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Phase Transition Effect on Preconsolidation Stress of Soil

Madhira R. Madhav* and Norihiko Miura†

Introduction

The phenomenon of consolidation of fine grained soils has received considerable attention ever since Terzaghi propounded his simple but in many ways a unique theory. Gibson et al. (1967), Olson and Ladd (1979), etc., consider variations of void ratio, compressibility, and permeability with effective stress, and finite strains, for normally consolidated (NC) soils for which these variations are significant and applicable. Many soils exist in situ in an overconsolidated state for various reasons. Soft clays exhibit pseudo-preconsolidation effect, and overconsolidation ratios in the range one to two for plasticity index ranging from zero to 100 due to aging (Bjerrum, 1972; Murakami, 1979). Barry and Nicholls (1982) present typical results (Fig.1) from oedometer test on a soft marine clay. The soil exhibits a preconsolidation stress, p_c or σ_c, of 63 kPa, and virgin compression, C_c, to recompression, C_s, index ratio of 18.5. Interestingly, the coefficients of consolidation in the reloading and virgin compression ranges are nearly constant (independent of effective stress) and respectively equal to 1.3×10^{-6} m/s and 5.5×10^{-8} m/s, giving a ratio of 23.6, which is comparable to the virgin to recompression index ratio.

Ariake clay in the Saga plain is a typical example of such soil, which is lightly over consolidated with OCR in the range 1.0 to 4.0 (Miura et al., 1988). Accurate determination of the preconsolidation stress is an important step in the prediction of long-term consolidation settlements and short term stability. The importance of knowing precisely the overconsolidation stress for the estimation of secondary compression in the quasi-overconsolidated

* Professor Emeritus, J.N.Technical University, Hyderabad 500 034, India.

† Emeritus Professor of Civil Engineering, Saga University, Saga 840-8501, Japan.

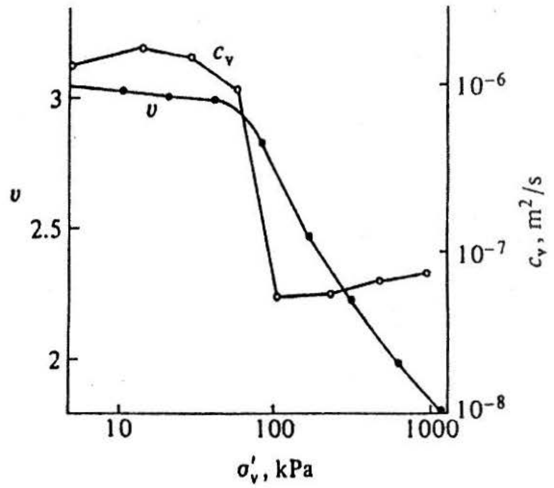

FIGURE 1 : Typical Results for a Soft Clay

region for a diluvial clay is also highlighted by Akai et al. (1991). Mesri and Rokhsar (1974), Tavenas et al. (1979), and Murakami (1980) have studied the consolidation of over consolidated (OC) and sensitive clays. Scott (1989) has proposed a theory representing the consolidation of OC sensitive soil as a phase change process assuming that the clay exhibits no compression up to preconsolidation stress, and beyond when it collapses to a new void ratio.

Following Casagrande (1936) many empirical methods based on void ratio, e, $-\log$ effective stress, σ', have been proposed for the estimation of the preconsolidation stress (Burmister 1951; Schmertmann 1955). Recently, curve fitting methods based on $\log e - \log \sigma'$ and $\log(1+e)$ versus $\log \sigma'$, are available (Oikawa, 1987; Jose et al., 1989; and Sridharan et al., 1991). However, no theoretical investigation on this aspect is available. In this paper, the consolidation of lightly OC soils as a phase change process is examined closely, considering the compressions in the OC and NC ranges. The resulting void ratio - effective stress relation and the estimated value of preconsolidation pressure for a soil layer of finite thickness are shown to be affected by the stress increment ratio (SIR), the stress level from which the stress increment is applied and the ratio of coefficients of consolidation in the reloading and virgin compression ranges.

Analysis

The soil (Fig.2a) is considered to be homogeneous, lightly over consolidated and to obey Darcy's law, and the problem to be one-dimensional. Each element of the soil follows the bilinear void ratio, e, $-\log$ effective stress, σ', relation (Fig.2b),

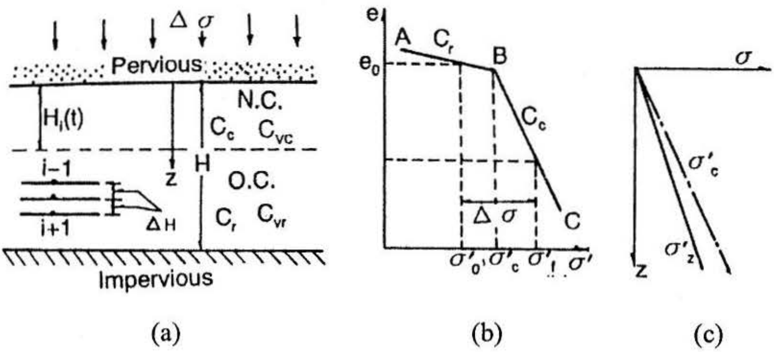

FIGURE 2 : Definition Sketch

$$e = e_0 - C_r \log(\sigma'/\sigma'_0) \quad \text{for } \sigma' < \sigma'_c \tag{1}$$

$$e = e_0 - C_r \log(OCR) - C_c \log(\sigma'/\sigma'_c) \qquad \text{for } \sigma' > \sigma'_c \tag{2}$$

where C_r = compression indices for the OC ranges of stress

C_c = compression indices for the NC ranges of stress

σ'_c = preconsolidation stress, and

OCR = σ'_c/σ'_0 – the over consolidation ratio.

It is assumed that in both OC and NC ranges of stress, the permeability of the soil decreases inversely with stress such that the respective coefficients of consolidation, C_{vr} and C_{vc}, remain constant in each range. If a total stress increment, $\Delta\sigma$, is applied on to the soil, all the elements of the soil follow the path AB (Fig.2b) initially since $\sigma'_0 < \sigma'_c$, and the rate of consolidation is governed by the coefficient of consolidation, C_{vr}. As the effective stress exceeds the preconsolidation stress, i.e. $\sigma' > \sigma'_c$, the element becomes NC, follows the path BC, and consolidates with a coefficient of consolidation, C_{vc}, which is much smaller than C_{vr}. The governing equations for the two phases are derived similar to Murakami (1980), and can be written as

for $z < H_i(t)$

$$\frac{\partial u}{\partial t} = C_{vc}\left\{\frac{\partial^2 u}{\partial z^2} + \frac{1}{\sigma'}\left(\frac{\partial u}{\partial z}\right)^2\right\} \tag{3}$$

for $z > H_i(t)$

$$\frac{\partial u}{\partial t} = C_{vr}\left\{\frac{\partial^2 u}{\partial z^2}+\frac{1}{\sigma'}\left(\frac{\partial u}{\partial z}\right)^2\right\} \tag{4}$$

where u = excess pore water pressure at depth, z, and time, t, and

$H_i(t)$ = depth to the interface between NC and OC phases of the soil.

The boundary conditions for drainage from the top surface alone are:

For $t > 0$,

$$u = 0 \text{ at } z = 0 \text{ and } \partial u/\partial z = 0 \text{ at } z = H \tag{5}$$

At $t = 0$,

$$u = \Delta\sigma \text{ for } 0 \le z \le H \tag{6}$$

The conditions at the interface at depth, $H_i(t)$, between the normally and overconsolidated phases are derived treating the soil as a two layered one:

For $z = H_i(t)$

$$u!_{NC} = u!_{OC} \text{ and } K_{NC}(\partial u/\partial z)!_{NC} = K_{OC}(\partial u/\partial z)!_{OC} \tag{7}$$

where K_{NC} and K_{OC} are the coefficients of permeabilities for the NC and OC phases on either side of the interface. However, since the interface corresponds to a point B, with an effective stress, σ'_c, and a void ratio, e_c, common to both the phases, the coefficient of permeabilities K_{NC} and K_{OC} can be taken to be equal. Consequently the pore pressure and its gradient are continuous across the interface but not the compressibilities. Equation 3 and 4 are solved using a finite difference approach by dividing the soil into N sub-layers of thickness, ΔH. Equations 3 and 4 in nondimensional form can be written as

$$W_{i,T+\Delta T} = W_{i,T} + \beta\begin{bmatrix} W_{i-1,T} - 2W_{i,T} + W_{i+1,T} \\ +f\cdot\left\{\left(W_{i+1,T} - W_{i-1,T}\right)/2\right\}^2 \end{bmatrix} \tag{8}$$

where
$$W = u/\Delta\sigma$$
$$T = C_{vc}\,t/H^2$$
$$Z = z/H$$
$$SIR = \Delta\sigma/\sigma'_0$$
$$\Delta H = H/N$$
$$f = SIR/\{1+SIR(1-W)\}$$
$$\beta = \beta_{NC} = \Delta T/(\Delta H)^2 \text{ for } i<I_i$$
$$b = \beta_{OC} = \beta_{NC}\cdot R_{CV} \text{ for } i>I_i$$
$$R_{CV} = C_{vr}/C_{vc} = 1/\mu$$
$$\mu = C_r/C_c\text{, and}$$
$$I_i = H_i(t)/\Delta H$$

For the interface, i.e. $i = I_i$, β_{NC} is replaced by β_{NC}/f_i to account for different compression indices on either side and $f_i = (1+R_{cv})/2$. The average pore pressure, W_{av}, is obtained as

$$W_{av,T} = U_{av}(T)/u_0 = \int_0^H u\cdot dz/u_0 = \sum_{i=1}^{N+1} a_i\cdot W_{i,T} \tag{9}$$

where $u_{av}(T)$ is the average pore pressure, and a_i is weighting parameters for Simpson's rule for the area under a curve. The degree of settlement, U_s, is defined as

$$U_s = \int_0^H (e_0-e)\,dz/H(e_0-e_f) \tag{10}$$

with

$$(e_0-e_f) = C_c\left[\mu\log(OCR)+\log\{(1+SIR)/OCR\}\right] \tag{11}$$

Equation 10 is evaluated numerically using Eqn.1 for $i > I_i$ and Eqn.2 for $i < I_i$. In order to compare different void ratio - log effective stress relations obtained from this analysis, e_0 of 2.7 at normalized initial effective stress, σ'_0, of 1.0, and a C_c value of 1.0 are assumed.

Results

Numerical results are obtained by solving Eqns.8 through 11, for given values of the parameters, SIR, OCR and μ. Initially, the value of N is varied

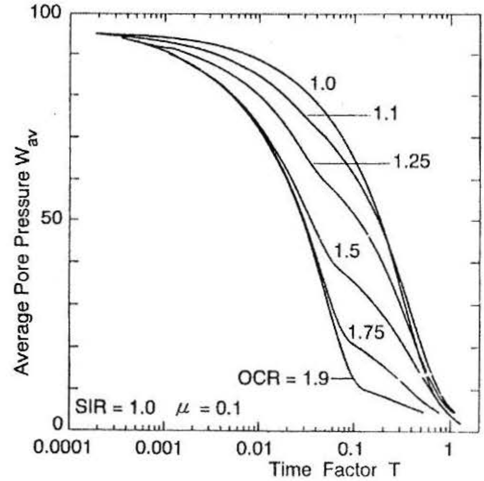

FIGURE 3 : Effect of OCR on W_{av} - T Relations

from 10 to 40. N equal to 20 is chosen as it gave results of sufficient accuracy and further increase in its value did not improve the results by any significant amount. A parametric study is conducted for: μ = 0.02 − 0.5; OCR = 1 − 2.5, SIR = 1.0 − 5.0.

The decrease of average pore water pressure, W_{av}, with time factor, T, for different OCR is depicted in Fig.3, for SIR of 1.0 and μ of 0.1. OCR of 1.0 implies normally consolidated soil and the average pore water pressure versus T relation agrees with the results of Davis and Raymond (1965) for σ'_f/σ'_i of 2.0. With increasing values of OCR, the dissipation of pore pressures is faster since the coefficient of consolidation in the OC phase of the soil is taken to be 10 times C_{vc}, the value for the NC phase. For OCR increasing from 1.0 to 1.9, T_{50} decreases from 0.197 to 0.027. With increasing values of OCR, the consolidation rate in the fully NC phase is smaller since the soil has already consolidated significantly in the OC phase. The average pore pressure, W_{av}, time factor, T, curves (Fig.4) are influenced by the stress increment ratio as well. The rate of dissipation of pore pressure is fast for reconsolidation in the reloading phase. The rate becomes zero when the interface between the NC/OC phases reaches the bottom of the clay layer indicated by the kink in the curves. For SIR equal to 1.0, this transition occurs at a time factor of 0.07 and W_{av} of about 40%. With increasing values of SIR, the transition occurs at higher values of W_{av}. For high values of SIR (> 5), the effect of OCR of 1.5 has negligible influence on the W_{av}-T curves, which tend to those of a NC soil. The recompression to virgin loading index ratio, μ, affects the rate of pore pressure dissipation only in the earlier part of the consolidation process (Fig.5). The smaller the value of m, the larger

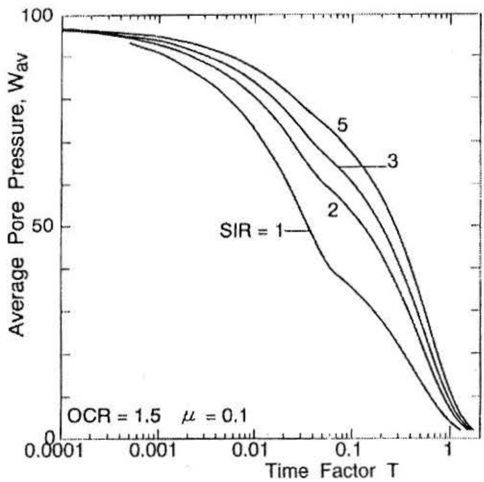

FIGURE 4 : Effect of SIR on W_{av} - T Relations

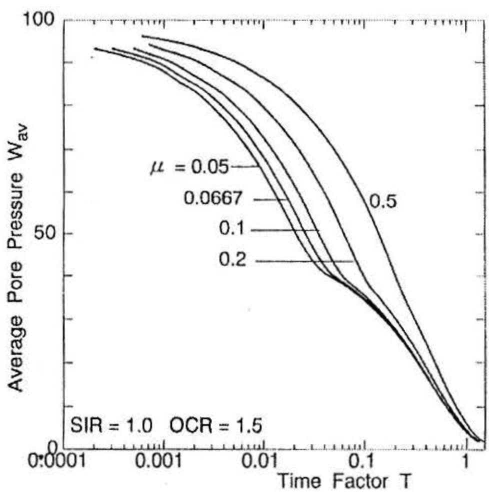

FIGURE 5 : Effect of μ on W_{av} - T Relations

the coefficient of consolidation in the OC phase, and the rate of pore pressure dissipation is thus faster. The curves tend to merge at later times as the soil becomes fully normally consolidated.

The degree of settlement, U_s, versus time factor, T, curves (Fig.6) also show a similar trend in that U_s values are more for higher values of OCR. For a NC soil (OCR = l), U_s vs. T relation is identical to the one given by Terzaghi's theory as has noted by Davis and Raymond (1965). For OCR

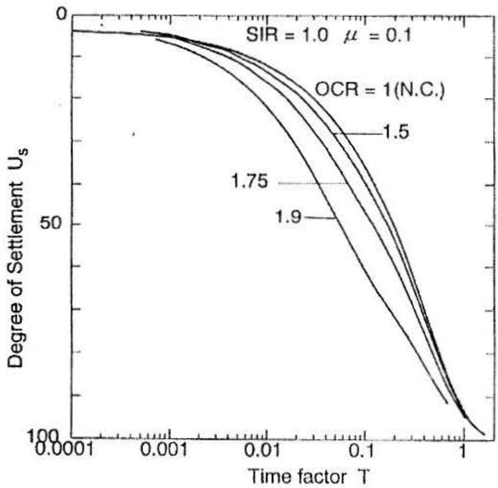

FIGURE 6 : Effect of OCR on U_s - T Relations

increasing from 1.0 to 1.9, T_{50} reduces by a factor of about 3.5, from 0.197 to 0.058. The curves converge at higher values of U_s as anticipated. It is also observed that for a given value of OCR equal to 1.5 and μ equal to 0.1, the stress increment ratio, SIR, has no influence on the U_s -T relation. Larger the value of SIR, the less the effect of OC phase and closer is the behavior of the soil to NC soil. Interestingly, but expectedly, the U_s-T curves are almost independent of the ratio μ (Fig.7). The mobilization of settlements is thus

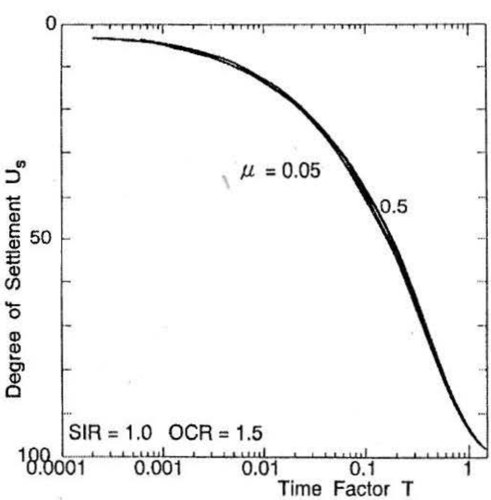

FIGURE 7 : Effect of μ on U_s - T Relations

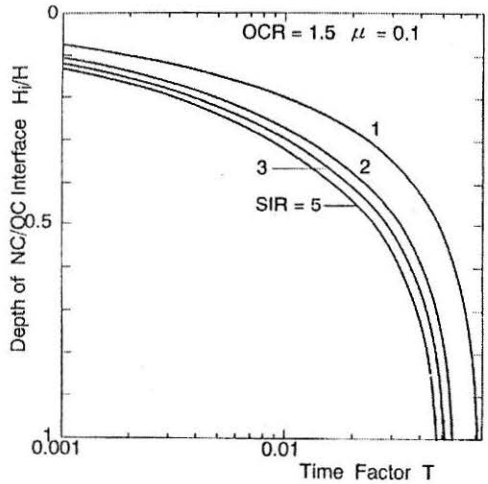

FIGURE 8 : Effect of SIR on NC/OC Interface Movement

unique and the degree of settlement - time factor relation follows Terzaghi's theory for all values of μ and also stress increment ratio.

The movement of the NC/OC interface with time factor is depicted in Fig.8. For OCR = 1.5 and μ = 0.1, the interface reaches the bottom of the clay layer at a time factor of 0.07. For higher values of SIR, the stress range in OC phase is smaller, and the movement of the interface is faster. The soil

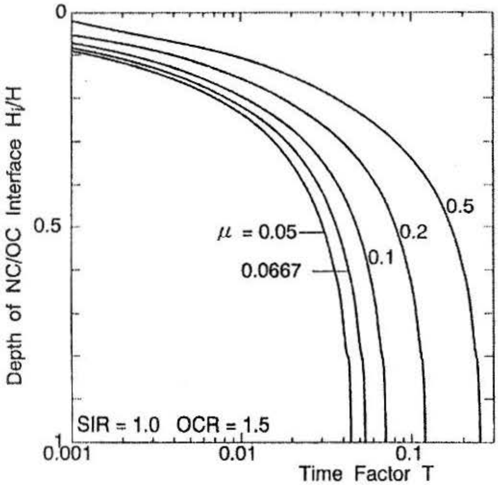

FIGURE 9 : Effect of μ on NC/OC Interface Movement

behaves as a fully NC soil for T > 0.047 for SIR = 5. The phase change from OC to NC is significantly affected by the ratio μ (Fig.9) as it indirectly controls the rate of pore pressure dissipation. For SIR equal to 1.0 and OCR = 1.5, the NC/OC interface traverses the full depth of the clay layer at T equal to 0.043, 0.07 and 0.25 for m equal to 0.05, 0.1 and 0.5, respectively.

The effect of bilinear e - $\log \sigma'$ relation valid for each element of the soil, on the overall e - $\log \sigma'$ response of a finite thickness of soil is studied through Figs.10 to 12. For the sake of comparison of various curves, $C_c = 1.0$, and $e_0 = 2.7$ at a normalized effective stress of 1.0, are chosen. The average void ratio versus log average effective stress curves are plotted only for the stress increment that straddles the preconsolidation stress, σ'_c. The remarkable effect of σ'_c on the e_{av} - $\log \sigma'_{av}$ curves is apparent from Fig.10. Higher the value of OCR, the flatter is the initial part of the e_{av} - $\log \sigma'_{av}$ curve. Once the full depth of the soil passes into the NC phase (BC in Fig.2b), the curves in Fig.10 are parallel to each other. For smaller values of OCR, the soil near the drainage surface of the soil goes from the OC phase into the NC phase at relatively smaller time factors and retards the consolidation of the OC phase of the soil below the NC/OC interface. The overall response of the soil is thus that of a soil that responds partly as an NC soil and partly as an OC soil. As a result, the preconsolidation stress is masked. Almost identical trends are exhibited in the $\log e_{av}$ - $\log \sigma_{av}$ and $\log(1+e_{av})$ - $\log \sigma_{av}$ relations. The Casagrande, $\log e$ - $\log \sigma'$ and $\log(1+e)$ - $\log \sigma'$ (Oikawa 1987; Jose et al., 1989; Sridharan et al., 1991) curve fitting methods are used to obtain the preconsolidation stress values for

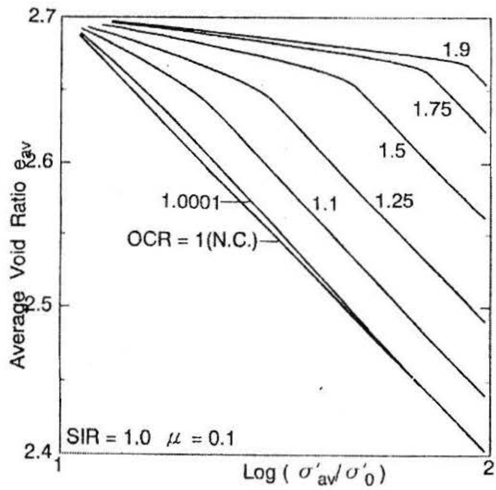

FIGURE 10 : Effect of OCR on e_{av} - $\log \sigma_{av}$ Relations

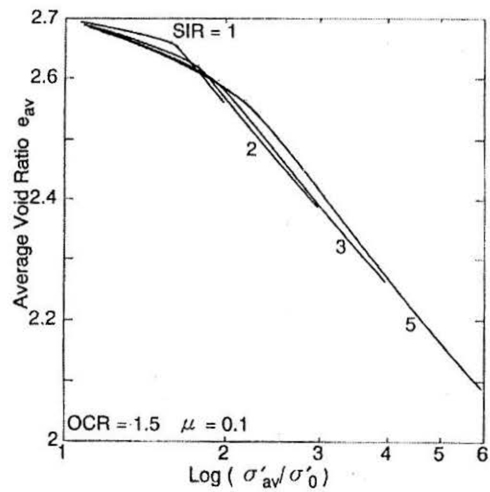

FIGURE 11 : Effect of SIR on e_{av} - $\log\sigma_{av}$ Relations

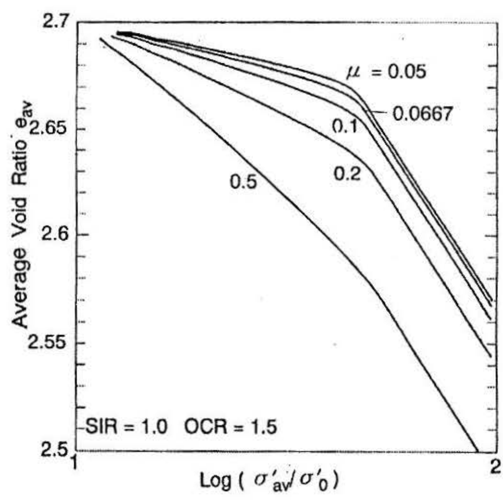

FIGURE 12 : Effect of m on e_{av} - $\log\sigma_{av}$ Relations

the numerically generated curves. Interestingly, all the three methods give nearly identical values of OCR, which are higher than the true OCR of the soil (Table 1). The interpreted OCR values are 15% to 0.5% higher than the true values of OCR for OCR increasing from 1.1 to 1.9 and for SIR of 1.0. In a standard oedometer test in which SIR is equal to 1.0, OCR will be more accurately predicted if σ'_c is farther from σ'_0 but slightly smaller than $\sigma'_0 + \Delta\sigma$.

TABLE 1 : Effect of Phase Change on Estimated OCR

	True OCR				
	1.1	1.25	1.5	1.75	1.9
OCR Estimated by Casagrande (1936)	1.25	1.38	1.58	1.80	1.92
$\log e - \log \sigma$	1.26	1.39	1.59	1.79	1.92
$\log(1+e) - \log \sigma$	1.25	1.38	1.58	1.78	1.91
$(OCR)_E/(OCR)_T$	1.14	1.11	1.06	1.02	1.008

The effect of the stress increment ratio, SIR, on the shape of the e_{av} vs. $\log \sigma'_{av}$ curves (Fig.11) is significant. With increasing values of SIR, the initial portion of the e_{av} vs. $\log \sigma'_{av}$ curves corresponding predominantly to OC phase, are steeper as a consequence of the rapid transition of the upper soil layers closer to the drainage face, into the NC phase. The portions of the curves for the NC phase ($\sigma'_{av} > \sigma'_c$) are nearly parallel to each other. The preconsolidation stresses for these curves are once again estimated by the three methods mentioned earlier. It is interesting to note (Table 2), that for a soil with a true OCR of 1.5, the values of OCR obtained using the curve fitting methods increase from 1.59 for SIR of 1.0 to 2.22 for SIR of 5.0. Thus, the stress increment ratio has a very significant effect on the σ'_c shown by the full thickness of a finite layer of soil as a result of the masking effect of the OC/NC phase transition. Thus, if the stress increment is large, as is likely in situ, compared to the initial effective stress, i.e. high SIR, the soil would exhibit normally consolidated behavior only after an apparently larger OCR implying over consolidated response while in fact the soil is passing through a phase transition.

Akai et al. (1991) report secondary compression behavior of a diluvial clay in the quasi-overconsolidation range ($\sigma'_0 < \sigma' < \sigma'_c$), to be similar to that in the normally consolidated region ($\sigma' > \sigma'_c$). It is possible that the estimated preconsolidation stress is in fact in excess of the true value because

TABLE 2 : Effect of SIR on Estimated OCR

	SIR			
	1.0	2.0	3.0	5.0
OCR Estimated by Casagrande (1936)	1.584	1.777	1.943	2.225
$\log e - \log \sigma$	1.585	1.778	1.940	2.210
$\log(1+e) - \log \sigma$	1.590	1.765	1.945	2.214
$(OCR)_E/(OCR)_T$	1.056	1.180	1.293	1.480

of the masking effect shown above, which means that part of the quasi-overconsolidation range may in fact belong to the normally consolidated region and hence the secondary compression behavior. This conjecture if accepted, validates the results obtained here. The ratio $\mu = C_r/C_c$ also reflecting the ratio of the coefficients of consolidation in the NC and OC phases, C_{vc}/C_{vr}, has prominent effect (Fig.12) on the e_{av} vs. $\log\sigma'_{av}$ relations for the full thickness of the consolidating soil. The curves demonstrate a sharper break once the soil passes fully into the NC phase, with decreasing value of μ. It is to be anticipated since smaller values of μ imply that C_r values are significantly smaller than C_c values. The masking effect is marginally more for soils with μ in the range 0.1 to 0.5. The estimated OCR values are close to 1.6 as against a true value of 1.5.

The progress of consolidation with depth and the exact preconsolidation stress can be monitored with the interconnected consolidometer tests proposed by Imai and Tang (1992). Alternatively, if the base pore pressure, W_b, is plotted (Fig.13) against time factor, T, the transition from OC to NC phase is clearly discernable. Mesri and Choi (1979) report case histories from Malaysia and Canada in which the pore pressure variations with time are very similar to the variation depicted in Fig.13. The pore pressure value at the transition where the slope of the curve is horizontal, is closely related to the true OCR. The phase transition is noted when W_b equals $(1+SIR-OCR)/SIR$. The base pore pressure when the soil attains fully the NC phase, is very sensitive (Fig.14) to the OCR also. Higher the value of OCR, smaller will be the pore pressure when the slope of the W_b - time factor curve becomes zero. The transition from OC to NC phases is clearly

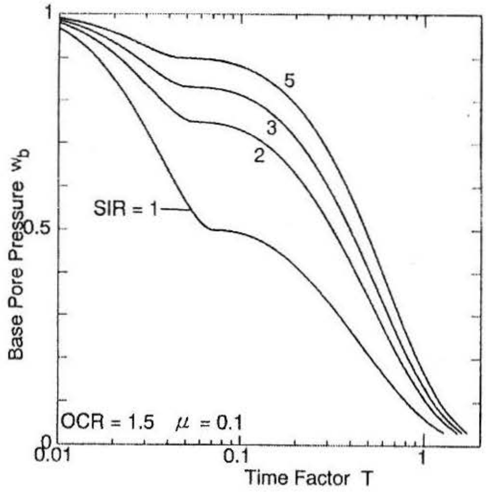

FIGURE 13 : Base Pore Pressures – Effect of SIR

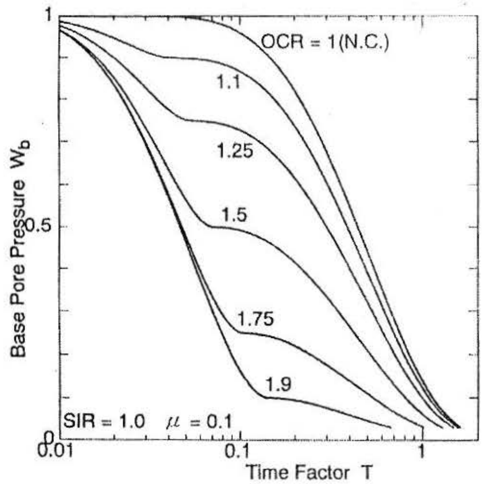

FIGURE 14 : Base Pore Pressures – Effect of OCR

discernable from these curves. Hence the true OCR can easily be obtained from the observed value of base pore pressure at the phase transition.

Conclusions

The phenomenon of consolidation of a lightly over consolidated soil wherein its behavior changes gradually with time and depth from OC to NC phases, is studied. The true void ratio - log effective stress relation of the soil is assumed to be bilinear with C_r and C_c being the slopes for the reloading and the virgin loading ranges. The coefficients of consolidation for the two ranges are inversely proportional to the respective values of compression indices. The degree of pore pressure dissipation versus time factor and degree of settlement versus time factor curves are sensitive to the over consolidation ratio of the soil. The average void ratio - log average effective stress relation for the whole soil of finite thickness, is very sensitive to both OCR and the stress increment ratios. The true OCR of the soil is clearly masked by the phase transition process of the consolidating soil. The OCR values predicted by the presently available curve fitting methods over predict the true OCR value by up to about 50% depending on the stress increment ratio used. For a better estimation of OCR from a routine oedometer test, the preconsolidation stress level should be farther from the initial stress but closer to the final stress in the stress increment that straddles it. Alternatively, the stress level at which the OC/NC phase transition takes place and the true value of the preconsolidation stress can be identified precisely from the variation of base pore pressure with time.

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Notation

C_c, C_r	=	Compression indices for virgin and re-loadings
C_{vc}, C_{vr}	=	Coefficients of consolidation
e, e_0	=	Void ratio and initial void ratio
f	=	a parameter
H	=	Thickness of clay layer
H_i	=	Depth to NC/OC interface
i	=	Variable
N	=	Number of sub-layers for finite difference approach
NC	=	Normally consolidated
OC	=	Over consolidated
OCR	=	Over consolidation ratio
SIR	=	Stress increment ratio
T	=	Time factor
t	=	time
U_s	=	Degree of settlement
u, u_{av}, u_0	=	pore pressures at time t, average, and initial
w_{av}	=	u/u_0 (= normalized pore pressure)
Z	=	z/H (= normalized depth)
z	=	Depth
β	=	$\Delta T/\Delta H$
μ	=	C_r/C_c
σ'	=	Effective stress
σ'_c	=	Preconsolidation stress
$\Delta\sigma$	=	Stress increment