Indian Geotechnical Journal, 35 (4), 2005

Analysis for Passive Earth Pressure – Catenary Arch in Soil

Rupa Sunil Dalvi*, S.S.Bhosale† and Prabhakar Jagannath Pise‡

Introduction

Arching is a universal phenomenon that involves transfer of pressure from the yielding part of soil to the adjoining part. The soil is said to arch over the yielding part of support. The state of stress within the zone of arching depends upon the amount of yield. As yielding increases, arching effect is gradually reduced. However, the effect of arching is permanent in character as the shear strength property of soil. Arching effect is less if the shear strength is less. Though arching phenomenon occurs in a number of geotechnical engineering problems, it has not received much attention.

Generally, using Rankine's and Coulomb's theory, earth pressure exerted by the soil on the retaining wall is calculated. Both of them assumed that the distribution of earth pressure is triangular and the pressure increases with depth. Many experimental results (Tsagarali, 1965 as well as Fang and Ishibhishi, 1986) showed that there is non-linearity of pressure distribution due to horizontal translation of the wall. Arching is considered for calculation of earth pressure in retaining wall by Handy (1985) and Harrop-Williams (1989). Arching is also observed in the silos by Janssen (1895) and buried structures by Getzler et al. (1968)

* Lecturer, Department of Civil Engineering, Pune Institute of Engineering and Technology, Shivajinagar, Pune – 411 005, India. E-mail: rupa.dalvi@indiatimes.com

† Lecturer, Department of Civil Engineering, Pune Institute of Engineering and Technology, Shivajinagar, Pune – 411 005, India.

‡ AICTE Emeritus Fellow and Professor, Pune Institute of Engineering and Technology, Shivajinagar, Pune – 411 005, India. (Former Professor, IIT Kharagpur) E-mail: pjpise@vsnl.net

Review of Previous Work

In 1936 Terzaghi performed experiments on sand with a yielding trap door and showed the zone of arching. Based on arching theory, Janssen (1895) derived the equation for pressure in the silos. Using similar approach Handy (1985) considered the trajectory of minor principal stress for the active earth pressure due to arching effect and derived non-linear pressure distribution. He assumed the shape of the arch as catenary. He has mentioned two stages of arching. In Stage I arching, rotation of principal stresses adjacent to the rough wall has been considered and found that lateral pressure on the retaining wall is much more than calculated by classical theory. In Stage II arching, he has considered horizontal translation of wall. Stage II arching gives non-linear pressure distribution with the center of pressure at 0.40 to 0.45 times the height of the wall.

Harrop-Williams (1989) examined the shape of arch theoretically and proved that the minor principal stress arch is very close to the circular arch. Paik and Salgado (2003) have proposed new formulation for the active earth pressure on rigid retaining wall undergoing horizontal translation. They also considered the arching effect and taken into account the effect of angle of shearing resistance (ϕ) and wall friction angle (δ) on the vertical stress. They assumed that the slip surface in the soil behind the wall is plane and makes an angle of $45+\phi/2$ with the horizontal. They proposed equations considering arching. The equations were applied to the test results of 5 rigid retaining walls with different heights.

Scope of Study

In this paper the effect of arching on the passive earth pressure in the non-cohesive backfill is considered. The backfill is assumed to rise upward in a catenary form due to arching. The coefficient of passive earth pressure has been derived making suitable assumptions. An illustrative example has been solved to show the effect on earth pressure distribution on retaining wall considering arching and without it.

Proposed Method of Analysis

Assumptions

Following assumptions have been made in the analysis.

1. The soil is cohesionless, semi infinite, homogeneous, isotropic and the backfill is horizontal.

2. The problem is a plane strain problem i.e. two-dimensional.

3. The soil mass is bounded between two parallel, un-yielding rough vertical walls. The walls are assumed to rotate towards the soil mass creating passive case.

4. The sliding surfaces are vertical and pass through the outer edge of the yielding wall.

5. The soil mass moves up in a curved path taken as catenary arch.

6. Full shear strength, s is mobilized on these vertical surface and it is expressed by Coulomb's empirical law, $s = c + \sigma \tan \phi$

7. The major and minor principal stresses have been considered to be constant along the length of the arch.

8. The ratio of horizontal to vertical pressure σ_h to σ_v is constant and $K = \sigma_h / \sigma_v$

Analysis

Figure 1 shows the model considered for representing arching in the soils. The soil mass is considered to be bounded between two parallel, un-yielding rough vertical walls as shown. B is the distance between the walls. When rotation of the walls takes place towards the soil mass, passive state is created and the soil moves in the upward direction.

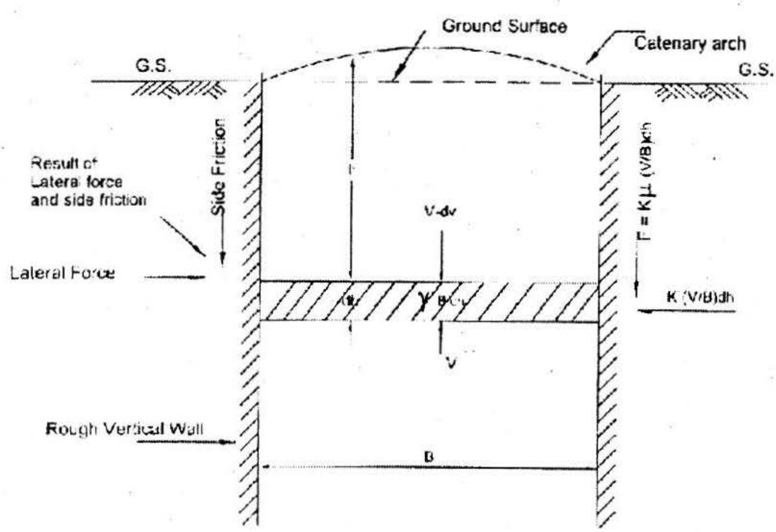


FIGURE 1 : Representation of Soil Arching

Consider a small strip of soil mass having thickness dh at a depth h below the ground surface in the soil mass. V is the vertical upward force acting on strip. The weight of the strip is $\gamma B\,dh$. The forces acting on the strip of soil mass have been shown in Fig.1. F is the frictional resistance acting in the downward direction as shown. This frictional resistance is equal to the lateral force times the coefficient of friction μ (i.e. $\mu = \tan\delta$), where δ is the angle of friction between soil and wall.

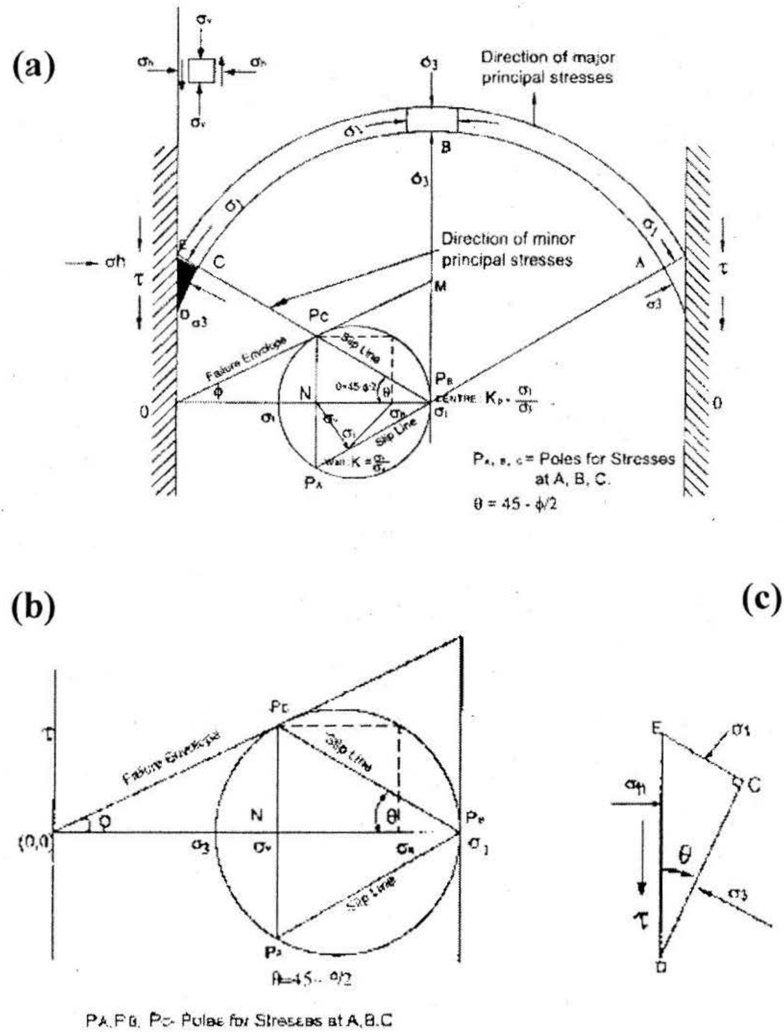


FIGURE 2 : a) Continuous Major Principal Arch (Trajectory of Major Principal Arch); b) Mohr's Circle showing Arching Stresses at Rough Wall; c) Stresses on Element at C

Major Principal Stress

The major principal plane and minor principal planes are shown in Fig.2a. The vertical and horizontal stresses at the wall are σ_v and σ_h. Inside the soil mass the trajectory of major principal stress (σ_1) gives continuous "tension arch" in the upward direction. Due to the catenary arch considered, the directions of the principal stress change along and normal to the arch. The major principal stress (σ_1) makes an angle of θ with the wall as shown at point C. Fig.2b shows Mohr's circle of stress for any point with failure envelope. The slip lines make an angle, $\theta = 45 - \phi/2$. The stress conditions at point C are modified and they are shown separately in Fig.2c.

Stresses in Arch

From force equilibrium on a triangular element, as at C, in Fig.2a, gives

$$\sigma_h = \sigma_3 \cos^2\theta + \sigma_1 \sin^2\theta \tag{1}$$

$$t = (\sigma_1 - \sigma_3)\sin\theta\cos\theta \tag{2}$$

Dividing Eqn.1 by σ_3 and considering the soil mass to be in a passive state

$$\sigma_1/\sigma_3 = K_p$$

where, K_p = Coefficient of passive earth pressure

From Eqns.1 and 2,

$$\sigma_h/\sigma_3 = \cos^2\theta + K_p \sin^2\theta \tag{3}$$

From Fig.2a and using geometrical relationships,

$$\sigma_1 - \sigma_h = \sigma_v - \sigma_3$$

Putting the value of σ_h, in Eqn.3,

$$(\sigma_1 - \sigma_v + \sigma_3)/\sigma_3 = \cos^2\theta + K_p \sin^2\theta$$

Solving the above and rearranging the terms,

$$\sigma_v/\sigma_3 = K_p \cos^2\theta + \sin^2\theta \tag{4}$$

Dividing Eqn.3 by Eqn.4.

$$K = \frac{\sigma_h / \sigma_3}{\sigma_v / \sigma_3}$$

Therefore,

$$K = \frac{\left(\cos^2\theta + K_p \sin^2\theta\right)}{\left(K_p \cos^2\theta + \sin^2\theta\right)} \tag{5}$$

Thus, K in Eqn.5 is related to the passive earth pressure coefficient and θ.

Shape of Arch

The arching element is bounded by surfaces representing principal planes. If the element is of uniform density, thickness, uniform weight and subjected to upward force the shape will be upward catenary as shown in Fig.2a. A curvilinear path which is formed by a perfectly flexible uniform string under its own weight traces a curve which is called as Common Catenary (Fig.3). The equation of the curve is given as,

$$y = c \cosh\frac{x}{c} \tag{6}$$

where c is known as parameter of the curve and it is symmetrical about y-axis. It does not pass through the origin when x = 0 and y = c ($\because$ cosh 0 = 1).

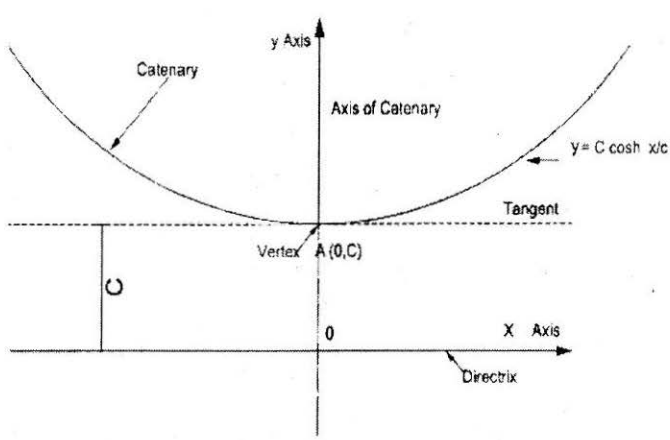


FIGURE 3 : Common Catenary

It cuts y-axis at A (0, c) and this point A (0, c) is called the vertex of the common catenary. Here x-axis is called as the "directix". It does not cut x-axis at all. Therefore,

$$\frac{dy}{dx} = c \cdot \sinh\left(\frac{x}{c}\right)\frac{1}{c} \tag{7}$$

$$\frac{dy}{dx} = \sinh\left(\frac{x}{c}\right) > 0 \quad \text{for } x > 0$$

$\therefore$ y increases as x increases from o to ∞

$$\frac{dy}{dx} = \sinh(0) = 0 \quad \text{for } x = 0$$

$\therefore$ the tangent at A (0, c) is parallel to the x-axis. Here AY is called the axis of common catenary.

$$y = \cosh x = \left(\frac{e^{x} + e^{-x}}{2}\right)$$

$$y = a\cosh\left(\frac{x}{a}\right) = a\left(\frac{e^{x/a} + e^{-x/a}}{2}\right)$$

$$y = \frac{a}{2}\left[\exp\left(\frac{x}{a}\right) + \exp\left\{-\left(-\frac{x}{a}\right)\right\}\right] \tag{8}$$

The equation for upward catenary for passive case considered in the analysis is as follows,

$$-y = \frac{-a}{2}\left[\exp\left(\frac{x}{-a}\right) + \exp\left\{-\left(\frac{x}{-a}\right)\right\}\right] \tag{9}$$

Differentiating Eqn.9, gives the slope of the upward catenary, i.e.

$$\begin{aligned}\frac{dy}{dx} &= \frac{-1}{2}\left[\exp\left(\frac{x}{-a}\right) - \exp\left(\frac{x}{-a}\right)\right] \\ &= \cot\theta\end{aligned} \tag{10}$$

When wall friction is fully developed, $\theta_{max} = \pm(45^\circ - \phi/2)$, and $x = \pm 1$ near the walls, enables to evaluate the shape parameter, a, of the catenary.

An illustrative example is solved for the parameters taken below:

When, $\phi = 10$,

$\theta_{max} = 40°$

$$\cot\theta = \frac{-1}{2}\left[\exp\left(\frac{x}{-a}\right) - \exp\left(\frac{x}{-a}\right)\right] \tag{11}$$

This gives the value of the parameter, a as 1.311.

Similarly for different values of ϕ, and using Eqn.11 the values of shape parameters are evaluated and tabulated in Table 1.

Using Eqn.9 and putting the values of the shape parameter, a, and distance from the axis of catenary to the wall, i.e. x = 0.10, 0.20, etc., different values of y are determined. Arch shapes for different values of ϕ are thus generated theoretically, and presented in graphical forms in Fig.4. For the clarity of arch shapes, $\phi = 0$, $\phi = 20$, $\phi = 40$ are shown in the left side of the axis of catenary and $\phi = 10$ and $\phi = 30$ are shown on the

TABLE 1 : Values of Shape Parameters (a)

Values of ϕ	Shape Parameters (a)
0	1.135
5	1.218
10	1.311
15	1.415
20	1.532
25	1.666
30	1.820
35	2.002
40	2.218
45	2.480
50	2.800

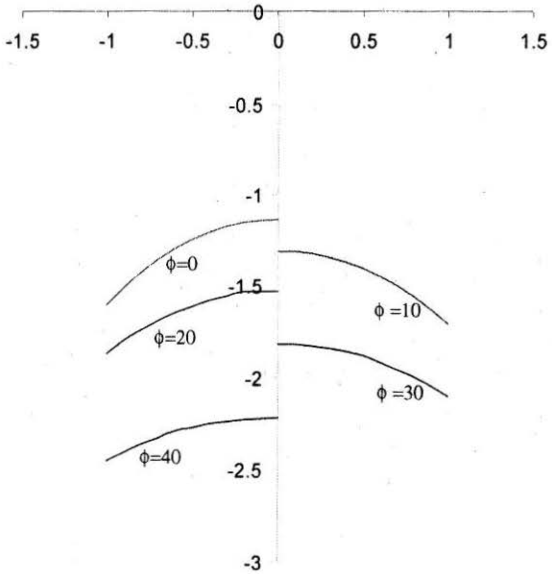


FIGURE 4 : Arch Shape for Different Values of ϕ

right side. From Fig.4 it is observed that as ϕ increases the shape of arch becomes flatter and flatter.

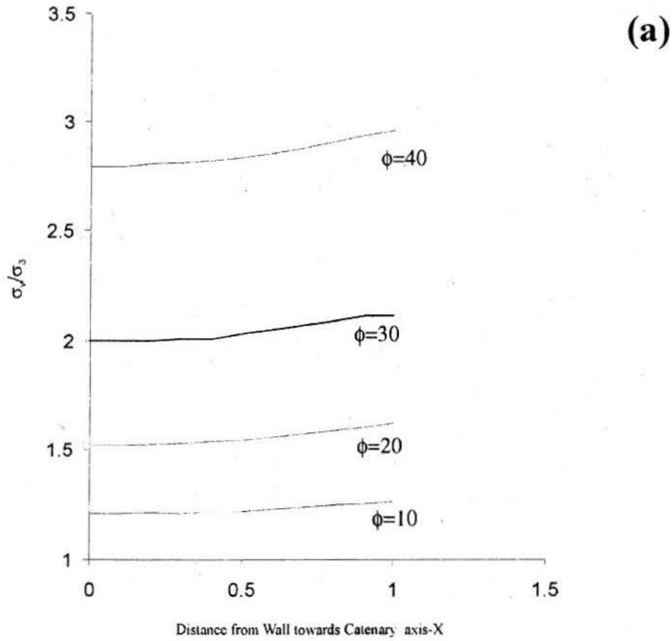
Vertical and Horizontal Stresses

The major stress direction is defined by Eqn.10. The principal stresses are resolved into horizontal and vertical stresses by using Eqns.3, 4 and 5. The values of θ and ϕ are substituted in Eqn.3 and Eqn.4 and the ratio of σ_h/σ_3 and σ_v/σ_3 are obtained. In Fig.5a the values of σ_v/σ_3 are plotted on y-axis and the distance from the wall towards the axis of the catenary are plotted on x-axis. Similarly in Fig.5b, the values of σ_h/σ_3 are plotted on y-axis and the distance from the wall towards the axis of the catenary are plotted on x-axis.

In case of wall or conduit problems instead of considering the vertical stress at the wall, average stress was considered by Marston (1895) and Handy (1985).

Average stress is equal to $\sigma_{av} = V/B$. Considering their approach the lateral stress ratio, σ_h/σ_{av} is evaluated.

The value of σ_{av} is obtained by averaging σ_v/σ_3 for ϕ = 10° and ϕ = 40° we get σ_{av} = 1.16. This horizontal to average vertical stress ratio is designated here as K_w.



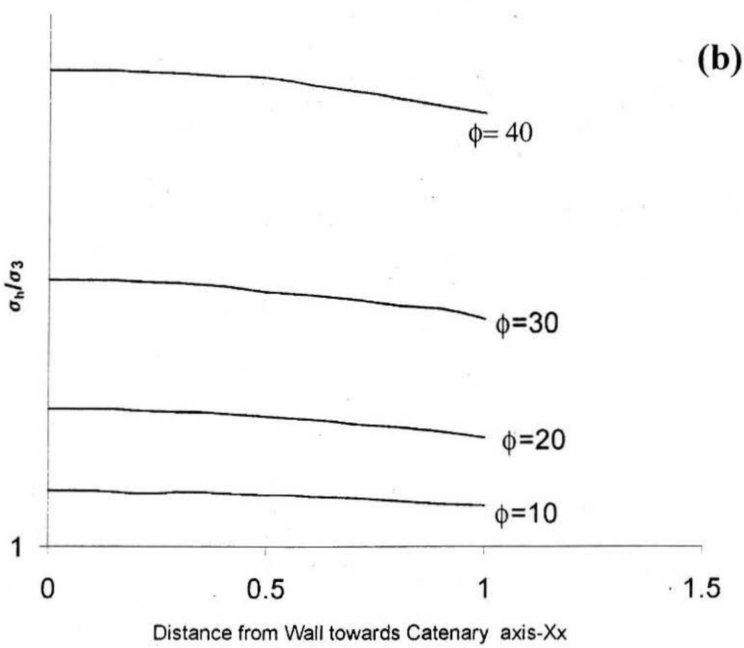


FIGURE 5 : a) Theoretical Vertical Stresses; b) Theoretical Horizontal Stresses

Equation 3 is written as

$$K_w = \frac{\sigma_h}{\sigma_{av}} = \cos^2\theta + K_p \sin^2\theta \tag{12}$$

If we put θ = 90° in Eqn.12 then the equation becomes

K_w = K_p for horizontal surface

$$K_w = \sigma_n = 1.16\left(\cos^2\theta + K_p \sin^2\theta\right) \tag{13}$$

Discussion of Results

1. Soil arching during passive state may be shown by trajectory of major principal stress considering upward catenary.

2. Due to rotation of principal stresses at the rough wall, the lateral and vertical stresses are modified. The ratio of lateral to average vertical stress is denoted by K_w, a new coefficient.

3. From Fig.5a, it is seen that vertical stress increases towards the axis of the catenary. It also increases with increase in the value of angle of shearing resistance. Similarly from Fig.5b it is observed that horizontal stress decreases towards the axis of catenary and it decreases with the increase in angle of shearing resistance.

Illustrative Example

To illustrate the effect of arching in passive case and Coulomb's earth pressure, an example for the soil-wall data given below has been solved;

Wall height = 2 m; ϕ = 32 and δ = 0.2 ϕ and 0.6 ϕ and γ_e = 18 kN/m^3. The results have been compared in Fig 6 which shows that the pressure distribution considering arching effect as well as, as per Coulomb are having linear variation.

For all δ-values, at any depth the Coulomb's pressure is more than that as predicted by considering arching effects. The difference depends on δ-values. It increases with increase in δ-value.

For δ = 0.2 ϕ the pressure due to arching effect is 8% less than the Coulomb's lateral pressure at base. However, for δ = 0.6 ϕ it is 52% less than the Coulomb's lateral pressure at base.

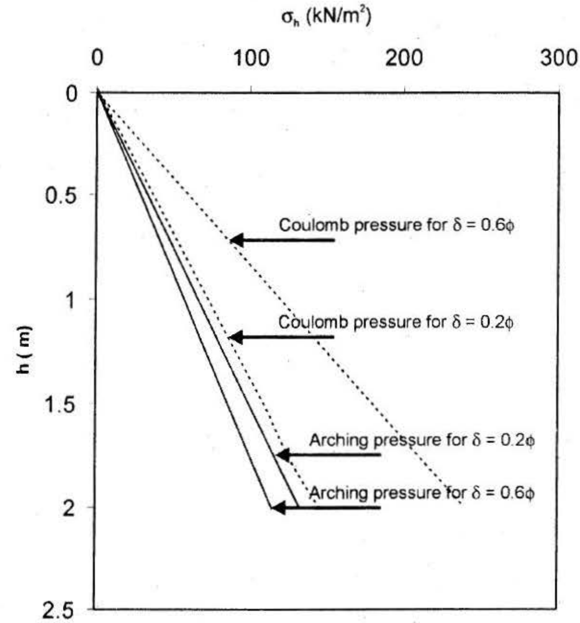


FIGURE 6 : Lateral Earth Pressure Distribution with Depth and Comparison with Coulomb Theory

Conclusion

Soil arching during passive state is shown by trajectory of major principal stress considering upward catenary.

Due to rotation of principal stresses at the rough wall, the lateral and vertical stresses are modified. The ratio of lateral to average vertical stress is denoted by K_w, a new coefficient due to arch.

Vertical stress increases and horizontal stress decreases towards the axis of the catenary. It also increases with increase in the value of angle of shearing resistance.

Lateral pressure on the retaining wall due to arching is less than the pressure predicted by Coulomb's analysis.

References

FANG, Y. and ISHIBHISHI, I. (1986) : "Static Earth Pressure with Various Wall Movements", *Journal of Geotechnical Engineering*, ASCE, 112, No.3, pp.313-333.

GETZER, Z., KOMOVNIK, A. and MAZURIK, A. (1968) : "Model Study on Arching above Buried Structures", *Journal of Geotechnical Engineering*, ASCE, pp.1123-1141.

HANDY, R.L. (1985) : "The Arch in Soil Arching", *Journal of Geotechnical Engineering*, ASCE, Vol.III, No.3, pp.302-317.

HARROP-WILLIAMS (1989) : "Geostatic Wall Pressures", *Journal of Geotechnical Engineering*, ASCE, 115(9), pp.1321-1325.

JANSSEN, H.A. (1895) : "Versche uber Getreidedruck in Silozellen", *Z. Ver. dut. Ingr.*, Vol.39, pp.1045 (Partial English Translation in Proceedings of the Institute of Civil Engineers, London, England, 1896, pp.553).

PAIK, K.H. and SALGADO, R. (2003) : "Estimation of Active Earth Pressure against Rigid Retaining Wall considering Arching Effects", *Geotechnique*, 53, No.7, pp.643-653.

TSAGARELI, Z.V. (1965) : "Experimental Investigation of the Pressure of a Loose Medium Retaining Wall with a Vertical Back Face and Horizontal Backfill Surface", *Journal of Soil Mechanics and Foundation Engineering*, ASCE, 91(4), pp.197-200.

Notations

a	=	mathematical coefficient in equation for Caternary
B	=	breadth of soil between the two vertical rough walls
H	=	height of wall
K	=	ratio of horizontal to vertical stress s_h/sv
K_p	=	passive earth pressure coefficient s_1/s_3
K_w	=	K at wall due to catenary arch
V	=	vertical force from soil weight
x, y	=	coordinates of catenary
θ	=	angle of major principal plane to the horizontal
δ	=	soil - wall friction angle
μ	=	wall friction coefficient
σ_1, σ_3	=	major and minor principal stresses
σ_{av}	=	average vertical stress
σ_h	=	horizontal stress
s	=	shear strength
ϕ	=	angle of shearing resistance of soil of soil
γ	=	soil unit weight