

Effect of Residual Stresses and Sequence of Excavation on Displacement of Tunnel by Finite Element Method

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In-situ stresses are developed in a rock mass during its formation and also due to later tectonic movements. Subsequently, this stress-state may undergo change as a result of loading or unloading due to the construction of structures, the excavation of rock mass, etc. The in-situ stress-state must be assessed before undertaking any excavation. In rock mechanics the above kind of in-situ stresses are referred to as the residual stresses. These stresses are important whenever cuts are made through a zone of residual stress system in the course of excavation into rock, underground or on surface. Because the locked up or residual stresses are released during excavation.

The residual or in-situ stresses constitute the initial condition, prior to the excavation of the tunnel. As the tunnel excavation progresses these stresses are released thereby affecting the stress distribution around the tunnel opening. The normal practice is to define the magnitude of the residual stress through the coefficient of lateral earth pressure at rest (K_0). Its value can be less than one or one or greater than unity. The field tests are available to ascertain its value. In addition the empirical expressions such as the one suggested by Hoek and Brown (1980) are available and the same can be used in absence of the test data.

The finite element analysis is conducted for the excavation of the tunnel in the materials with known residual stresses. For this purpose, the entire

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continuum is idealized in an appropriate manner. It is assumed that, the excavated element is removed from the idealization, at its nodes the actions that are equal to the element end actions are applied. This means the residual idealization would be subjected to the nodal forces, without the stiffness of the excavated element being available. In this manner not only the influence of the excavation gets evaluated but the same also provides facilities for simulating the total construction sequence.

In the present analysis square and circular shapes of openings are considered. The software has been employed for the parametric study comprising of:

- 1) shape of opening,
- 2) range of K_0 ,
- 3) range of overburden pressures and
- 4) varieties of construction sequences for the opening being excavated.

With the assumption of material density of 20 kN/m^3 (2.0 t/m^3), the pressure resulting from depths in the range 45 m, 90 m and 135 m are considered and the values of K_0 are in the range 0.333 to 3.0 (Fig.1).

Thus the primary aim of investigation is to study the influence of construction sequence on the deformations of the excavated medium. In this process it is assumed that the stand-up time of the excavated profile is infinite hence the opening is assumed to be unsupported during the excavation process. Both the symmetrical and unsymmetrical construction sequences are simulated. With a view to investigate the influence of various kinds of

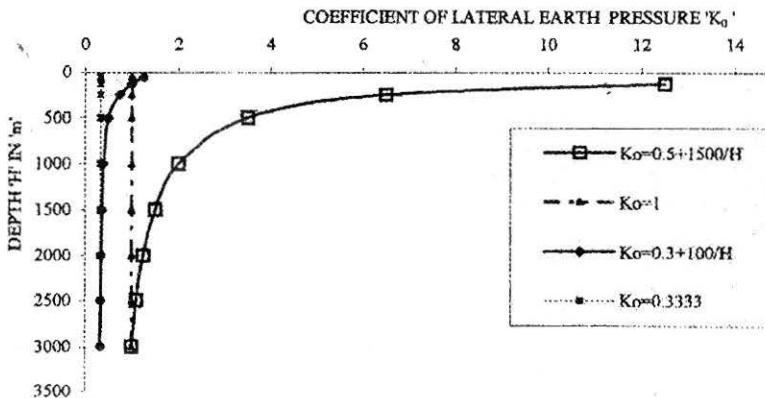


FIGURE 1 : Range of Values for K_0 vs. Depth

excavation sequences the problem of square opening of the size $6\text{ m} \times 6\text{ m}$ and circular opening with diameter 6 m in a semi-infinite medium at a depth of 90 m is reported here.

Design of tunnel needs consideration to large number of parameters and their interactions. In case of tunnels driven at great depths (i.e. deep tunnels, in which $H/D > 1.5$, where ' H ' is the depth to the longitudinal centerline of the tunnel and ' D ' is the outside diameter of the tunnel) the important area of investigation affecting a decision making process for a large number of tunnels is related to the influence of residual stresses or in-situ stresses vis-à-vis the excavation sequence. The investigation presented herein is concerned with this particular aspect of the problem, so that rational decision could be taken regarding the excavation process.

In exceptionally soft and loose ground the methods shield tunnelling will afford good results. In full face tunnelling without supports the sequence of excavation discussed by Szechy (1970) is from crown to invert of the tunnel opening. The excavation possibilities in parts has also been discussed giving the combination of various methods such as Belgian method, German method, Italian method, centre cut method and various mining methods. American method, English method, Austrian method, shield tunnelling method, needle beam method etc. have been discussed in IS 5878 (Part III-1972).

Brown (1987) discussed the stresses around an opening before and after excavation by boundary element method, as well as linked boundary element- distinct element scheme, and shown that the computational schemes provide efficient methods for analysis of the state of stress, and induced displacement around underground excavations. Ramamurthy (2001) described the stress measurement and their effect on design of rock structures.

Scope of Work

Geotechnical formations are millions of years old and wherever they are encountered, their personality is a function of entire geomorphological history of the formations. As far as the problem of tunnel design in such formation is concerned the most important parameter is related to the residual stresses (or in-situ stresses) developed due to sequence of excavation. Considerable developments have taken place in the measurement of the residual stresses. For example a field test referred to as flat jack test, pressure cells, extensometers, stress relieving methods and stress compensating methods etc. are available of measurement of stresses. The aim of such a test is to establish the coefficient of lateral pressure (K_0) in the geotechnical medium. Now, lateral earth pressure ' σ_H ' is given by Eqn.1,

$$\sigma_H = K_0 \sigma_v \quad (1)$$

where σ_v = existing overburden pressure at the point.

Once the value of K_0 is available the finite element solution technique is employed wherein the influence of K_0 vis-à-vis construction sequence could be tackled. It is employed for the parametric study comprising shape of opening, range of K_0 , range of overburden pressures and varieties of construction sequence for the opening being excavated in stages. Thus the study involves following set of parameters.

Shape of opening: Two shapes of tunnel have been considered for the purpose of analysis. These are square tunnel 6 m × 6 m sides and circular tunnel of 6m diameter.

Overburden pressure: With the assumption of material density of 20 kN/m³ (2.0 t/m³), the pressure resulting from depths of 45 m, 90 m and 135 m are considered for the analysis.

Range of K_0 : Hoek and Brown (1980) have given the empirical relationship between K_0 (ratio of average horizontal stress to vertical in-situ stress) and depth of overburden, H (in m), by Eqn.2.

$$0.3 + 100/H \leq K_0 \leq 0.5 + 1500/H \quad (2)$$

It appears from the literature survey that, the upper and lower values of K_0 for the geotechnical materials encountered in the field are as shown in Fig.1. Coupled this with the few studies reported by Amirsoleymani (1988) and Rame Gouda (1988) it is observed that our requirements would be served by assuming the values of K_0 as 0.333, 0.5, 1.0, 2.0 and 3.0.

Construction Sequence

The primary aim of investigation is to study the influence of construction sequence on the excavations, deformations and stresses in the medium. The opening is therefore assumed to be unsupported during the excavation process. Both the symmetrical and unsymmetrical construction sequences are simulated.

As mentioned earlier the problem of residual stresses and construction sequence has been studied by considering only two shapes of opening, viz. circular and square. The plane strain finite element method is used for obtaining the displacements and stresses at various points. Symmetrical as well as non-symmetrical construction sequences are considered and their effect on displacements and stresses is observed. For the investigation purpose

of square opening, total 5 types of construction sequences are considered. For each type the vertical pressure considered is $\sigma_v = 1.8 \text{ MPa}$ (180 t/m^2) and the values of K_0 considered are 0.333, 0.5, 1.0, 2.0 and 3.0. Thus, the number of cases studied for square opening are about 20. For each type single stage as well as multiple stage (4 or 8, as per type) excavation is considered which results in total 40 cases.

For circular opening the effective vertical pressure considered is $\sigma_v = 0.9 \text{ MPa}$ (90 t/m^2), 1.8 MPa (180 t/m^2) and 2.7 MPa (270 t/m^2). In this case also the values of K_0 considered are 0.333, 0.5, 1.0, 2.0 and 3.0, resulting in additional 15 cases.

Finite Element Methodology

The plane strain finite element analysis is employed for the proposed work. The method of analysis is too well known to need any more detailed description, hence only the salient features governing the problem under consideration are discussed herein. The analysis involves application of first order isoparametric triangular and quadrilateral elements for simulation of the geotechnical material. Residual stresses constitute the initial condition prior to excavation of tunnel. As the tunnel excavation progresses, there is stress release affecting the stress distribution around the opening. While conducting the finite element analysis for the excavation of tunnels with the known residual stresses, following process is adopted.

- a) To begin with the entire continuum is idealized in an appropriate manner, as illustrated in Fig.2a, which gives details of idealization scheme for the square opening $A'B'C'D'$.

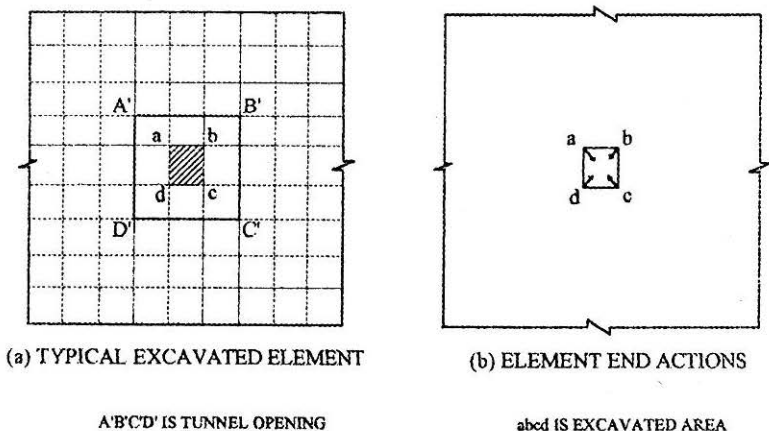


FIGURE 2 : Schematic Simulation for Tunnel Excavation

b) Assuming that, the shaded element 'a-b-c-d' as shown in Fig.2a is removed by excavation, the same element is removed from the idealization, but at its nodes the actions that are equal to the element end actions are applied (Fig.2b). This means the residual idealization would be subjected to the nodal forces, without the stiffness of the excavated element being available.

The end actions of the element being removed are derived by a technique dealing with the problem of initial stresses as discussed by Zienkiewicz (1989). Accordingly, the nodal force vector $[F_e^r]$ representing the element end action is given by Eqn.3,

$$[F_e^r] = \int \int_A [B]^T [\sigma^r] dx dy \tag{3}$$

Here $[B]$ = Element strain transformation matrix and $[\sigma]$ = vector sigma as defined by Eqn.4.

$$[\sigma^r] = \begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{pmatrix} = \begin{pmatrix} K_0 \sigma_v \\ \sigma_v \\ 0 \end{pmatrix} \tag{4}$$

wherein (σ_x, σ_y) = normal stresses in x and y direction respectively,
 τ_{xy} = shear stress over xy-plane,
 σ_v = overburden pressure,
 K_0 = coefficient of lateral earth pressure.

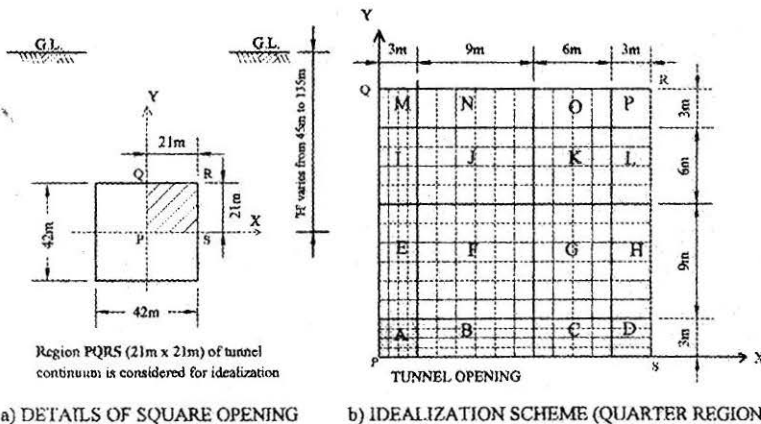
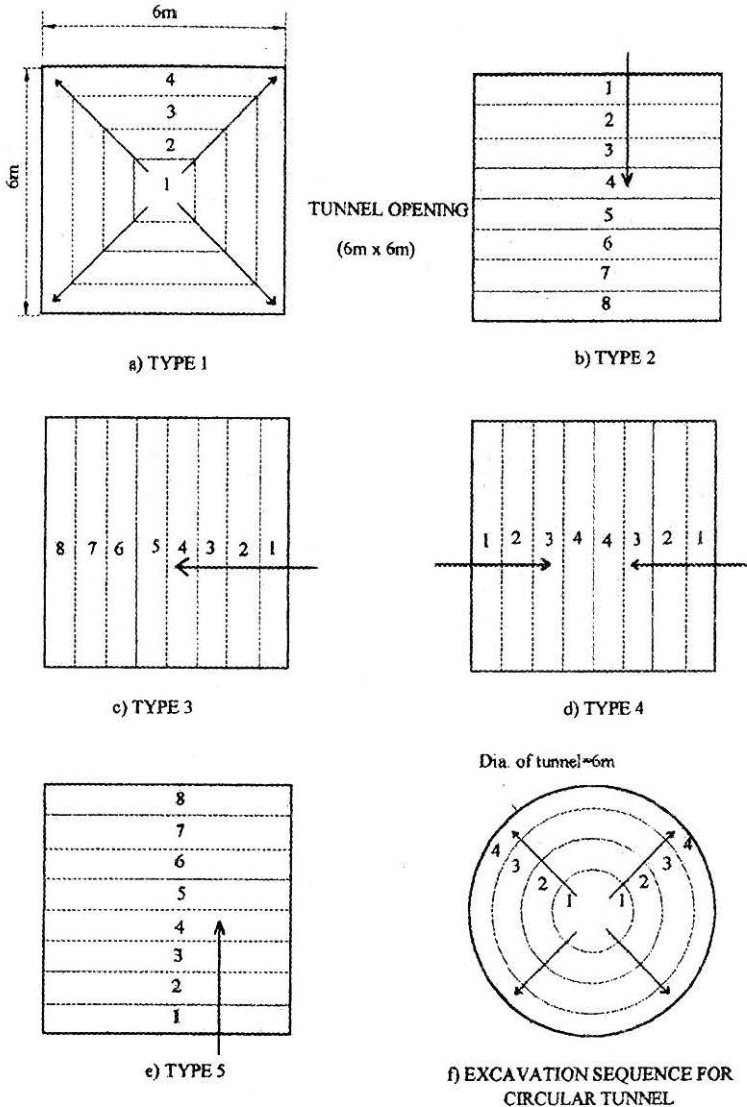


FIGURE 3 : Idealization Scheme for Square Opening (Quarter Region)

In this manner not only the influence of excavation gets evaluated but at the same time it also provides facilities for simulating a construction sequence. In this connection it should be noted that depending upon number of excavation stages a step by step sequential analysis would be warranted.



NOTE: i) ARROWS INDICATE DIRECTION OF PROGRESS OF EXCAVATION IN STAGES.

ii) NOS. 1,2,3... INDICATE STAGES OF PROGRESS OF EXCAVATION.

FIGURE 4 : Progress of Excavation in Various Types and Stages

Investigation for Square Tunnel

In this section effect of construction sequence on displacement of crown and displacement of midpoint of side wall of opening is studied. Further the stress pattern is also presented. For excavation of square opening the sequence considered is as explained in Figs.4a to 4e.

Data for Analysis

With a view to investigate the influence of various kinds of excavation sequences the problem of square opening of the size $6\text{ m} \times 6\text{ m}$ in a semi-infinite medium at a depth of 90 m is considered. The overburden has density of 20 kN/m^3 (2 t/m^3), consequently for a small region surrounding the opening it could be assumed that the vertical stress at every point in the region is $\sigma_v = 20 \times 90 = 1800\text{ kN/m}^2$ (180 t/m^2). The other properties considered are $E_s = 2.85 \times 10^7\text{ kN/m}^2$ ($0.285 \times 10^7\text{ t/m}^2$), $\mu_s = 0.30$. Where E_s is modulus of elasticity of geotechnical material and μ_s is Poisson's ratio. The horizontal stress (σ_H) thereby is $K_0 \times \sigma_v$.

Idealization Scheme

In Fig.3a the details regarding the continuum considered for the analysis are shown whereas Figure 3b illustrates the idealization scheme for the square opening. The details regarding zones, elements and nodes are given in Table 1. Assuming that the influence of the excavation reaches up to three times dimension of the opening, the finite element idealization for the opening with the surrounding material is undertaken. Total number of nodes are 841 and total number of elements 784. The elements, as used earlier are four noded isoparametric elements.

Sequence of Excavation

The opening is proposed to be created through five different types of construction sequences as explained in Figs.4a to 4e.

Type 1 : Excavation is carried out in a symmetrical manner from the center to the periphery of the opening. (Fig.4a).

Type 2 : Excavation is carried out from the crown of the opening to the base of the opening. (Fig 4b).

Type 3 : Excavation is carried out from right towards left of the opening. (Fig.4c).

Type 4 : Excavation is carried out from two sides to inward in a symmetrical manner. (Fig.4d).

TABLE 1 : Details of Idealization Scheme (Refer Fig. 3)

Zone	Total No. of Elements	Equal No. of Horizontal Divisions	Equal No. of Vertical Divisions
A	16	4	4
B	24	6	4
C	16	4	4
D	4	1	4
E	24	4	6
F	36	6	6
G	24	4	6
H	6	1	6
I	16	4	4
J	24	6	4
K	16	4	4
L	4	1	4
M	4	4	1
N	6	6	1
O	4	4	1
P	1	1	1

Type 5: Excavation is carried out from base of the opening to the crown. (Fig.4e).

Presentation of Results and Discussion

As discussed in above section the finite element methodology is adopted to get the deflections and stresses at the nodal points, through the software. The details are presented in this section. The displacement profile for the square opening is presented in Fig.5 whereas the corresponding values of displacement of the tunnel for single stage excavation (full cross section of the opening excavated) and multistage excavation (excavation progresses in stages as explained in Fig.4) are given in Table 2. Some displacements show -ve values, which indicates crown displacement in downward direction and displacement of sidewall towards center of the opening. Further the values are normalized with reference to the values of displacement for $K_0 = 1$ condition.

Movement of the Crown

The primary study aims at the establishment of the deflected profile of the opening periphery. From the Fig.5 and Fig.6 it is clear that the crown displacement show negligible effect of K_0 . It is further noticed that the

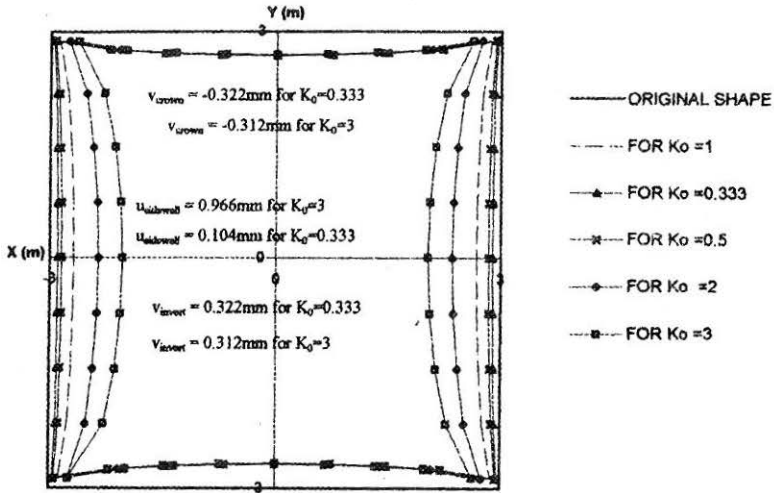


FIGURE 5 : Displacement Profile for Square Opening

peripheral deflections at the symmetrical as well as non-symmetrical excavation have the same tendency of crown displacement. The trend continues to manifest even with the entire range of value of K_0 considered in the investigation.

It is interesting however, to compare the values of the deflections and their variation with K_0 , with that due to $K_0 = 1$. The results for the same are presented in Fig.6. It may be noted from the figure that irrespective of the

TABLE 2 : Displacement Values for Square Opening (for $\sigma_v = 1.8$ MPa)
For Type I, II, III, IV and V (Refer Figs.4a - 4e, Fig.5 and Fig.6)

K_0	v_{crown} [*] (mm)	$u_{sidewall}$ [*] (mm)	v_{invert} (mm)	Normalized Displacements (w.r.t. $K_0 = 1$)		
				v_{crown}	$u_{sidewall}$	v_{invert}
1	-0.319	-0.319	0.319	1	1	1
0.333	-0.322	-0.104	0.322	1.009	0.326	1.009
0.5	-0.321	-0.158	0.321	1.006	0.495	1.006
2	-0.316	-0.643	0.316	0.990	2.016	0.990
3	-0.312	-0.966	0.312	0.978	3.028	0.978

* -ve sign indicates crown displacement in downward direction and displacement of sidewall towards center of the opening.

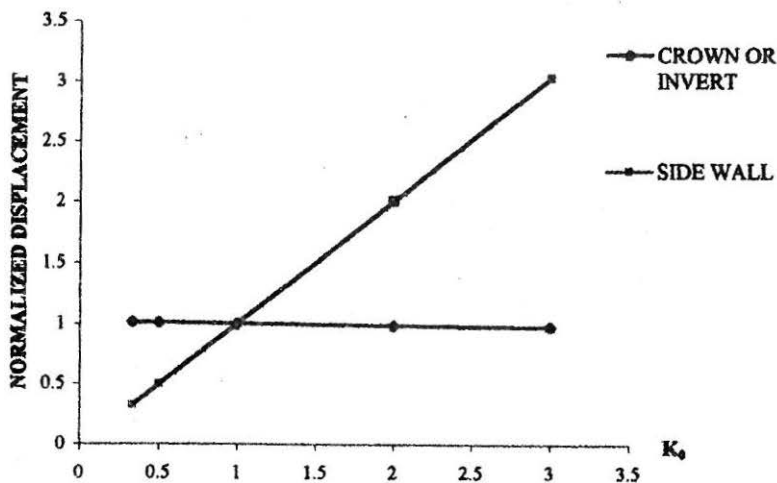


FIGURE 6 : Effect of K_0 on Normalized Displacements for Square Tunnel

type of construction sequence, the crown develops negligible tendency of the downward movements for all values of K_0 .

Movement of the Sidewall

The values of horizontal displacement of side wall ($u_{sidewall}$) are presented in Table 2 and the displaced profile is shown in Fig.5. It is obvious from the figure that for $K_0 = 1$, the crown displacement and the displacement of side wall have same numerical value (0.319 mm). This is due to the symmetrical loading conditions and symmetry of the opening. Further, the deflected profile of the opening periphery suggests that for the range of K_0 values with increasing trend, the excavation have tendency to induce relatively more order of distress. The trend continues to manifest even with the entire range of value of K_0 considered in the investigation.

The values of the deflections and their variation with K_0 , have been compared with that due to $K_0 = 1$. The results for the same are presented in Fig.6. It may be noted from the figure that irrespective of the type of construction sequence the displacement of side wall develops a linearly increasing tendency for all values of K_0 . The tendency of side walls is towards inward movements. This is due to the effect of deep seated tunnel. In case of shallow tunnel this may not occur.

The results of displacements for single as well as multiple stage excavation are same hence the stresses must be same around the opening.

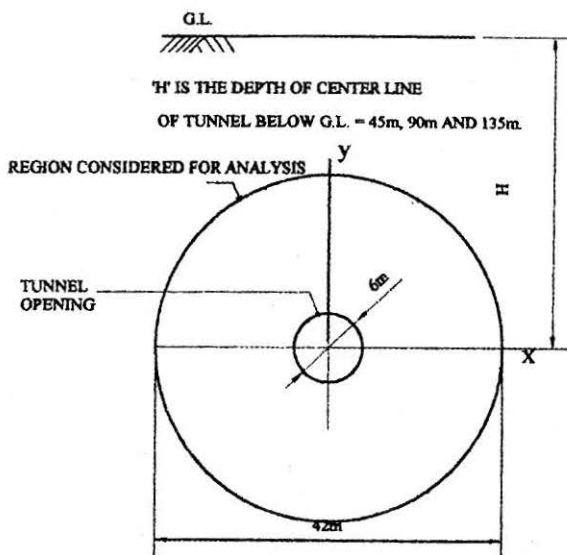


FIGURE 7 : Tunnel Opening in Continuum

Investigation for Circular Tunnel

Similar to the analysis performed for square opening, the analysis of circular opening is also conducted. The diameter of the opening is taken as 6m and further it is assumed that the influence of the excavation is up to a distance of three times diameter (Fig.7). Accordingly the idealization scheme is as shown in Fig.8.

Sequence of Excavation

As illustrated in Fig.4f the sequence of excavation for circular tunnel of diameter 6 m is from center towards outward direction. The numbers 1, 2, 3, 4 in Fig.4f denotes various stages of excavation, in sequential manner.

Idealization for Circular Opening

The location of tunnel opening in the continuum under consideration is explained in Fig.7 whereas Fig.8 illustrates the idealization scheme for the circular opening. The elements, as used earlier are three noded and four noded isoparametric elements. The finite element mesh consists of radial and circumferential markings wherein the radial markings are at 22.5° giving rise to 4 divisions to the circle. For circumferential markings the graded mesh is provided as per the details shown in Fig.8.

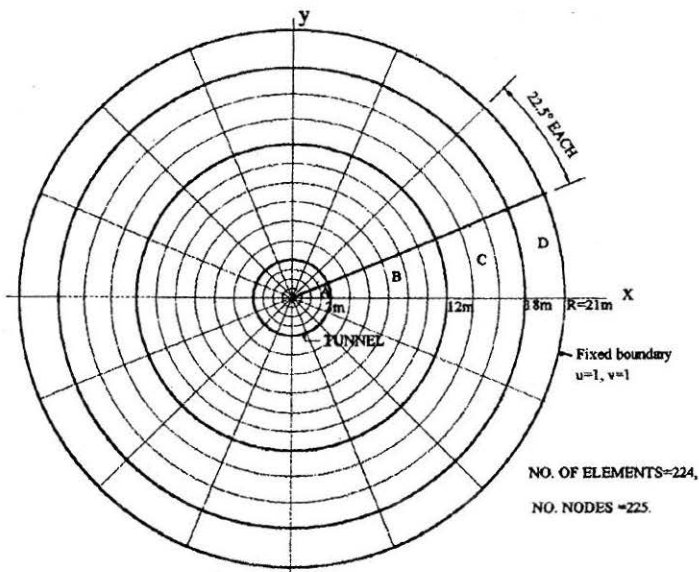


FIGURE 8 : Idealization for Circular Opening

Zone A: Circles with radius 0.75 m, 1.5 m, 2.25 m, and 3.0 m.

Zone B: Circles with radius 4.5 m, 6.0 m, 7.5 m, 9.0 m, 10.5 m, and 12.0 m.

Zone C: Circles with radius 14.0 m, 16.0 m, and 18.0 m.

Zone D: Circle with radius 21.0 m.

Thus the idealization resulted in total elements = 224 and total nodes = 225.

Presentation of Results and Discussion

For circular tunnel the values of displacements observed at crown, sidewall and invert for overburden stress σ_v of 0.9 MPa (90 t/m²), 1.8 MPa (180 t/m²) and 2.7 MPa (270 t/m²) corresponding to depths 45 m, 90 m and 135 m respectively are presented in Tables 3, 4 and 5. The values of displacement of the tunnel profile for single stage excavation and multistage excavation are same. In the Tables 3, 4 and 5 some displacements show -ve values, which indicates crown displacement in downward direction and displacement of sidewall towards center of the opening. Further the values are normalized with reference to the values of displacement at $K_0 = 1$. It is observed that the displacement of crown and invert for the particular value of K_0 is same but opposite in direction. This may be due to the effect of

TABLE 3 : Displacement Values for Circular Opening (for $\sigma_v = 0.9$ MPa)

K_0	v_{crown}^* (mm)	$u_{sidewall}^*$ (mm)	v_{invert} (mm)	Normalized Displacements (w.r.t. $K_0 = 1$)		
				v_{crown}	$u_{sidewall}$	v_{invert}
1	-0.111	-0.111	0.111	1	1	1
0.333	-0.120	-0.0278	0.120	1.081	0.250	1.081
0.5	-0.118	-0.0485	0.118	1.063	0.437	1.063
2	-0.0971	-0.235	0.0971	0.875	2.117	0.875
3	-0.0834	-0.359	0.0834	0.751	3.234	0.751

TABLE 4 : Displacement Values for Circular Opening (for $\sigma_v = 1.8$ MPa)

K_0	v_{crown}^* (mm)	$u_{sidewall}^*$ (mm)	v_{invert} (mm)	Normalized Displacements (w.r.t. $K_0 = 1$)		
				v_{crown}	$u_{sidewall}$	v_{invert}
1	-0.221	-0.221	0.221	1	1	1
0.333	-0.240	-0.0555	0.240	1.085	0.251	1.085
0.5	-0.235	-0.0971	0.235	1.063	0.439	1.063
2	-0.194	-0.470	0.194	0.877	2.126	0.877
3	-0.167	-0.719	0.167	0.755	3.253	0.755

TABLE 5 : Displacement Values for Circular Opening (for $\sigma_v = 2.7$ MPa)

K_0	v_{crown}^* (mm)	$u_{sidewall}^*$ (mm)	v_{invert} (mm)	Normalized Displacements (w.r.t. $K_0 = 1$)		
				v_{crown}	$u_{sidewall}$	v_{invert}
1	-0.332	-0.332	0.332	1	1	1
0.333	-0.359	-0.0833	0.359	1.08	0.251	1.08
0.5	-0.353	-0.146	0.353	1.06	0.440	1.06
2	-0.291	-0.705	0.291	0.876	2.123	0.876
3	-0.250	-1.08	0.250	0.753	3.253	0.753

* -ve sign indicates crown displacement in downward direction and displacement of sidewall towards center of the opening.

deep seated tunnel. In this section the effect of K_0 on deformation of circular tunnel, variation of stress and the effect of depth of overburden on crown displacement of tunnel is discussed.

Effect of K_0 on Deformation of Circular Opening

As in case of square opening the observation that the excavation from center to outward is practically more advantageous, a similar study in case of circular opening should constitute the most important problem for investigation. Accordingly the analysis with four stages of excavation is performed. In Fig.9 the deflected profile of the periphery of circular tunnel is presented, whereas, in Fig.10 the influence of K_0 on the relative deformations are indicated. As in case of square opening it is once again observed that with increasing K_0 values the tendency for inward movement of the sidewall is more markedly indicated whereas there is negligible effect on crown displacement. (Fig.10)

Variation of Stress

The maximum stress contours around the opening for $K_0 = 1$ are in the manner of circumferential curves almost parallel to the periphery of excavation. It is a common practice to adopt shield tunnelling technique in soft strata for circular sections. As far as the effect of excavation in such technique is concerned, it is as good as, the effect of removal of all the material in a single stage. For such case with $K_0 = 1$ the displaced configuration of the periphery

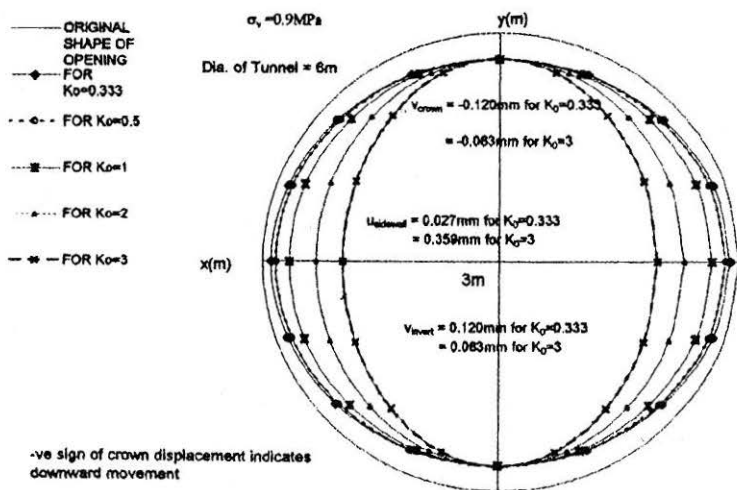


FIGURE 9 : Effect of K_0 on Deformation of Circular Tunnel

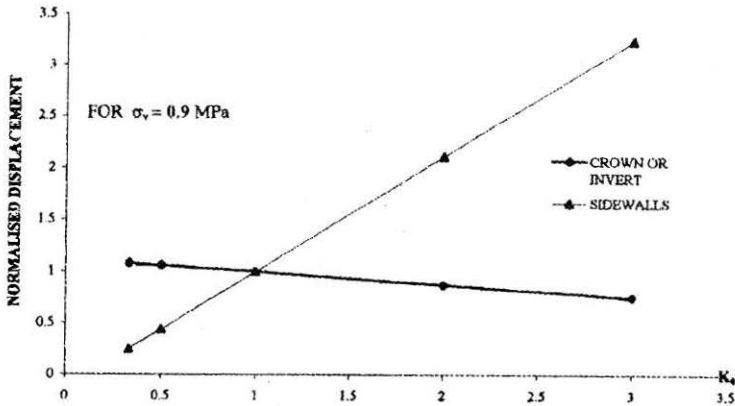


FIGURE 10 : Effect of K_0 on Normalized Displacements of Circular Tunnel for Single and Multiple Stages of Excavation

is as shown in Fig. 9. It is observed that the stress contours around circular opening are in the form of concentric circles around the opening for $K_0 = 1$. Further for other values of K_0 the trend is similar for square and circular opening, as described by Hoek and Brown (1980). As mentioned earlier, the results of displacements for single as well as multiple stage excavation are same, hence the stresses must be same around the opening.

Effect of Depth of Overburden on Crown Displacement of Tunnel

As mentioned earlier the above investigation is performed for a depth of overburden of 45 m, 90 m and 135 m leading to the overburden stress $\sigma_v = 0.90$ MPa, 1.8 MPa and 2.7 MPa respectively. Results of investigation are presented in Tables 3, 4 and 5 respectively. The results of only crown displacement have been presented in Table 6. The effect of depth on crown

TABLE 6 : Effect of Depth on Crown Displacement of Circular Tunnel (Refer Fig.11 and Fig.12)

Depth (m)	Crown Displacement ' v_{crown} ' (mm)*				
	$K_0 = 1$	$K_0 = 0.333$	$K_0 = 0.5$	$K_0 = 2$	$K_0 = 3$
45	-0.111	-0.120	-0.118	-0.097	-0.083
90	-0.221	-0.240	-0.235	-0.194	-0.167
135	-0.332	-0.359	-0.353	-0.291	-0.250

* -ve sign indicates crown displacement in downward direction and displacement of sidewall towards center of the opening.

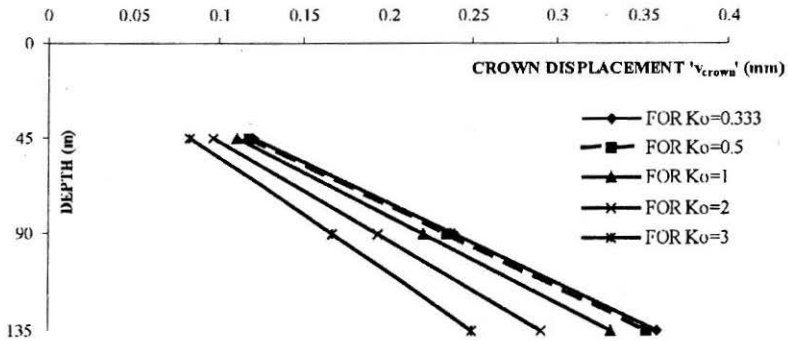


FIGURE 11 : Effect of Depth on Crown Displacement of Circular Tunnel

displacement of circular tunnel is presented graphically by Fig.11. As expected the entire response was found to be linearly proportional to the overburden stress. Considering the effect of depth, the crown displacement is less susceptible to the higher values of K_0 (3), whereas for lower values of K_0 (0.333) the effect on crown displacement is more. (Fig.11).

Comparison between Square and Circular Opening

The area of square opening is about 21.46% more than that of circular opening under consideration. Table 2 and Table 6 gives the crown displacement values for the square and circular opening for overburden pressure $\sigma_v = 1.8$ MPa.

Figure 12 shows almost negligible effect of K_0 on crown displacement for the two shapes of opening. It is further noted that the crown displacement of square opening for $K_0 = 1$ and overburden pressure 1.8 MPa is about 44%

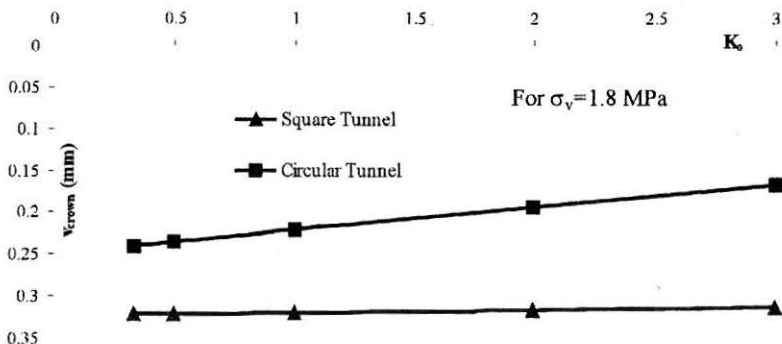


FIGURE 12 : Effect on Crown Displacement of Tunnel

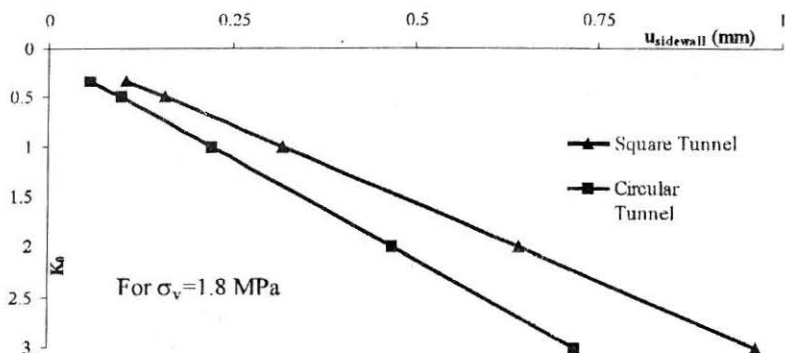


FIGURE 13 : Effect on Sidewall Displacement of Tunnel

more as compared to circular opening. Further for the range of K_0 under consideration, the variation of crown displacement is about 34% to 86% more for the square opening (Fig.12 and Table 6).

The side wall displacement of square opening is about 50% (for $K_0 = 0.333$) to about 70% (for $K_0 = 3$) of the circular opening (Fig.13 and Table 7).

Conclusions

Based on the above data the investigation on the residual stresses and their influence on construction sequence of the tunnel excavation, following important and practically useful conclusions are reported.

1. The excavation from center of opening to outward in a symmetrical manner is most advantageous from the viewpoint of the distress due to excavation.

TABLE 7 : Effect of Depth on Sidewall Displacement of Circular Tunnel
(Refer Fig.13)

Depth (m)	Sidewall Displacement ' $u_{sidewall}$ ' (mm)*				
	$K_0 = 1$	$K_0 = 0.333$	$K_0 = 0.5$	$K_0 = 2$	$K_0 = 3$
45	-0.111	-0.0278	-0.0485	-0.235	-0.359
90	-0.221	-0.0555	-0.0971	-0.470	-0.719
135	-0.332	-0.0833	-0.146	-0.705	-1.08

* -ve sign indicates crown displacement in downward direction and displacement of sidewall towards center of the opening.

2. The response is highly sensitive to a value of coefficient of lateral earth pressure K_0 , however the same is observed to be linearly proportional to the value of K_0 .
3. Though only square and circular openings are considered here the program employed could deal with any shape of opening as well as any type of construction sequence.
4. The sequence of construction do not affect the final deformed surface, for the entire range of K_0 .
5. For 6m x 6m square opening and 6m diameter circular opening representing about 21% variation in cross sectional area the results of crown and side wall displacement vary in magnitude about 30% -80% and about 60% respectively.
6. As far as side wall is concerned the normalised displacements increase with increase in value of K_0 , whereas the displacement at the crown reduces with K_0 . This is true for both the openings under consideration.
7. Fig. 5 and Fig. 9 represent the typical deformed profiles with range of K_0 values for the square and circular opening which indicates that the displacements as expected try to close the opening with a more pronounced effect in the lateral direction.
8. From the excavation of circular and square tunnel opening in single and multiple stages allow the important conclusion that, in the elastic range of loading the displacements of opening at various points are same for single and multiple stage excavation. Similarly the stress pattern remains the same for the two conditions of excavations for single and multiple stages excavation.

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Notations

$[B]$	=	element strain transformation matrix
D	=	diameter of the tunnel
E_s	=	modulus of elasticity of geotechnical material
$[F_e^r]$	=	nodal force vector
H	=	depth of overburden from the longitudinal centerline of tunnel (in m)
K_0	=	coefficient of lateral earth pressure at rest
$u_{sidewall}$	=	horizontal displacement of side wall
v_{crown}	=	vertical displacement of crown
v_{invert}	=	vertical displacement of invert
μ_s	=	Poisson's ratio of geotechnical material
$[\sigma]$	=	stress vector
σ_H	=	lateral earth pressure
σ_v	=	existing overburden pressure at the point
σ_x, σ_y	=	normal stresses in x and y direction respectively,
τ_{xy}	=	shear stress over xy-plane,