# Uplift Capacity of Single Piles Embedded in Sand 

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## Introduction

Structures supported on piles are very often subjected to large lateral loads due to wind or wave loads and the resulting moments induce tension in some of the piles. Unlike the prediction of ultimate load carrying capacity of piles under compressible loads the same under pull out force is an area which is least studied. Resistance to uplift is due to the shaft friction developed between the pile shaft and the surrounding soil. Mohan et al. (1963), Vesic (1970), Rao and Venkatesh (1985), Joshi et al. (1992), Nicola and Randolph (1993), O'Neill (2001), Ramasamy et al. (2004) etc. have shown that pull-out shaft friction is significantly less than the push-in shaft friction. Few theories have been developed to find the net uplift capacity of a bored pile (Meyerhof 1973; Chattopadhyay and Pise 1986) and validated through experimental measurements. The above theories differ mainly in their assumptions with regard to the shape and extent of the failure surface. Meyerhof (1973) theory assumes failure along the pile-soil interface. Chattopadhyay and Pise (1986) assumes a curved failure surface within the soil, but the predicted extent of failure surface at the ground level is unreasonably high for deep piles in dense soil. As such, using limit equilibrium method an attempt has been made here to estimate the uplift capacity of a single pile embedded in sand for different placement density more accurately. The theory is based on the assumption of a curved failure surface initiating along the pile shaft at the tip of the pile and gradually progressing upwards and outwards. The lateral extent of the failure surface depends on pile length-to-diameter ratio, pile roughness and shear strength of soil. The predictive capability of the theory is then tested with the experimental results.

## Analysis

## Failure surface

A vertical pile of diameter, $d$, and length, $L$, is assumed to be embedded in a soil medium having an angle of shearing resistance $(\phi)$ and unit weight $(\gamma)$.

[^0]During uplift of the pile, an axisymmetric solid body of revolution of soil will also move with it, resulting in a curved surface, as suggested by Chattopadhyay and Pise (1986) and shown in Figure 1. The movement is resisted by the mobilized shear strength of the soil along the failure and the weight of the soil and the pile. In the limiting equilibrium condition, ultimate uplift capacity of the pile is attained.


Fig. 1 Pile and Failure Surface
The following assumptions have been made about the resulting failure surface.

1. The shape and extent of the failure surface depend on the slenderness ratio ( $\lambda=\mathrm{L} / \mathrm{d}$ ), the angle of shearing resistance ( $\phi$ ) of the soil, and pile-soil friction angle ( $\delta$ ). For $\delta>0$, the inclination of the failure surface with the horizontal at the ground surface approaches (45- $\phi / 2$ ) (Balla 1961; Meyerhof and Adams 1968; Chattopadyay and Pise 1986) and for $\delta=0$, the failure surface coincides with the interfacial plane between the pile and soil.
2. For piles with pile-soil friction angle $\delta \geq 0$, under ultimate uplift force, $\mathrm{P}_{\mathrm{u}}$, the resulting failure surface initiates tangentially to the pile surface at the tip of the pile and moves through the surrounding soil (Balla 1961; Meyerhof and Adams 1968).
3. When $\phi \rightarrow \phi_{\text {max }}, \delta \rightarrow \phi$ and $\lambda \rightarrow \infty$ the lateral extent of the failure surface at the ground surface approaches its maximum value. The maximum value of angle of friction ( $\phi_{\max }$ ) for practical purposes has been assumed to be $50^{\circ}$ (Chattopadhyay and Pise 1986).

Assuming linear variation of angle that the tangent at any point on the failure surface makes with the horizontal, from $90^{\circ}$ to $\left(45^{\circ}-\phi / 2\right)$ over the length of the pile the slope of the failure surface at a height $Z$ above the pile tip (Figure 1) can be written as

$$
\begin{equation*}
\frac{d Z}{d x}=\tan \left\{90-\left(\frac{45+\phi / 2}{L}\right) Z\right\} \tag{1}
\end{equation*}
$$

To satisfy the above mentioned assumptions (such as when $\delta \rightarrow 0 \Rightarrow \frac{\mathrm{dZ}}{\mathrm{dx}} \rightarrow \infty$ and $\delta \rightarrow \phi_{\text {max }}, \mathrm{x}$ at ground surface approaches maximum value provided $\phi \rightarrow \phi_{\max }$ and $\lambda \rightarrow \infty$ ) Equation (1) is modified as
$\frac{d Z}{d x}=\tan \left\{90-\left(\frac{45+\phi / 2}{L}\right) Z\right\} \exp \left\{\beta\left(1-\left(\frac{Z}{L}\right)^{\frac{1}{\lambda}}\right)\right\}$
where $\beta=(\lambda)^{C} \frac{\left(\phi_{\max }-\phi\right)}{2 \delta}$
where C is a constant.
Equation (2) satisfies all the above boundary conditions such as at $Z=0$, $\frac{d Z}{d x}=\infty$ and at $Z=L, \frac{d Z}{d x}=\tan \left(45-\frac{\phi}{2}\right)$.

In order to determine the failure surface profile Equation (2) has to be integrated. As direct integration is complicated, the solution has been conveniently obtained by the numerical method of integration.

## Ultimate uplift capacity

With the pile and the proposed failure surface shown in Figure 1, it has been assumed that in the limiting equilibrium condition, ultimate uplift capacity of the pile is attained when the mobilized shear strength of the soil along the failure surface and the weights of the body of the soil and pile balance the applied uplift force. A circular wedge of thickness $\Delta Z$ at a height $Z$ above the tip of the pile has been considered. Forces acting on the wedge has been shown in Figure 2 (a). For evaluating the mobilized shear resistance $\Delta T$ along the failure surface of length $\Delta L$, at limiting condition it has been assumed that $\Delta T=\Delta R \tan \phi$, in which $\Delta R$ is normal force acting on the failure surface of the wedge. Further the coefficient of lateral earth pressure (K) within the wedge has been taken as (1$\sin \phi$ ) from Figure $2(b)$. The $\Delta R$ is given as
$\Delta R=\Delta Q \cos \theta+K \Delta Q \sin \theta$
where $\Delta Q=\gamma\left(L-Z-\frac{\Delta Z}{2}\right) \Delta L$

$$
\begin{align*}
& \Delta R=\gamma\left(L-Z-\frac{\Delta Z}{2}\right)(\cos \theta+K \sin \theta) \frac{\Delta Z}{\sin \theta}  \tag{6}\\
& \Delta T=\gamma\left(L-Z-\frac{\Delta Z}{2}\right)(\cos \theta+K \sin \theta) \frac{\Delta Z \tan \phi}{\sin \theta}
\end{align*}
$$


(a)

Fig. 2 Free Body Diagram of the Wedge
Considering the vertical equilibrium of the circular wedge,

$$
\begin{equation*}
\left(P_{1}-P_{2}\right)-q_{1}\left(\pi x_{1}^{2}-\Delta A_{\mu^{\prime}}\right)+q\left(\pi x^{2}-\Delta A_{\mu^{\prime}}\right)-\Delta W-2 \pi\left(\frac{x_{1}+x}{2}\right) \Delta T \sin \theta=0 \tag{8}
\end{equation*}
$$

where
$q=\gamma(L-Z) ; q_{1}=\gamma(L-Z-\Delta Z) ; x_{1}=x+\Delta x$ and $\Delta A_{p}=$ area of pile
Substituting the value of $\Delta \mathrm{T}$ from Equation (7) in Equation (8) and simplifying

$$
\begin{align*}
\frac{\Delta P}{\Delta Z}= & \pi \gamma(L-Z)(2 x+\Delta x) \frac{\Delta x}{\Delta Z}-\pi \gamma\left(x^{2}+\Delta x^{2}+2 x \Delta x\right) \\
& +\Delta A_{p} \gamma_{p}+\pi \gamma x^{2}+\frac{\Delta T}{\Delta Z} 2 \pi x \sin \theta \tag{9}
\end{align*}
$$

In the limit, Equation (9) can be written as
$\frac{d P}{d Z}=\pi \gamma(L-Z)(2 x) \frac{d x}{d Z}+A_{p} \gamma_{p}+\gamma(L-Z)(\cos \theta+K \sin \theta) \tan \phi(2 \pi x)$
Then Equation (2) is substituted in Equation (10) and simplified as

$$
\begin{equation*}
\frac{d P}{d Z}=2 \pi \gamma L d\left[\frac{x}{d}\left(1-\frac{Z}{L}\right) \frac{1}{M_{1}}+\frac{x}{d}\left(1-\frac{Z}{L}\right) M\right]+A_{p} \gamma_{p} \tag{11}
\end{equation*}
$$

where $M_{1}=\tan \left\{90-\left(\frac{i \delta+\phi / 2}{L}\right) Z\right\} \exp \left\{\beta\left(1-\left(\frac{Z}{L}\right)^{\frac{1}{2}}\right)\right\}$ and

$$
\begin{equation*}
M=(\cos \theta+K \sin \theta) \tan \phi \tag{12}
\end{equation*}
$$

The gross uplift capacity of the pile $\mathrm{P}_{\mathrm{u}}$ (Gross) is given by

$$
\begin{equation*}
P_{n}(\text { Gross })=\int_{0}^{\prime} \frac{d P}{d Z} d Z=\int_{0}^{\prime} \pi \gamma L d\left\{\frac{2 x}{d}\left(1-\frac{Z}{L}\right)\left(\frac{1}{M_{1}}+M\right)\right\} d Z+A_{p} \gamma_{p} L \tag{13}
\end{equation*}
$$

Net uplift capacity $\mathrm{P}_{\mathrm{u}}(\mathrm{Net})$ is as follows
$P_{u}($ Net $)=P_{u}$ (Gross) - Weight of the pile
$P_{n}(N e t)=\int_{0}^{L} \pi \gamma L d\left\{\frac{2 x}{d}\left(1-\frac{Z}{L}\right)\left(\frac{1}{M_{1}}+M\right)\right\} d Z$
$P_{u}($ Net $)=A \gamma \pi d L^{2}$
in which $\mathrm{A}=$ net uplift capacity factor $=\frac{1}{L} \int_{0}^{t} \frac{2 x}{d}\left(1-\frac{Z}{L}\right)\left(\frac{1}{M_{1}}+M\right)$
The net uplift capacity factor depends on slenderness ratio $\lambda$, angle of shearing resistance of soil $\phi$, angle of soil pile friction $\delta$.

## Limitation and Merits of the present Model

The theory is based on limit equilibrium approach and strictly speaking does not satisfy the conditions laid down by mathematical theory of plasticity. Chattopadhyay and Pise (1986) model predicts well for short and medium piles $(\mathrm{L} / \mathrm{d} \leq 30)$. However, present model predicts well the ultimate resistance of single piles embedded in different placement density of sand and for different length to diameter ratios of piles.

## Experimental Investigation

Tests on model piles were conducted in a steel tank of size 990 mmx 975 $\mathrm{mm} \times 970 \mathrm{~mm}$. The tank was sufficiently large to take care of the effect of the edges of the tank on the test results as the zone of influence of the pile due to loading is reported to be in the range of 3-8 pile diameter (Kishida 1963).

Model piles were prepared from mild steel rod of $20 \mathrm{~mm} \times 20 \mathrm{~mm}$ square cross section. The length of embedment of pile, $L$ in sand bed was $200 \mathrm{~mm}, 400$ $\mathrm{mm}, 600 \mathrm{~mm}$ and 800 mm resulting L/d as 10, 20, 30 and 40 respectively. The model piles were embedded in homogeneous dry sand bed composed of uniformly graded Ennore sand having uniformity coefficient 1.71 and specific gravity 2.65 . The maximum and minimum dry unit weights of the sand were found to be 16.2 and $14.74 \mathrm{kN} / \mathrm{m}^{3}$ respectively. Sand was poured uniformly in the tank by using rainfall technique to prepare loose, medium dense and dense bed. The technique of sand placement plays an important role in the process of achieving reproducible density. After proper placement of the pile, sand was poured in the tank continuously through the slot hopper keeping the height of fall as $200 \mathrm{~mm}, 300 \mathrm{~mm}$ and 400 mm respectively to achieve sand beds of loose, medium dense and dense state. Placement density of sand was checked at the end of each test by a dynamic penetrometer. Depth of penetration was recorded for $1,2,3,4,5,6,7$ numbers of blows. The depth of penetration recorded at different locations in the tank was practically same for all the tests for similar
number of blows and it indicates a uniform placement density in tank for all the tests. To obtain strength parameters, specimens of the sand used in this study were prepared to the same unit weight as available in the model tank. They were tested in direct shear and peak value of $\phi$ is determined. The sample M.S. plate made of the same material as that of the model piles is used in direct shear test to get soil-pile interface friction angle ( $\delta$ ). The details of the soil properties and corresponding soil-pile friction angles are given Table 1.

Piles were subjected to tensile loading through a pulley arrangement with a flexible wire whose one end is attached with the pile cap and the other end with a loading pan over which dead loads are gradually placed in stages. A schematic diagram of the complete experimental set-up with the loading system and pile in place and ready for test is shown in Figure 3. Two dial gauges with magnetic base having sensitivity of 0.01 mm were used to measure the displacement placing them on the pile cap at $180^{\circ}$ apart and equidistant from load axis.

TABLE 1: Details of Soil Properties

| Soil property | Loose bed | Medium dense bed | Dense bed |
| :--- | :--- | :--- | :--- |
| Relative density $\left(\mathrm{D}_{\boldsymbol{\prime}}\right)$ | $34.4 \%$ | $54.3 \%$ | $69 \%$ |
| Unit weight $\left(\gamma_{\mathrm{r}}\right), \mathrm{kN} / \mathrm{m}^{3}$ | 15.4 | 15.8 | 16.1 |
| Angle of internal friction $(\phi)$ | $34^{\circ}$ | $38^{\circ}$ | $41^{\circ}$ |
| Pile-soil friction angle $(\delta)$ | $22^{\circ}$ | $26^{\circ}$ | $28^{\circ}$ |



Fig. 3 Experimental Set-up

## Presentation of Test Results

The ultimate resistance of single piles under axial pull has been estimated from the load-axial displacement diagrams. It is taken as the load at
which the piles move out of the soil. In such conditions the pull versus axial movement curves become parallel to the displacement axis and maintains continuous displacement increase without any further increase in pull (Meyerhof 1973; Chattopadhayay and Pise 1986). In this manuscript, double tangent method has been used to determine the ultimate load. Typical loaddisplacement curves are shown in Figure 4 for dense soil. From these curves the gross ultimate uplift capacity of the pile was determined using double tangent method (as shown in Figure 4 for a typical curve corresponding to $\mathrm{L} / \mathrm{d}=$ 40) and there after subtracting the weight of the pile the net ultimate capacity of the pile was found. The pile head displacement required to mobilize the ultimate uplift resistance as estimated from the above figures reveal that these values generally corresponds to about 2 to $5 \%$ of pile diameter.


Fig. 4 Load-Deflection Curves for Dense Soil

## Results and Discussion

The experimental results obtained from the present investigation and several others available in the literature on the subject have been compared with the predictions made with two earlier models (Meyerhof 1973; Chattopadhyay and Pise 1986) and also with the present model. The obtained results are then compared with each other and presented as follows. Predictions are made using the present model by assuming different trial values of constant (C) in Equation (3). From several trials so made it was observed that at $C$ equal to 1.9 the predicted values are in very good agreement with the experimental results. Therefore, the same is used for further predictions.

For a typical value of $\lambda=20, \phi=40^{\circ}$, and $\delta=10^{\circ}, 20^{\circ}, 30^{\circ}$ and $40^{\circ}$, failure surfaces have been evaluated and shown in Figure 5. It is seen that the extent of the failure surface is maximum when $\delta=\phi=40^{\circ}$, while it diminishes with a decrease in $\delta$. When $\delta=0$, it coincides with pile-soil interface.


Fig. 5 Variation of Failure Surface for Different Values of 8 ( $\lambda=20$ and $\phi=400$ )

In Figures 6-8, the predicted and the measured data are plotted to show the quality of the predictions. In Table 2 a quantitative comparison of the same is shown. Figure 6 shows the comparisons between Meyerhof (1973) predictions with the measured net uplift capacity of a pile. From this figure a good agreement between both can be observed as evident from the fact that most of the data points lie very close or around the ideal line. For most of the cases at higher $\phi$ values the theory is over estimating the value. From the accompanied table (Table 2) showing a quantitative comparative study it can be seen that the absolute relative errors between the predicted values ( 25 data out of 32 ) lie in general with in the range of 2 to $45 \%$ but in some cases the errors are as high as 55 to $90 \%$.

From Figure 7 it is observed that the model proposed by Chattopadhyay and Pise (1986) under estimates the net uplift capacity when L/d ratio is 30 and above. From Table 2, it is found that for $60 \%$ of the data, error is more for the above said L/d ratio. However, the rest of the data (20 out of 32 ) are close to the ideal line with error less than $45 \%$.

The comparison of the values of the uplift resistance computed by using the present model with the values as measured in the laboratory scale model tests are presented in Figure 8. It is seen that in this case 95\% of the data lies on and around the ideal line. From Table 2 it is seen that for $90 \%$ data (29 out of 32 ) the error is less than $45 \%$. For better appreciation of the relative predicting capability of the above models a cumulative frequency table for the data corresponding to the percentage of errors is presented in Table 3. From this table it can be observed that the predictions made with the present model are better than the other two models discussed above because 24 data points are having error less $\tan 30 \%$ for present model and were as for Meyerhof's model it is 19 data points and Chattopadhyay and Pise's model only 16 data points are having error less than $30 \%$.

To check further the predictive capability of the present model under field condition also, the following study has been undertaken.

Ismael and Klyam (1979) and Ismael and Al-Sanand (1986) conducted field test to measure the uplift capacity of piles. Using the present approach the values of the uplift capacity of those piles for the given site conditions were estimated and compared with the measured values as follows.

## Ismael and Klyam (1979)

Reported a full-scale pull out test of a cylindrical pier of diameter 1.2 m and length of 6.4 m embedded in a soil medium composed of compact fine to medium sand with some silt and traces of clay. The average standard penetration number ( N ) reported was 20 and $\phi=34^{\circ}$. Submerged unit weight was $11 \mathrm{kN} / \mathrm{m}^{3}$. For theoretical prediction, $\delta=27^{\circ}$, i.e., $80 \%$ of the value of $\phi$ was used (Potyondy 1961). The value of the predicted gross uplift capacity of the pier using the present model is 1065 kN which is close to the measured value of 890 kN with an error of $-19.7 \%$.


Das (1983) exp.results
Chattopadhyay and Pise (1986)
Series (ii)
$\rightarrow L d=B$
$\rightarrow-L d=12$
$\rightarrow-L d=16$
$\rightarrow-L d=24$

$$
\begin{aligned}
& \text { Series (iii) } \\
& \rightarrow\llcorner/=8 \\
& \rightarrow\llcorner/ d=12
\end{aligned}
$$

exp. Results
$\rightarrow-L / d=105$

* $-L / d=16$
- L L $ل=26$
Chattopadhyay (1994)
exp. Results
$--L d=15.78$
$-x-L d=238$
$-x-L d=31.57$

$$
\begin{array}{lc}
\text { Dash and Pise (2003) exp results } \\
\text { Loose bed } & \text { Dense bed } \\
--L / d=\theta & -L d d=8 \\
--L d=16 & --L d=16 \\
--L d=24 & -L d=24
\end{array}
$$

$$
\begin{aligned}
& \text { Loose bed } \\
& \rightarrow L / d=10 \\
& -L / d=20 \\
& -L / d=30 \\
& -L / d=4 D
\end{aligned}
$$

Present exp.results
Medium dense bed
$-L d=10$
$-L / d=20$
$-L d=30$
$\rightarrow-L d=40$

Dense bed

- $\quad \mathrm{L} d=10$
- $L / d=20$
- $L T=30$
- $L d=40$

Fig. 6 Measured Vs Predicted (Meyerhof's Model 1973) Net Uplift Capacity


Fig. 7 Measured Vs Predicted (Chattopadhyay and
Pise's Model 1986) Net Uplift Capacity


Fig. 8 Measured Vs Predicted (Present Model) Net Uplift

TABLE 2: Predicted and Measured Net Uplift Capacity of the Pile

| L/d Ratio | Expt. results ( N ) | Meyerhof's Model(1973) |  | Chattopadhyay and Pise's Model (1986) |  | Proposed Model |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Das (1983) Expt. results, Series (ii): $\gamma=15.79 \mathrm{kN} / \mathrm{m}^{3}, \phi=34^{\circ}, \delta=30.5^{\circ}, \mathrm{d}=0.0254 \mathrm{~m}$, $\mathrm{D}_{\mathrm{t}}=47.6 \%$ |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
| 8 | 27 | 25.7 | 4.8 | 40.7 | -50.7 | 28 |  |
| 12 | 62 | 57.8 | 6.8 | 75 | -21.0 | 56.7 | 8.5 |
| 16 | 114 | 102.8 | 8.8 | 108.2 | 5.1 | 92 | 19.3 |
| 24 | 180 | 230.8 | -28.2 | 170.8 | 5.1 | 173.8 | 3.4 |
| Series (iii): $\gamma=16.88 \mathrm{kN} / \mathrm{m}^{3}, \phi=40.5^{\circ}, \delta=39.2^{\circ}, \mathrm{d}=0.0254 \mathrm{~m}, \mathrm{D}_{\mathrm{t}}=72.9 \%$ |  |  |  |  |  |  |  |
| 8 | 60 | 61.2 | -2.0 | 108 | -80.0 | 57.1 | 4.8 |
| 12 | 165 | 137.7 | 16.5 | 249 | -50.9 | 133.5 | 19.1 |
| 16 | 325 | 245.4 | 24.5 | 421.5 | -29.7 | 236.2 | 27.3 |
| 24 | 650 | 551.0 | 15.2 | 791.5 | -21.8 | 502.2 | 22.7 |
| Chattopadhyay and Pise (1986) Expt. results: $\gamma=16.0 \mathrm{kN} / \mathrm{m}^{3}, \phi=41^{\circ}, \delta=34^{\circ}$, $\mathrm{d}=0.019 \mathrm{~m}$ |  |  |  |  |  |  |  |
| 10.5 | 74 | 34.78 | 61.3 | 70 | 5.4 | 38.5 | 48.0 |
| 16 | 127 | 80.36 | 36.6 | 149.3 | -17.6 | 90 | 29.1 |
| 26 | 256 | 217.4 | 15.0 | 298.3 | -16.5 | 211.2 | 17.5 |
| Chattopadhyay (1994) Expt. results: $\gamma=17.0 \mathrm{kN} / \mathrm{m}^{3}, \phi=40^{\circ}, \delta=25^{\circ}, \mathrm{d}=0.019 \mathrm{~m}$ |  |  |  |  |  |  |  |
| 15.78 | 40 | 57 | -42.5 | 81 | -102.5 | 60.4 | -51.0 |
| 23.8 | 70 | 128.5 | -83.57 | 127.8 | -82.6 | 111.3 | -59.0 |
| 31.57 | 130 | 228 | -75.38 | 165 | -26.9 | 184 | -41.5 |
| Dash and Pise (2003) Expt. results: $\gamma=15.0 \mathrm{kN} / \mathrm{m}^{3}, \phi=30^{\circ}, \delta=21^{\circ}, \mathrm{d}=0.025 \mathrm{~m}$, $D_{t}=35 \%$ |  |  |  |  |  |  |  |
| 8 | 12.2 | 11.3 | 7.37 | 14.4 | -18.0 | 14.7 | -20.5 |
| 16 | 65.2 | 45.2 | 44.2 | 33.4 | 48.8 | 47 | 27.9 |
| 24 | 86.7 | 101.75 |  | 57.4 | 33.8 | 88.5 | -2.1 |
| $\gamma=16.4 \mathrm{kN} / \mathrm{m}^{3}, \phi=38^{\circ}, \delta=29^{\circ}, \mathrm{d}=0.025 \mathrm{~m}, \mathrm{D}_{\mathrm{f}}=80 \%$ |  |  |  |  |  |  |  |
| 8 | 47.6 | 33.9 | 28.7 | 57.2 | -20.2 | 36 | 24.4 |
| 16 | 142 | 135.6 | 4.5 | 166.2 | -17.0 | 125.3 | 11.8 |
| 24 | 271 | 305 | -12.5 | 262.5 | 3.1 | 244 | 10.0 |
| Present Expt. results: $\gamma=15.4 \mathrm{kN} / \mathrm{m}^{3}, \phi=34^{\circ}, \delta=22^{\circ}, \mathrm{d}=0.02 \mathrm{~m}, \mathrm{D}_{\mathrm{f}}=34.4 \%$ |  |  |  |  |  |  |  |
| 10 | 12.6 | 16.7 | -32.5 | 16.1 | -27.8 | 15 | -19.0 |
| 20 | 70 | 66.9 | $4.5$ | 34.6 | 50.6 | 45.6 | 34.9 |
| 30 | 108.6 | 150 | $38$ | 54.7 | 49.6 | 87.3 | 19.6 |
| 40 | 242.4 | 267 | $-10.1$ | 80 | 67.0 | 137.6 | 43.2 |
| $\gamma=15.8 \mathrm{kN} / \mathrm{m}^{3}, \phi=38^{\circ}, \quad \delta=26^{\circ}, \mathrm{d}=0.02 \mathrm{~m}, \mathrm{D}_{\mathrm{t}}=54.3 \%$ |  |  |  |  |  |  |  |
| 10 | 17.3 | 29.3 | -69.4 | 35.7 | -106.4 | 24.7 | -42.8 |
| 20 | 112 | 117 | -4.5 | 86.3 | 22.9 | 79.7 | 28.8 |
| 30 | 181.6 | 263 | -44.8 | 127 | 30.1 | 149.5 | 17.7 |
| $\gamma=16.1 \mathrm{kN} / \mathrm{m}^{3}, \phi=41^{0}, \delta=28^{\circ}, \mathrm{d}=0.02 \mathrm{~m}, \mathrm{D}_{\mathrm{t}}=69 \%$ |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 10 | 25 | 40 | -60 | 61.0 | -144.0 | 35.0 | -40.0 |
| 20 | 128 | 148 | -15.6 | 178.7 | -39.6 | 128.0 | 0.0 |
| 30 | 200 | 332 | -66.0 | 278.7 | -39.4 | 250.0 | -25.0 |
| 40 | 395 | 591 | -49.63 | 358.6 | 9.2 | 390.0 | 1.3 |

## Ismael and Al-Sanad (1986)

Conducted full-scale pull out tests of bored piles of diameter 0.5 m in dense calcareous soils at three sites in Kuwait. For all the three sites, the pile and soil properties are as follows;

Site 1: $L=6.3 \mathrm{~m}, \phi^{\prime}=43^{\circ}$ and measured $P_{u}=1027 \mathrm{kN}$

Site 2: $L=6.8 \mathrm{~m}, \phi^{\prime}=43^{\circ}$ and measured $P_{u}=962 \mathrm{kN}$
Site 3: $L=9.2 \mathrm{~m}, \phi^{\prime}=42^{\circ}$ and measured $P_{u}=1980 \mathrm{kN}$
The $\gamma^{\prime}$ and average ratio of $\tan \delta / \tan \phi$ reported to be $11.25 \mathrm{kN} / \mathrm{m}^{3}$ and 0.65 respectively for all the three sites. The predicted ultimate gross uplift capacity using the present model for site 1,2 and 3 are 811 kN (error $=21 \%$ ), $940 \mathrm{kN}($ error $=2.3 \%)$ and $1407 \mathrm{kN}(\mathrm{error}=28.9 \%)$ respectively.

## Applicability to practice

For driven piles in sand, in field, the soil is compacted by displacements and vibrations resulting in change in the of value of $\phi$. This effect has to be considered before using the model for design purpose. The angle of shearing resistance gets modified. They are explained by Kishida (1967) for a single pile and successfully applied by Chattopadhyay and Pise (1987) for driven piles.

TABLE 3: Comparison of Absolute Error Distribution
(CP = Chattopadhyay and Pise's Model)

| Absolute <br> error (\%) | Number of data points |  |  | Cumulative data points |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Meyerhof's <br> Model <br> $(1973)$ | CP <br> $(1986)$ | Proposed <br> Model | Meyerhofs <br> Model <br> $(1973)$ | CP <br> $(1986)$ | Proposed <br> Model |
| $0-5$ | 5 | 3 | 5 | 5 | 3 | 5 |
| $5-10$ | 4 | 2 | 3 | 9 | 5 | 8 |
| $10-15$ | 2 | 0 | 8 | 11 | 5 | 16 |
| $15-20$ | 4 | 4 | 3 | 15 | 9 | 19 |
| $20-25$ | 1 | 4 | 2 | 16 | 13 | 21 |
| $25-30$ | 3 | 3 | 3 | 19 | 16 | 24 |
| $30-35$ | 1 | 2 | 1 | 20 | 18 | 25 |
| $35-40$ | 2 | 2 | 1 | 22 | 20 | 26 |
| $40-45$ | 2 | 0 | 1 | 24 | 20 | 27 |
| $45-50$ | 2 | 2 | 1 | 26 | 22 | 28 |
| $>50$ | 6 | 10 | 4 | 32 | 32 | 32 |

## Conclusions

Based on the studies reported above the following conclusions are drawn.

1. A semi empirical method for predicting the uplift capacity of vertical piles embedded in sand with assumed curved failure surface through soil has been proposed. The effect of various parameters like length to diameter ratio ( $\lambda$ ), angle of shearing resistance of the soil $(\phi)$ and pile-soil friction angle ( $\delta$ ) on the uplift capacity are incorporated in the proposed model.
2. It is found that use of the proposed model leads to predictions of the uplift capacity of single piles embedded in sand that are in reasonably good agreement with the experimental values and, as such, can be used with confidence.
3. Predictive capabilities of the some of the available methods are also checked vis-a-vis the presently developed models and the experimental
data and can be ranked as follows in the order with maximum and minimum errors being shown against each method.

Model-1
Meyerhof (1973)
Chattopadhyay and Pise (1986)
: Error varying from -59\% to 48\%
: Error varying from $-83 \%$ to $44 \%$
: Error varying from $-144 \%$ to $67 \%$

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## Notations

The following symbols are used in this paper.
$A=$ Net uplift capacity factor
C $=$ Constant
$D_{r}=$ Relative density
D = Pile diameter
$\mathrm{Ks}=$ Lateral earth pressure coefficient
$\mathrm{L}=$ Embedded length of pile
$\mathrm{P}_{\mathrm{u}}=$ Ultimate uplift capacity of pile
$P_{n u}=$ Net ultimate uplift capacity of pile
$x=$ Lateral extent of the failure surface
$\Delta z=$ Thickness of wedge element
$\phi \quad=$ Angle of internal friction of the soil
$\theta=$ Angle of failure surface with horizontal
$\delta=$ Pile-soil friction angle
$f=$ Unit skin friction
$\gamma=$ Unit weight of the soil
$\lambda=$ Slenderness ratio (L/d).


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