Indian Geotechnical Journal, 34 (2), 2004

Torsional Vibration of Foundations Resting on A Soil Layer Over Rigid Base

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Introduction

The study of the dynamic response of foundations resting on or embedded in layered soil subjected to torsional dynamic loading is an important aspect in the design of machine foundations and dynamic soil-structure interaction problems. This torsional dynamic loading arises when asymmetric horizontal forces resulting from windstorms, earthquake shaking, horizontal movements of the antennas of radar towers and operation of reciprocating engines, act on a superstructure. Thus, the determination of resonant frequency and maximum torsional rotation is an important aspect in the design of foundations under torsional loading. One of the key steps in the current methods of dynamic analysis of a foundation soil system to predict resonant frequency and amplitude under machine type loading is to estimate the dynamic impedance functions (spring and dashpot coefficients) of an associated rigid but massless foundation. With the help of these functions the amplitude of vibration is calculated using the equation of motion of a single degree of freedom oscillator.

Since the contribution of Reissner (1937), the harmonic response of torsionally excited foundations has been the subject of numerous investigations (Reisssner and Sagoci, 1944; Collins, 1962; and Weissmann, 1971). The fundamental problem considered in these investigations has been

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the analysis of the steady state response of a rigid, circular massless plate bonded to an elastic half-space and excited by a harmonically varying moment about an axis normal to the disk. Gazetas (1991) presented formulae and charts for impedances of surface and embeded foundations for all modes of vibration, which can be readily used by the practicing engineers. Ahmad and Gazetas (1992a, 1992b) presented simple expressions and charts for stiffness and radiation damping of arbitrary shaped embedded foundations particularly in torsional mode of vibration.

In most of the studies the soil medium below the foundation was assumed to be a homogeneous elastic half-space. In reality, however, soils are rarely homogeneous. The presence of a hard rock at shallow depth is one of the common features of soil in natural state. Arnold et al. (1955) and Bycroft (1956) evaluated the torsional vibration response of a circular footing on the surface of an elastic layer underlain by a rigid base. They showed that the presence of a thin layer tends to increase the resonant frequency and amplitude compared to the half-space values. Gazetas (1983), Apsel and Luco (1987), Aviles and Perez-Rocha (1996) to name a few, also considered the effect of layering or nonhomogenity in their analyses under torsional mode. Most of these works were confined to analytical or semi-analytical type.

The cone model was originally developed by Ehlers (1942) to represent a surface disk under translational motions and later for rotational motion (Meek and Veletsos, 1974; Veletsos and Nair, 1974). Later Meek and Wolf (1992a) presented a simplified methodology to evaluate the dynamic response of a base mat on the surface of a homogeneous half-space. The cone model concept was extended to a layered cone to compute the dynamic response of a footing or a base mat on a soil layer resting on a rigid rock Meek and Wolf (1992b) and on flexible rock, Wolf and Meek (1993). Wolf and Meek (1994) have found out the dynamic stiffness coefficients of foundations resting on or embedded in a horizontally layered soil using cone frustums. Also Jaya and Prasad (2002) studied the dynamic stiffness of embedded foundations using the same cone frustums. The major drawback of cone frustums method is that the damping coefficient can become negative, which is physically impossible. Pradhan et al. (2003) computed impedance functions of foundations resting on layered soil using wave propagation in cones, which eliminates the drawback of cone frustum method. The details of the use of cone models in foundation vibration analysis are summarized in Wolf (1994).

For foundation vibration analyses simple models, which fit the size and economics of the project and require no sophisticated computer code are better suited. For instance the cone models, which provide conceptual clarity with physical insight and is easier for the site engineer to understand. Most of the published results using cone model are confined to the determination of the dynamic response of the foundation in the form of impedance

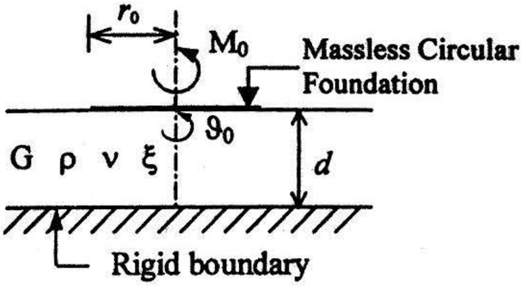

FIGURE 1 : Massless Foundation on Soil Layer under Torsional Interaction Moment

functions. To the best of author's knowledge no literature is available with regard to the study of frequency-amplitude response of the foundation using cone model, though resonant frequency and amplitude are the two main design parameters. Hence, in the present investigation the dynamic response (frequency-amplitude) of the foundation resting on a soil layer underlain by a rigid base under torsional harmonic excitation is found out using wave propagation in cones. The validity of the model is checked by comparing the predicted resonant frequency with reported analytical and experimental results. The foundation response is also studied varying widely the parameters like mass ratio, depth of the layer and hysteretic material damping ratio.

Dynamic Response of the Foundation

To study the dynamic response of a machine foundation resting on the surface of a soil layer underlain by a rigid base, a rigid massless "associated" foundation of radius r_0 is addressed for torsional degree of freedom (Fig.1). The layer with depth d has the shear modulus G, Poisson's ratio ν, mass density ρ and hysteretic material damping ratio ξ. The interaction moment M_0 and the corresponding rotational displacement ϑ_0 are assumed to be harmonic. The dynamic behaviour of the massless foundation (disk) is expressed by the dynamic impedance

$$\bar{K}(a_0) = \frac{M_0}{\vartheta_0} = K\left[k(a_0) + ia_0c(a_0)\right] \tag{1}$$

where, $k(a_0)$ = normalized spring coefficient

$c(a_0)$ = normalized damping coefficient

$a_0 = (\omega r_0/c_s)$
= dimensionless frequency

$$c_s = \sqrt{G/\rho}$$

= shear wave velocity of the layer

$$K = (16/3)Gr_0^3$$

= static stiffness coefficient of the rigid disk on homogeneous half-space with material properties of the layer

Using the equations of dynamic equilibrium, the dynamic rotational amplitude of the massive foundation with mass m and subjected to a harmonic moment M is expressed as

$$|\vartheta_0| = \left|\frac{M}{K\left[k(a_0)+ia_0c(a_0)-\frac{3}{16}B_\vartheta a_0^2\right]}\right| \tag{2}$$

where, $|\vartheta_0|$ = dynamic rotational displacement amplitude under the foundation resting on the layer.

$|M|$ = torsional moment amplitude

$B_\vartheta = \dfrac{I_\vartheta}{\rho r_0^5}$, the mass ratio, with $I_\vartheta = \dfrac{m r_0^2}{2}$

In general, $|M|$ can be assumed to be constant or equal to $m_e e x \omega^2$, which is created by the vibratory machine where m_e is the unbalanced mass, e is the eccentricity and x is the horizontal-moment arm of the unbalanced mass from the center of rotation.

Expressing the dynamic rotational displacement amplitude given in Equation (2) in the non-dimensional form

$$\left|\frac{\vartheta_0 G r_0^3}{M}\right| = \left|\frac{16}{3}\left[k(a_0)+ia_0c(a_0)\right]-B_\vartheta a_0^2\right|^{-1} \tag{3}$$

Wave Propagation in Cones

Fig.2a shows wave propagation in cones beneath the disk of radius r_0 resting on a layer underlain by a rigid base under torsional harmonic excitation M_0. The shear waves emanate beneath the disk and propagate at velocity c equal to shear wave velocity c_s. These waves reflect back and forth at the rigid base and free surface, spreading and decreasing in amplitude. Let the rotational displacement of the (truncated semi-infinite) cone, modeling a disk with same torsional moment M_0 on a homogeneous half-space with the material properties of the layer be denoted as $\bar{\vartheta}$ with the

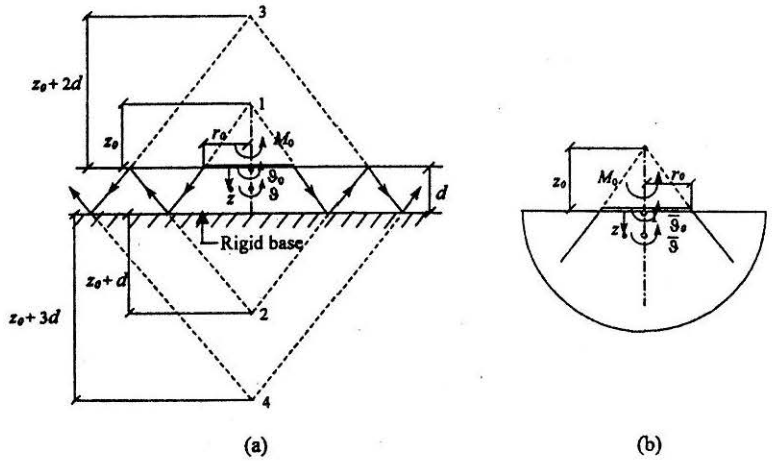

FIGURE 2 : (a) Wave Propagation in Cones for the Layer, (b) Cone Model for the Half-Space

value $\bar{\vartheta}_0$ under the disk, Fig.2b, the parameters of which are given in Table 1. This rotational displacement $\bar{\vartheta}_0$ is used to generate the rotational displacement of the layer ϑ with its surface value ϑ_0, Fig.2a. Thus, $\bar{\vartheta}_0$ can also be called as the generating function. The first downward wave propagating in a cone with apex 1 (height z_0 and radius of base r_0), which may be called as the incident wave and its cone will be the same as that of the half-space, as the wave generated beneath the disk does not know if at a specific depth a rigid interface is encountered or not. Thus the aspect ratio defined by the ratio of the height of cone from its apex to the disk is made equal for cone of the half-space and first cone of the layer. Since the incident wave and subsequent reflected waves propagate in the same medium (layer), the aspect ratio of the corresponding cones will be same. Thus knowing the height of the first cone, from the geometry, the height of other cones corresponding to subsequent upward and downward reflected waves are found as shown in Fig.2a. The rotational displacement amplitude of the first downward incident wave (propagating in a cone with apex 1) at a depth z, which is inversely proportional to the square and cube of the distance from the apex of the cone and expressed in frequency domain as

$$\bar{\vartheta}(z,\omega) = \left[\frac{1}{\left(1+\frac{z}{z_0}\right)^2} + \left\{\frac{1}{\left(1+\frac{z}{z_0}\right)^3} - \frac{1}{\left(1+\frac{z}{z_0}\right)^2}\right\} \cdot \frac{1}{1+i\omega\frac{z_0}{c}}\right] \cdot e^{-i\omega\frac{z}{c}} \cdot \bar{\vartheta}_0(\omega) \qquad \text{..... (4)}$$

The rotational displacement of the incident wave at rigid base equals

$$\bar{\vartheta}(d,\omega) = \left[\frac{1}{\left(1+\frac{d}{z_0}\right)^2} + \left\{\frac{1}{\left(1+\frac{d}{z_0}\right)^3} - \frac{1}{\left(1+\frac{d}{z_0}\right)^2}\right\} \cdot \frac{1}{1+i\omega\frac{z_0}{c}}\right] \cdot e^{-i\omega\frac{d}{c}} \cdot \bar{\vartheta}_0(\omega) \qquad \text{..... (5)}$$

Enforcing the boundary condition that the rotation at rigid base vanishes, the rotational displacement of the first reflected upward wave propagating in a cone with apex 2 (vide Fig.2a) is given by

$$\bar{\vartheta}(2d-z,\omega) = \left[\frac{1}{\left(1+\frac{2d-z}{z_0}\right)^2} + \left\{\frac{1}{\left(1+\frac{2d-z}{z_0}\right)^3} - \frac{1}{\left(1+\frac{2d-z}{z_0}\right)^2}\right\} \cdot \frac{1}{1+i\omega\frac{z_0}{c}}\right] \cdot e^{-i\omega\frac{2d-z}{c}} \cdot \bar{\vartheta}_0(\omega) \qquad \text{..... (6)}$$

At the free surface the rotational displacement of the upward wave derived by substituting z = 0 in Eqn.6 equals

$$\bar{\vartheta}(2d,\omega) = \left[\frac{1}{\left(1+\frac{2d}{z_0}\right)^2} + \left\{\frac{1}{\left(1+\frac{2d}{z_0}\right)^3} - \frac{1}{\left(1+\frac{2d}{z_0}\right)^2}\right\} \cdot \frac{1}{1+i\omega\frac{z_0}{c}}\right] \cdot e^{-i\omega\frac{2d}{c}} \cdot \bar{\vartheta}_0(\omega) \qquad \text{.... (7)}$$

Enforcing compatibility of the amplitude and of elapsed time of the reflected wave's rotational displacement at the free surface, the rotational displacement of the downward wave propagating in a cone with apex 3 is formulated as

$$\bar{\vartheta}(2d+z,\omega) = \left[\frac{1}{\left(1+\frac{2d+z}{z_0}\right)^2} + \left\{\frac{1}{\left(1+\frac{2d+z}{z_0}\right)^3} - \frac{1}{\left(1+\frac{2d+z}{z_0}\right)^2}\right\} \cdot \frac{1}{1+i\omega\frac{z_0}{c}}\right] \cdot e^{-i\omega\frac{2d+z}{c}} \cdot \bar{\vartheta}_0(\omega) \qquad \text{.... (8)}$$

In this pattern the waves propagate in their own cones and their corresponding rotational displacements are found out. The superposition of all the down and up waves gives the resulting rotational displacement in the layer

$$\vartheta(z,\omega) = \left[\frac{1}{\left(1+\frac{z}{z_0}\right)^2} + \left\{\frac{1}{\left(1+\frac{z}{z_0}\right)^3} - \frac{1}{\left(1+\frac{z}{z_0}\right)^2}\right\} \cdot \frac{1}{1+i\omega\frac{z_0}{c}}\right] \cdot e^{-i\omega\frac{z}{c}} \cdot \bar{\vartheta}_0(\omega)$$
$$+ \sum_{j=1}^{\infty}(-1)^j \left[\frac{1}{\left(1+\frac{2jd-z}{z_0}\right)^2} + \left\{\frac{1}{\left(1+\frac{2jd-z}{z_0}\right)^3} - \frac{1}{\left(1+\frac{2jd-z}{z_0}\right)^2}\right\} \cdot \frac{1}{1+i\omega\frac{z_0}{c}}\right] \cdot e^{-i\omega\frac{2jd-z}{c}} \cdot \bar{\vartheta}_0(\omega)$$
$$+ \sum_{j=1}^{\infty}(-1)^j \left[\frac{1}{\left(1+\frac{2jd+z}{z_0}\right)^2} + \left\{\frac{1}{\left(1+\frac{2jd+z}{z_0}\right)^3} - \frac{1}{\left(1+\frac{2jd+z}{z_0}\right)^2}\right\} \cdot \frac{1}{1+i\omega\frac{z_0}{c}}\right] \cdot e^{-i\omega\frac{2jd+z}{c}} \cdot \bar{\vartheta}_0(\omega) \quad \text{..... (9)}$$

where, j is the number of impingement of waves at the rigid boundary.

At the free surface the rotational displacement of the foundation is obtained by setting z = 0 in Eqn.9.

$$\vartheta_0(\omega) = \left[1 + 2\sum_{j=1}^{\infty}(-1)^j \left[\frac{1}{\left(1+\frac{2jd}{z_0}\right)^2} + \left\{\frac{1}{\left(1+\frac{2jd}{z_0}\right)^3} - \frac{1}{\left(1+\frac{2jd}{z_0}\right)^2}\right\} \cdot \frac{1}{1+i\omega\frac{z_0}{c}}\right] \cdot e^{-i\omega\frac{2jd}{c}}\right] \cdot \bar{\vartheta}_0(\omega) \quad \text{..... (10)}$$

$$\bar{\vartheta}_0(\omega) = H(\omega)\vartheta_0(\omega) \quad (11)$$

where,

$$H(\omega) = \left[1 + 2\sum_{j=1}^{\infty}(-1)^j \left[\frac{1}{\left(1+\frac{2jd}{z_0}\right)^2} + \left\{\frac{1}{\left(1+\frac{2jd}{z_0}\right)^3} - \frac{1}{\left(1+\frac{2jd}{z_0}\right)^2}\right\} \cdot \frac{1}{1+i\omega\frac{z_0}{c}}\right] \cdot e^{-i\omega\frac{2jd}{c}}\right]^{-1} \quad \text{..... (12)}$$

$H(\omega)$ given by Eqn.12 may be called as rotational displacement transfer function, the value of which at $\omega = 0$ gives the static stiffness of the layer normalized by the static stiffness of the homogeneous half-space with material properties of the layer. In numerical evaluation of the above transfer function, the summation of series over j is worked out up to a finite term as the displacement amplitude of the waves vanish after a finite number of impingement.

Dynamic Impedance

The interaction moment rotation relationship for a massless disk resting on homogeneous half-space using the cone model can be written as

$$M_0(\omega) = K\{k(\omega) + i\omega c(\omega)\}\bar{\vartheta}_0(\omega) \tag{13}$$

where, $k(\omega)$ and $c(\omega)$ are the stiffness and damping coefficients normalized by the static stiffness of homogeneous half-space K, the values of which are given in Table 1 as functions of non dimensional frequency a_0.

TABLE 1 : Parameters of Semi-Infinite Cone Modeling Disk on Homogeneous Half-Space under Torsional Motion (Wolf, 1994)

Cone Parameters	Parameter expressions
Aspect Ratio z_0/r_0	$\frac{9\pi}{32}$
Static stiffness coefficient, K	$\frac{3\rho c^2 I_0}{z_0}$
Polar moment of inertia, I_0	$(\pi/2)(r_0)^4$
Normalized spring coefficient, $k(a_0)$	$1-\frac{1}{3}\frac{a_0^2}{\left(\frac{r_0 c}{z_0 c_s}\right)^2 + a_0^2}$
Normalized damping coefficient, $c(a_0)$	$\frac{z_0 c_s}{3 r_0 c} \cdot \frac{a_0^2}{\left(\frac{r_0 c}{z_0 c_s}\right)^2 + a_0^2}$
Dimensionless frequency, a_0	$\frac{\omega r_0}{c_s}$
Appropriate wave velocity, c	$c = c_s$ for all values of Poisson's ratio (ν)

Using Eqn.11 in Eqn.13, we have the interaction moment rotation relationship for the layer-rigid base system

$$M_0(\omega) = K\{k(\omega)+i\omega c(\omega)\}H(\omega)\cdot\vartheta_0(\omega) \tag{14}$$

From Eqn.14, the dynamic impedance equals

$$\bar{K}(\omega) = \frac{M_0(\omega)}{\vartheta_0(\omega)} = K\{k(\omega)+i\omega c(\omega)\}H(\omega) \tag{15}$$

Results and Discussions

Comparison of Results

In the static case, the normalized stiffness of the layer K_L/Gr_0^3 with K_L, the static stiffness of circular disk on the layer rigid base system is found using the model for different values of d/r_0 and compared against the constant value (16/3 for $d/r_0 \geq 1.25$) given by Gazetas (1983), which shows a very good agreement (Fig.3). This figure indicates that the computed values of normalized stiffness decreases with increase in depth of the layer up to $d/r_0 = 4$, beyond which it remains constant. Thus from this static analysis it is observed that the layer-rigid base system under torsional load behaves as a homogeneous half-space with material properties of the layer

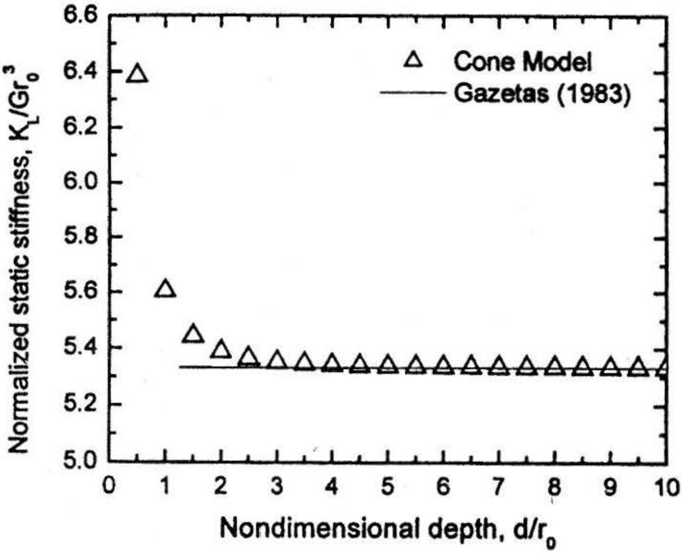

FIGURE 3 : Comparison of Normalized Static Stiffness

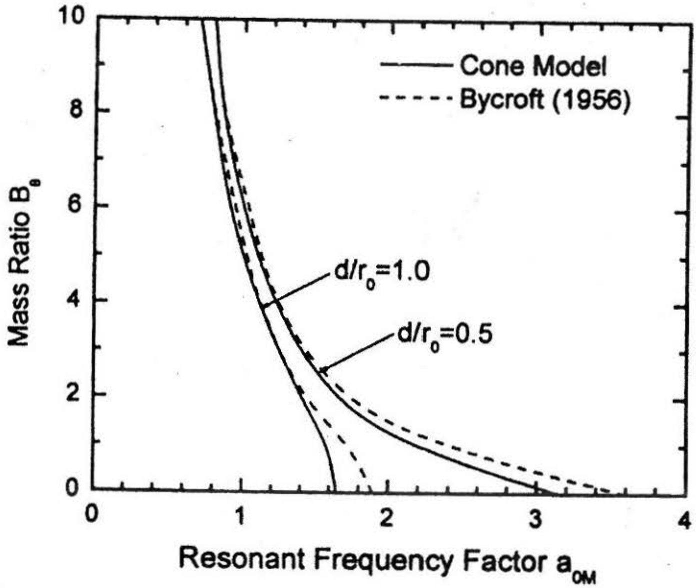

FIGURE 4 : Comparison of Predicted Resonant Frequency with Bycroft's Results

when $d/r_0 \geq 4$. An indication of the cause of this behaviour of rigid circular foundation subjected to torsional loading can be obtained by observing the depth of the 'zone of influence' (known as 'pressure bulb'). Thus, from Gazetas (1983), in a homogeneous half-space the horizontal shear stresses in circumferential directions due to linear distributed torsional surface stresses become less than 10% of the maximum applied shear traction at depth greater than $0.75\ r_0$.

Also, the validity of the model is checked by comparing the predicted resonant frequencies with the published analytical as well as experimental results. It is seen that the variation of non-dimensional resonant frequency with mass ratio is in good agreement with the results of Bycroft (1956) for $d/r_0 = 0.5$ and 1.0 as shown in Fig.4, as the maximum difference is found to be within 5%.

The computed resonant frequencies are also compared with the experimental results of Arnold et al. (1955) with variation in mass ratios for two depths of the layer ($d/r_0 = 0.47$ and 0.97) .The comparison shows a good accuracy which can be observed from Fig.5.

Thus, the model based on one-dimensional wave propagation in cones not only provides physical insight and adequate accuracy in the analysis of

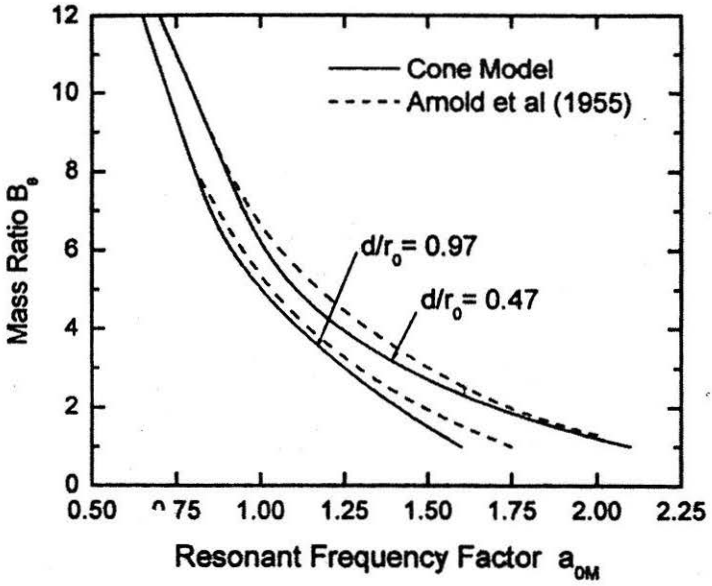

FIGURE 5 : Comparison of Predicted Resonant Frequency with Experimental Results

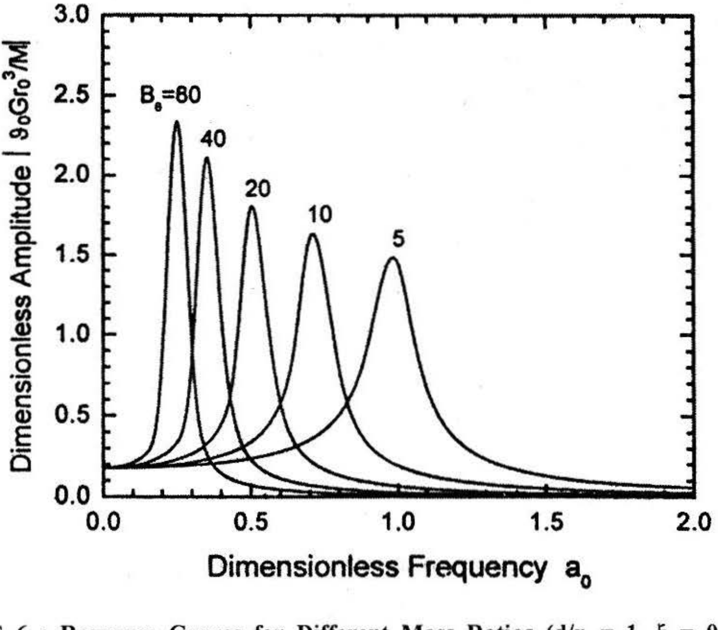

FIGURE 6 : Response Curves for Different Mass Ratios ($d/r_0 = 1$, $\xi = 0.05$)

torsional vibration of foundation on soil layer but also very simple to use. Hence, in a project where finance is a constraint, the cone model can substitute rigorous models, which are based on three-dimestional elastodynamics and require sophisticated computer codes.

Parametric Study

A parametric study is conducted widely varying the influencing parameters such as mass ratio B_ϑ, dimensionless depth of layer d/r_0, and hysteretic material damping ratio ξ. The results are presented in the form of dimensionless graphs, which may prove to be useful in understanding the response of foundation resting on a layer underlain by a rigid base subjected to harmonic torsional excitation.

Figure 6 presents a plot of the response of the foundation for different mass ratio, B_ϑ at $d/r_0 = 1$. An increase in the amplitude and decrease in resonant frequency is observed with increase in mass ratio.

The effect of the depth of the layer on the dynamic response is presented in Fig.7, considering marerial damping ratio $\xi = 5\%$. It is observed that both resonant frequency and maximum amplitude decrease with increase in the depth of the layer. Also it is observed that when $d/r_0 \geq 4$, the layer-

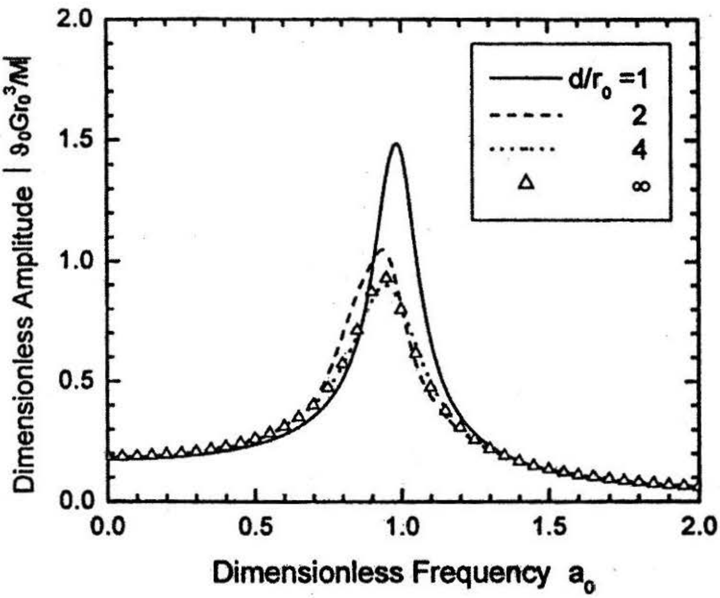

FIGURE 7 : Effect of d/r_0 Ratio on the Response of the Foundation ($B_\vartheta = 5$, $\xi = 0.05$)

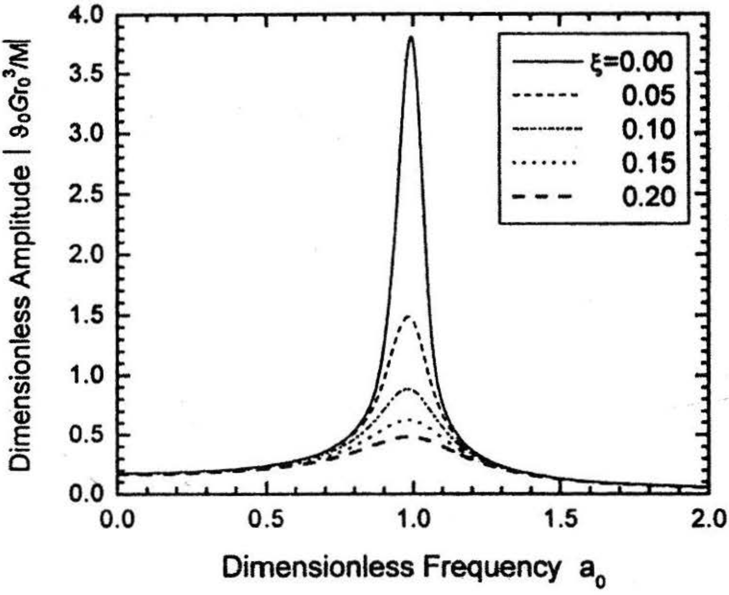

FIGURE 8 : Effect of Material Damping Ratio on the Response of the Foundation ($d/r_0 = 1$, $B_\vartheta = 5$)

rigid base system behaves as a half-space and thus the half-space theory may be used. These results are consistent with the conclusion derived earlier regarding the depth of 'pressure bulb' of a statically loaded foundation.

Figure 8 shows the effect of material damping ratio (hysteretic) on the response of the foundation at $d/r_0 = 1$. A decrease in the amplitude is observed with increase in material damping ratio, which is more pronounced in the frequency range 0.75 to 1.25 times the resonant frequency. For example, it can be observed from Fig.8 that presence of 5% material damping reduces the rotational amplitude of vibration by 60% at resonance. But no distinct change in the resonant frequency is observed with increase in material damping ratio.

Conclusions

In contrast to rigorous methods, which address the very complicated wave pattern consisting of body waves and generalized surface waves working in wave number domain, the procedure based on one dimensional wave propagation in cones with reflection at interface (rigid base) and free surface considers only one type of body wave (shear wave) for the torsional degree of freedom considered. The sectional property of the cones increases in the direction of wave propagation downward as well as upwards. Physical insight with conceptual clarity thus results. The method based on wave propagation in cones is well suited for foundation vibration analysis, as good accuracy results and the procedure is easier to use.

Based on the parametric studies, the following conclusions can be drawn.

1. With increase in the depth of the layer the static stiffness decreases and reaches a value of static stiffness of homogeneous half-space at $d/r_0 = 4$. Also the resonant frequency and resonant amplitude decrease with increase in the depth of the layer and at $d/r_0 = 4$, both resonant frequency and resonant amplitude approach the half-space values.

2. With increase in mass ratio the resonant frequency decreases and resonant amplitude increases.

3. With increase in material damping ratio the resonant amplitude decreases, but no remarkable change in resonant frequency is observed.

Results of parametric studies presented in the form of dimensionless plots, provide a clear understanding of the torsional dynamic response of the foundation resting on a soil layer underlain by rigid base.

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Notations

d = depth of the soil layer

r_0 = radius of circular foundation or radius of equivalent circle for non circular foundation

z_0 = height of the apex of semi-infinite truncated cone from the disk.

a_0	=	dimensionless frequency $\left(\omega r_0/c_s\right)$
c	=	appropriate wave velocity in soil layer
c_s	=	shear wave velocity in soil layer
G	=	shear modulus of soil
m	=	mass of the foundation or total vibrating mass (mass of foundation plus machine) in case of machine foundation
m_e	=	unbalanced mass (on machine)
e	=	eccentricity of unbalanced mass
x	=	horizontal-moment arm of the unbalanced mass from the center of rotation
M_0	=	harmonic interaction moment
$\bar{\vartheta}_0$	=	rotational harmonic surface displacement for homogeneous half-space
ϑ_0	=	rotational harmonic surface displacement for the layer
$\lvert\vartheta_0\rvert$	=	rotational displacement amplitude for the layer
M	=	harmonic torsional moment on foundation
$\lvert M\rvert$	=	torsional moment amplitude on the foundation
$\bar{K}(a_0)$	=	dynamic impedance
K	=	static stiffness coefficient of the disk on homogeneous half-space
K_L	=	static stiffness coefficient the disk on layer
$k(a_0)$	=	normalized stiffness coefficient
$c(a_0)$	=	normalized damping coefficient
I_ϑ	=	$\dfrac{m r_0^2}{2}$
	=	mass moment of inertia of the foundation about the axis of rotation
B_ϑ	=	$\dfrac{I_\vartheta}{\rho r_0^5}$
	=	nondimensional mass ratio

$I_0 = \frac{\pi}{2} r_0^4$

= Polar moment of inertia

$\left|\frac{\vartheta_0 G r_0^3}{M}\right|$ = nondimensional rotational amplitude

ν = Poisson's ratio of the soil

ρ = mass density of soil

ξ = hysteretic material damping ratio

ω = circular frequency of excitation