_t (dE/dx) \tag{12}$$

If a resultant force at a boundary face (x = d), is represented by the components E_d and T_d, the overall directional equilibrium of the free body requires

$$\int_0^d dT = T_d \text{ and } \int_0^d dE = E_d \tag{13}$$

At the boundary (Figure 3) $T_d = E_d = 0$, therefore from Equation (11)

$$\int_0^d (P+t) \tan\alpha \, dx - \int_0^d \tau \cos^{-2}\alpha \, dx = 0 \tag{14}$$

Now, introducing τ_f =F τ in Equation (10)

$$\tau_f / F = [(P + t) \tan\phi/F] / [1+ (\tan\alpha \tan\phi/F)]$$

$$\tau_f = [(P + t) \tan\phi] / [1+ (\tan\alpha \tan\phi)/F] \tag{15}$$

Putting the value of τ from Equation (10) into Equation (14)

$$\int_t^d (p+t) \tan\alpha \, dx = \frac{1}{F} \int_0^d \tau_f \cos^{-2}\alpha \, dx$$

Therefore for finite differences,

$$F = (\sum \tau_f \cos^{-2}\alpha \, \Delta x) / (\sum (p + t) \tan\alpha \, \Delta x) \tag{16}$$

Equation (16) gives the working formula for calculating the factor of safety of slip surface against a specified load. The value of τ_f can be determined from Equation (15), but as it is evident from Equation (15) it is necessary to assume an initial value of F for calculating the value of τ_f.

Examination of all the sketched out failure surfaces of two dimensional model footing test reveal that the experimental failure surfaces comprise of straight lines and curve which resemble logarithmic spirals. A typical failure surface has been shown in Figure 4. Hence in the present analysis the failure surface was assumed to be the slip surface as suggested by Terzaghi (1943).

The soil mass enclosed by the slip surface was divided into number of slices. Number of slices to be used in the computation markedly affects the results and the number of slices depends on the kind of slip surfaces considered in the analysis. For the log spiral composite surface, this number is in the vicinity of 20 (Bhattacharya and Pan 2000). The portion below the footing was divided into an integral number of slices of equal width. The remaining portion was also divided into large number of slices of equal width, the only slice of unequal width being the last one (Figure 5). In all the cases considered in the study, the number of slices has been kept in the vicinity of 20.

The pressure on the footing required for calculation of factor of safety available for the assumed slip surface was taken as that given by Terzaghi (1943)

$$q_{ult} = p \, Nq + 0.5 \, \gamma \, B \, N\gamma \tag{17}$$

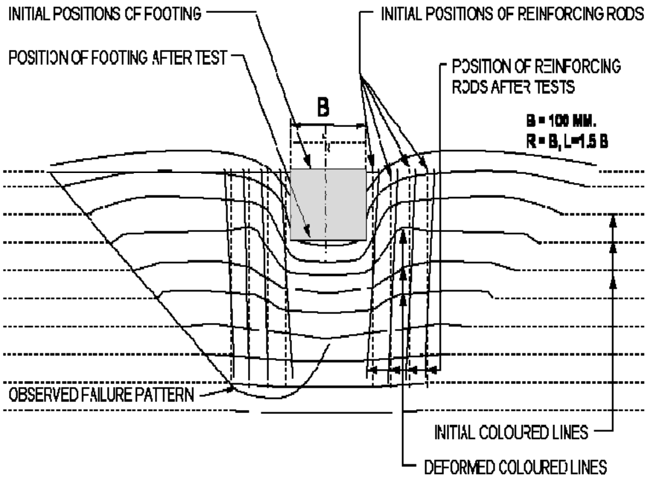

Fig. 4 Failure Pattern of Reinforced Sand Subgrade

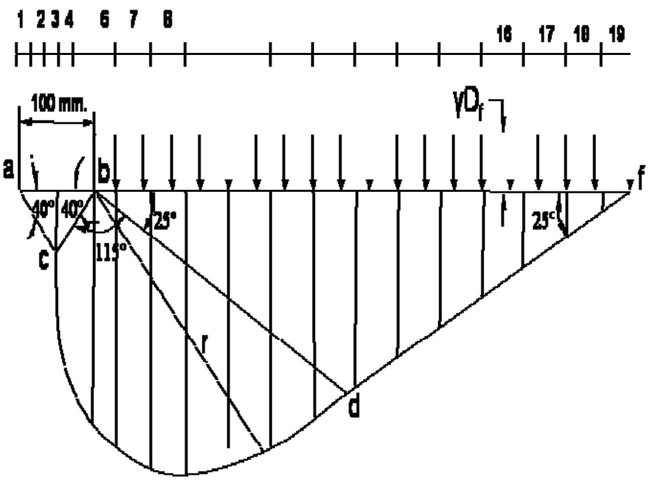

Fig. 5 Slices in the Slip Surface

The value of surcharge load on the slices was obtained as that of corresponding to the surcharge depth of soil at failure, which was found from the experimental observation. The self weight of the slices was known as the density of the subgrade is known. The quantity ‘t’ has been taken equal to zero in the present analysis as a starting point because it does not seriously affect the components. Shields and Tolunay (1973) also reported that the passive earth pressure coefficient will be lowest, hence it would be on conservative side, if it is assumed that vertical shear force at the interface of slices are lost and for such case‘t’ = 0.

The unreinforced subgrade is first analysed with an initial assumed value of F (= 1.5) for calculating τ_f from Equation (15). The quantities necessary to determine F by Equation (16) are calculated for all the slices. The value of ‘p’ is calculated from the weight of the slices, surcharge and from the extent of superimposed footing load acting on the slice. Using finite difference method, the procedure is continued till the difference between the two successive values of F are negligible (restricted to 0.001 in the present analysis). The minimum value of F corresponds to critical failure surface for the unreinforced sand subgrade for the assumed type of failure.

Presence of reinforcing rods in the subgrade provides extra passive resistance against the movement of soil grains about the slip surface and thus contributes additional factor of safety. The resistance offered by the reinforcing rod is assumed to be due to the lateral passive force and is computed through Brom’s (1965) relation

$$E_s = 1.5\,\gamma\, d\, L^2\, K_p \qquad (18)$$

where E_s = Lateral force per rod,

γ = Unit weight of the soil,

d = Diameter of the reinforcing element,

L = Length of reinforcing element

K_p= Coefficient of Rankine passive earth pressure = $(1+\sin\phi)/(1-\sin\phi)$ or the structural capacity of the reinforcing element, whichever is smaller in the present investigation.Equation (16) always gives lesser values.

Total resistance from all reinforcements is equal to $E = \sum(E_s N_r/b)$ (19)

where N_r = Total number of reinforcing element on one side of the center line of the footing,

b = Width of the sand box

The increase in factor of safety due to the presence of reinforcing elements have been obtained by adding the total resistance of the elements to the numerator of Equation (16) and thus the factor of safety due to presence of reinforcement is obtained as

$$F_R= (\sum\tau_f \cos^{-2}\alpha\ \Delta x + E) / (\sum (p + t) \tan\alpha\ \Delta x) \qquad (20)$$

The ultimate bearing capacities of the unreinforced and reinforced subgrades were obtained by multiplying the initial assumed loads with the minimum factors of safety from stability analysis and the theoretical bearing capacity ratios were computed and are compared with experimentally obtained values of bearing capacity ratio for two dimensional model footing tests. Bearing Capacity Ratio (BCR) is the ratio of bearing capacity of the reinforced and unreinforced soil respectively. It is important to realize that the pressure bulb as also the failure wedge at the stage of bearing capacity form in shorter direction (Analogous to the bending of a one way slab in shorter direction). Therefore the analysis for continuous footing of two dimensional cases can be extended to hold good without much error for rectangular footing by taking ‘B’ as width (the smaller of two plan dimension) or for a square footing by taking ‘B’ as the side of square (Kurian 1994). The same working formula derived above has been extended to three dimensional case also where values from both x and y directions have been considered. The only parameter which changes in this case are the total resistance (E) offered by reinforcement due to change in the number of reinforcement wire / elements. Now this theoretical bearing capacity ratio is again compared with the experimental results obtained from three dimensional cases, where reinforcements were provided on all the four sides laterally around the footing but not directly below the footing base.

RESULT AND DISCUSSION

The effect of providing vertical reinforcements beyond / around the footing base in the soil was to increase its bearing capacity compared to the case where no reinforcement was provided. BCR is a non-dimensional parameter. Therefore in Figure 6, BCR is plotted against another non-dimensional parameter X = R/B, where R is the extent of reinforcement in terms of footing width (B). In this particular case, length of reinforcement (L = 1.5B) has been maintained constant whereas the spacing of reinforcement has been varied (S = 0.1B, 0.13B, 0.17B and 0.25B). Similarly Figures 7 and 8 show typical plots between BCR and X for three dimensional case. In Figure 7, width of footing (B) = 100 mm and length of reinforcement (L) =1.5B whereas the corresponding value of B and L in Figure 8 is 75 mm 2.0B. Examination of these three figures clearly reveals that as X (= R/B) increases, both theoretical and experimental bearing capacity ratio increases. Again as the spacing of reinforcement decreases, theoretical and experimental bearing capacity ratio increases. This observation is similar for all the cases considered in the study. Based upon the results of study within the range of variables L, R, and S used in this investigation, it was observed that for the best improvement in bearing capacity, the combination of parameters would be as given in Table 3.

The experimental results obtained in this investigation are similar to the findings reported earlier by investigators like Verma and Char (1986), Mahmoud and Abdrabbo (1989), Long et al. (1990) and Puri et al. (2005).

TABLE 3 Optimum Combination of Reinforcement Parameters

Parameters	Three dimensional test		Two dimensional test
Footing Width (B)	75mm	100mm	100mm
Reinforcement Parameters			
Length (L)	2B	1.5B	1.5B
Spacing (S)	0.09B	0.06B	0.1B
Extent (R)	2B	2B	2B

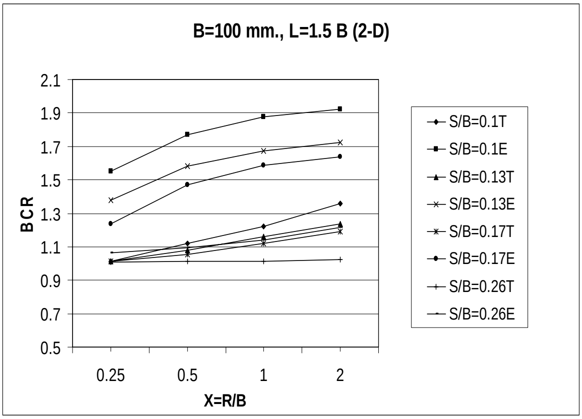

Fig. 6 Bearing Capacity Ratio vs R/B Ratio (2-D) for Constant ‘L’

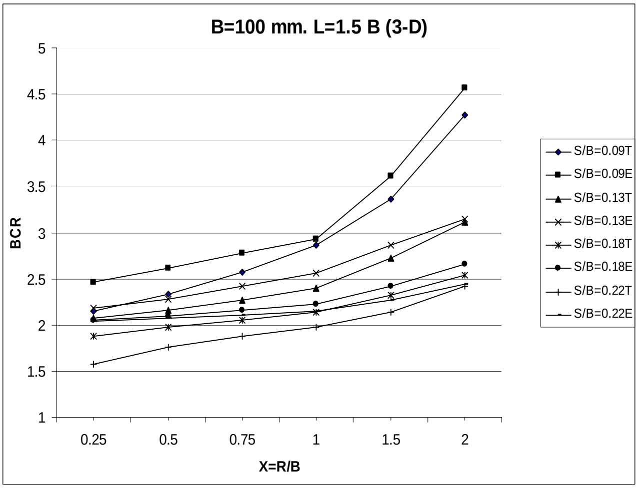

Fig. 7 Bearing Capacity Ratio vs R/B Ratio (3-D) for Constant 'L'

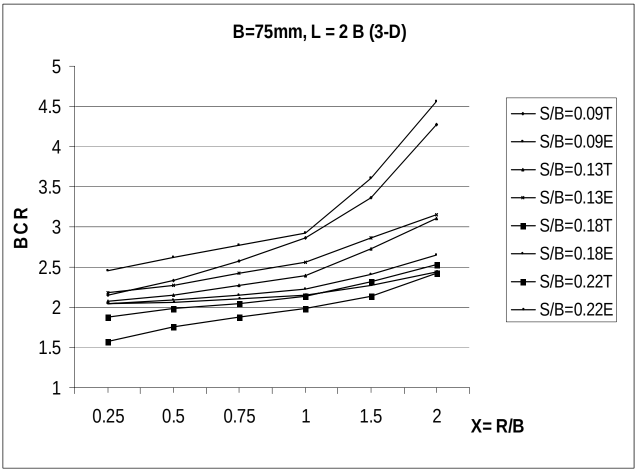

Fig. 8 Bearing Capacity Ratio vs R/B Ratio (3-D) for Constant 'L'

It can also be observed from Figures 6 - 8 that the theoretical bearing capacity ratio is always lower than the experimental bearing capacity ratio. The difference is more pronounced in case of two dimensional tests and it may be due to the fact that soil movements have been restrained in one direction.

In three dimensional tests it is being observed that the difference between theoretical and experimental value decreases significantly if length of reinforcement (L) increases from one and half times to two times footing width (Figure 9). The experimental observation of the present study is similar to the findings of Mahmoud and Abdrabbo (1989) where the authors reported that when L/B < 2.0, length of reinforcing elements provided only partial confinement.

It can also be observed from Figure 8 that when extent of reinforcement (R) and length of reinforcement (L) is equal to twice the width of footing, theoretical and experimental bearing capacity ratio almost coincides for many cases.

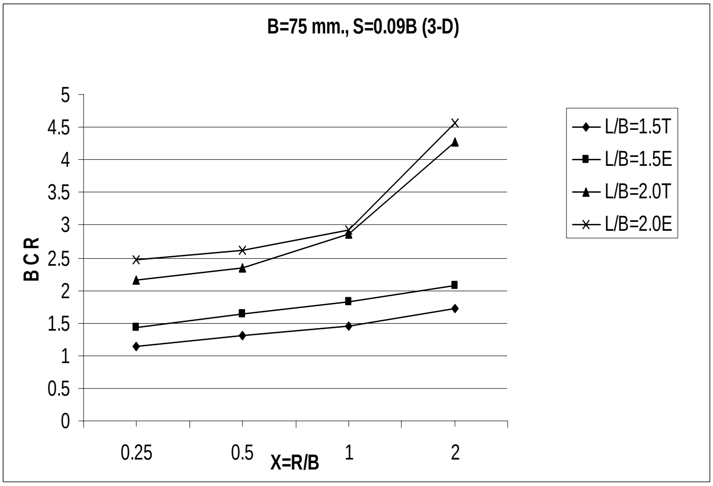

Fig. 9 Bearing Capacity Ratio vs R/B Ratio (3-D) for Constant 'S'

The difference in experimental and theoretical value can be attributed to factors such as increase in density of sand (due to installation of reinforcing elements), change in stress distribution within the soil, anchoring effect due to increase in the length of reinforcing elements and rotation of reinforcements from its initial position at failure.

Rotation of reinforcement during failure and anchoring effect are two most important parameters which can affect the result significantly. Any small displacement can generate sufficient friction at the soil –wire interface, which in turn mobilizes the tensile force in the reinforcing elements (Juran et al. 1983). The tangential components of this tensile force directly resists shear and the normal component increases the confining stress on the shear plane. This particular aspect has not been considered in theoretical analysis. Again as the length of reinforcement increases, the larger portion get anchored in the lower soil mass, which don't fail and hence do not move with the sliding slip surface. Though length of reinforcing element has been considered while calculating passive resistance in theoretical analysis but it does not take care of anchoring effect.

Conclusion

While computing BCR through stability analysis it has been assumed that there is movement of soil particles laterally in the plastic zone. Hence if reinforcing elements are put in soil subgrade, it will exert passive resistance against the movement of soil which has been estimated through Equation (20). This concept seems reasonable considering experimental evidence, but the computed values are invariably lower than experimental values. Improved corelation may be achieved by modifying the coefficients in Equation (20) and through a more refined analysis when further substantial data under different conditions are available.

Installation of Vertical reinforcing elements provided beyond / around the footing base increases the load carrying capacity of the subgrade, and therefore can be used for strengthening the existing footing. Bearing capacity ratio increases with the increase in the density of reinforcement.

The ratio of diameter of the reinforcing rods to the particles for a field installation is likely to be different, further investigation on a bigger model or numerical analysis is necessary.

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