

## Reliability Analysis of Piled Raft Foundation

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### Introduction

**D**ue to rapid growth in the urban population and limited open space makes town planer to go for high rise buildings. The role of foundation engineer is very important in the design of multistoried building, where he has to take care of both the bearing capacity and settlement of foundation. It is a common practice in foundation design to consider the use of footings or rafts to support a structure, if these shallow foundations are not adequate, the recourse is to use pile foundation in which piles are designed to carry entire load. Despite, piles taking all the loads, it is usual for the raft or cap to be a part of foundation system either for the need to provide a basement below the structure or for distributing the load from the superstructure to the pile group. In the past few years, there has been an increasing recognition that the strategic use of piles could reduce raft's total and differential settlements (Franke, 1991; Clancy and Randolph, 1996; Poulos, 2002), thereby leading to a considerable economy without compromising the safety and performance of the foundation. Such foundation makes use of both the raft and the piles, and is referred to as pile-enhanced raft or piled raft.

Piled raft foundation (PRF) is a good economic decision for high-rise buildings because the bearing capacities of the raft and the piles are optimally utilized. The PRF acts as a composite structure consisting of three bearing elements: piles, raft and the subsoil. In comparison with the pile foundation, the PRF allows to use the individual piles up to 100% of their bearing capacity. If the piles are used along with the raft, piles will act as settlement reducers first and eventually act as load carrying members.

Geotechnical design is performed under considerable degree of inherent uncertainty, without complete knowledge of the underlying ground conditions,

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material properties or applied loads. Conventionally, foundation engineer strikes a balance between reliability and economy by using a combination of precedent knowledge, judgment and analysis, and express this balance primarily in terms of a factor of safety. The traditional deterministic design approach, utilizing the global factor of safety, adopts conservative assumptions in the face of uncertainty. As the demand for a rational treatment of uncertainty in geotechnical engineering has increased, the use of probabilistic methodology has gained importance. Probabilistic reliability methods employ the use of statistics and probability theory in order to formally include and quantify uncertainties in design.

In recent times, there is an increasing interest in adopting reliability-based design methods in civil engineering profession. These methods are intended to quantify the reliability and are used to develop balanced designs that are both reliable and less expensive. Standards and design rules for piled raft foundations incorporating uncertainties are not available in its complete form. In view of this, the problem of the design of piled raft foundations becomes more and more important and hence, it is necessary to evaluate the safety concept for the piled raft foundation in the framework of probability theory. The objective of the paper is to evaluate systematically the effects of uncertainty on the PRF settlement using improved Hasofer-Lind and Rackwitz-Fiessler (iHLRF) algorithm which is the First Order Reliability Method (FORM) in the literature. Using inverse reliability formulation of iHLRF algorithm and setting target reliability index for specified allowable settlement, the required design length of the pile is evaluated.

### **Piled Rafts: Load-Settlement Behaviour**

In the design of a piled raft foundation, the main criterion to be considered is the settlement. Researchers in the last decade have proved that the piles are taking a substantial load along with the reduction in the settlement (Randolph, 1994; Clancy and Randolph, 1996). In piled raft foundation the total load is taken by the raft through contact with soil and the remaining load is transferred to the piles through skin friction. The load transfer from pile to soil in case of piled raft is somewhat different from that of regular piles. A number of simplified methods for analyses have been developed to estimate the load-settlement behaviour of piled rafts. A well-known equivalent raft method is one such approach, in which the loading is assumed to be applied at some distance below the raft and usually over a larger area to reflect the load transfer along the piles.

Randolph (1994) developed convenient approximate equations for estimating the stiffness of a piled raft system and the load sharing between the piles and the raft. The method is restricted to linear behaviour of the piled raft system, i.e., the initial portion of the load-settlement curve. The other approaches with similar concepts have also been reported in the literature.

Poulos and Davis (1980) developed a simplified hand calculation method for constructing the overall load-settlement curve up to failure. Elastic solutions were used for the initial stiffness of the piled raft and of the raft alone. A trilinear load-settlement curve was obtained, reflecting the three main portions of the relationship shown in Fig. 1

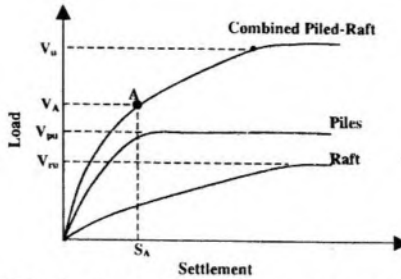


Fig. 1 Typical Load-Settlement Curve for Piled Raft (Poulos and Davis, 1980)

A method which combines and extends the approaches of Randolph (1994) and Poulos and Davis (1980) includes the following aspects:

- Estimation of the load sharing between the raft and the piles using the approximate solution of Randolph.
- Hyperbolic load-deflection relationships for the piles and the raft, thus providing a more realistic overall load-settlement response for the piled raft system than the original tri-linear approach of Poulos and Davis (1980).

In Fig. 1 the point A represents the applied load  $V_A$ , for which the pile capacity is fully mobilized. Up to this point, both the piles and the raft share the load, and the total settlement ( $S_t$ ) for the applied load  $V$  can be expressed as

$$S_t = \frac{V}{K_{pr}} \quad \text{for } V < V_A \quad (1)$$

where  $V$  = total vertical applied load and  $K_{pr}$  = axial stiffness of piled raft system.

Beyond the point A, additional load must be carried by the raft, and hence the settlement can be expressed as

$$S_t = \frac{V_A}{K_{pr}} + \frac{V - V_A}{K_r} \quad \text{for } V > V_A \quad (2)$$

where  $V_A$  = applied load at which pile capacity is mobilized and  $K_r$  = axial stiffness of the raft. The load  $V_A$  can be estimated from

$$V_A = \frac{V_{pu}}{\beta_p} \quad (3)$$

where  $V_{pu}$  = ultimate capacity of piles and  $\beta_p$  = proportion of load carried by piles.

The approximate expressions described by Randolph (1994) can be used for  $K_{pr}$  in Eq. (1) and  $\beta_p$  in Eq. (3):

$$K_{pr} = XK_p \quad (4)$$

where  $K_p$  denotes the stiffness of the pile group alone and  $X$  is the multiplying factor depends on stiffness ratio ( $K_r/K_p$ ) between the raft and the pile.

For fairly large number of piles, the expressions for  $X$ ,  $\beta_p$  and  $\alpha$  are as follows

$$X \approx \frac{1 - 0.6(K_r/K_p)}{1 - 0.64(K_r/K_p)} \quad (5)$$

$$\beta_p = 1/(1 + \alpha) \quad (6)$$

$$\alpha \approx \frac{0.2}{1 - 0.8(K_r/K_p)} \left( \frac{K_r}{K_p} \right) \quad (7)$$

If it is assumed that the pile and raft load-settlement relationships are hyperbolic, then the secant stiffnesses of the piles ( $K_p$ ) and the raft ( $K_r$ ) are expressed as

$$K_p = K_{pi}(1 - R_{fp}V_p/V_{pu}) \quad (8)$$

$$K_r = K_{ri}(1 - R_{fr}V_r/V_{ru}) \quad (9)$$

where  $K_{pi}$  = initial tangent stiffness of the pile group,  $R_{fp}$  = hyperbolic factor for the pile group,  $V_p$  = load carried by the piles,  $V_{pu}$  = ultimate capacity of the piles,  $K_{ri}$  = initial tangent stiffness of the raft,  $R_{fr}$  = hyperbolic factor for the raft,  $V_r$  = load carried by the raft, and  $V_{ru}$  = ultimate capacity of the raft.

The load carried by the pile and raft are given by the following equations:

$$V_p = \beta_p V \leq V_{pu} \quad (10)$$

$$V_r = V - V_p \quad (11)$$

where  $V$  denotes the total vertical applied load.

By substituting and simplifying the above equations, the following expression can be obtained for the load-settlement relationship of the piled raft system.

For  $V \leq V_A$ :

$$S_i = \frac{V}{XK_{pi} \left( 1 - \frac{R_{fp} \beta_p V}{V_{pu}} \right)} \quad (12)$$

Equation (12) provides the means for estimating the average load-settlement relationship for the piled raft. As  $K_i$  and  $K_p$  vary with the applied load level, the parameters  $X$  and  $\beta_p$  will also generally vary. Thus, it may be necessary to carry out an iterative or incremental analysis, starting with the initial stiffnesses  $K_n$  and  $K_p$ . This approach is very amenable to perform calculations using either the spreadsheets or MATHCAD.

### First Order Reliability Method

Probability theory has obtained its popularity in many research areas; and, stochastic analysis techniques, which are based on probability theory, have been widely used in civil engineering systems to model and propagate uncertainties. There are several well-developed methods for reliability analysis using probability theory: First-Order Reliability Method (FORM), Second-Order Reliability Method (SORM), Monte Carlo Simulation (MCS), Stochastic Finite Element Method (SFEM), and so on. To assess the reliability of a structure, a limit state function  $g$ , depending on load effects is defined as follows

- $g(S) > 0$  defines the safe state of the structure.
- $g(S) \leq 0$  defines the failure state. In reliability context, it does not necessarily mean the breakdown of the structure, but the fact that certain requirements of serviceability or safety limit states have been reached or exceeded.

Here  $S$  is the settlement limit state equation. The values of  $S$  satisfying  $g(S) = 0$  define the limit state surface in the original space. The mapping of the limit state function onto the standard normal space by using the probabilistic transformation is described by

$$g(S) \equiv g(S(X)) = g(S_0 u^{-1}(Y)) = G(Y) \quad (13)$$

where  $X$  is the vector of random variables and  $Y = u(X)$ . Hence the probability of failure can be rewritten as

$$P_f = \int_{G(Y) \leq 0} \varphi(u) du \quad (14)$$

where  $\varphi(u)$  denotes the standard normal probability density function (PDF) of  $Y$ , i.e.,

$$\varphi = \frac{1}{(2\pi)^{n/2}} \exp\left(-\frac{1}{2}\|u\|^2\right) \tag{15}$$

This PDF has two interesting properties, namely it is rotationally symmetric and decays exponentially with the square of the norm  $\|u\|$ . Thus the points making significant contributions to the integral, Eq. (14) are those with nearest distance to the origin of the standard normal space. This leads to the definition of the reliability index (Ditlevsen and Madsen, 1996) as shown in Fig. 2.

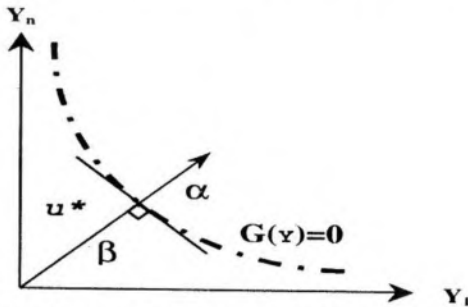


Fig. 2 Geometrical Definition of the Design Point

$$\beta = \alpha^T \cdot y^* \tag{16-a}$$

$$u^* = \operatorname{argmin} \{ \|u\| \mid G(u) \leq 0 \} \tag{16-b}$$

This quantity is obviously invariant under changes in parameterization of the limit state function, since it has an intrinsic definition, i.e., the distance of the origin to the limit state surface. The solution  $u^*$  of the constrained optimization problem, Eq. (16-b), is called the design point or the most likely failure point in the standard normal space. When the limit state function  $G(Y)$  is linear in  $u$ , it is easy to show that

$$p_f = \Phi(\beta) \tag{17}$$

where  $\Phi()$  is the standard normal cumulative distribution function (CDF).

When  $G(Y)$  is non-linear, the first-order approximation method (FORM) consists in

- evaluating the reliability index  $\beta$  by solving Eq. (16),
- obtaining an approximation of the probability of failure by :

$$p_f \approx p_{f1} = \Phi(-\beta) \tag{18}$$

Geometrically, this is equivalent to replacing the failure domain by the halfspace outside the hyperplane tangent to the limit state surface at  $\mathbf{u} = \mathbf{u}^*$ . Generally speaking, FORM becomes a better approximation when  $\beta$  is large.

### Determination of the Design Point

The classical reliability method requires determination of the design point, and is defined as the point on the limit state surface closest to the origin in the standard normal space. The constrained optimization problem, Eq. (16), is equivalent to

$$\mathbf{u}^* = \operatorname{argmin} \left\{ Q(\mathbf{u}) = \frac{1}{2} \|\mathbf{u}\|^2 \quad | \quad G(\mathbf{u}) \leq 0 \right\} \quad (19)$$

By introducing the Lagrangian of the problem as

$$\mathcal{L}(\mathbf{u}, \lambda) = \frac{1}{2} \|\mathbf{u}\|^2 + \lambda G(\mathbf{u}) \quad (20)$$

Equation (19) reduces to solving:

$$(\mathbf{u}^*, \lambda^*) = \operatorname{argmin} \mathcal{L}(\mathbf{u}, \lambda) \quad (21)$$

Assuming sufficient smoothness of the functions involved, the partial derivatives of  $\mathcal{L}$  have to be zero at the solution point. Hence

$$\mathbf{u}^* + \lambda^* \nabla G(\mathbf{u}^*) = 0 \quad (22)$$

$$G(\mathbf{u}^*) = 0 \quad (23)$$

The positive Lagrange multiplier  $\lambda^*$  is obtained from Eq. (22) then substituted in the same equation. This yields the first-order optimality conditions as

$$\|\nabla G(\mathbf{u}^*)\| \cdot \mathbf{u}^* + \|\mathbf{u}^*\| \cdot \nabla G(\mathbf{u}^*) = 0 \quad (24)$$

This means that the normal to the limit state surface at the design point should point towards the origin. Hasofer and Lind (1974) suggested an iterative algorithm to solve Eq. (21), which was later used by Rackwitz and Fiessler (1978) in conjunction with probability transformation techniques. This algorithm, referred to as HLRF in the sequel, generates a sequence of points  $\mathbf{u}_i$  from the recursive rule :

$$\mathbf{u}_{i+1} = \frac{\nabla G(\mathbf{u}_i)^T \cdot \mathbf{u}_i - G(\mathbf{u}_i) \nabla G(\mathbf{u}_i)}{\|\nabla G(\mathbf{u}_i)\| \|\nabla G(\mathbf{u}_i)\|} \quad (25)$$

Eq. (25) can be given the following interpretation: at the current iterative point  $\mathbf{u}_i$ , the limit state surface is linearized, i.e., replaced by the trace in the  $\mathbf{u}$ -space of the hyperplane tangent to  $G(\mathbf{Y})$  at  $\mathbf{u} = \mathbf{u}_i$ . Eq. (25) is the solution of this linearized

optimization problem, which corresponds to the orthogonal projection of  $u_i$  onto the trace of the tangent hyperplane.

As the limit state function and its gradient is usually defined in the original space, it is necessary to make use of a probabilistic transformation. The Jacobian of the transformation is used in the following relationship:

$$\nabla_u G(u) = \nabla_x g(X) J_{X,u} \quad (26)$$

The HLRF algorithm has been widely used due to its simplicity. However, it may not converge in some cases, even for rather simple limit state functions. Der Kiureghian and de Stefano (1991) have shown that it certainly diverges when a principal curvature of the limit state surface at the design point satisfies the condition  $|\beta k_i| > 1$ . Thus several modified versions of this algorithm have been developed (Abdo and Rackwitz, 1990; Liu and Der Kiureghian, 1990). The latter reference presents a comprehensive review of general purpose optimization algorithms, including the gradient projection method (GP), the augmented Lagrangian method (AL), the sequential quadratic programming method (SQP), the HLRF and the modified HLRF (mHLRF). All these algorithms have been implemented in the computer program CALREL (Liu et al., 1989) for comparison purposes, and tested with several limit state functions. Although the modified mHLRF was an improvement over the original HLRF, no proof of its convergence could be derived. In this study, improved HLRF algorithm is used to evaluate the reliability index of the piled raft foundation and a brief explanation about this algorithm is presented in the next section.

### The Improved HLRF Algorithm

Zhang and Der Kiureghian (1995, 1997) proposed an improved version of HLRF denoted by iHLRF. It is based on the following recast of the HLRF recursive definition [Eq. (25)]:

$$u_{i+1} = u_i + \lambda_i d_i \quad (27)$$

$$\text{with } \lambda_i = 1 \quad (28)$$

$$d_i = \frac{\nabla G(u_i)^T \cdot u_i - G(u_i)}{\|\nabla G(u_i)\|} \frac{\nabla G(u_i)}{\|\nabla G(u_i)\|} - u_i \quad (29)$$

In the above equations,  $d_i$  and  $\lambda_i$  are the search direction and the step size respectively. The original HLRF can be improved by computing an optimal step size  $\lambda_i \neq 1$ . For this purpose, a merit function  $m(u)$  is introduced. At each iteration, after computing [Eq. (29)], a line search is carried out to find  $\lambda_i$  such that the merit function is minimized, i.e.:



$$\lambda_i = \operatorname{argmin} \{m(u_i + \lambda d_i)\} \quad (30)$$

This non-linear problem is not easy to solve. It is replaced by the problem of finding a value  $\lambda$ , such that the merit function is sufficiently reduced (if not minimal). The so-called Armijo rule (Luenberger, 1986) is an efficient technique. It reads

$$\lambda_i = \max_{k \in N} \left\{ b^k \left[ m(u_i + b^k d_i) - m(u_i) \right] \leq -ab^k \|\nabla m(u_i)\|^2 \right\}, \quad a, b > 0 \quad (31)$$

Zhang and Der Kiureghian (1995, 1997) proposed the following merit function:

$$m(u) = \frac{1}{2} \|u\|^2 + c|G(u)| \quad (32)$$

This expression has two properties:

- The HLRF search direction  $d$  [Eq. (29)] is a descent direction for it, that is  $d$  satisfies:  $\forall u, \nabla m(u)^T \cdot d \leq 0$  provided

$$c > \frac{\|u\|}{\|\nabla G(u)\|}$$

- It attains its minimum at the design point provided the same condition on  $c$  is fulfilled.

Both properties are sufficient to ensure that the global algorithm defined by Eqs. (27), (29) and (31) is unconditionally convergent (Luenberger, 1986). The flowchart of iHLRF is given in Fig. 3 and is coded in MATLAB to obtain the design points and reliability index for the piled raft foundation.

### Reliability Analysis of Piled Raft

In this study, a probabilistic reliability analysis is carried out using First-Order Reliability Method (FORM) which uses improved Hasofer-Lind and Rackwitz-Fiessler (iHLRF) formulation. This formulation is used to evaluate the reliability index ( $\beta$ ) and probability of failure ( $p_f$ ) of the piled raft foundation. MATLAB is used to program the analysis procedure of the iHLRF formulation. The main steps involved in the reliability analysis of piled raft foundation are given below:

1. The spreadsheet is prepared considering the above equations. For the total vertical applied load, the load at which the piles are fully mobilized is determined first. In the analysis, the loads up to  $V_A$  (Fig. 1) are only considered.
2. The values from Step 1 are used as inputs to the MATLAB code to evaluate the reliability index and probability of failure for different values of  $V$  and  $V_{pu}$ .

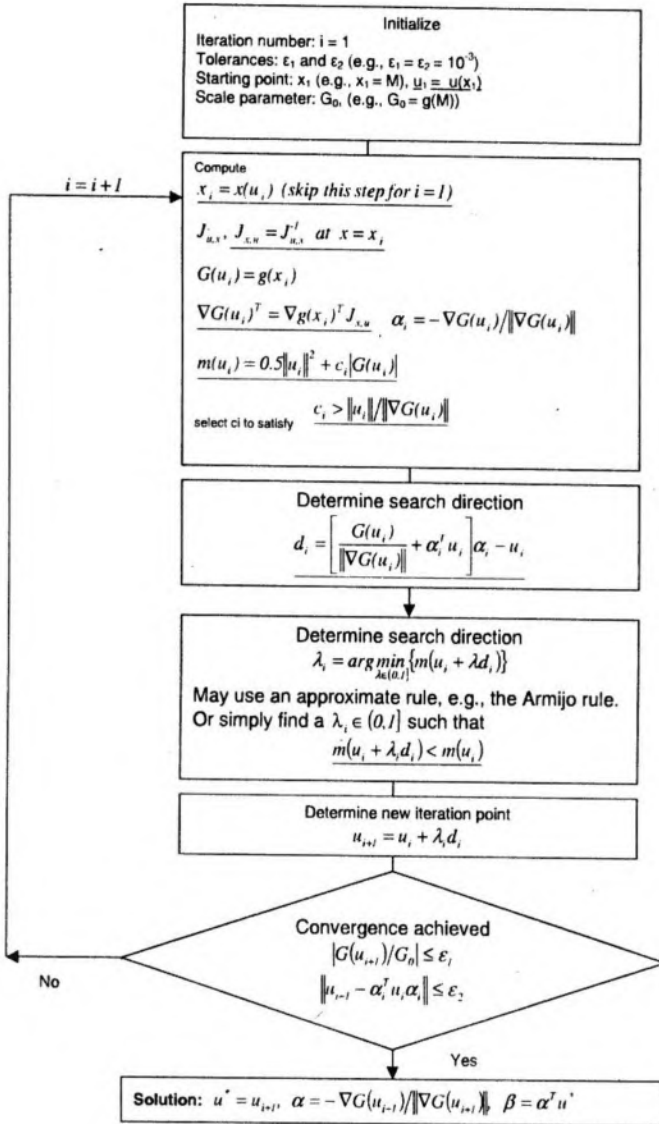


Fig. 3 Flowchart for iHLRF

**Numerical Example**

The following numerical example is used to illustrate the applicability of the FORM for the reliability analysis of piled raft foundation. Fig. 4 shows the schematic plan of the piled raft foundation system. The other relevant data are given below (Hemsley, 2000):

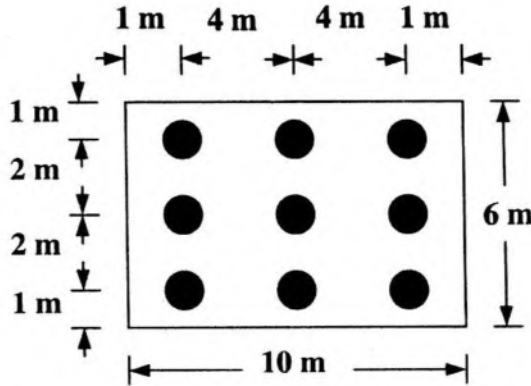


Fig. 4 Typical Piled Raft

Diameter of pile = 0.6 m, length of pile = 15 m, thickness of raft = 0.5 m, shaft friction = 60 kPa, end bearing capacity = 900 kPa, axial capacity of each pile = 1.95 MN,  $K_n = 420$  MN/m,  $K_{pi} = 651$  MN/m,  $V_{nu} = 36$  MN,  $R_{ip} = 0.5$ ,  $R_{ir} = 0.75$ , and allowable settlement,  $S_a = 0.1$  m.

By using the values given in Table 1 for  $V$ , the values of  $\beta_p$ ,  $K_n$ ,  $K_{pi}$ ,  $V_{pu}$ , and  $V_A$  are determined according to Step 1 and the results are presented in Table 2. The values,  $K_n = 420$  MN/m,  $K_{pi} = 651$  MN/m,  $V_{nu} = 36$  MN,  $R_{ip} = 0.5$ ,  $R_{ir} = 0.75$  and  $S_a = 0.1$  m are kept unaltered in the analysis. For reliability analysis, the parameters and their distributions given in Table 1 and results of Table 2 are used in Step 2 to evaluate the reliability index and probability of failure. The results of the reliability analysis are given in Table 3 and the variation of  $p_f$  with  $V_{pu}$  is shown in Fig. 5.

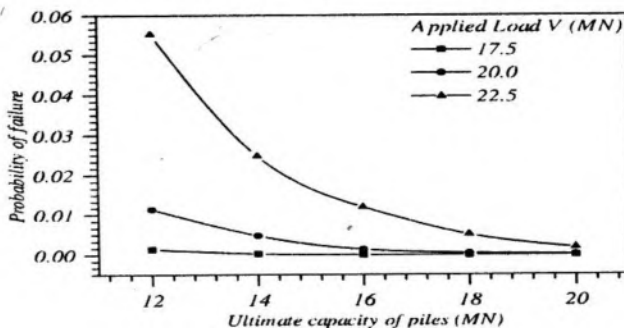


Fig. 5 Variation of  $p_f$  with  $V_{pu}$

TABLE 1: Parameters and their Distributions Used in the Analysis

| Variable  | Distribution | Mean (MN)  | COV (%) |
|-----------|--------------|--|---------|
| $V$       | Lognormal    | 17.5, 20, 22.5   | 10      |
| $V_{pu}$  | Lognormal    | 12, 14, 16, 18, 20   | 10      |
| $\beta_p$ | Normal       | {One particular value at each of the $V_{pu}$ from spreadsheets} | 10      |
| $K_n$     | Normal       | {One particular value at each of the $V_{pu}$ from spreadsheets} | 10      |
| $K_{pi}$  | Normal       | {One particular value at each of the $V_{pu}$ from spreadsheets} | 10      |

**TABLE 2: Parameters Obtained from Spreadsheets and Used in FORM**

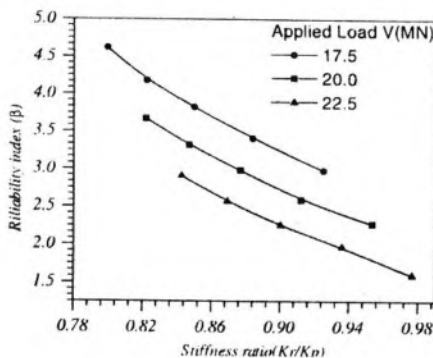
| $V_{pu}$<br>(MN) | V = 17.5 MN |        |        | V = 17.5 MN |        |        | V = 17.5 MN |        |        |
|------------------|-------------|--------|--------|-------------|--------|--------|-------------|--------|--------|
|                  | $\beta_p$   | $K_r$  | $K_p$  | $\beta_p$   | $K_r$  | $K_p$  | $\beta_p$   | $K_r$  | $K_p$  |
| 12.00            | 0.58        | 369.80 | 399.73 | 0.55        | 356.32 | 373.72 | 0.53        | 342.02 | 350.24 |
| 14.00            | 0.62        | 374.36 | 423.53 | 0.60        | 362.36 | 397.28 | 0.57        | 349.49 | 373.37 |
| 16.00            | 0.65        | 377.64 | 444.32 | 0.63        | 366.88 | 418.48 | 0.61        | 355.26 | 394.64 |
| 18.00            | 0.68        | 380.07 | 462.26 | 0.66        | 370.30 | 437.25 | 0.64        | 359.74 | 413.88 |
| 20.00            | 0.69        | 381.93 | 477.69 | 0.68        | 372.94 | 453.72 | 0.66        | 363.24 | 431.08 |

**TABLE 3: Results of First-Order Reliability Method**

| $V_{pu}$<br>(MN) | Reliability index ( $\beta$ ) |           |             |
|------------------|-------------------------------|-----------|-------------|
|                  | V = 17.5 MN                   | V = 20 MN | V = 22.5 MN |
| 12.0             | 2.9817                        | 2.2773    | 1.5954      |
| 14.0             | 3.4075                        | 2.5878    | 1.9632      |
| 16.0             | 3.8278                        | 2.9863    | 2.2568      |
| 18.0             | 4.1795                        | 3.3188    | 2.5706      |
| 20.0             | 4.6197                        | 3.6711    | 2.9066      |

**Results and Discussion**

Table 3 presents the variation of reliability index for the increased ultimate load carrying capacity of the pile ( $V_{pu}$ ) for different applied loads on a piled raft system. From the table it can be observed that, as the applied load is increased for a particular value of  $V_{pu}$ ,  $\beta$  decreases by 50–80 %. As the value of  $V_{pu}$  approaches towards the applied load, the reliability index increased significantly and the probability of failure approaches to zero (Fig. 5). These trends are observed due to the fact that as the applied load is increased for the constant  $V_{pu}$ , the raft will take more load resulting in increase in the settlement and as the  $V_{pu}$  is increased for the constant applied load, load taken by the piles keep on increasing, ultimately reducing the settlement.



**Fig. 6 Variation of  $\beta$  with  $K_r/K_p$**

Fig. 6 shows the variation of  $\beta$  with stiffness ratio of raft to pile ( $K_r/K_p$ ) for different values of the applied loads. When the stiffness ratio  $K_r/K_p$  is increased to 0.12,  $\beta$  decreases by about 30 %; this is due to the fact that the stiffness ratio increases due to decrease in pile length or increase in raft thickness. In both the cases, the load contributed by the raft increases resulting in increase in the settlement. The variation of  $\beta$  with portion of load transferred to the piles is shown in Fig. 7 which supplements the results of Table 3. As the load carried by the piles is increased by 26 %, the increase in reliability index is found to be 66 %.

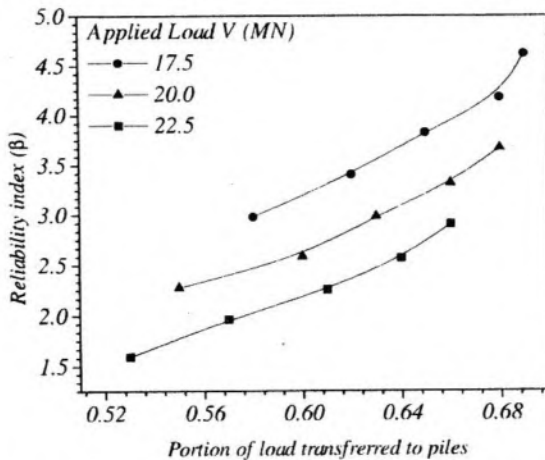


Fig. 7 Variation of  $\beta$  with Portion of Load Transferred to Piles

### Inverse Reliability Analysis

Reliability based analysis is not only the tool to analyze the present design for obtaining  $\beta$  and  $p_u$ , but it can also be used to get the design parameters which will satisfy the required  $\beta$  and  $p_u$  for the specified allowable settlement. Length of the pile is a major parameter which governs the design, so in order to consider this design parameter in the extended analysis for clayey soils, the following expression is used to determine the ultimate load capacity of the pile.

$$V_{pu} = 9A_p s_u + pLms_u \quad (33)$$

where  $A_p$  = cross sectional area of the pile,  $s_u$  = undrained shear strength,  $p$  = perimeter of the pile and  $m$  = adhesion factor.

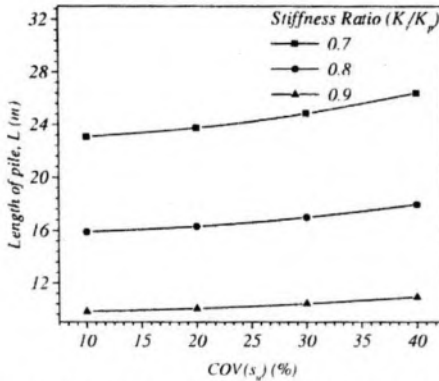
Substituting Eq. (33) into Eq. (12), the settlement of the piled raft system can be obtained by considering the length as the independent parameter. An inverse reliability code developed in MATLAB is used to perform the reliability analysis. In this analysis, for the pre-specified settlement with target reliability index, the length is treated as a dependent parameter and  $K_r/K_p$  is treated as a random variable. A target

reliability index of 2 has been set and is adequate for serviceability criterion – 50 years of the design life span with the allowable settlement of 0.05 m. The parameters and their corresponding probability distributions given in Table 4 are used in the inverse reliability analysis.

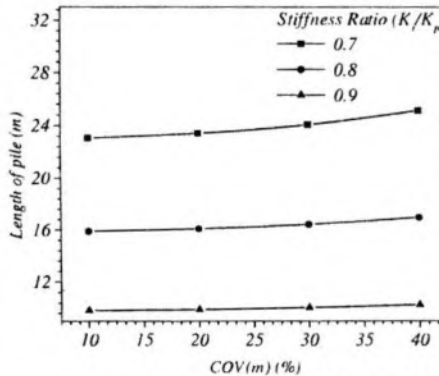
**TABLE 4: Parameters Used in Inverse Reliability Analysis**

| Variable    | Distribution | Mean          | COV (%) |
|-------------|--------------|---------------|---------|
| V (MN)      | Lognormal    | 25            | 10      |
| $s_u$ (MPa) | Lognormal    | 0.2           | 10 – 40 |
| m           | Normal       | 0.5           | 10 – 40 |
| $K_s/K_p$   | Normal       | 0.7, 0.8, 0.9 | 5       |
| D (m)       | Normal       | 0.3, 0.4, 0.5 | 10      |

**Results and discussion**



**Fig. 8 Variation of L with Undrained Shear Strength ( $s_u$ )**



**Fig. 9 Variation of L with Adhesion Factor (m)**

Figs. 8 and 9 show the variation of length of the piles, respectively, with COV( $s_u$ ) and COV(m). The figures depict that, as the COV( $s_u$ ) and COV(m) are varied from 10 to 40%, there is a slight increase in the required design length, for a

particular stiffness ratio. But for the constant  $COV(s_u)$ , design length of the pile drastically reduces from 26 to 10 m and with constant  $COV(m)$  it reduces from 24 to 9 m, with increase in the stiffness ratio from 0.7 to 0.9.

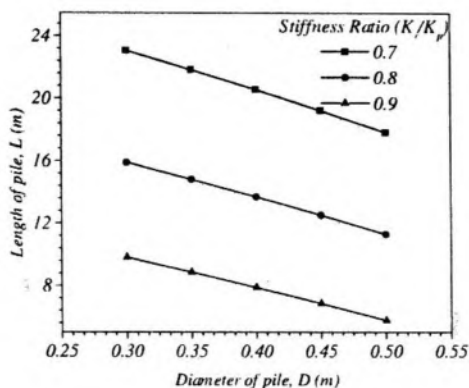


Fig. 10 Variation of L with Diameter D

Fig.10 shows the variation of length of the pile with variation in the diameter for different  $K_r/K_p$  ratios. From the figure it can be inferred that as the diameter of the pile is increased from 0.3 to 0.5 m, the design length of the pile is reduced by 50 to 60% for a particular stiffness ratio. This signifies that as the stiffness ratio is kept constant, the design length is reduced for the increased diameter. But as the stiffness ratio increases from 0.7 to 0.9, there is a drastic reduction in the design length of the pile, i.e., 50 to 70 %. One could infer here is that the settlement of the piled raft foundation mainly depends on the stiffness ratio of the pile and the raft, because, when the raft stiffness is more, all the piles will not operate at their ultimate capacity and the piled raft will act as a block. This information is more useful in the design of piled raft foundations.

## Conclusions

The following conclusions are drawn from the study presented in the paper. As the vertical applied load on the piled raft system increases, the  $\beta$  decreases for a particular value of the ultimate load capacity of the pile. For the serviceability limit state with a given allowable settlement, when the ultimate load capacity of the pile approaches to the vertical applied load, the probability of failure is nearly equal to zero. For the particular values of  $COV(s_u)$  and  $COV(m)$ , as the  $K_r/K_p$  increases there is a corresponding decrease in the requirement of the design pile length. For a particular value of  $K_r/K_p$ , as the  $COV(s_u)$  and  $COV(m)$  increase, the required design pile length is almost the same. For the increased diameter, the pile length requirement decreases for the target reliability index of 2 with an allowable settlement of 0.05m. It is concluded that the reliability-based analysis could be used

to better evaluate the various sources of uncertainties associated with piled raft design and use this information to develop robust design.

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