Indian Geotechnical Journal, 34 (2), 2004

Effect of Dilatancy Angle on Uplift Resistance of Shallow Anchors

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Introduction

Only limited information is available in literature to estimate the uplift resistance of anchors buried in non-associated soils; the research on this topic has been performed by Rowe and Davis (1982a, 1982b) and Vermeer and Sutjiadi (1985). These studies demonstrate that the uplift resistance of anchors reduces extensively with decrease in dilatancy angle of soil mass. It is a known fact that the assumption of an associated flow rule over-predicts volume increase during shear than that is observed in most of soils (Rowe, 1971; Zienkiewicz et al., 1975). For an associated material, the upper bound theorem of limit analysis is usually employed to solve the stability problems in soil mechanics (Chen, 1975; Chen and Liu, 1990). For an associated flow rule the stress distributions along various rupture surfaces do not appear in energy rate expressions. Whereas, for a non-associated flow rule material it is necessary to have the knowledge of stresses along the rupture surfaces so as to find the rate of dissipation of internal energy. Dreshcher and Detournay (1993) have established the expressions for equivalent values of c and ϕ to be used in the solution for an associated material so as to find directly the collapse load for a given value of ψ for coaxial flow rule material. In the present investigation, the vertical uplift resistance of strip anchors has been obtained for a general $c-\phi$ soil with the value of ψ ranging from 0 to ϕ. The solution has been obtained with the consideration of force equilibrium and energy balance. The kinematic admissibility of the chosen collapse mechanism has also been addressed. The computed results have been compared with the various available theories and published experimental data.

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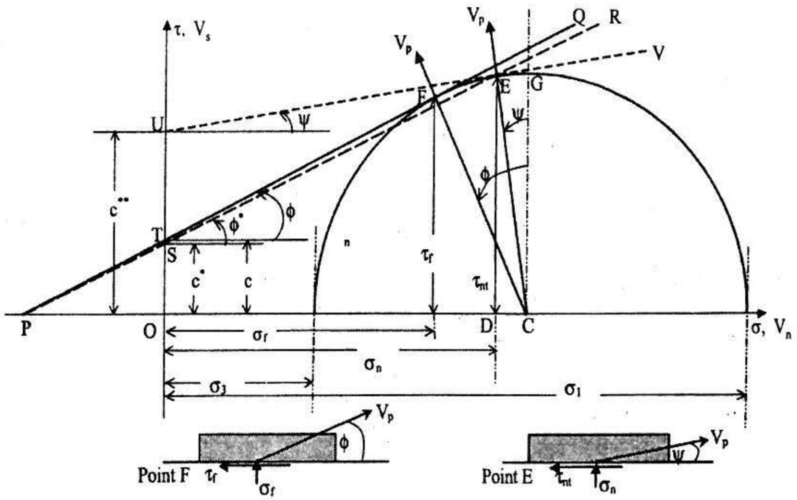


FIGURE 1 : Stress Relationships and the Direction of the Velocity Jump on the Rupture Plane for Associated and Non-Associated Flow Rules

Flow Rule

If the soil mass is assumed to follow Mohr-Coulomb yield condition, then for an associated flow rule, the plastic strain rate vector becomes normal to the yield surface PQ. It is indicated in Fig.1 by showing an arrow of the direction of the resultant velocity jump, V_p, at the point F. For an associated flow rule the direction of V_p makes an angle ϕ with the plane of shear. Whereas with non-associated co-axial flow rule, the direction of the V_p, indicated by an arrow at the point E, is normal to the line UV and it makes an angle ψ with the plane of shear (Drescher and Detournay, 1993); in Fig.1 V_n and V_s represents the normal and shear velocities.

For the Mohr-Coulomb yield line (PQ):

$$\tau_f = \sigma_f \tan\phi + c \qquad (1)$$

where τ_f and σ_f are the magnitudes of shear and normal stresses acting on the failure plane, indicated by the point F on the Mohr circle.

Along the line UV:

$$\tau_{nt} = c^{**} + \sigma_n \tan\psi \qquad (2)$$

where τ_{nt} and σ_n are the shear and normal stresses on a plane represented by the point E on the Mohr-circle. From Fig.1, it can be shown that the value of c^{**} becomes a function of σ_n, c, ϕ and ψ; the expression for c^{**} was derived and it is given below:

$$c^{**} = \frac{\left[c\cos\phi + \sigma_n\left(\sin\phi + \tan^2\psi\sin\phi - \frac{\tan\psi}{\cos\psi}\right)\right]}{(\sec\psi - \tan\psi\sin\phi)} \tag{3}$$

For $\psi = \phi$, it can be checked that the value of the c^{**}, irrespective of σ_n, becomes simply equal to c. Alternatively, the shear and normal stresses on the plane marked by the point E on the Mohr circle, can also be defined by means of a line PR, having the same starting point (P) as that of the yield line PQ.

For the line PR:

$$\tau_{nt} = c^* + \sigma_n \tan\phi^* \tag{4}$$

wherein the values of the c^* and ϕ^* become independent of σ_n and can be shown to be related by the following expressions (Drescher and Detournay, 1993):

$$c^* = \eta c \tag{5a}$$

$$\phi^* = \tan^{-1}(\eta\tan\phi) \tag{5b}$$

$$\eta = \frac{\cos\psi\cos\phi}{1 - \sin\psi\sin\phi} \tag{5c}$$

However, in this case the direction of the V_p at the point E is not normal to the line PR; however, it makes an angle ψ with the plane of shear.

Energy Expressions

If the normal and shear stresses on a velocity discontinuity surface are defined by σ_n and τ_{nt}; the direction of V_p makes an angle ψ with the direction of shear, as shown in Fig.1 by means of a point E on the Mohr circle, then the expression for the rate of the dissipation of the specific energy dE (energy per unit area) along such a velocity discontinuity surface will become:

$$dE = \tau_{nt} V_p \cos\psi - \sigma_n V_p \sin\psi \tag{6}$$

This expression is valid for a general non-associated flow rule. For an associated flow rule, $\psi = \phi$, $\tau_{nt} = \tau_f$, $\sigma_n = \sigma_f$ and $\tau_f = c + \sigma_f \tan\phi$; on substituting these conditions in above equation, it can be seen that,

$$dE = c V_p \cos\phi \tag{7}$$

For a non-associated flow rule, as mentioned earlier, there are two ways of defining the relationship between τ_{nt} and σ_n, which are Eqns.(2) and (4). Substituting first the relationship (2) in Eqn.(6),

$$dE = c^{**} V_p \cos\psi \tag{8}$$

Though, the stress σ_n does not appear in the above energy expression, however, the value of c^{**} itself is a function of unknown σ_n (refer Eqn.3).

On the other hand, when the relationship (4) is substituted in Eqn.6, it can be seen that

$$dE = V_p \cos\psi \left[c^* + \sigma_n\left(\tan\phi^* - \tan\psi\right)\right] \tag{9}$$

From Eqns.8 and 9, it becomes clear that in case of a non-associated flow rule, it is essential to know the value of σ_n in order to determine the rate of the dissipation of internal energy along any velocity discontinuity surface.

Uplift Resistance from Statical Force Equilibrium

The failure mechanism was similar to that employed by Murray and Geddes (1987) and Kumar (1997) for an associated flow rule material. The chosen collapse mechanism is shown in Fig.2. The mechanism consists of a central triangular block, having its base aligned along with the anchor plate, surrounded by two adjoining triangular blocks on its either side. The anchor plate is assumed to be perfectly rough and no relative movement was permitted between the plate and the overlying block OFG; it should be mentioned that Rowe and Davis (1982a, 1982b) using the displacement FE analysis indicated that the vertical uplift resistance of horizontal anchors remains unaffected by the roughness of the anchor. On account of the symmetry, only half of the collapse mechanism was considered in the analysis. For given width (b) and depth (d) of the anchor from the ground, the chosen mechanism can be fully defined by means of a single independent variable, namely the horizontal inclination, β, of the rupture line OS as shown in Fig.2. The mechanism involves two rigid blocks OGS and OFG

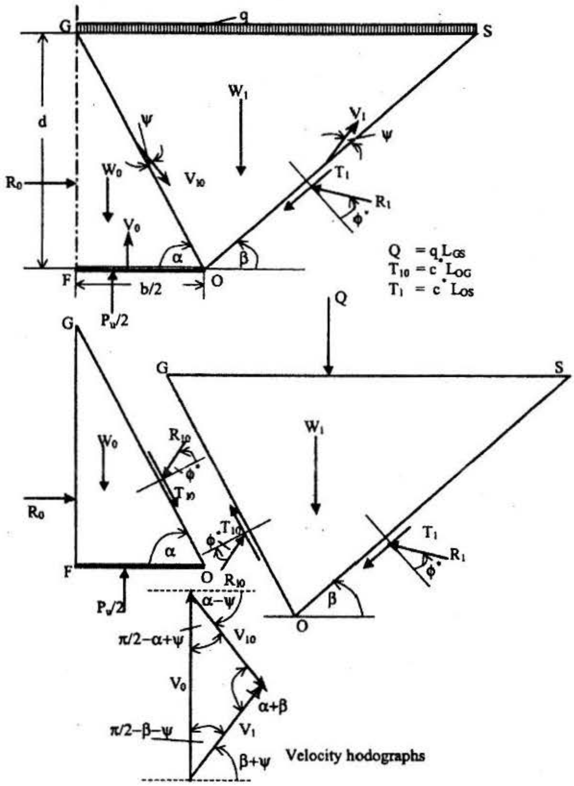


FIGURE 2 : Free Body Diagram and Velocity Hodograph for the Collapse Mechanism

with the rupture lines OG and OS. For a non-associated flow rule material the relationship between the normal and the shear stresses along a rupture line is governed either by Eqns.2 or 4. The computations were carried out on the basis of both these equations, and it was found that in all the cases both the relationships provide exactly the same answer. However, the solution with the relationship (2) requires trial and iteration, on account of the dependency of c^{**} on unknown σ_n. Whereas the use of the expression (4) involves no iterations, and the limit load can be directly obtained. The forces acting on two blocks based on the Eqn.4 are shown in Fig.2. Various unknown forces are given below:

1 The magnitude of the collapse load, P_u; its direction is specified (vertical).

2 The magnitude of the reaction R_0 along the axis of symmetry FG; its direction is known (horizontal).

3 The magnitude of R_{10}, the frictional component of the reaction along the rupture line OG. Its direction is known from Eqn.4, that is, the direction of R_{10} makes an angle ϕ^* with the normal to the line OG; limit state is assumed along line OG.

4 The magnitude of R_1, the frictional component of the reaction along the rupture line OS. Its direction is again known on the basis of Eqn.4; R_1 makes an angle ϕ^* with the normal to line OS.

It should be noted that the cohesion components of the reactions (i) along OG, $T_{10} = c^* L_{OG}$, (ii) and along OS, $T_1 = c^* L_{OS}$; wherein L_{OG} and L_{OS} are the lengths OG and OS, respectively. The values of T_{10} and T_1 will become automatically known for a given collapse mechanism. In other words, the chosen mechanism involves four unknowns, the magnitudes of which can be determined from four available force equilibrium equations of static (two for each block) even without the consideration of the energy balance; the obtained magnitudes of R_1, R_{10}, R_0 and P_u, are given below:

$$R_1 = \frac{\left[\begin{array}{l} W_1 + Q + T_1\left\{\sin\beta - \cot\left(\alpha-\phi^*\right)\cos\beta\right\} \\ \qquad - T_{10}\left\{\sin\alpha + \cot\left(\alpha-\phi^*\right)\cos\alpha\right\}\end{array}\right]}{\cos\left(\beta+\phi^*\right) + \cot\left(\alpha-\phi^*\right)\sin\left(\beta+\phi^*\right)} \tag{10}$$

$$R_{10} = \frac{R_1 \sin\left(\beta+\phi^*\right) + T_1\cos\beta + T_{10}\cos\alpha}{\sin\left(\alpha-\phi^*\right)} \tag{11}$$

$$R_0 = R_{10}\sin\left(\alpha-\phi^*\right) - T_{10}\cos\alpha \tag{12}$$

$$P_u = 2\left[W_0 + R_{10}\cos\left(\alpha-\phi^*\right) - T_{10}\sin\alpha\right] \tag{13}$$

In the above expressions, W_0 and W_1 are the weights of the blocks OFG and OGS; α is the horizontal inclination of the rupture line OG, that is, $\alpha = \tan^{-1}(2d/b)$; and $Q = q L_{GS}$ where, q is the applied surcharge pressure and L_{GS} is the length of the line GS. The magnitude of P_u can then be minimized with respect to variation of β. While doing the optimization it was ensured that the tensile stress on any of the rupture surface does not exceed $c\cot\phi^*$. It was checked by imposing the condition that $R_1 \geq -c^* \cdot L_{OS}/\sin\phi^*$ and $R_{10} \geq -c^* \cdot L_{OG}/\sin\phi^*$; it should be noted that R_1 and R_{10} are positive for compressive force. The reaction R_0 along the line GF for all the admissible mechanisms was seen to remain compressive in all the cases.

Velocity Hodograph and Kinematically Admissible Conditions

At collapse, the central block OFG moves vertically upward with the velocity V_0 same as that of the anchor. The block OGS moves with the velocity V_1; its relative velocity with respect to the block OFG is V_{10}. Since the flow rule is non-associated, the velocities V_1 and V_{10} are inclined at angle ψ with the rupture lines OS and OG, respectively. The velocity hodograph are also drawn in Fig.2. It can be seen that

$$V_1 = \frac{\cos(\alpha-\psi)}{\sin(\alpha+\beta)} V_0 \tag{14}$$

$$V_{10} = \frac{\cos(\beta+\psi)}{\sin(\alpha+\beta)} V_0 \tag{15}$$

For a mechanism to be kinematically admissible, the block OGS with respect to the block OFG should move downward and outward. It will be true provided the magnitude of V_{10}, with the direction as shown in Fig.2, remains always positive. For a positive V_{10}, it can be seen from Eqn.15 that the value of β should remain always $\leq \pi/2-\psi$; it should be noted that the value of $(\alpha+\beta)$ can not become greater than or equal to 180°. For the type of mechanism considered in this paper, the value of α can not become greater than or equal to 90°; on this basis it can be seen from Eqn.14 that the magnitude of V_1, with the direction as shown in Fig.2, will always remain positive.

The magnitude of P_u should, therefore, be minimized with respect to kinematically admissible variation of the β, that is, for $0 < \beta \leq \pi/2-\psi$.

Uplift Resistance from Energy Balance

An upper bound estimate of the uplift resistance can also be determined by equating the rate of work done by external and body forces to the rate of dissipation of internal energy. On this basis it can be shown that the magnitude of P_u will be given by the following expression:

$$P_u = \frac{2\left[W_0 V_0 + (W+Q) V_1 \sin(\beta+\psi) + E_{OG} + E_{OS}\right]}{V_0} \tag{16}$$

In the above equation E_{OG} and E_{OS} are the rate of the dissipation of internal energy along the rupture lines OG and OS. The expressions for obtaining E_{OG} and E_{OS}, based on Eqn.9, are presented below:

$$E_{OG} = V_{10} \cos\psi \left[T_{10} + R_{10} \cos\phi^{*} \left(\tan\phi^{*} - \tan\psi \right) \right] \tag{17}$$

$$E_{OS} = V_{1} \cos\psi \left[T_{1} + R_{1} \cos\phi^{*} \left(\tan\phi^{*} - \tan\psi \right) \right] \tag{18}$$

For a given collapse mechanism, the $T_1 = c^{*}L_{OS}$; $T_{10} = c^{*}L_{OG}$; R_1 and R_{10} were computed from Eqns.10 and 11. Therefore, by using Eqn.16, the magnitude of the uplift resistance can be computed on the basis of the energy balance. The magnitude of P_u is then minimized with respect to variation of β from 0 to $\pi/2-\psi$.

Solution

Computations have invariably shown that for any collapse mechanism, the magnitude of the uplift resistance computed entirely from the equations of force equilibrium becomes exactly the same as that from the energy balance. In all the cases, it was seen that the magnitude of P_u becomes minimum for the maximum kinematic admissible value of the variable β, that is, for $\beta = \pi/2-\psi$. For the critical collapse mechanism, the final expression for P_u is presented below:

$$P_{uc} = \gamma d \left(b + d\tan\psi \right) + q\left(b + 2d\tan\psi \right) + 2c^{*}d + 2R_{1c}\cos\psi\cos\phi^{*}\left(\tan\phi^{*} - \tan\psi \right) \tag{19}$$

where R_{1c} is the value of R_1 for $\beta = \pi/2-\psi$; the expression for R_{1c} is given below:

$$R_{1c} = \frac{\left[\left(0.5b + d\tan\psi \right)\left(0.5\gamma d + q \right) - c^{*}d\cot\left(\alpha - \phi^{*} \right)\left\{ \tan\psi + \cot\alpha \right\} \right]}{\sin\left(\psi - \phi^{*} \right) + \cot\left(\alpha - \phi^{*} \right)\cos\left(\psi - \phi^{*} \right)} \quad ..... (20)$$

Uplift Factors

The magnitude of the ultimate uplift pressure associated with the critical collapse mechanism, $p_{uc} = P_{uc}/b$, was expressed by the following equation:

$$p_{uc} = c f_c + q f_q + \gamma b f_\gamma \tag{21}$$

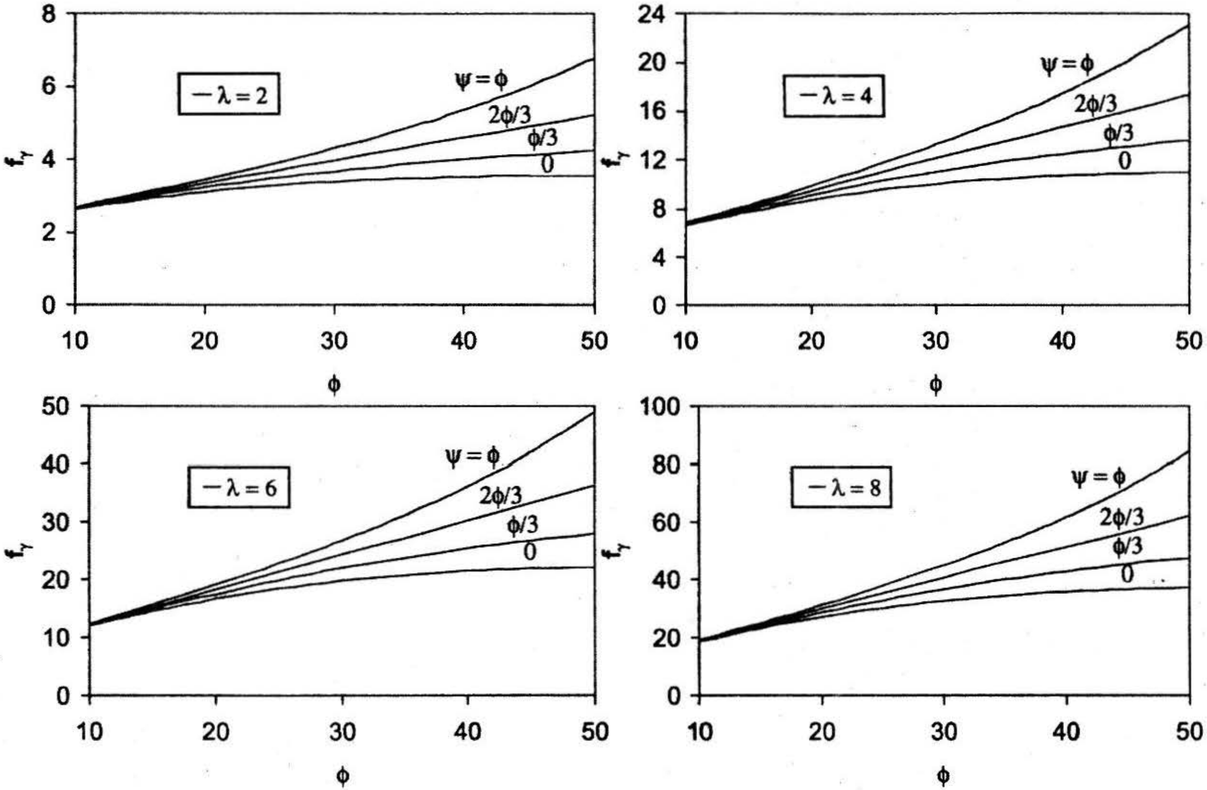


FIGURE 3 : The Variation of the Uplift Factor f_γ with ψ, ϕ and λ

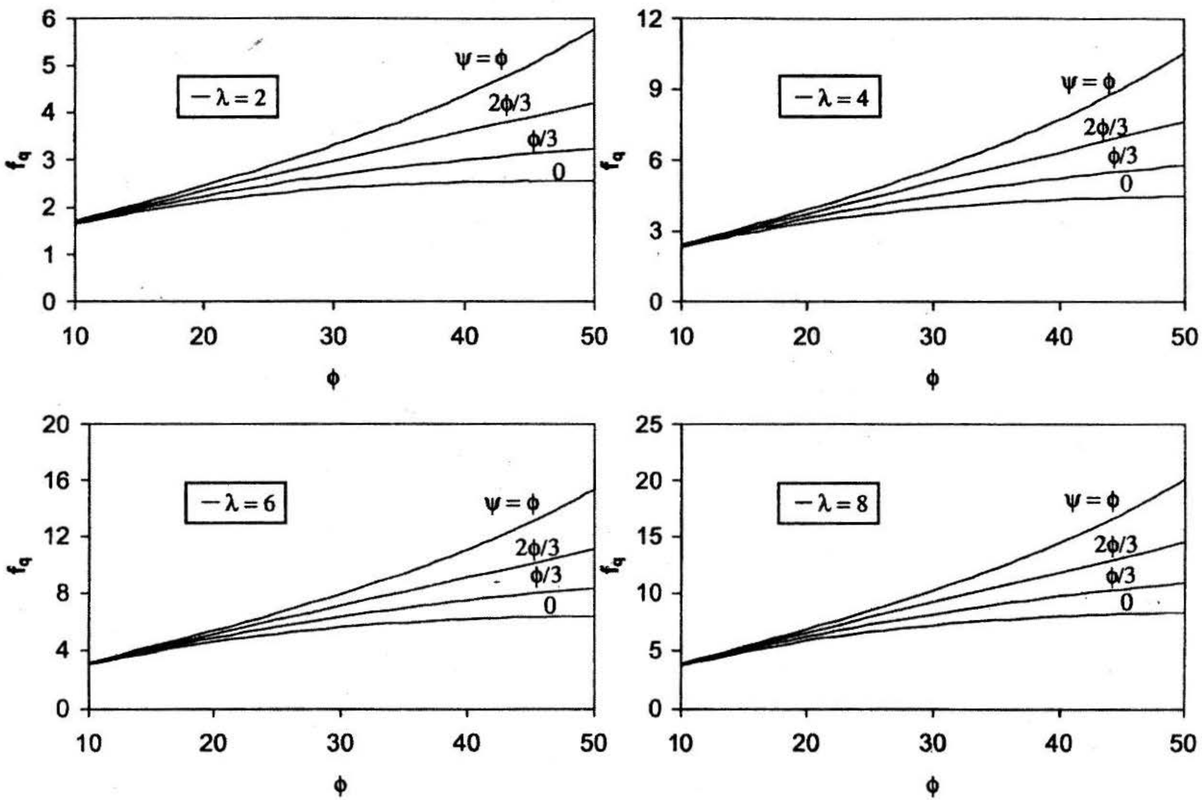


FIGURE 4 : The Variation of the Uplift Factor f_q with ψ, ϕ and λ

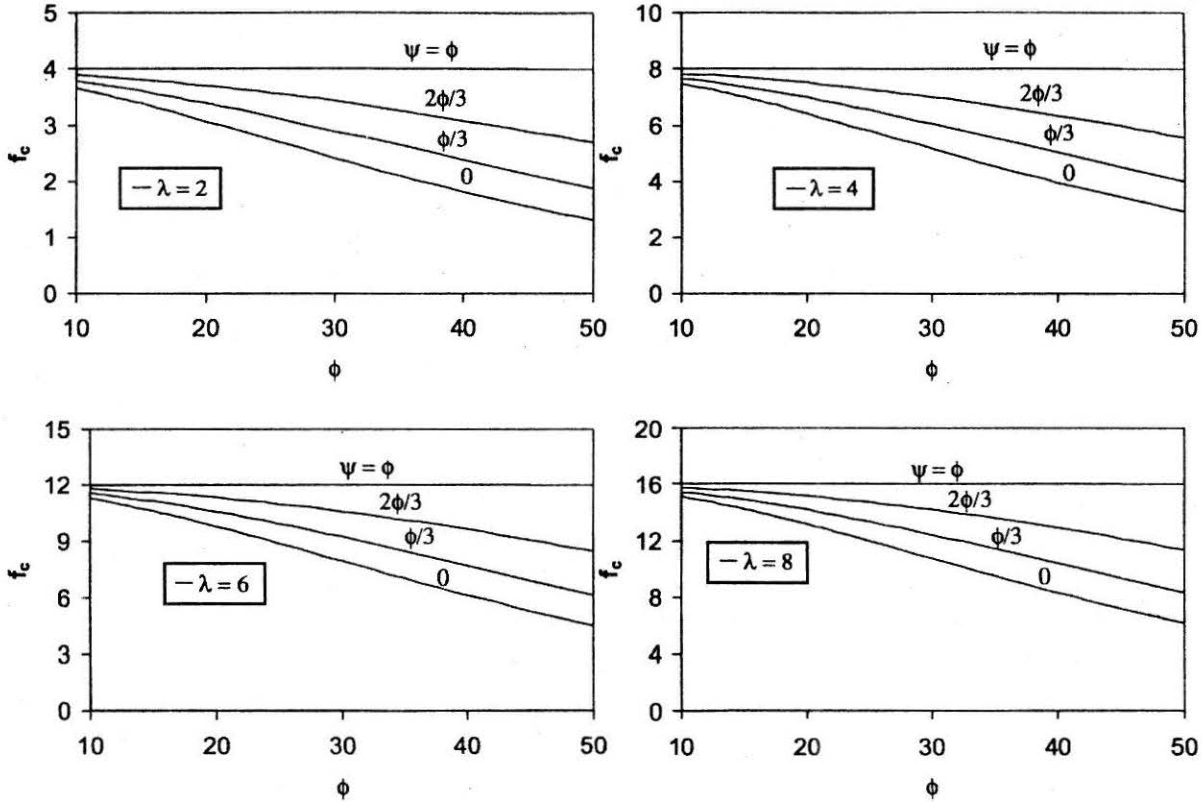


FIGURE 5 : The Variation of the Uplift Factor f_c with ψ, ϕ and λ

In the above equations f_c, f_q and f_γ are the uplift factors due to the components of cohesion (c), surcharge pressure (q) and unit weight (γ), respectively. The uplift factors are functions of ϕ, ψ and λ; where embedment ratio, $\lambda = d/b$.

The values of the uplift factors were obtained for values of $\psi = 0 - \phi$, $\phi = 10 - 50°$, and $\lambda = 2 - 8$. The variation of the uplift factors has been illustrated in Figs.3 to 5. It can be seen that the uplift factors decrease quite extensively with the reduction in dilatancy angle. The reduction of P_u with ψ is found more predominant especially for higher values of ϕ. The values of the uplift factors become higher as the embedment ratio of the anchor is increased. The factors f_q and f_γ increase further with increases in the friction angle, ϕ. However, the uplift factor f_c for $\psi = \phi$ becomes constant with respect to variation in ϕ. Whereas, for $\psi < \phi$, the magnitude of the f_c decreases with increase in the ϕ value; this observation is on account of the fact that for a given ψ value although ϕ^* increases with increase in ϕ but the magnitude of the c^* reduces with the increase in ϕ.

Table 1 provides a comparison of the uplift factors obtained with and without the consideration of the kinematic admissibility. It can be seen that if the issue of kinematic admissibility is not satisfied the uplift factors become more conservative and the corresponding value of β_{cr} becomes even greater than $\pi/2-\psi$.

Comparisons

With existing theories

The uplift factors were compared with the available solutions of (i) Rowe and Davis (1982a, 1982b) using FEM; (ii) Vermeer and Sutjiadi (1985) on the basis of limit equilibrium; (iii) Murray and Geddes (1987) using upper bound limit analysis, but with the use of c^* and ϕ^* as given by Drescher and Detournay (1993) for non-associated flow rule material; (iv) Subba Rao and Kumar (1994) using the method of characteristics; and (v) Meyerhof and Adams (1968) using the limit equilibrium. The comparison of all the results is shown in Tables 2 to 4. It can be seen that for the associated flow rule, the magnitudes of the uplift factors become exactly the same as reported earlier by Murray and Geddes (1987). However, for the non-associated flow rule materials with $\psi = 0$, the magnitudes of the obtained uplift factors become lower than those found simply by substituting c^* and ϕ^*, in place of c and ϕ, as per the recommendation of Drescher and Detournay (1993) in the existing solution for the associated flow rule. The difference between the two is on account of the fact that while the true critical rupture surface makes an angle ψ with the vertical (present analysis); whereas the critical rupture surface simply on the use of c^* and ϕ^* makes

TABLE 1 : Effect of Kinematic Admissibility on Uplift Factors

	λ	ψ/ϕ	ϕ (in degrees)	Kinematic admissibility not considered		Kinematic admissibility considered	
				Uplift factors	β_{cr}	Uplift factors	β_{cr}
Uplift Factor, f_c	3	0	30	3.35	99.46	3.81	90
			45	2.08	99.46	2.50	90
		1	30	3.51	99.46	6.00	60
			45	1.92	99.46	6.00	45
	7	0	30	8.96	94.09	9.35	90
			45	5.90	94.09	6.27	90
		1	30	9.58	94.09	14.00	60
			45	5.96	94.09	14.00	45
Uplift Factor, f_q	3	0	30	2.93	99.46	3.20	90
			45	3.08	99.46	3.50	90
		1	30	3.03	99.46	4.46	60
			45	2.92	99.46	7.00	45
	7	0	30	6.17	94.09	6.40	90
			45	6.90	94.09	7.27	90
		1	30	6.53	94.09	9.08	60
			45	6.96	94.09	15.00	45
Uplift Factor, f_γ	3	0	30	5.90	99.46	6.30	90
			45	6.13	99.46	6.74	90
		1	30	6.04	99.46	8.20	60
			45	5.88	99.46	12.00	45
	7	0	30	25.10	94.09	25.90	90
			45	27.65	94.09	28.93	90
		1	30	26.36	94.09	35.29	60
			45	27.88	94.09	56.00	45

TABLE 2 : Comparison of the Uplift Factor, f_γ

λ	ϕ (in degrees)	Uplift Factor, f_γ					
		Present Analysis	Use of c^* and ϕ^* in Murray and Geddes (1987)	Vermeer and Sutjiadi (1985)	Rowe and Davis (1982b)	Subba Rao and Kumar (1994)	Meyerhof and Adams (1968)
		$\psi = 0$ ($\psi = \phi$)	$\psi = 0$ ($\psi = \phi$)	$\psi = 0$ ($\psi = \phi$)	$\psi = 0$ ($\psi = \phi$)		
3	15	5.09 (5.41)	5.33 (5.41)	5.33 (5.41)	5.58 (5.69)	4.85	5.29
	30	6.30 (8.20)	7.50 (8.20)	7.50 (8.20)	7.71 (8.71)	6.82	7.94
	45	6.74 (12.00)	9.36 (12.00)	9.36 (12.00)	9.87 (13.72)	7.57	11.55
5	15	10.91 (11.70)	11.47 (11.70)	11.47 (11.70)	12.15 (12.51)	9.97	11.36
	30	14.50 (19.43)	17.50 (19.43)	17.50 (19.43)	18.55 (22.44)	16.69	18.71
	45	15.95 (30.00)	22.68 (30.00)	22.68 (30.00)	24.30 (40.34)	21.45	28.75
7	15	18.67 (20.13)	19.68 (20.13)	19.68 (20.13)	21.00 (21.74)	16.09	19.47
	30	25.90 (35.29)	31.50 (35.29)	31.50 (35.29)	32.97 (42.86)	29.74	33.88
	45	28.93 (56.00)	41.65 (56.00)	41.65 (56.00)	44.52 (85.48)	42.04	53.55

an angle ϕ^* (which is always $\geq \psi$) with the vertical. The analysis of Vermeer and Sutjiadi (1985) provides exactly the same values of f_γ as obtained by substituting c^* and ϕ^* in the solution of the associated flow rule. Similar to the present finding, it has been demonstrated earlier by Vermeer and Sutjiadi that the critical rupture surface makes an angle ψ with the vertical. However, their pullout resistance is higher as compared to the present analysis on account of their assumption of the stresses along the rupture surface based on Eqn.1 rather than Eqn.2 or 4. For the associated flow rule, the obtained solution is quite close to the limit equilibrium solution

TABLE 3 : Comparison of the Uplift Factor, f_q

λ	ϕ (in degrees)	Uplift Factor, f_q			
		Present Analysis	Use of c^* and ϕ^* in Murray and Geddes (1987)	Subba Rao and Kumar (1994)	Meyerhof and Adams (1968)
		$\psi = 0$ ($\psi = \phi$)	$\psi = 0$ ($\psi = \phi$)		
3	15	2.39 (2.61)	2.55 (2.61)	1.84	2.53
	30	3.20 (4.46)	4.00 (4.46)	2.93	4.29
	45	3.50 (7.00)	5.24 (7.00)	4.04	6.70
5	15	3.36 (3.68)	3.59 (3.68)	2.25	3.55
	30	4.80 (6.77)	6.00 (6.77)	4.24	6.49
	45	5.38 (11.00)	8.07 (11.00)	6.69	10.50
7	15	4.33 (4.75)	4.62 (4.75)	2.53	4.56
	30	6.40 (9.08)	8.00 (9.08)	5.36	8.68
	45	7.27 (15.00)	10.90 (15.00)	9.18	14.30

of Meyerhof and Adams (1968). Whereas the solution of Subba Rao and Kumar (1994) on the basis of the method of characteristics is closer to the present answer with $\psi = 0$. For all values of ψ, the FEM analysis of Rowe and Davis (1982a, 1982b) provides higher values of the uplift factors. For $\psi = 0$, the uplift factor f_c of Rowe and Davis (1982a) decreases with increases in ϕ for $\lambda = 3$, and it shows a mixed trend of variation with ϕ for $\lambda = 5$ and 7; on the other hand the present f_c values decreases with increase in ϕ for all λ. For $\psi = \phi$, as shown earlier, the present values of the factor f_c become independent of ϕ; the same trend has been shown earlier by Murray

TABLE 4 : Comparison of the Uplift Factor, f_c

λ	ϕ (in degrees)	f_c				
		Present Analysis	Use of c^* and ϕ^* in Murray and Geddes (1987)	Rowe and Davis (1982b)$^+$	Subba Rao and Kumar (1994)	Meyerhof and Adams (1968)
		$\psi = 0$ $(\psi = \phi)$	$\psi = 0$ $(\psi = \phi)$	$\psi = 0$ $(\psi = \phi)$		
3	15	5.20 (6.00)	5.80 (6.00)	5.56 (-)	3.16	6.00
	30	3.81 (6.00)	5.20 (6.00)	4.89 (-)	3.35	6.00
	45	2.50 (6.00)	4.24 (6.00)	3.67 (-)	3.08	6.00
5	15	8.82 (10.00)	9.66 (10.00)	7.33 (-)	4.65	10.00
	30	6.58 (10.00)	8.66 (10.00)	8.20 (-)	5.62	10.00
	45	4.38 (10.00)	7.07 (10.00)	6.67 (-)	5.87	10.00
7	15	12.44 (14.00)	13.52 (14.00)	7.78 (-)	5.71	14.00
	30	9.35 (14.00)	12.12 (14.00)	9.67 (-)	7.59	14.00
	45	6.27 (14.00)	9.90 (14.00)	8.89 (-)	8.25	14.00

$^+$ Rowe and Davis did not report the f_c values for $\psi = \phi$

and Geddes (1987) and the limit equilibrium solution of Meyerhof and Adams (1968). Rowe and Davis (1982a, 1982b) did not report the f_c values for an associated flow rule material.

With published experimental data

For an associated flow rule material, the theoretical results on the basis of the upper bound analysis, with the similar collapse mechanism, have been

TABLE 5 : Comparison of the Experimental Results of Rowe and Davis (1982b) with the Present Analysis for Strip Anchors in Sand

λ	Average ultimate uplift pressure (p_u) in kPa for strip anchor with width (b) = 51 mm			
	$\gamma = 14.90$ kN/m^3 $\phi_p = 35.2°$, $\psi = 4°$ c = q = 0		$\gamma = 15.27$ kN/m^3 $\phi_p = 36.63°$, $\psi = 10°$ c = q = 0	
	Experimental Data of Rowe and Davis (1982b)	Present Analysis	Experimental Data of Rowe and Davis (1982b)	Present Analysis
1	1.11	1.020	-	1.092
2	2.99	2.739	-	2.980
3	5.74	5.155	6.03	5.665
4	9.09	8.270	-	9.146
5	14.45	12.083	15.65	13.424
6	20.45	16.593	-	18.499
7	26.45	21.802	-	24.371
8	33.33	27.709	37.60	31.039

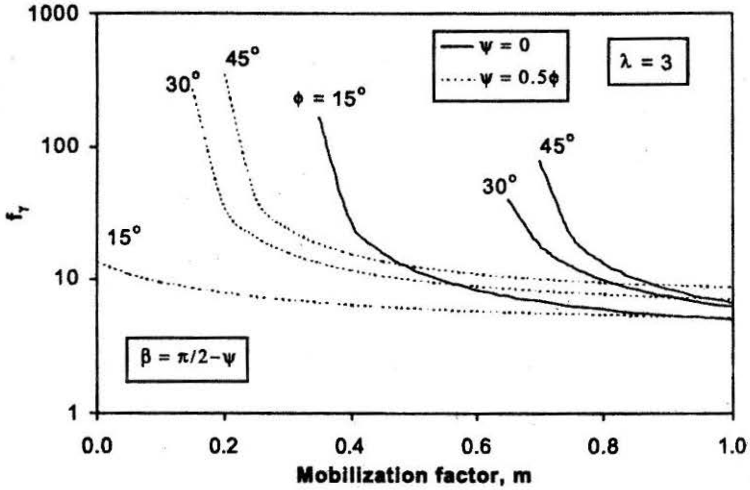
already shown to compare well with the experimental data (Murray and Geddes, 1987; Kumar, 1997). For non-associated flow materials, the results obtained from the present study were compared with experimental data of Rowe and Davis (1982a) for strip anchors in sand. The comparison of the results is shown in Table 5. In this table, for each embedment ratio, the reported experimental uplift resistance is an average value of 2 to 5 tests. It can be seen that the theory compares reasonably well with the experimental data; the difference between the two ranges from about 6.1% to 18.9%. The theoretical predictions were found to be slightly conservative in all the cases; the difference between the theory and experiments increases with increase in the value of the embedment ratio.

Remarks

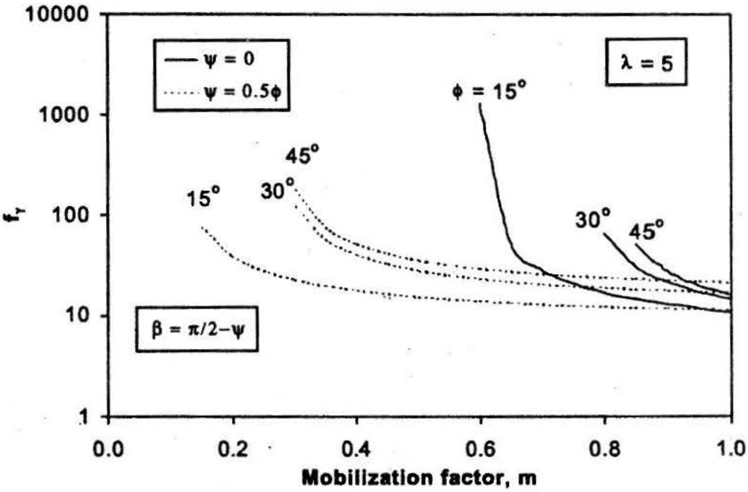
(i) The analysis will be valid only for shallow anchors where the rupture surface extends up to the ground surface. It was shown previously by Meyerhof and Adams (1968) and Meyerhof (1973) that the local shear failure of deep circular anchors under vertical uplift anchors occurs if the embedment ratio of the anchors exceeds about 4 for clays and loose sands. The magnitude of the critical embedment ratio increases with relative density to about 8 for dense sands. For strip anchors, the magnitude of critical embedment ratio is about 8 for clays and roughly 50% higher than the specified limits for circular anchors in sands.

(ii) The soil mass lying below the surface of the anchor has been assumed not to offer any resistance to uplift. For anchors embedded in saturated cohesive soils, the soil mass below the anchor may also contribute towards the uplift resistance (Davie and Sutherland, 1977; and Sutherland, 1988). In all such cases, the obtained solution will be conservative.

(iii) Since the optimal solution corresponds to $\beta = \pi/2 - \psi$ in all the cases, the magnitude of the slip velocity V_{10} becomes equal to zero (Eqn.15). For the critical collapse mechanism the entire soil mass within OFGS (Fig.2) moves vertically upward as a single rigid unit with the velocity same as that of the anchor itself. It indirectly indicates that the assumed velocity discontinuity line OG becomes inactive for the critical collapse mechanisms. It should, however, be mentioned that the all uplift factors have been computed with the consideration of the limit state along the line OG. The assumption of the limit state can be justified along a line provided it remains a path of velocity discontinuity/rupture. For a limit state along OG, the reaction R_{10} would make an angle ϕ^* with the normal to line OG and T_{10} will become equal to $c^* L_{OG}$. If the limit state is not specified along OG, then let (i) R_{10} makes an angle δ with the normal to line OG, and (ii) T_{10} be equal to $mc^* L_{OG}$; where m is a factor for the mobilization of the shear resistance along the line OG, and $\delta = \tan^{-1}(m \tan\phi^*)$. The value of m could obviously vary between 0 and 1. For the critical collapse mechanism ($\beta = \pi/2 - \psi$), the variation of P_u with changes in m was examined comprehensively with consideration of the force equilibrium of the blocks OFG and OGS. While doing this exercise, a mechanism was defined to be statically admissible provided it gives positive value of the pullout resistance and also the maximum tensile stress along the lines OS and OG does not exceed $c^* \cot\phi^*$ and $mc^* \cot\delta$, respectively. The condition of maximum tensile stress was checked by assuring that $R_1 \geq -c^* L_{OG}/\sin\phi^*$ and $R_{10} \geq -mc^* L_{OG}/\sin\delta$. The variation of the uplift factors with changes in m for different cases is shown in Figs.6 to 8. It should be noted that all the curves start with some minimum value of m; the collapse mechanisms between m = 0 and this minimum value of m were found to be always statically inadmissible. It should be seen that the values of all the uplift factors become minimum for m = 1. For all the statically admissible collapse mechanisms for values of m less than 1, the uplift resistance was always found to be higher. In other words, the results obtained with the



(a)



(b)

FIGURE 6 : The Variation of the Uplift Factor f_γ with m for $\beta = \pi/2-\psi$

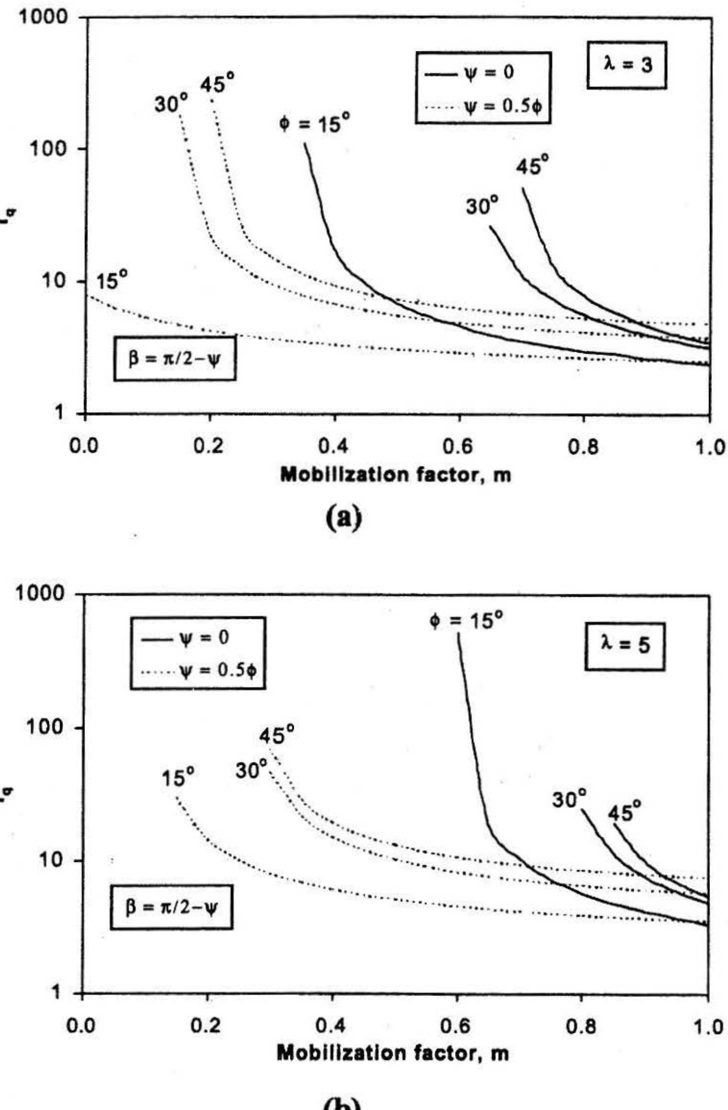
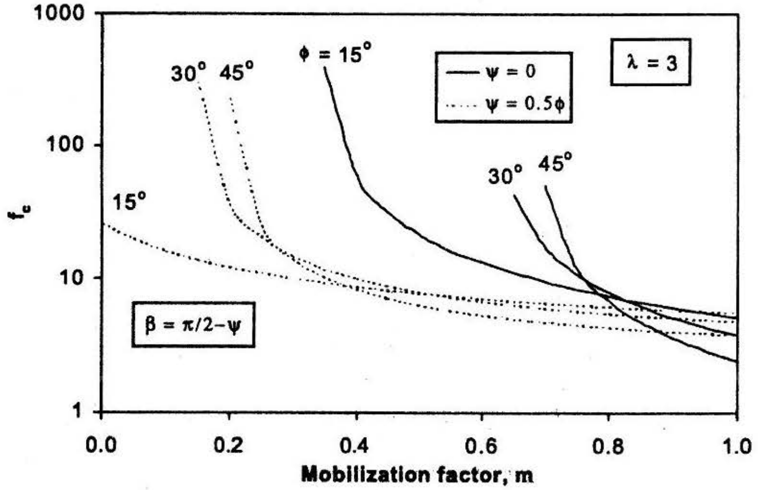
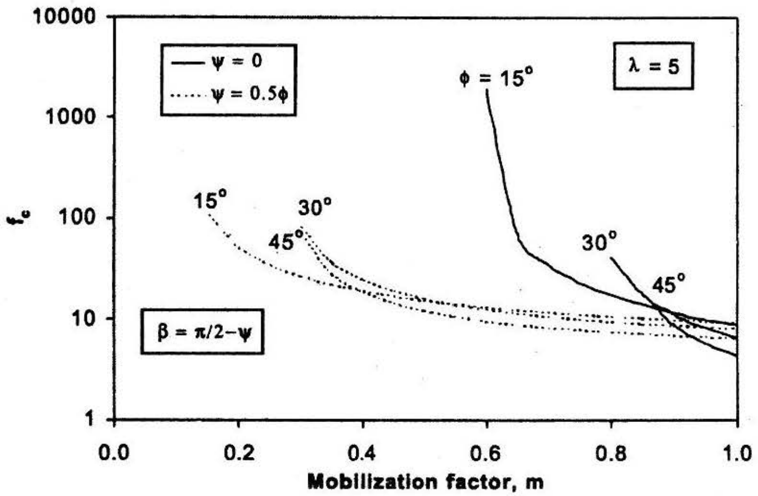


FIGURE 7 : The Variation of the Uplift Factor f_q with m for $\beta = \pi/2-\psi$



(a)



(b)

FIGURE 8 : The Variation of the Uplift Factor f_c with m for $\beta = \pi/2-\psi$

assumption of the limit state along the line OG will invariably provide a conservative estimate of the uplift resistance.

(iv) It is now clear that the uplift resistance of anchors is not only a function of soil friction angle but also depends on its angle of dilatancy. The strength of the soil mass can be characterized by its peak shear strength parameters (c and ϕ) and the angle of dilatancy (ψ). The angle of dilatancy can be obtained by using the following expression (Davis, 1968):

$$\frac{d\varepsilon_{vp}}{d\varepsilon_{1p}} = \frac{(1+\sin\psi)}{(1-\sin\psi)} - 1 \qquad (22)$$

where, $d\varepsilon_{vp}$ and $d\varepsilon_{1p}$ are incremental changes in the volumetric strain and the major principal strain at failure, respectively; $d\varepsilon_{vp}$ and $d\varepsilon_{1p}$ are taken as positive for the increase in volume, and the compressive strain, respectively.

Therefore, by determining the shear strength parameters and the dilatancy angle corresponding to the peak strength of the soil mass, the present study can be used to compute the uplift resistance of shallow anchors.

Conclusions

The solution for finding the uplift resistance of anchors embedded in associated and non-associated flow soil mass has been presented. The uplift resistance reduces very significantly with decrease in ψ. The uplift resistance simply from the force equilibrium provides exactly the same answer as that obtained from the use of energy balance. The obtained results for an associated flow rule material compare favorably with the theories reported in literature. However, for a non-associated flow rule material the obtained pullout resistance is found to be lower as compared to the existing theories and published experimental data.

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