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Slope Stability Coefficients for Steep Homogeneous Soil Slopes

Sitaram Nayak* and G. Padmaja**

Introduction

Many civil engineering projects such as earthen dams, railway cuttings, embankments etc. requires knowledge of slope stability analysis. Failure of slopes may lead to loss of human life as well as economical loss. Various procedures of stability analysis are, in general, divided into two major classes: (i) Mass procedure: In this case, the mass of the soil above the surface of sliding is taken as a unit. This procedure is useful when the soil that forms the slope is assumed to be homogeneous, although this is not the case in most of the natural slopes. (ii) Method of slices: In this procedure, the soil above the surface of sliding is divided into a number of vertical parallel slices. The stability of each slice is calculated separately. This is a versatile technique in which the nonhomogenity of the soils and pore water pressure can be taken into consideration. It also accounts for the variation of the normal stress along the potential failure surface. Extensions to the generalized method of slices (that is commonly used in slope stability analysis) were suggested by Chen and Morgenstern (1983). In addition a numerical procedure has been developed for exploring formally the bounds of the factor of safety within the limits of physical admissibility. Chen and Shao (1988) studied the feasibility of using the optimization methods to search for the minimum factor of safety in slope stability analysis. Davidon-Fletcher-Powel (DFP) method was employed to determine the minimum factor of safety and its associated critical slip surface. Dov Leshchinsky and Huang (1992) proposed a generalized variational slope stability analysis, which is an efficient method. No guessing of a force related function is needed in this method. An alternative slope analysis based on the discrete element method was proposed by Chang (1992) to avoid the assumptions regarding the inclination and location of the interslice forces. The expressions for the factor of safety were presented in a manner that allows most of the current limit equilibrium based method of slices to be studied within a single framework. Bhattacharya and Basudhar (1999) proposed a new method of solution, in which the limit equilibrium formulation has been cast as a nonlinear programming problem and solved by using the sequential unconstrained minimization technique also known as the penalty function method. This method does not force to satisfy conditions like the force and the moment equilibrium conditions at each step during the process of minimization. Bhattacharya and Basudhar (2000) modified their earlier approach by avoiding

* Assistant Professor, Dept. of Civil Engg., National Institute of Technology, Surathkal, Karnataka –575 025.

** Formerly Post Graduate Student, Dept. of Civil Engg., National Institute of Technology, Surathkal, Karnataka –575 025.

the assumption on the shape of failure. surface. From these slope stability analysis methods, values of factor of safety are not readily available for the practising engineers. Each method invites running of the program for the specific case and thereby obtains the factor of safety. In this work, Bishop and Morgenstern method (1960) has been used to determine the stability coefficients for different values of $c/\gamma H$, ϕ and i, which can be readily used by the practicing engineers to calculate the factor of safety.

Bishop and Morgenstern Method

Bishop and Morgenstern (1960) provided tables based on Bishop method (1955), for the calculation of factor of safety (FS) for few specific cases. They covered $c/\gamma H$ up to 0.10 and slope angle i up to 26.57°. But in actual cases slope angle steeper than 26.57° are quite common and hence the approach is used to establish slope stability coefficients for steep slope angles.

Consider a slope AB of vertical height H as shown in Figure 1. A trial slip circle AC has centre at O and has radius r. The forces acting on nth slice are shown in Figure 2. From moment equilibrium and force equilibrium in vertical direction Bishop (1955) derived expression for the factor of safety (FS) as

$$FS = \frac{\sum_{n=1}^{n=p} \left(cb_n + (W_n - u_n b_n)\tan\phi\right)\frac{1}{m_{\alpha(n)}}}{\sum_{n=1}^{n=p} W_n \sin\alpha_n} \tag{1}$$

where $m_{\alpha(n)} = \cos\alpha + \frac{\tan\phi \sin\alpha_n}{FS}$

$$W_n = \gamma b_n z_n \tag{2}$$

where

z_n = average height of the *n*th slice

b_n = width of nth slice

α_n = Angle made by the tangent with the horizontal at the base of nth slice

$u_n = h_n \gamma_w$

The pore pressure ratio

$$r_u = \frac{u_n}{\gamma z_n} = \frac{h_n \gamma_w}{\gamma z_n} \tag{3}$$

Substituting Equations (2) and (3) into Equation (1) and simplifying

$$FS=\left[\frac{1}{\sum_{n=1}^{n=p}\frac{b_n}{H}\frac{z_n}{H}\sin\alpha_n}\right]\times\sum_{n=1}^{n=p}\left\{\frac{\frac{c}{\gamma H}\frac{b_n}{H}+\frac{b_n}{H}\frac{z_n}{H}\left[1-r_u\right]\tan\phi}{m_{\alpha(n)}}\right\} \tag{4}$$

The factor of safety based on the above equation was expressed in the form

$$\mathrm{FS} = \mathrm{m}' - \mathrm{n}'\mathrm{r_u} \tag{5}$$

where m', n' are stability coefficients and are given by

$$m' = \frac{\sum\left\{\left[\frac{\left(\frac{cb_n}{\gamma H^2}\right)+\left(\frac{b_n z_n \tan\phi}{H^2}\right)}{m_{\alpha(n)}}\right]\right\}}{\sum\frac{b_n z_n \sin\alpha_n}{H^2}} \tag{6}$$

and

$$n' = \frac{\sum\left\{\frac{b_n z_n \tan\phi / H^2}{m_{\alpha(n)}}\right\}}{\sum\frac{b_n z_n \sin\alpha_n}{H^2}} \tag{7}$$

Bishop and Morgenstern developed design tables (values of m′ and n′) for c/γH = 0.025, 0.05, 0.075 and 0.1 and i = 11.31°(5H:1V), 14.04°(4H:1V), 18.26°(3H:1V) and 26.57°(2H:1V). But in actual practice, slopes steeper than 26.57° are quite common and range of c/γH from 0.025 to 0.1 will not be sufficient. Hence there is a need for developing tables of m′ and n′ values for all practical cases of c/γH, i and ϕ. Range of c/γH, i and ϕ considered in this study are c/γH = 0.0, 0.025, 0.05, 0.075, 0.1, 0.3, 0.5, 0.75 and 1.0; i = 26.57°, 45°, 60° and 75° and 0° ≤ ϕ ≤ 40°. A program has been developed in C language to determine the stability coefficients, namely, *m*′ and *n*′ and also the failure surface details.

Results and Discussion

Values of m′ and n′ for different values c/γH, for different ϕ values and slope angle, i were found out and typical values are presented in Tables 1 to 5. Table1 represents the values of m′ and n′ when c/γH = 0.0 and i = 26.57°, 45°, 60° and 75° for 0° ≤ ϕ ≤ 40°. Similarly Tables 2, 3, 4 and 5 represent the values of m′ and n′ for i = 26.57°, 45°, 60° and 75°, when 0° ≤ ϕ ≤ 40° for c/γH = 0.05, 0.1, 0.5 and 1.0 respectively. The values of m′ and n′ for ϕ = 0° correspond to base failure for i = 26.57° and 45°, whereas for i > 53°, toe failure takes place even if ϕ = 0° (Terzaghi and Peck 1967; Das 1998).

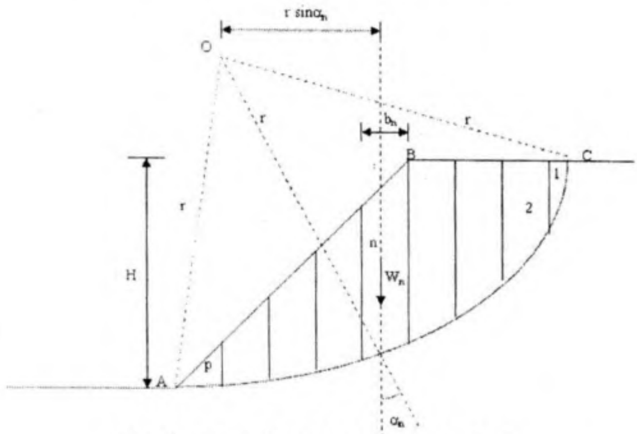

Fig. 1 Trial Failure Surface For Analysis

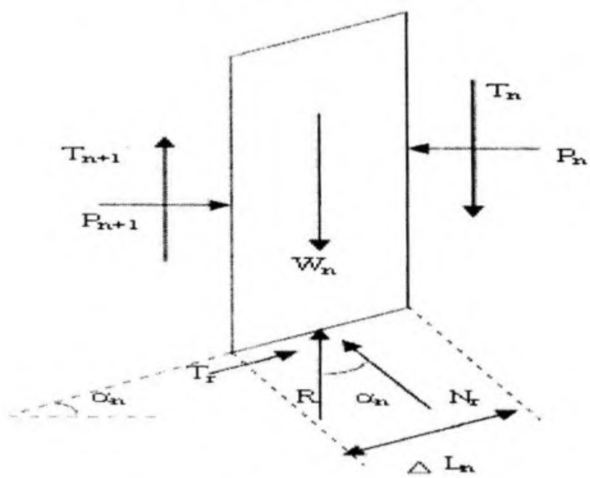

Fig. 2 Forces Acting on the nth Slice

Variation of m′ and n′ with c/γH

Variation of m′ and n ′ with c/γH for different ϕ values (slope angle, $i=60^0$) has been presented in Figures 3 and 4. From Figure 3, we can conclude that variation of m′ with c/γH is linear. As c/γH increases, m′ increases and also m′ increases with increase in ϕ value. From Figure 4, it is found that n′ increases with increase in c/γH and it also increases with ϕ value, but increase is in nonlinear fashion.

Variation of m′ and n′ with slope angle, i

Variation of m′ versus i and n′ versus i (c/γH = 0.5) considering various values of ϕ are presented in Figures 5 and 6. It is found that m′ decreases with increase in the slope angle, i and it increases with increase in ϕ value. Similarly n′ also decreases with increase in i and increases with increase in ϕ.

Factor of Safety

Results (m′ and n′ values) have been presented in the form of tables (Tables 1 to 5) from which the factor of safety can be calculated directly using Equation (5). From Tables 1 to 5, corresponding to the field conditions, the values of m′ and n′ can be taken directly and the factor of safety can be calculated using Equation (5). Two illustrative problems given below highlight the same.

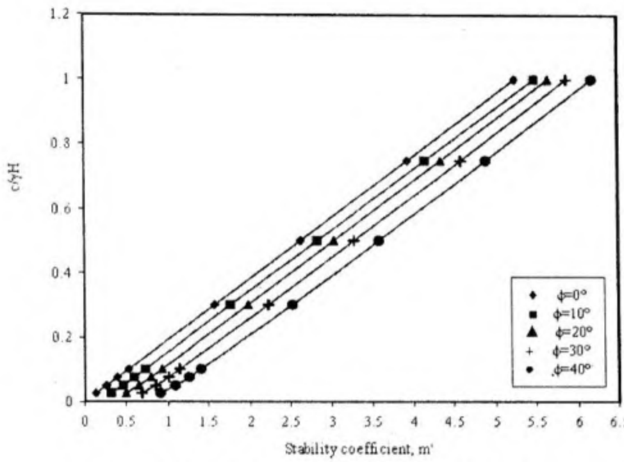

Fig. 3 Variation of m′ with c/γH for i = 60°

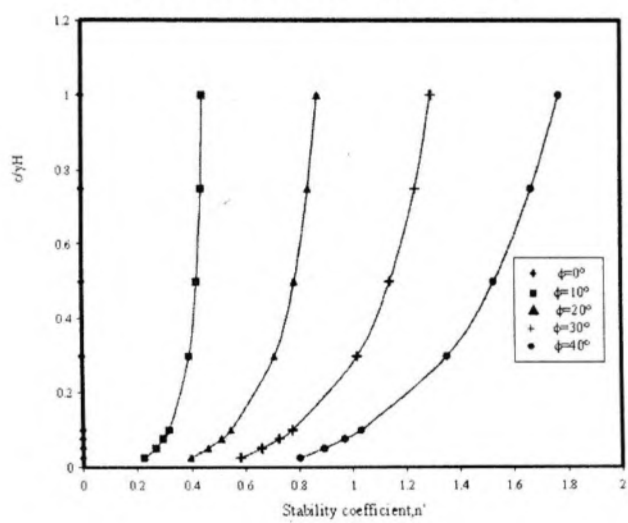

Fig. 4 Variation of n′ with c/γH for i = 60°

Failure Surface

Extent of failure surface at the top of the slope is denoted by x_e (Figure 7). For various conditions, extent of failure surface is represented in the non dimensional form (with respect to height of the slope) and is provided as x_e/H in Tables 1 to 5. For $\phi = 0°$ and $i < 53°$, base failure occurs (Terzaghi and Peck 1967; Das 1998). For $i = 26.57°$ and $\phi = 0°$, extent of failure surface at the top of the slope is expressed in terms of x_e/H and it is found to be 1.138. x_e/H for $i = 45°$ and $\phi = 0°$ is found to be 1.389. These details are of great use in deciding the extent of failure and knowing the soil mass involved in the failure. Precautions can be taken not to apply any surcharge load (at the top of the slope) within this failure zone. Also high slopes can be divided into 2-3 small slopes by providing horizontal projections (berms/benches) in between them. Extent of failure surface details help in deciding the minimum width of horizontal projection required in between the slopes (= x_e), so as to make each slope to behave independently. Surcharge load effect is not considered in the entire analysis. In addition to gravity loads, if the slopes are also subjected to surcharge loads, stability coefficients reported in Tables 1 to 5 shall not be used.

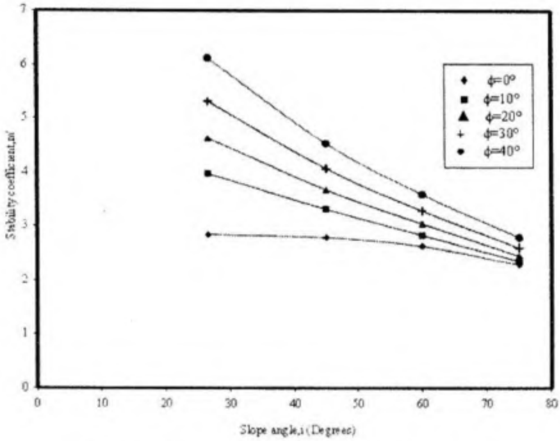

Fig. 5 Variation of m′ with i for c/γH = 0.5

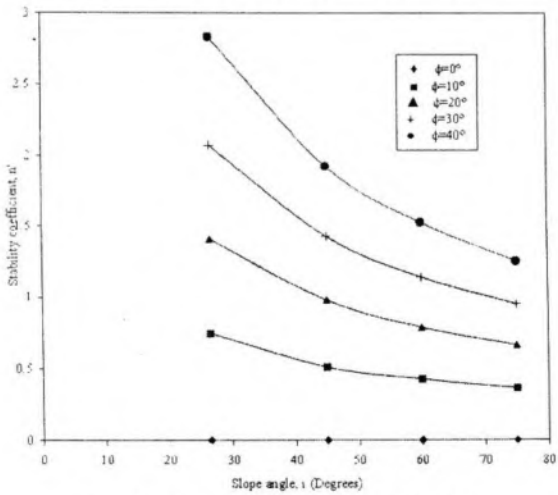

Fig. 6 Variation of n′ with i for c/γH=0.5

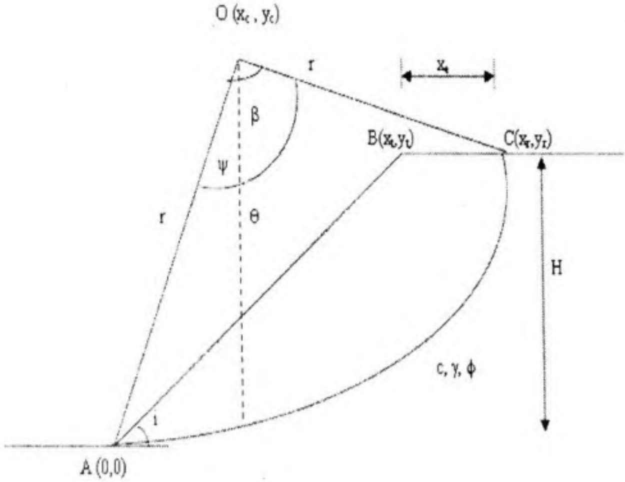

Fig. 7 Reference Figure for Toe Failure

TABLE 1: Slope Stability Coefficients m′ and n′ for c/γH = 0.0

i / ϕ	26.57° (2H:1V)			45° (1H:1V)			60° (0.58H:1V)			75° (0.27H:1V)		
	m′	n′	x_e/H	m′	n′	x_e/H	m′	n′	x_e/H	m′	n′	x_e/H
5°	0.242	0.201	0.369	0.137	0.110	0.295	0.094	0.080	0.215	0.065	0.060	0.285
10°	0.438	0.392	0.213	0.246	0.241	0.185	0.177	0.151	0.169	0.131	0.125	0.226
15°	0.631	0.581	0.160	0.351	0.342	0.135	0.256	0.245	0.119	0.200	0.190	0.190
20°	0.829	0.742	0.139	0.462	0.435	0.130	0.336	0.322	0.124	0.268	0.251	0.161
22.5°	0.931	0.820	0.128	0.514	0.490	0.120	0.380	0.360	0.116	0.292	0.291	0.118
25°	1.038	0.970	0.109	0.573	0.550	0.101	0.416	0.413	0.095	0.340	0.325	0.094
27.5°	1.145	1.080	0.101	0.630	0.615	0.098	0.461	0.461	0.090	0.379	0.365	0.088
30°	1.258	1.190	0.095	0.696	0.675	0.091	0.502	0.510	0.085	0.420	0.410	0.081
32.5°	1.375	1.310	0.091	0.763	0.725	0.087	0.558	0.551	0.080	0.462	0.445	0.078
35°	1.501	1.439	0.088	0.828	0.802	0.082	0.602	0.612	0.078	0.506	0.505	0.074
40°	1.779	1.700	0.082	0.980	0.950	0.079	0.714	0.685	0.076	0.603	0.590	0.071

TABLE 2: Slope Stability Coefficients m′ and n′ for c/γH = 0.050

i / ϕ	26.57° (2H:1V)			45° (1H:1V)			60° (0.58H:1V)			75° (0.27H:1V)		
	m′	n′	x_e/H	m′	n′	x_e/H	m′	n′	x_e/H	m′	n′	x_e/H
0°	0.314	0.000	1.138	0.288	0.000	1.389	0.262	0.000	0.833	0.228	0.000	0.860
5°	0.622	0.294	0.566	0.458	0.197	0.400	0.363	0.158	0.360	0.281	0.129	0.357
10°	0.868	0.522	0.421	0.599	0.340	0.320	0.461	0.268	0.266	0.350	0.215	0.276
15°	1.101	0.738	0.344	0.732	0.470	0.280	0.557	0.369	0.220	0.420	0.293	0.239
20°	1.334	0.957	0.316	0.864	0.597	0.240	0.652	0.463	0.176	0.494	0.370	0.209
22.5°	1.452	1.062	0.280	0.932	0.662	0.240	0.701	0.511	0.164	0.532	0.410	0.199
25°	1.574	1.181	0.282	1.000	0.728	0.210	0.750	0.561	0.153	0.572	0.449	0.187
27.5°	1.699	1.296	0.245	1.071	0.796	0.210	0.802	0.611	0.141	0.612	0.491	0.179
30°	1.829	1.421	0.248	1.143	0.864	0.200	0.855	0.662	0.143	0.655	0.534	0.172
32.5°	1.963	1.544	0.225	1.218	0.936	0.180	0.909	0.716	0.139	0.700	0.579	0.161
35°	2.104	1.679	0.214	1.297	1.013	0.178	0.968	0.774	0.145	0.747	0.627	0.159
40°	2.410	1.968	0.190	1.467	1.174	0.169	1.093	0.894	0.142	0.850	0.730	0.144

TABLE 3: Slope Stability Coefficients m′ and n′ for $c/\gamma H = 0.1$

ϕ \ i	26.57° (2H:1V)			45° (1H:1V)			60° (0.58H:1V)			75° (0.27H:1V)		
	m′	n′	x_e/H	m′	n′	x_e/H	m′	n′	x_e/H	m′	n′	x_e/H
0°	0.627	0.000	1.163	0.576	0.000	1.389	0.525	0.000	0.833	0.457	0.000	0.862
5°	0.973	0.328	0.692	0.761	0.229	0.421	0.623	0.183	0.432	0.499	0.153	0.421
10°	1.248	0.591	0.571	0.919	0.399	0.328	0.727	0.318	0.356	0.564	0.260	0.358
15°	1.507	0.831	0.470	1.067	0.550	0.342	0.829	0.436	0.298	0.634	0.352	0.314
20°	1.765	1.072	0.431	1.215	0.697	0.321	0.933	0.548	0.255	0.709	0.440	0.280
22.5°	1.893	1.189	0.388	1.289	0.768	0.296	0.986	0.606	0.249	0.748	0.483	0.262
25°	2.028	1.314	0.392	1.365	0.844	0.289	1.040	0.661	0.225	0.798	0.528	0.249
27.5°	2.162	1.437	0.349	1.442	0.918	0.276	1.097	0.721	0.220	0.830	0.573	0.247
30°	2.304	1.573	0.353	1.523	0.996	0.271	1.154	0.777	0.204	0.873	0.619	0.230
32.5°	2.448	1.704	0.312	1.605	1.076	0.253	1.214	0.837	0.193	0.919	0.668	0.220
35°	2.602	1.851	0.313	1.692	1.159	0.249	1.276	0.899	0.182	0.967	0.718	0.220
40°	2.929	2.153	0.278	1.878	1.340	0.235	1.411	1.031	0.158	1.073	1.073	0.197

TABLE 4: Slope Stability Coefficients m′ and n′ for $c/\gamma H = 0.5$

ϕ \ i	26.57° (2H:1V)			45° (1H:1V)			60° (0.58H:1V)			75° (0.27H:1V)		
	m′	n′	x_e/H	m′	n′	x_e/H	m′	n′	x_e/H	m′	n′	x_e/H
0°	2.831	0.000	1.138	2.785	0.000	1.389	2.624	0.000	0.833	2.283	0.000	0.867
5°	3.628	0.394	0.948	3.124	0.261	0.764	2.742	0.214	0.707	2.328	0.196	0.552
10°	3.963	0.745	0.843	3.308	0.508	0.512	2.822	0.426	0.513	2.343	0.364	0.506
15°	4.293	1.081	0.789	3.486	0.752	0.477	2.920	0.610	0.476	2.387	0.520	0.476
20°	4.624	1.409	0.765	3.669	0.979	0.443	3.030	0.788	0.452	2.444	0.663	0.446
22.5°	4.788	1.569	0.699	3.768	1.098	0.443	3.089	0.877	0.442	2.476	0.733	0.432
25°	4.960	1.731	0.688	3.860	1.206	0.409	3.149	0.965	0.425	2.512	0.806	0.416
27.5°	5.136	1.911	0.677	3.964	1.326	0.409	3.213	1.054	0.408	2.551	0.878	0.416
30°	5.313	2.073	0.636	4.062	1.430	0.376	3.280	1.143	0.402	2.592	0.949	0.403
32.5°	5.501	2.255	0.625	4.170	1.544	0.376	3.349	1.234	0.393	2.635	1.020	0.390
35°	5.697	2.443	0.614	4.277	1.651	0.344	3.423	1.329	0.378	2.680	1.092	0.374
40°	6.113	2.835	0.566	4.524	1.924	0.344	3.584	1.528	0.360	2.786	1.254	0.362

TABLE 5: Slope Stability Coefficients m′ and n′ for $c/\gamma H = 1.0$

ϕ \ i	26.57° (2H:1V)			45° (1H:1V)			60° (0.58H:1V)			75° (0.27H:1V)		
	m′	n′	x_e/H	m′	n′	x_e/H	M′	n′	x_e/H	m′	n′	x_e/H
0°	5.662	0.000	1.138	5.571	0.000	1.389	5.249	0.000	0.833	4.567	0.000	0.861
5°	6.909	0.410	1.005	6.064	0.270	0.814	5.370	0.220	0.763	4.637	0.192	0.768
10°	7.261	0.795	0.937	6.251	0.527	0.727	5.486	0.447	0.530	4.656	0.396	0.535
15°	7.613	1.167	0.843	6.441	0.781	0.512	5.561	0.664	0.513	4.664	0.574	0.506
20°	7.968	1.537	0.843	6.638	1.047	0.477	5.656	0.877	0.486	4.692	0.747	0.482
22.5°	8.152	1.724	0.764	6.737	1.185	0.477	5.709	0.979	0.476	4.711	0.834	0.478
25°	8.341	1.918	0.767	6.840	1.337	0.477	5.764	1.083	0.458	4.737	0.925	0.465
27.5°	8.531	2.103	0.767	6.943	1.462	0.443	5.822	1.189	0.452	4.766	1.015	0.453
30°	8.728	2.306	0.738	7.054	1.612	0.443	5.886	1.298	0.452	4.798	1.105	0.440
32.5°	8.934	2.505	0.690	7.172	1.768	0.409	5.955	1.413	0.440	4.833	1.195	0.427
35°	9.153	2.719	0.689	7.284	1.890	0.409	6.028	1.527	0.425	4.871	1.286	0.417
40°	9.609	3.159	0.663	7.545	2.205	0.376	6.190	1.771	0.402	4.960	1.481	0.408

Illustrative Examples

Problem 1: A homogeneous soil slope of 8 m height makes an angle of 60^0 with the horizontal. The soil properties are: $c = 15$ kN/m^2, $\phi = 25^0$, $\gamma = 18.75$ kN/m^2. Pore pressure ratio is 0.15. Estimate the factor of safety and extent of critical failure surface.

Solution: $i = 60^0$, $H = 8$ m, $r_u = 0.15$, $c = 15$ kN/m^2, $\phi = 25^0$, $\gamma = 18.75$ kN/m^2.

$c / (\gamma H) = 15 / (18.75 \times 8) = 0.1$

For $c / (\gamma H) = 0.1$ and $\phi = 25^0$, referring Table 3
$m' = 1.040$, $n' = 0.661$ and $x_e/H = 0.225$
Therefore, Factor of safety, $FS = m' - n' r_u$
$= 1.040 - 0.661 \times 0.15$
$= 0.941$ (unsafe)
Extent of failure surface, $x_e = 0.225$ H
$= 0.225 \times 8$
$= 1.80$ m (from top edge of the slope)

Problem 2: A homogeneous soil slope of 5 m height makes an angle of 45^0 with the horizontal. The soil properties are: $c = 30$ kN/m^2, $\phi = 20^0$, $\gamma = 20$ kN/m^2. Pore pressure ratio is 0.20. Estimate the factor of safety and extent of critical failure surface.

Solution: $i = 45^0$, $H = 5$ m, $r_u = 0.20$, $c = 50$ kN/m^2, $\phi = 20^0$, $\gamma = 20$ kN/m^2.
$c / (\gamma H) = 50 / (20 \times 5) = 0.5$
For $c / (\gamma H) = 0.5$ and $\phi = 20^0$, referring Table 4
$m' = 3.669$, $n' = 0.979$ and $x_e/H = 0.443$
Therefore, Factor of safety, $FS = m' - n' r_u$
$= 3.669 - 0.979 \times 0.20$
$= 3.473$ (safe)
Extent of failure surface, $x_e = 0.443$ H
$= 0.443 \times 5$
$= 2.215$ m (from top edge of the slope)

Conclusions

Slope stability analysis by Bishop and Morgenstern method has been carried out considering $c/\gamma H$ = 0.0, 0.025, 0.05, 0.075, 0.1, 0.3, 0.5, 0.75 and 1.0; i = 26.57°, 45°, 60° and 75° and ϕ = 0°, 5°, 10°, 15°, 20°, 22.5°, 25°, 27.5°, 30°, 32.5°, 35° and 40°. Based on the work presented in this paper, the following conclusions are drawn:

1) Stability coefficients m′ and n′ and extent of failure surface are provided in Tables 1 to 5 for various conditions, which can be directly made use for finding out factor of safety and extent of failure surface for the field conditions.
2) Stability coefficient m′ increases linearly with increase in $c/\gamma H$ values. As ϕ increases, the stability coefficient m′ increases.
3) Stability coefficient n′ increases nonlinearly with increase in $c/\gamma H$ values. As ϕ increases, the stability coefficient n′ also increases.
4) As the slope angle i increases, the stability coefficients m′ and n′ decrease.
5) Extent of failure surface decreases with increase in ϕ and increases with increase in $c/\gamma H$.

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Notations

The following notations are used in this paper.

α_n = Angle made by the horizontal with the tangent at the base of nth slice
b_n = Width of nth slice
c = Unit cohesion
γ = Unit weight of soil
H = Vertical height of the slope
i = Angle made by the slope angle with the horizontal
m', n' = stability coefficients
ϕ = Angle of internal friction
r_n = Pore pressure ratio for the nth slice
r_u = Pore pressure ratio
W_n = Weight of the nth slice
z_n = average height of the *n*th slice.