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The Effect of Prestressing Force and Interfacial Friction on the Settlement Characteristics of Beams on Reinforced Granular Beds

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Introduction

The use of geosynthetics in Civil Engineering construction has increased phenomenally over the years. Modeling and analysis of foundations reinforced with geosynthetics is one of the interesting topics in geotechnical engineering. Extension of some of the lump parameter models (Winkler, 1867; Filonenko-Borodich, 1940; Hetenyi, 1946; Pasternak, 1954; Kerr, 1964 and 1965) used in soil-structure interaction studies has been made to study the behavior of reinforced soils (Madhav and Poorooshasb, 1988; Ghosh and Madhav, 1994a and 1994b; Shukla and Chandra, 1994a, 1994b and 1994c; Yin, 1997 and 2000). These studies have resulted in improved understanding and better predictions regarding the behavior of reinforced beds. Mechanical models have found wide acceptability in predicting the flexural response of such foundations. The same is adopted in this study. It is observed from the available literature that reinforcing elements are considered to be rough membrane and very few analyses take care of the bending resistance of the reinforcing elements (Fakher and Jones, 2001). Geomats, geomattresses etc. when used as reinforcement do offer some bending resistance, which needs to be considered in analyzing foundation reinforced with such elements. Recently such a study has been undertaken by Maheshwari et al. (2004), to find the response of reinforced foundation beds considering the bending resistance offered by the embedded reinforcement. This study considers the reinforcing beam to be smooth and frictionless.

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However, in reality, the reinforcement generally offers some frictional resistance, which is mobilized on both upper and lower faces. As such, in this study an attempt has been made to analyze the reinforced earth beds by taking into account the friction between the soils and reinforcing element, introduction of prestressing force in the upper and lower beam and considering the resistance offered due to bending of the reinforcing elements. Various parametric studies to find the effect of coefficient of interfacial friction between the reinforcement and neighboring soil and prestressing force in the upper and lower beam, on the behavior of such foundations have been carried out.

Statement of the Problem

Figure 1 shows a shallow strip footing idealized (Fig.2) as an elastic beam (flexural rigidity E_1I_1) of length, $2l_1$, resting on the surface of compacted sand layer overlying a natural loose soil deposit and acted upon by a column load (concentrated load) Q_t at the middle of the footing. The reinforcing layer idealized as beam (flexural rigidity E_2I_2) of length $2l_2$ is placed on the surface of the original loose soil layer and the thickness of the compacted sand cover is 'h'. The reinforcing layer is assumed to be rough and it is able to resist bending. The soil above and below the lower beam is idealized by Winkler springs with spring constants k_1 and k_2 respectively. The values of the unit weight of the corresponding soil layers are γ_1 and γ_2 respectively. The prestressing force in the upper and lower beam are T_1 and T_2 respectively while the lower reinforcing beam also experiences a resultant tensile force T_3 arising due to friction executed by the surrounding sand as the beam deflects due to the external loads. Analysis is to be carried out to find the flexural response of the upper beam as well as the lower reinforcing beams. The effect of normalized prestressing force in the beams (T_1' and T_2') and interfacial frictional coefficient (μ) on the flexural behavior of upper and lower beams is to be studied.

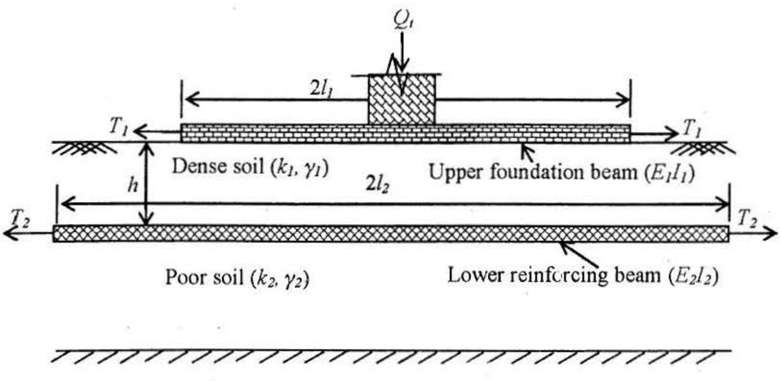

FIGURE 1 : Definition Sketch of the Problem

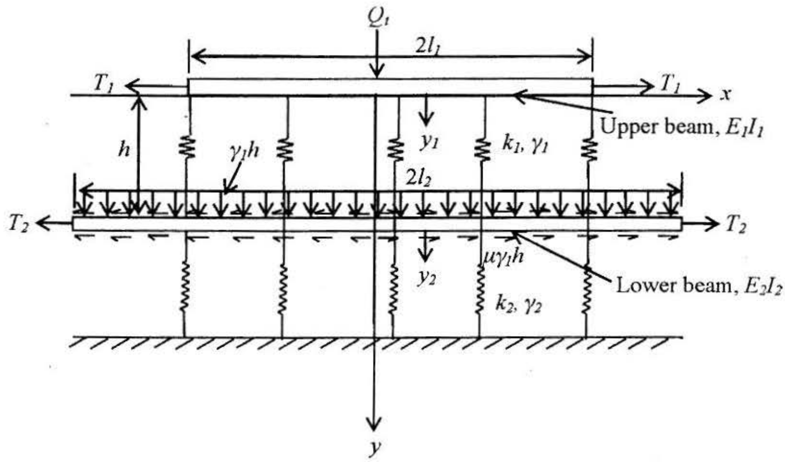

FIGURE 2 : Idealization of the Problem

Analysis

Physical Modeling

To accomplish the above objectives, the idealization of the problem, development of a general mathematical model and the corresponding solution are presented as follows:

The idealization of the problem and the co-ordinate axes x and y are shown in Fig.2. To take care of the effect of the thickness of the upper layer of soil on the reinforcing layer, surcharge, $\gamma_1 h$, over the entire length of lower beam is considered. Due to symmetry, only one half ($x \geq 0$) of the model is considered. The deflection ordinates of the upper and lower beams are denoted as y_1 and y_2 respectively. The prestressing forces in upper and lower beam are T_1 and T_2 respectively and T_3 is the mobilized tension in the lower beam due to friction between the beam and neighboring soil. It is assumed that the prestressing force given in the lower beam remains constant with time.

Mathematical Modeling

The distributed pressure in the foundation under the upper beam is expressed as $p_1 = k_1(y_1 - y_2)$, and under the lower beam, as $p_2 = k_2 y_2$.

The governing differential equations for the upper and lower beams for $0 \leq x \leq l_1$, are as follows,

$$E_1I_1\frac{d^4y_1}{dx^4}-T_1\frac{d^2y_1}{dx^2} = -p_1 = -k_1(y_1-y_2) \qquad (1)$$

and

$$E_2I_2\frac{d^4y_2}{dx^4}-(T_2+T_3)\frac{d^2y_2}{dx^2} = \gamma_1 h-(p_2-p_1) = \gamma_1 h-(k_1+k_2)y_2+k_1y_1 \qquad \dots (2)$$

The pressures p_1 and p_2 are considered positive when accompanied by positive (downward) deflection. From Eqn.1, one can write,

$$y_2 = \frac{E_1I_1}{k_1}\frac{d^4y_1}{dx^4}-\frac{T_1}{k_1}\frac{d^2y_1}{dx^2}+y_1 \qquad 0\le x\le l_1 \qquad (3)$$

Differentiating the above equation four times, one gets,

$$\frac{d^4y_2}{dx^4} = \frac{E_1I_1}{k_1}\frac{d^8y_1}{dx^8}-\frac{T_1}{k_1}\frac{d^6y_1}{dx^6}+\frac{d^4y_1}{dx^4} \qquad 0\le x\le l_1 \qquad (4)$$

Combining Eqns.2 and 4, the following equation can be obtained,

$$\begin{aligned}
&\frac{d^8y_1}{dx^8}-\left[\frac{T_1}{E_1I_1}+\frac{1}{E_2I_2}(T_2+T_3)\right]\frac{d^6y_1}{dx^6}\\
&+\frac{k_1}{E_1I_1E_2I_2}\left[\left(1+\frac{k_2}{k_1}\right)E_1I_1+\frac{T_1}{k_1}(T_2+T_3)+E_2I_2\right]\frac{d^4y_1}{dx^4}\\
&-\frac{1}{E_1I_1E_2I_2}\left[k_1(T_1+T_2+T_3)+T_1k_2\right]\frac{d^2y_1}{dx^2} \qquad (5)\\
&+\frac{k_1k_2}{E_1I_1E_2I_2}y_1 = \frac{\gamma_1 hk_1}{E_1I_1E_2I_2}, \qquad 0\le x\le l_1
\end{aligned}$$

For the reinforcing lower beam the flexural behavior of the beam, beyond the length l_1 and up to the length l_2, will be governed by the following equation,

$$\frac{d^4y_2}{dx^4}-\frac{1}{E_2I_2}(T_2+T_3)\frac{d^2y_2}{dx^2}+\frac{k_2}{E_2I_2}y_2 = \frac{\gamma_1 h}{E_2I_2} \qquad l_1\le x\le l_2 \qquad (6)$$

where $T_3 = 2\mu\gamma_1 h l_1 \quad 0 \le x \le l_1$ and

$$T_3 = 2\mu\gamma_1 h\left(l_2 - l_1\right) \quad l_1 \le x \le l_2$$

Equations 3 and 5 are the governing differential equations for the proposed model up to the length l_1, while beyond length l_1 and up to the length l_2 Eqn.6 governs the response of the model. R_1 and R_2 are the characteristic lengths of the upper and lower beams respectively defined as,

$$R_1 = \sqrt[4]{\frac{E_1 I_1}{k_1}} \quad \text{and} \quad R_2 = \sqrt[4]{\frac{E_2 I_2}{k_2}}$$

The governing differential equations are non-dimensionalized in terms of the following non-dimensional parameters (Matlock and Reese, 1960),

Non-dimensional deflection of the upper beam, $y_1' = \dfrac{y_1 E_1 I_1}{Q_1 R_1^3}$

Non-dimensional deflection of the lower beam, $y_2' = \dfrac{y_2 E_2 I_2}{Q_1 R_2^3}$

Relative flexural rigidity of the beams, $R = \dfrac{E_1 I_1}{E_2 I_2}$

Relative stiffness of the soil layers, $r = \dfrac{k_1}{k_2}$.

Non-dimensional unit weight of the upper soil layer, $\gamma_1' = \dfrac{\gamma_1 R_1^2}{Q_1}$

Non-dimensional depth of placement of reinforcement, $h' = \dfrac{h}{R_1}$

Non-dimensional prestressing force in upper beam, $T_1' = \dfrac{T_1}{Q_1}$

Non-dimensional prestressing force in lower beam, $T_2' = \dfrac{T_2}{Q_1}$

Non-dimensional mobilized tension in lower reinforcing beam, $T_3' = \dfrac{T_3}{Q_1}$

Non-dimensional flexural rigidity of upper beam, $I_1' = \dfrac{E_1 I_1}{Q_1 R_1^2}$

Non-dimensional flexural rigidity of lower beam, $I_2' = \dfrac{E_2 I_2}{Q_1 R_1^2}$

Non-dimensional co-ordinate along the length of the beams, $z = \dfrac{x}{R_1}$

Non-dimensional length of the upper beam, $z_1 = \dfrac{l_1}{R_1}$

Non-dimensional length of the lower beam, $z_2 = \dfrac{l_2}{R_1}$

Using the above non-dimensional parameters the governing differential equations (3) and (5) can be written as in non-dimensional forms as follows,

$$y_2' = \frac{1}{R}\left(\frac{R}{r}\right)^{3/4}\left(\frac{d^4 y_1'}{dz^4} - \frac{T_1'}{I_1'}\frac{d^2 y_1'}{dz^2} + y_1'\right), \quad 0 \le z \le z_1 \tag{7}$$

and

$$\frac{d^8 y_1'}{dz^8} + a\frac{d^6 y_1'}{dz^6} + b\frac{d^4 y_1'}{dz^4} + c\frac{d^2 y_1'}{dz^2} + dy_1' = \gamma_1' h' R, \quad 0 \le z \le z_1 \tag{8}$$

where
$$a = \frac{1}{I_1'}\left[T_1' + R\left(T_2' + T_3'\right)\right],$$

$$b = 1 + \frac{\left(T_2' + T_3'\right)T_1' R}{\left(I_1'\right)^2} + R + \frac{R}{r},$$

$$c = \frac{1}{I_1'}\left[\left(T_1' + T_2' + T_3'\right)R + T_1'\frac{R}{r}\right] \text{ and}$$

$$d = \frac{R}{r}$$

The roots of the auxiliary equation of Eqn.8 are obtained by using standard software. Numerical values of constants a, b, c and d for various physically possible parameters are calculated and the eight roots of the auxiliary equation of Eqn.8 are obtained. These eight roots are found to be complex in nature and are of the form $\pm\alpha_1 \pm \beta_1 i$ and $\pm\alpha_2 \pm \beta_2 i$. The numerical values of α_1, β_1, α_2 and β_2 corresponding to various set of parameters a, b, c and d, are obtained and thus, the general solution of Eqn.8 can be written as,

$$\begin{aligned} y_1' = {} & e^{\alpha_1 z}\left(C_1 \cos\beta_1 z + C_2 \sin\beta_1 z\right) + e^{-\alpha_1 z}\left(C_3 \cos\beta_1 z + C_4 \sin\beta_1 z\right) \\ & + e^{\alpha_2 z}\left(C_5 \cos\beta_2 z + C_6 \sin\beta_2 z\right) + e^{-\alpha_2 z}\left(C_7 \cos\beta_2 z + C_8 \sin\beta_2 z\right) \\ & + \gamma_1' h' r \end{aligned}$$

for $0 \le z \le z_1$. (9)

Using the above mentioned non-dimensionalized parameters; Eqn.6 can be written as follows,

$$\frac{d^4 y_2'}{dz^4} - A\frac{d^2 y_2'}{dz^2} + B^2 y_2' = \gamma' h' \left(\frac{R}{r}\right)^{3/4}, \quad z_1 \le z \le z_2 \qquad (10)$$

where $A = \frac{R}{I_1'}\left(T_2' + T_3'\right)$ and

$B^2 = R/r$

The range of numerical values chosen to study this problem always satisfies the condition $2B - A \ge 0$. The roots of the auxiliary equation of Eqn.10 in this case are,

$$m_{1,2,3,4} = \pm\alpha_3 \pm \beta_3 i$$

where $\alpha_3 = \frac{\sqrt{2B + A}}{2}$ and

$\beta_3 = \frac{\sqrt{2B - A}}{2}$

The general solution of Eqn.10 is,

$$\begin{aligned} y_2' = {} & e^{\alpha_3 z}\left(C_9 \cos\beta_3 z + C_{10} \sin\beta_3 z\right) + e^{-\alpha_3 z}\left(C_{11} \cos\beta_3 z + C_{12} \sin\beta_3 z\right) \\ & + \gamma_1' h' \left(\frac{R}{r}\right)^{-1/4} \end{aligned}$$

for $z_1 \le z \le z_2$. (11)

In case of some other range of parameters where $2B-A \geq 0$ is not satisfied, the solution of Eqn.10 will be different. This case is not considered in the present study. The response of the upper and lower beams can be written as,

$$\left.\begin{aligned}
y_1' &= e^{\alpha_1 z}\left(C_1 \cos\beta_1 z + C_2 \sin\beta_1 z\right) + e^{-\alpha_1 z}\left(C_3 \cos\beta_1 z + C_4 \sin\beta_1 z\right) \\
&\quad + e^{\alpha_2 z}\left(C_5 \cos\beta_2 z + C_6 \sin\beta_2 z\right) + e^{-\alpha_2 z}\left(C_7 \cos\beta_2 z + C_8 \sin\beta_2 z\right) \\
&\quad + \gamma_1' h' r \qquad \left(0 \leq z \leq z_1\right) \\
&\qquad\qquad \text{and} \\
y_2' &= \frac{1}{R}\left(\frac{R}{r}\right)^{3/4}\left(\frac{d^4 y_1'}{dz^4} - \frac{T_1'}{I_1'}\frac{d^2 y_1'}{dz^2} + y_1'\right) \qquad \left(0 \leq z \leq z_1\right) \\
&= e^{\alpha_3 z}\left(C_9 \cos\beta_3 z + C_{10} \sin\beta_3 z\right) + e^{-\alpha_3 z}\left(C_{11} \cos\beta_3 z + C_{12} \sin\beta_3 z\right) \\
&\quad + \gamma_1' h'\left(\frac{R}{r}\right)^{-1/4} \qquad \left(z_1 \leq z \leq z_2\right)
\end{aligned}\right\} \quad (12)$$

Boundary and Continuity Conditions

The twelve constants of integration appearing in Eqn.12 are to be evaluated using appropriate and sufficient number of boundary and continuity conditions. These boundary conditions for the present problem are as follows,

For the upper beam, at the point of application of load, i.e., at $x = 0$, slope of the deflected shape of beam is zero and shear force is $Q_t/2$. At the edge of upper beam, i.e., at $x = l$, the bending moment and shear force are zero as beam end is a free end. For the lower beam, which is within the foundation soil, at point $x = 0$, slope of deflected shape of the beam and shear force are zero and at $x = l_2$, bending moment and shear force are zero. At $x = l_1$ the continuity of deflection, slope, bending moment and shear force in the lower beam is taken into account. From these boundary and continuity conditions the following non-dimensional equations can be obtained,

For upper beam,

$$\left.\begin{aligned}
&\text{At } z = 0, \quad \frac{dy_1'}{dz} = 0 \text{ and } \frac{d^3 y_1'}{dz^3} = \frac{1}{2} \\
&\text{At } z = z_1, \quad \frac{d^2 y_1'}{dz^2} = 0 \text{ and } \frac{d^3 y_1'}{dz^3} = 0
\end{aligned}\right\} \quad (13a)$$

For lower beam,

$$\text{At } z = 0,\ \frac{dy_2'}{dz} = 0 \text{ and } \frac{d^3y_2'}{dz^3} = 0$$

$$\text{At } z = z_1,\ y_2'\Big|_{z_1-\varepsilon} = y_2'\Big|_{z_1+\varepsilon},\ \frac{dy_2'}{dz}\Big|_{z_1-\varepsilon} = \frac{dy_2'}{dz}\Big|_{z_1+\varepsilon} \qquad (13b)$$

$$\text{At } z = z_1,\ \frac{d^2y_2'}{dz^2}\Big|_{z_1-\varepsilon} = \frac{d^2y_2'}{dz^2}\Big|_{z_1+\varepsilon},\ \frac{d^3y_2'}{dz^3}\Big|_{z_1-\varepsilon} = \frac{d^3y_2'}{dz^3}\Big|_{z_1+\varepsilon}$$

where, ε tends to zero.

$$\text{At } z = z_2,\ \frac{d^2y_2'}{dz^2} = 0 \text{ and } \frac{d^3y_2'}{dz^3} = 0$$

Using the above conditions, one can get a set of twelve linear equations which are solved by using Cholesky decomposition scheme to get the unknown constants C_1 to C_{12}. Using these constants in the appropriate expressions, the deflection, bending moment and shear force of the upper and lower beam can be found out. It is interesting to note that the Hetenyi's (1946) model is a special case of the present model where both the upper and lower beams are of equal length and are infinite and there is no prestressing force.

Results and Discussions

In this section the proposed model is validated and parametric studies are done to quantify the effect of various parameters, like prestress in the upper and lower beams and interfacial friction, on the response of the reinforced bed. The parameters chosen are:

Ratio of non-dimensional length of the upper beam and lower beam, z_r	= 2.5,
Concentrated load (Q_t)	= 100 kN,
parameter γ_1'	= 0.083.

The range of parameters used in the study is chosen based on the physically possible parameters (Selvadurai 1979) is as follows:

Non-dimensional prestressing force in

upper beam ($T_1' = T_1/Q_t$) = 0 – 0.2.

Non-dimensional prestressing force in lower beam ($T_2' = T_2/Q_t$) = 0 – 0.2.

Interfacial friction coefficient (μ) = 0 – 1.0.

The results from the present study are compared to that obtained by Hetenyi's solution (1946) for infinite beams on elastic foundations by taking appropriate values of the parameters. Thus, Hetenyi's solution is a degenerated case of the presently developed general foundation model. Various Parametric studies are carried out to show the effect of normalized prestressing force in the beams (T_1' and T_2') and interfacial frictional coefficient (μ), on the flexural behavior of the reinforced foundation.

Keeping all other parameters constant, the effect of interfacial friction coefficient on the deflections of upper and lower beams is shown in Table 1. It is observed that the interfacial friction coefficient μ does not affect the response of upper and lower beam significantly for the range of parameters considered in the analysis. The deflection decreases as the frictional coefficient increases and this decrease is around 1.5% and 2% for upper and lower beam respectively for the increase in interface friction coefficient μ from 0 to 1. This decrease is negligible from practical point of view so for all practical purposes the interfacial friction can be neglected for the analysis of beams on reinforced beds.

Figure 3 shows the variation of normalized deflection of ground surface with normalized distance from the center of the beam for various non-dimensional values of prestressing force (T_1' = 0 to 0.2) in the upper beam, all other parameters are kept constant as shown in the figure. It is observed that the maximum deflection occurs at the center of the upper beam which

TABLE 1 : Effect of Interface Friction Coefficient

$z_r = 2.5$, $T_1' = T_2' = 0.1$, $R = 20$, $\gamma' = 0.083$, $I_1' = 10$ and $h_1' = 0.5$

Interface friction coefficient (μ)	Non-dimensional maximum deflection of upper beam (y_1')	Non-dimensional maximum deflection of lower beam (y_2')
0	1.635	0.1946
0.25	1.628	0.1936
0.50	1.622	0.1925
0.75	1.616	0.1916
1.00	1.61	0.1906

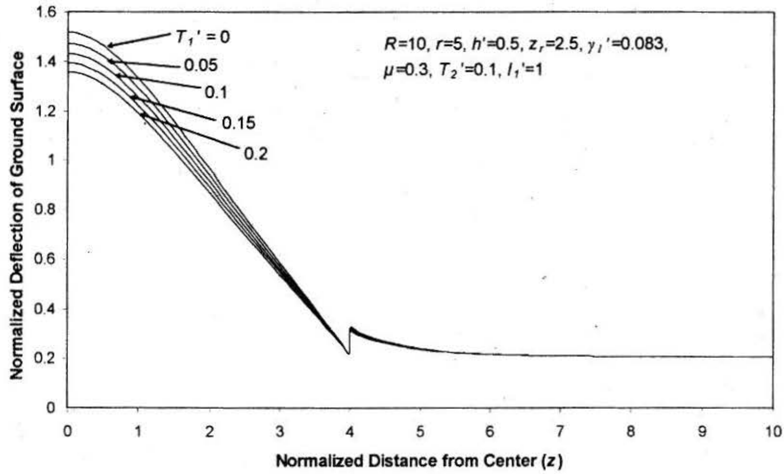

FIGURE 3 : Effect of Prestressing Force in Upper Beam on Deflection of Ground Surface

gradually decreases along the length of the beam, a discontinuity is observed in the deflection pattern at the edge of upper beam due to the foundation model (Winkler's model) considered for the analysis. The maximum normalized deflection occurring at the center of the beam, decreases by 10-11% as the non-dimensional prestressing force in the upper beam, T_1' increases from 0 to 0.2.

Figure 4 shows the effect of non-dimensional prestressing force in the upper beam on the normalized deflection of the lower beam for the same parameters as that in case of upper beam. The maximum deflection occurs at the center of the beam, decreases gradually and becomes almost constant towards the edge of the beam. The maximum normalized deflection of the lower beam decreases to the extent of 10% for the corresponding increase in the non-dimensional prestress T_1' from 0 to 0.2 for the various parameters as shown in figure.

Figures 5 and 6 show the effect of non-dimensional prestressing force in the lower beam, T_2', on the deflection of upper and lower beam respectively, all other soil and beam parameters are kept constant as was done in earlier. The maximum deflection for the upper beam (Fig.5) occurs below the load, i.e., at the center of the beam, which decreases gradually towards the edge of the beam. At the center of the beam the normalized deflection decreases by 4% for the corresponding increase in the non-dimensional prestressing force in lower beam from 0 to 0.2. The point $z = 2.6$ is observed as the critical point where the normalized deflection is independent of any variation in the non-dimensional prestress. Up to the

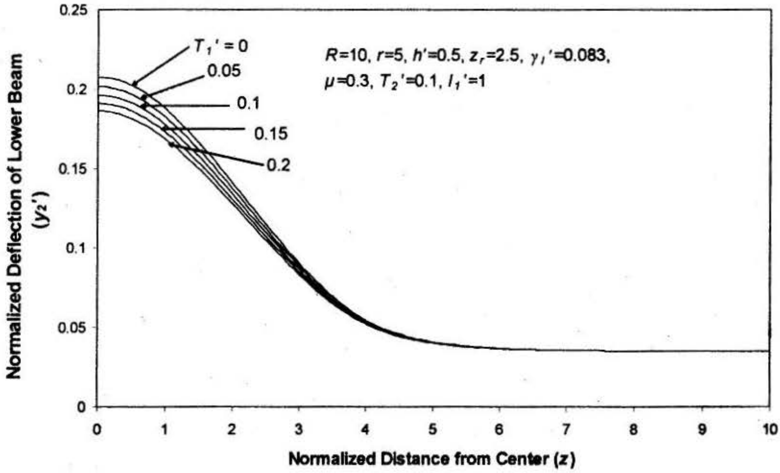

FIGURE 4 : Effect of Prestressing Force in Upper Beam on Deflection of Lower Beam

point z = 0 to 2.6, the normalized deflection decreases for any increase in the non-dimensional prestressing force but beyond this point the deflection pattern gets changed and the normalized deflection is less for lower values of non-dimensional prestressing force in the lower beam.

The effect of variation in non-dimensional prestressing force in lower

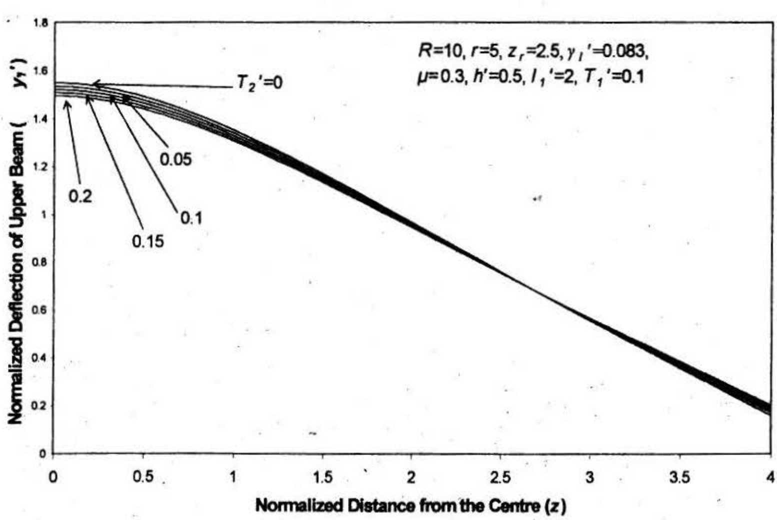

FIGURE 5 : Effect of Prestressing Force in Lower Beam on Deflection of Upper Beam

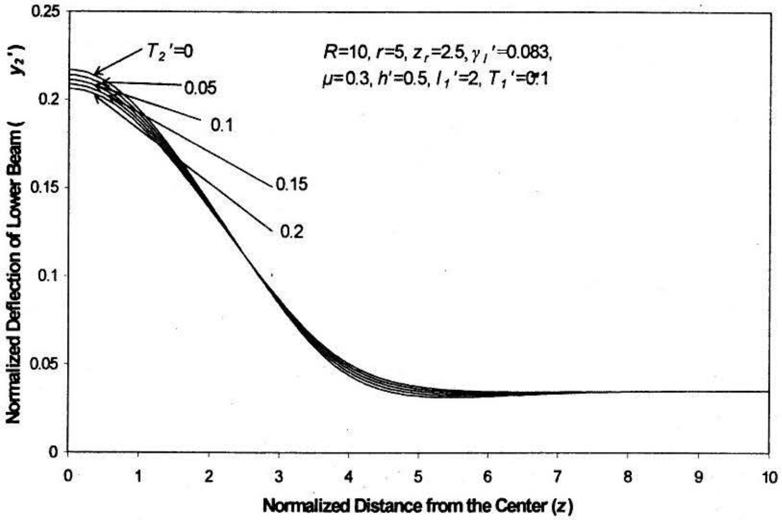

FIGURE 6 : Effect of Prestressing Force in Lower Beam on Deflection of Lower Beam

beam on the normalized deflection of lower beam is shown in Fig.6. In the case of lower beam also the maximum deflection occurs below the loading point (at the center of the beam) and gradually decreases and becomes constant towards the edge of the beam. It is observed that in case of lower beam the critical point about which the deflection pattern gets reversed is z = 2.5. At this point the deflection is same for all the values of non-dimensional prestress considered in the study. The maximum deflection reduces by around 5% for the corresponding increase in the normalized prestress from 0 to 0.2. From point z = 0 to z = 2.5 the deflection decreases as prestressing force increases, between z = 2.5 and z = 6.5 the deflection is less for lesser values of prestressing force and beyond the point z = 6.5 the deflection of the beam is almost constant.

Conclusions

A general model and solution procedure is presented here for the analysis of beams on reinforced beds, which incorporates prestressing force in the beams and the interfacial friction. The form of the solution may vary depending on the nature of the roots of the governing differential equations. As such, the presented solutions are valid for the complex roots. Various parametric studies are carried out to observe the effect of introducing the prestressing force in the beams and the developed interfacial friction between lower beam and neighboring soil. Parametric studies reveal that the interface friction between the lower reinforcing beam and neighboring soil has almost negligible effect (1-2% change) on the response of proposed model for the range of parameters considered in the analysis. So for all practical purposes this can be neglected in the analysis part. The maximum normalized deflection of the upper and lower beam reduces approximately by 10% when the non-dimensional prestressing force in the upper beam is increased from 0 to 0.2 for the range of other parameters considered. The reduction in the normalized deflection of the beams is found to be of the order of 4-5% for the corresponding increase in the non-dimensional prestressing force in the lower beam from 0 to 0.2.

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Notations

E_1I_1	=	Flexural rigidity of upper beam
E_2I_2	=	Flexural rigidity of lower beam
h	=	Depth of placement of lower beam
h'	=	Non-dimensional depth of placement of lower beam
I_1'	=	Non-dimensional flexural rigidity of upper beam
I_2'	=	Non-dimensional flexural rigidity of lower beam
k_1	=	Modulus of subgrade reaction of upper layer of soil
k_2	=	Modulus of subgrade reaction of lower layer of soil
l_1	=	Half length of the upper beam
l_2	=	Half length of lower beam
p_1	=	Distributed pressure in the foundation under the upper beam
p_2	=	Distributed pressure in the foundation under the lower beam
Q_t	=	Load applied on the upper beam
R	=	Relative flexural rigidity of beam (E_1I_1/E_2I_2)
R_1	=	Characteristic length of upper beam $\left(R_1 = \sqrt[4]{\frac{E_1I_1}{k_1}}\right)$
R_2	=	Characteristic length of lower beam $\left(R_2 = \sqrt[4]{\frac{E_2I_2}{k_2}}\right)$

r	=	Relative stiffness of soil (k_1/k_2)
T_1	=	Prestressing force in upper beam
T_2	=	Prestressing force in lower beam
T_3	=	Mobilized tension in lower reinforcing beam
T_1'	=	Non-dimensional prestressing force in upper beam
T_2'	=	Non-dimensional prestressing force in lower beam
T_3'	=	Non-dimensional mobilized tension in lower reinforcing beam
x	=	Co-ordinate along the length of the beams
y	=	Co-ordinate perpendicular to the length of beams
y_1	=	Deflection of upper beam
y_2	=	Deflection of lower beam
y_1'	=	Non-dimensional deflection of upper beam
y_2'	=	Non-dimensional deflection of lower beam
z	=	Non-dimensional distance along the length of beam
z_1	=	Non-dimensional half length of upper beam
z_2	=	Non-dimensional half length of lower beam
z_r	=	Ratio of length of upper and lower beams (l_1/ l_2)
γ_1	=	Unit weight of upper layer of soil
γ_2	=	Unit weight of lower layer of soil
γ_1'	=	Non-dimensional unit weight of upper layer
μ	=	Frictional coefficient between lower beam and neighboring soil