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Parametric Study of Slope Stability Analysis using AutoLISP

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Introduction

The stability analysis of earth slopes is a complex process requiring consideration of numerous factors to determine the factor of safety associated with the critical slip surface. The success of stability analysis depends on the determination of the critical slip surface corresponding to the field conditions, such as, type of failure, drainage condition, non-homogeneity, tension crack, hard stratum below base and many others, that give realistic factor of safety for the slope.

AutoLISP, which is a built-in programming language in AutoCAD, appears to be perfectly suited to solve the semi-graphical problem of Slope stability analysis. There are several special features in AutoLISP, which make it a powerful tool in solving problems such as slope stability analysis. Some of these features are as follows:

i) Any geometrical construction can be made by specifying the coordinates of the required points with reference to a predefined point in rectangular or polar coordinate system.

ii) The angle made by a straight line joining predefined end points with the reference axis can be computed using the *angle* command.

iii) The area enclosed by any geometrical figure, bounded by straight lines joining predefined points can be computed, by the *area* command.

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These and many other graphical features combined with the computational feasibility, as in C, make AutoLISP a unique programming language in solving semi graphical-computational problems such as slope stability analysis.

The objective of the paper is to introduce AutoLISP as a powerful and convenient tool to carry out slope stability analysis. A program has been developed in AutoLISP for slope stability analysis that effectively considers all the necessary parameters. With the help of this program, the effect of these parameters on the factor of safety has been examined and the results are presented in this paper. Work concerning the detailed study of the effect of various parameters on the factor of safety does not appear to have been published so far in the widely consulted literature.

Theoretical Considerations

Methods of Stability Analysis

Limit equilibrium and limit analysis are the two basic methods available for slope stability analysis. Even though it does not satisfy all conditions of global equilibrium, the slice method of limit equilibrium analysis, proposed by Bishop (1955), is found to give results similar to those obtained from other refined methods of limit equilibrium analysis (Yu et al., 1998). Partly because of this and partly because of its simplicity, Bishop's method has been widely used for predicting slope stability under both drained and undrained conditions.

In limit equilibrium method, the factor of safety with regard to slope stability is estimated by examining the conditions of equilibrium when incipient failure is postulated along a failure surface, and then comparing the strength necessary to maintain equilibrium with the available strength.

Bishop's Simplified Method of Slices

In the simplified Bishop's method, it is assumed that the resultant of the forces acting upon the sides of any slice is horizontal (Wilun and Krzysztof, 1980). The factor of safety is computed from the relation

$$F = \frac{\sum_{i=1}^{n} \left\{ c_i' b_i + \left(W_i - u_i b_i \right) \tan\phi_i' \right\} \Big/ \left\{ 1 + \frac{\tan\phi_i' \tan\alpha_i}{F} \right\} \cos\alpha_i}{\sum_{i=1}^{n} W_i \sin\alpha_i} \qquad (1)$$

where c_i = Effective average cohesion at the base of i^{th} slice

b_i = Width of i^{th} slice

W_i = Weight of soil in the i^{th} slice

u_i = Average pore pressure at the base of the i^{th} slice

ϕ_i' = Effective average friction angle of soil at the base of the i^{th} slice

α_i = Slope of the tangent to the trial slip surface at the mid point of the base of the i^{th} slice

n = number of slices, i varying from 1 to n.

Because the factor of safety appears on both sides of the Eqn.1, it is determined by a trial and error method, the convergence of trials is very rapid.

Shape of Failure Surface

The errors in a slope stability analysis are not so much in the shape of the assumed failure surface but in the soil properties and the search for the critical failure location (Bowles, 1984). Celestino and Duncan (1981) and Spencer (1981) found that in analyses, where the slip surfaces were allowed to take any shape, the critical slip surface found by the search was essentially circular, although Chen (1970) and Baker and Garber (1977) maintained that the critical slip surface is actually a log spiral. In any case, the difference between the minimum factor of safety for critical circle and the minimum factor of safety for critical log spiral is too small to be of practical importance (Duncan, 1996).

Fellenius' Method of Locating Critical Center

Since for any particular problem, a large number of trial surfaces can be assumed, it is necessary to determine the critical slip circle, for which the factor of safety has the minimum value. This is usually achieved by analyzing a sufficiently large number of trial surfaces until the most critical one is found.

In order to reduce the number of trials, Fellenius (1936) suggested a method that serves as a guide to locate the critical center with a few trials. The most critical circle passes through the toe of the slope a) when $\phi > 3°$ and b) when the slope angle $\beta > 53°$ irrespective of the value of ϕ. The most critical circle intersects the slope in front of the toe if $\phi < 3°$ and $\beta < 53°$ (Wilun and Krzysztof, 1980). In a cohesive soil, the center o1 for toe failure case can be located at the intersection of two lines drawn from the

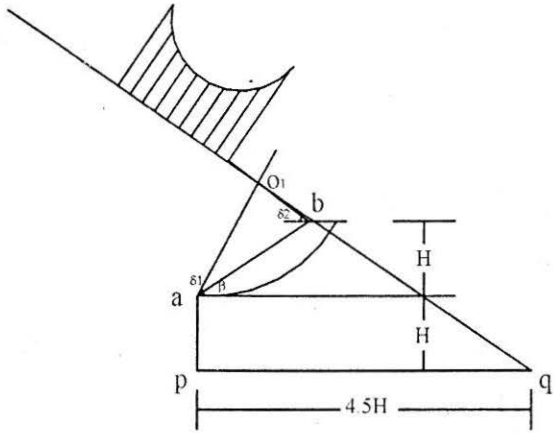


FIGURE 1 : Fellenius and Jumikis Methods of Location of Critical Center

ends a and b of the slope at angles δ_1 and δ_2 (Fig.1). The angles δ_1 and δ_2 vary with the slope angle β as given in Wilun and Krzysztof (1980).

Jumikis (1962) further extended this method to the case of a homogeneous $c - \phi$ soil. After obtaining the center o1 as for a $\phi_u = 0$ soil, a point q is located at a distance (4.5H, −H) from the toe of the slope. The center of the critical circle o is then assumed to lie on the extension of the line q-o1 and the factor of safety obtained are plotted to get a locus (Fig.1), from which the minimum factor of safety can be read.

Determination of pore water pressure

The method of determination of pore water pressure, u_i, in Eqn.1 for a given slice depends on the type of problem under consideration. In the steady seepage condition, the pore water pressure is determined from the flow net i.e., from the knowledge of piezometric head at a given point. For stability analysis, points mid-way along the slip surface of each slice element are considered. It is convenient in computer analysis to express the pore water pressure at these points in terms of the pore pressure ratio, r_u, and the corresponding total vertical stress (Bishop and Morgenstern, 1960).

$$r_u = \frac{\gamma_w h_{wi}}{\gamma hi} = \frac{u_i}{\gamma hi} \tag{2}$$

where hi = height of the i^{th} slice

h_{wi} = piezometric head midway along slip surface of the i^{th} slice.

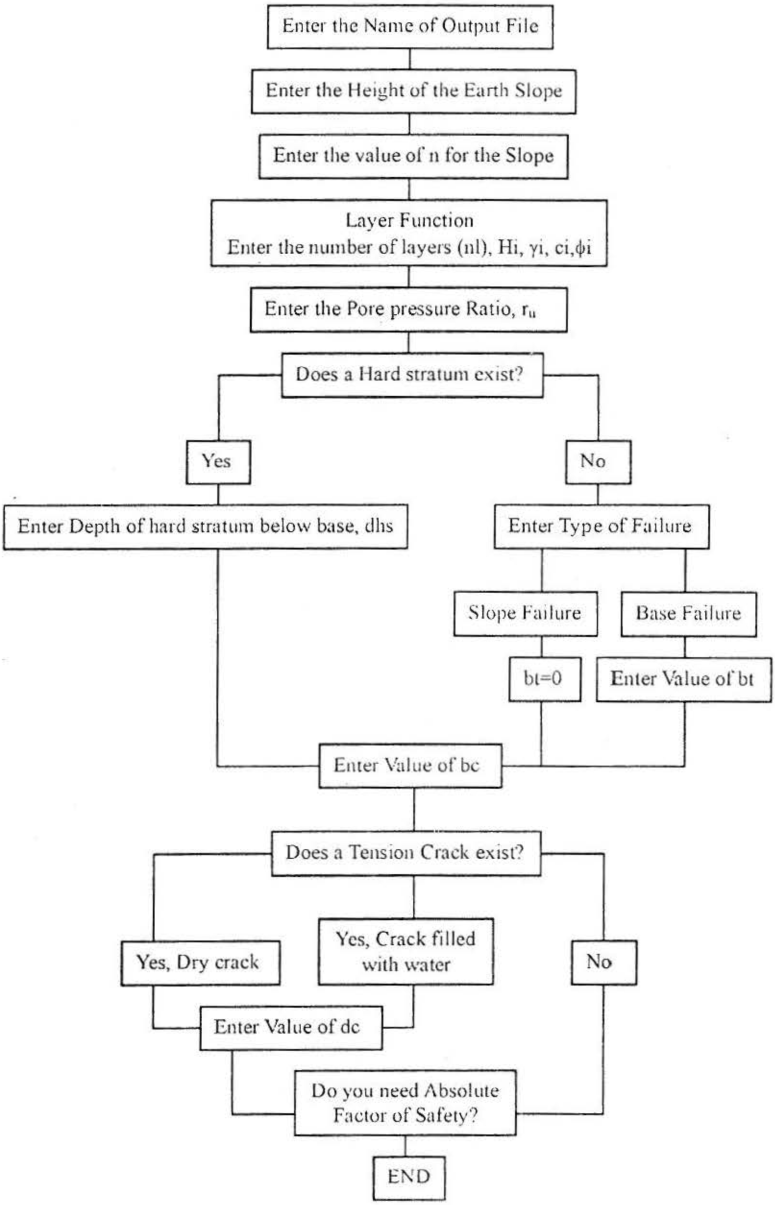


FIGURE 2 : Flow Chart of the AutoLISP Program

γ = Saturated unit weight of the soil

γ_w = Unit weight of water

AutoLISP Program For Slope Stability Analysis

A computer program is developed in AutoLISP to conduct stability analysis by Bishop's simplified method of slices. In this program, the factor of safety is determined for each of 441 trial centers at 1m interval, which form the corners of a 20 m × 20 m square grid. From each trial center, a number of slip surfaces at a radius interval of 1m are considered. Fig.2 shows the flow chart of the program. The program may be explained under the following heads.

Options Provided in the Program

Following are the options provided in the program.

1. Type of failure: The program considers all possible slip surfaces corresponding to slope failure. In addition, base failure is also considered depending on the choice of the user. In case a hard stratum exists at any depth below the base, the condition of base failure is automatically included in the program. In other cases, the user has to specify for including such a condition.

2. Drainage Condition: The effect of drainage condition has been considered in the program by means of an average pore pressure ratio r_u, the value of which is to be specified by the user.

$$u_i = r_u \gamma h_i \tag{3}$$

where h_i is the average height of the i^{th} slice

3. Hard Stratum: The effect of a hard stratum below base is considered by limiting the radius of the trial slip surface to such a value that the slip surface is tangential to the surface of the hard stratum.

4. Tension Crack: The effect of tension crack is considered in the program by ensuring that the slip circle passes through the base of the crack. For a dry tension crack, the slip surface is made to terminate at the base of the crack. For a crack filled with water, the slip circle is made to pass through the bottom of the crack and the moment of the corresponding hydrostatic pressure about the trial center is considered in computing the factor of safety.

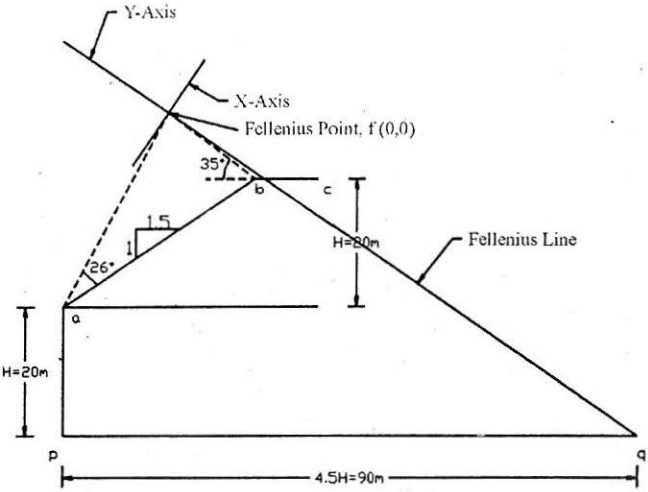


FIGURE 3 : Axes of Reference to define Location of Trial Centers

In the above two cases, all possible slip surfaces starting with the minimum radius have also been considered at each trial center to find the minimum factor of safety.

5. Non-homogeneous soil slope: The program has the option to consider the variation of soil properties with depth. A maximum of five layers are considered in the program, each with different soil properties such as γ, c and ϕ. In computing the shear strength of the soil along the bottom of a slice, the program uses those values of c and ϕ, depending on the layer in which the bottom of the slice falls. To compute the weight of a slice, the cross sectional area of the slice per m length, is multiplied with equivalent unit weight of the soil γ_e.

Geometrical Construction

The required geometrical construction for slope stability analysis viz., construction of slope, drawing a trial slip surface, dividing the wedge of soil into slices etc., is done using AutoLISP commands. After obtaining the necessary input data from the user, the slope ab and the Fellenius line qf are constructed. The extension of the Fellenius line beyond the Fellenius point, f, is used as the Y-axis. A normal is constructed to the Y-axis through the Fellenius point and is used as the X-axis as shown in Fig.3. The location of trial centers is specified with reference to the Fellenius point as origin and the coordinates are specified along these axes. Fig.4 shows the grid of squares, having sides 1m parallel to the X and Y-axes, the corners of which have been used as trial centers. The coordinates of the trial centers have a

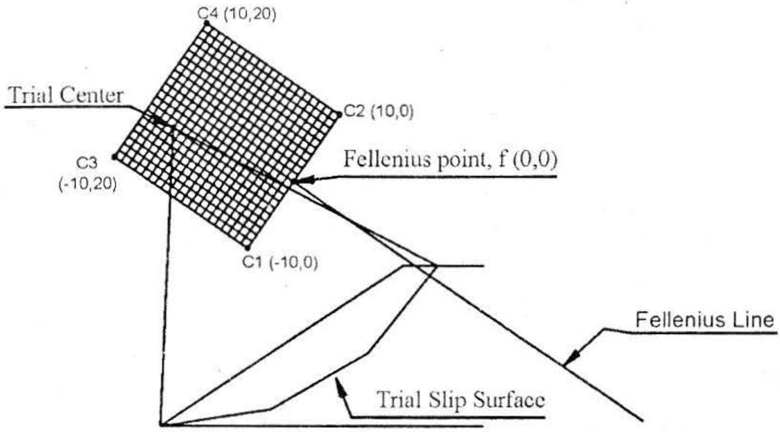


FIGURE 4 : Square Grid describing Locations of Trial Centers

range of x = -10 to 10 m and y = 0 to 20 m. The stability analysis starts with the trial center at (-10, 0) and proceeds to (-9, 0), (-8, 0) and so on, until all the trial centers on X0 line are covered (an X0 line is a line parallel to X-axis having a Y-coordinate of 0). When the stability analysis at all the trial centers on X0 line is completed, the control shifts to (-10, 1) to carry out the analysis at the trial centers on X1 line. The stability analysis is thus carried out in the program at 441 trial centers, on 21 X lines with 21 centers on each X line.

An important point with respect to the coordinate system followed in this paper is to be noted. A relative coordinate system with Fellenius point as origin and X and Y axes, as shown in Fig.3, is used to specify the location of trial centers. In all other cases, the program uses point p as origin and with X and Y axes along horizontal and vertical directions.

Radius of trial slip surface

Radius of trial slip surface is one of the most important parameters controlling the factor of safety. It depends on:

i) Type of failure: toe, slope or base

ii) Existence or otherwise of a hard stratum below the base and its depth

iii) Existence or otherwise of a tension crack, its type (dry or filled with water) and depth

iv) Maximum possible intercept of slip circle in front of toe (b_t) and crest (b_c)

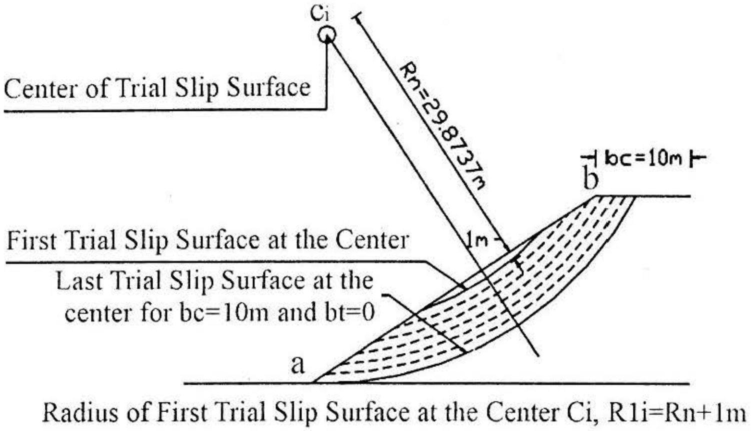


FIGURE 5 : Trial Slip Surfaces at a Typical Trial Center

The program takes into consideration all these factors in fixing the minimum and maximum radius of slip surface at each trial center.

Minimum Radius of Trial Slip Surface at Any Center

A perpendicular is dropped from a given trial center c_i onto the slope ab as shown in Fig.5. This normal distance Rn is incremented by 1 m to get the radius of the first trial slip surface at that center. The slope stability analysis at this center is carried out using this trial slip surface. The radius of the successive slip surfaces at the center are obtained by incrementing the radius of the preceding slip surface by 1m, until the radius does not exceed the maximum radius (r_{max}) at that trial center. When the radius of the slip surface exceeds r_{max}, the control shifts to the next trial center and the stability analysis at the new trial center is carried out in the same manner.

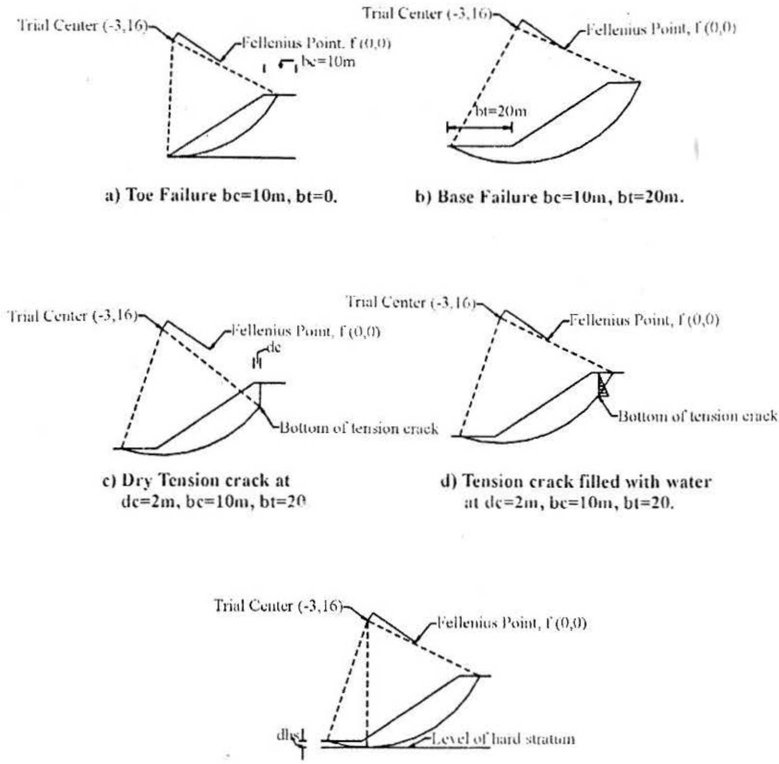
Maximum Radius of Trial Slip Surface at Any Trial Center

The maximum radius of the trial slip surface at each trial center is controlled by the following conditions:

i) Maximum permissible intercept (b_t) of the slip surface in front of the toe.

ii) Maximum permissible intercept (b_c) of the slip surface in front of the crest.

whichever condition gives lesser value of r_{max}. This is illustrated in Figs.6a and 6b.



a) Toe Failure bc=10m, bt=0.

b) Base Failure bc=10m, bt=20m.

c) Dry Tension crack at dc=2m, bc=10m, bt=20

d) Tension crack filled with water at dc=2m, bc=10m, bt=20.

e) Hard Stratum at dhs=2m, bc=10m, bt=20.

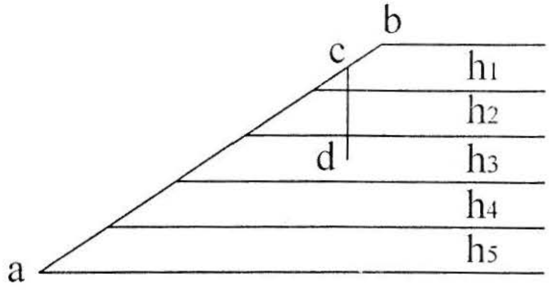
FIGURE 6 : Maximum Radius of Slip Circle at any Trial Center under different Conditions

If a tension crack exists, the maximum radius will be equal to the radial distance of the trial center from the base of the crack, because the slip surface passes through the base of the crack (Figs.6c and 6d).

If a hard stratum exists at any depth (d_{hs}) below the toe of slope, the vertical distance of the level of hard stratum from the trial center will be the maximum radius of the slip surface, because the slip surface tends to be tangential to 'the surface of the hard stratum (Fig.6e).

Equivalent Unit Weight For Layered soils

For non-homogeneous soil slopes, the weight of soil slice is computed by multiplying the area of the slice with the equivalent unit weight for the slice. The equivalent unit weight is calculated separately for each slice for every trial slip surface, as the weighted average of the density of the soil over the height of the slice. For the typical slice shown in Fig.7, for which



c=mid point of top of the slice
d=mid point of bottom of the slice

FIGURE 7 : Equivalent Unit Weight for a Typical Slice

(by − cy) < h1 and (by − dy) < h1 + h2 + h3, the equivalent unit weight is computed from the following relation

$$\gamma_e = \frac{(\gamma_1 h_{e1} + \gamma_2 h_{e2} + \gamma_3 h_{e3})}{h_{et}} \tag{4}$$

where

h_{e1} = height of the slice in the layer 1

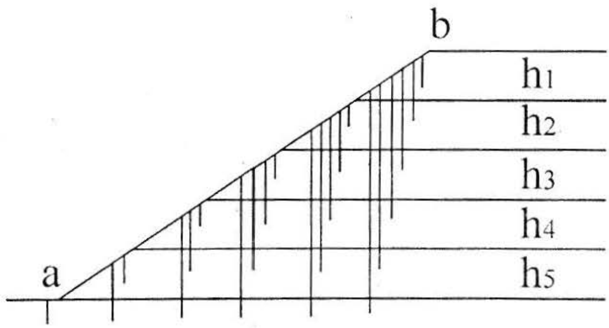
h_{e2} = height of the slice in the layer 2 = h_2

h_{e3} = height of the slice in the layer 3

h_{et} = total height of the slice = $c_y - d_y$

γ_1 = unit weight of the soil in layer 1

γ_2 = unit weight of the soil in layer 2



NOTE: Vertical lines represent center lines of the slices

FIGURE 8 : Possible Cases of Slices Extending across Different Layers

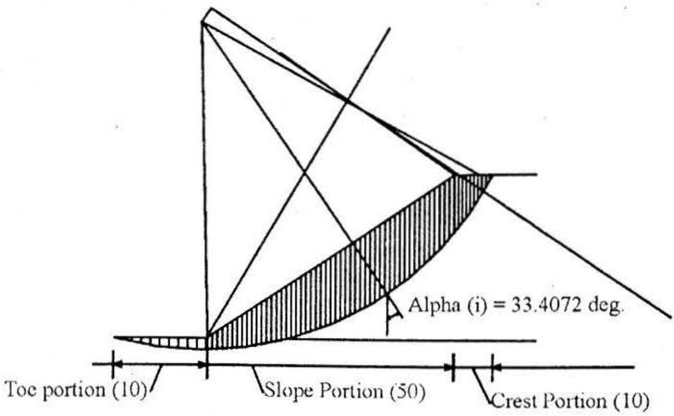


FIGURE 9 : Division of soil wedge into slices

γ_3 = unit weight of the soil in layer 3

by, cy, dy = y-coordinates of the points b, c and d respectively with respect to point p as origin.

All possible cases of the slices extending across different layers have been considered in the program as shown in Fig.8 and expressions similar to the one in Eqn.4 are used to compute the equivalent unit weight for each slice.

Slope Stability Analysis

At each trial center, and for a given slip surface, the wedge of soil above the slip surface is divided into 10 + 50 + 10 slices in the toe, slope and crest portions respectively as shown in Fig.9. The area of each slice is determined by using Area command. It is multiplied with density of soil to get the weight of the slice W_i per unit length. The angle (α_i) made by the line joining the bottom mid point of the slice and trial center with the vertical (Fig.9) is determined by the angle command. The normal (N_i) and tangential (T_i) components of W_i are computed. The effective normal component (N_i') is determined by subtracting the total pore pressure over the width of the slice. The procedure is repeated for all the slices and the factor of safety is determined using Eqn.1 by trial and error method.

Output of the Program

When the user enters the required input data, the program is executed, computing the factor of safety at 441 trial centers, considering all possible slip surfaces at each center. The command window of the AutoCAD indicates the progress of the program execution by printing the coordinates of the trial

Table 1 : Sample Slope Data Used in the Paper

H = 20 m	n = 1.5	γ = 18 kN/m^3	c = 20kN/m^2
ϕ = 30°	r_u = 0	bc = 10 m	bt = 0

Table 2 : Sample of an Output File

(-3 16)

Radius (m)	Factor of safety
29.4265	3.97592
30.4265	2.502
31.4265	2.03141
32.4265	1.80257
33.4265	1.68685
34.4265	1.63139
35.4265	1.60026
36.4265	1.68336
37.4265	1.75894

center at which the stability analysis has been completed. The output of the program is printed in the output file giving the coordinates of the trial center, the radii of all slip surfaces at the center and their respective factor of safety. A sample output for the slope, whose data is listed in Table 1, at a typical (critical) trial center (–3, 16), is shown in Table 2. The minimum factor of safety in all these trials is determined and is printed at the end of the output file and also in the command window.

The program also gives a sketch of the earth slope in the Drawing Area of AutoCAD (similar to the one in Fig.9) showing the critical slip surface and the critical center.

Results and Discussions

Effect of Radius of Slip Surface on Factor of Safety

Figures 10a and 10b present the factor of safety as a function of radius of slip surface at different trial centers for the slope whose data is given in Table 1. The points on each curve indicate the number of possible slip surfaces with a radius interval of 1 m. In Fig.10a, the number of points (read slip surfaces) obtained at the center (−10, 20) is only 2, where as this is 6 at the

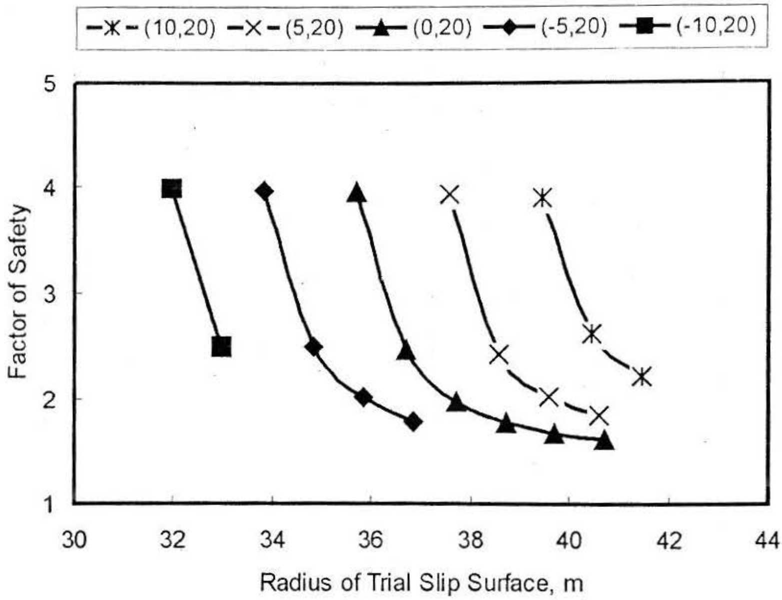


FIGURE 10a : Effect of Radius of Slip Surface on Factor of Safety on X20 Line

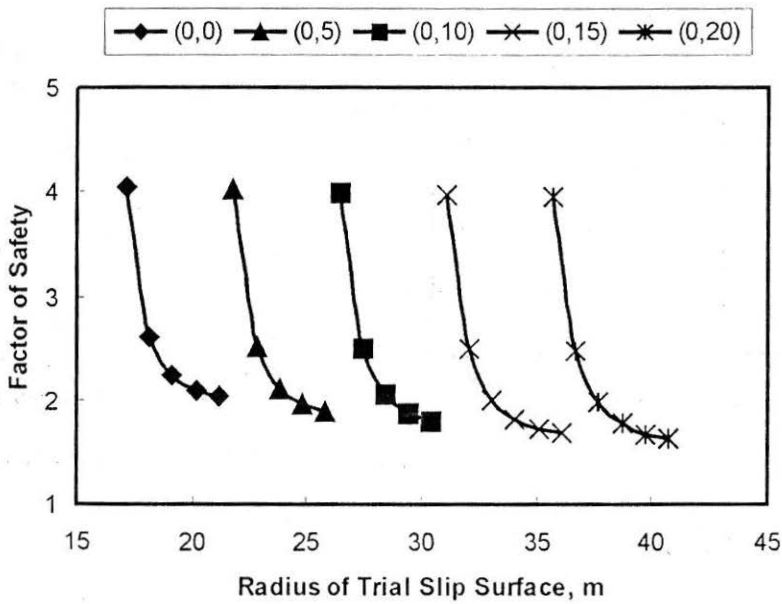


FIGURE 10b : Effect of Radius of Slip Surface on Factor of Safety on Y0 Line

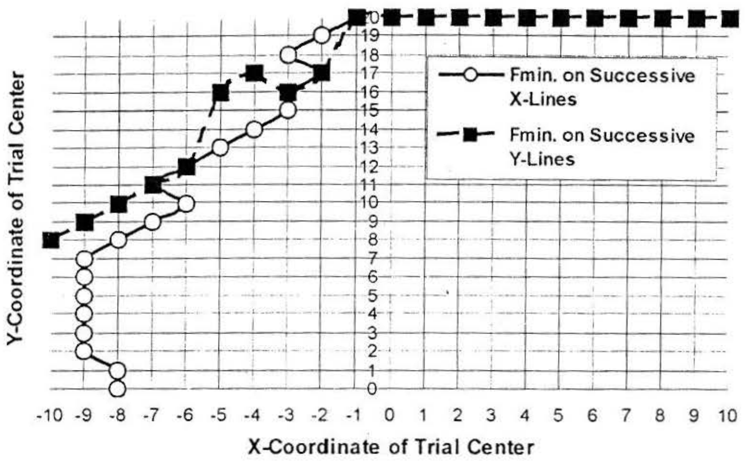


FIGURE 11 : Locus of Minimum Factor of safety on X and Y Lines

center (0, 20), indicating that only 2 and 6 trial slip surfaces are obtained for the slope at these centers satisfying the limits of r_{min} and r_{max}. For an earth slope having the same geometric and soil properties, and from a given trial center, an increase in radius of slip surface decreases the factor of safety.

From Fig.10, it is apparent that the minimum radius of the slip surface at different trial centers has a wide range and it depends on the relative location of the trial center with respect to the slope. On observing the minimum factors of safety at different trial centers, it is evident that a higher radius does not necessarily give the minimum factor of safety. Apparently, there are other parameters, such as the relative location of trial center, which offset the influence of radius in some cases.

Locus of Minimum Factor of Safety

The locus of minimum factor of safety on successive X-lines and Y-lines for the slope data in Table 1 is shown in Fig.11. The locus of minimum factor of safety on X-lines remains far from and on left of the Fellenius line on X_0 to X_7 lines (an X_7 line is a line parallel to X-axis with a Y-coordinate of 7 m). Thereafter, it gradually approaches the Fellenius line. The minimum factor of safety on X_{20} line occurs exactly on the Fellenius line. It may be noted that the locus of minimum factor of safety on X-lines remains completely on the left of the Fellenius line.

The locus of minimum factor of safety on Y-lines, located on the left of Fellenius line, shifts away from the Fellenius point, as the Y-lines approach the Fellenius line. The minimum factor of safety on Y_{-10} line occurs at a

Table 3 : Locus of Minimum Factor of Safety on X and Y Lines

Coordinates of center corresponding to F_{min} on X and Y lines	(−7, 11)	(−6, 12)	(−3, 16)	(−2, 17)	(−1, 20)
Factor of safety	1.627	1.619	1.600	1.602	1.610

Y-coordinate of 8 and that on Y_0 line occurs at a Y-coordinate of 20. The locus completely remains on the extreme end of Y-lines for the Y-lines on the right of the Fellenius line.

The intersection of the locus of minimum factor of safety on X-lines and Y-lines represents the minimum factor of safety on both these lines and from Fig.11, these points and their respective values of factor of safety may be noted as shown in Table 3. The minimum of all these values of factor of safety gives the absolute minimum factor of safety for the slope, which is 1.600 and the corresponding trial center is the critical center i.e., (−3, 16).

Effect of Search Parameters on Factor of Safety

Following search parameters have been used to locate the critical slip surface:

a) Lx and Ly – Size of entire grid in X and Y directions

b) ΔX_t and ΔY_t – Increment to the X and Y Coordinates of the Trial center

c) Rand – Increment to the radius of slip surface at each trial center

Refinement in stability analysis may be achieved and the true critical slip surface may be located by using suitable values of the search parameters. An attempt is made to study the effect of variation in the search parameters by comparing the corresponding minimum factor of safety and the results are presented in this section.

Effect of Size of grid Lx and Ly

Table 4 presents the minimum factor of safety obtained for different sizes of the grid for different heights of slope, keeping all other parameters same as in Table 1. From Table 4, it is evident that the critical center lies within the limits of −H to H in X-direction and 0 to 2H in Y-direction and further increase in grid size gives consistently the same factor of safety for the height of slope of 2 to 20 m.

Table 4 : Effect of Grid Size on Factor of Safety

Height of Slope, H (m)	Size of Grid		Factor of Safety	Coordinates of Critical Center
	Lx	Ly		
2.0	–H to H	0 to H	5.390	(–0.2H, 0.4H)
	–H to H	0 to 2H	5.390	(–0.2H, 0.4H)
	–2H to 2H	0 to 4H	5.390	(–0.2H, 0.4H)
5.0	–H to H	0 to H	3.005	(0.2H, 0.6H)
	–H to H	0 to 2H	3.005	(0.2H, 0.6H)
	–2H to 2H	0 to 4H	3.005	(0.2H, 0.6H)
20.0	–H to H	0 to H	1.6031	(–0.1H, H)
	–H to H	0 to 2H	1.6076	(0.2H, 1.4H)
	–2H to 2H	0 to 4H	1.6076	(0.2H, 1.4H)

$\Delta X_t = \Delta Y_t = 0.2H$; rand = 1 m

Table 5 : Effect of ΔX_t and ΔY_t on Factor of Safety

Height of Slope, H (m)	Increment to Coordinates of Trial Centre		Factor of Safety	Coordinates of Critical Center
	ΔX_t	ΔY_t		
2.0	0.2H	0.2H	5.390	(–0.2H, 0.4H)
	0.1H	0.1H	5.383	(–0.1H, 0.5H)
	0.05H	0.05H	5.319	(–0.1H, 0.25H)
	0.025H	0.025H	5.319	(–0.1H, 0.25H)
5.0	0.2H	0.2H	3.005	(–0.2H, 0.6H)
	0.1H	0.1H	2.979	(–0.3H, 0.2H)
	0.05H	0.05H	2.976	(–0.15H, 0.7H)
	0.025H	0.025H	2.976	(–0.15H, 0.7H)
20.0	0.2H	0.2H	1.6076	(0.2H, 1.4H)
	0.1H	0.1H	1.603	(–0.1H, H)
	0.05H	0.05H	1.60057	(–0.05H, 1.2H)
	0.025H	0.025H	1.60057	(–0.05H, 1.2H)

$L_x = -H$ to H; $L_y = 0$ to 2H; rand = 1 m

Table 6 : Effect of Rand on factor of safety

Height of slope, H (m)	Rand	Factor of safety	Coordinates of critical center
2	0.5H	5.319	(–0.1H, 0.525H)
	0.2H	5.198	(–0.25H, 0.25H)
	0.1H	5.198	(–0.25H, 0.25H)
	0.05H	5.198	(–0.25H, 0.25H)
5	0.5H	2.976	(–0.15H, 0.7H)
	0.2H	2.9097	(–0.15H, 0.5H)
	0.1H	2.9097	(–0.15H, 0.5H)
	0.05H	2.9097	(–0.15H, 0.5H)
20	0.5H	1.7115	(–0.075H, 0.5525H)
	0.2H	1.6246	(–0.15H, 0.7H)
	0.1H	1.60057	(0.05H, 1.2H)
	0.05H	1.60057	(–0.15H, 0.8H)

$\Delta X_t = \Delta Y_t = 0.05H$; $L_x = -H$ to H; $L_y = 0$ to $2H$

Effect of ΔX_t and ΔY_t

Table 5 presents the factor of safety for different increments to the coordinates of trial centers, keeping all other parameters same as in Table 1. Observing the data in Table 5, it is apparent that refinement in the search for critical slip surface is possible by decreasing the values of ΔX_t and ΔY_t i.e., placing the trial centers as closely as possible. Also trial centers closer than 0.05H give the same factor of safety for all heights. Thus, $\Delta X_t = \Delta Y_t = 0.05H$ appear to be the optimum value for the location of critical slip surface for the soil properties considered in Table 1.

Effect of Rand

Table 6 presents effect of rand on factor of safety for different heights of slopes. From Table 6, it is apparent that the factor of safety decreases with decrease in rand upto a rand of 0.05H. Further decrease in rand does not cause refinement in the search for critical slip surface and hence corresponding factor of safety remains the same.

Effect of Directional Angles on factor of safety

The directional angles δ_1 and δ_2 are used to locate the Fellenius point from the slope. The Fellenius point, in turn, decides the location of all the

Table 7 : Influence of Directional Angles on the Factor of Safety

Directional Angles (deg.)		Factor of Safety	Critical Center
δ_1	δ_2		
10	37	1.645	(-6, 20)
20	37	1.606	(-3, 20)
28	37	1.600	(-3, 16)
40	37	1.600	(-2, 10)
28	50	1.601	(-8, 15)
28	20	1.600	(-6, 20)
28	10	1.601	(-6, 20)

trial centers in the stability analysis. Fellenius recommended different sets of directional angles depending on the inclination of the slope. The effect of these directional angles on the factor of safety has been investigated for the slope data shown in Table 1 and the results are presented in Table 7.

From Table 7, it is apparent that increase in δ_1 decreases factor of safety marginally, while the variation of δ_2 has even lesser apparent effect on factor of safety. A higher value of δ_1 locates Fellenius point and hence all trial centers farther from the slope. It has been observed that the factor of safety decreases with increase in Y-coordinate of trial center. It is therefore desirable to use a higher value of δ_1 in the analysis, irrespective of the inclination of the slope. In any case, a considerable change in the directional angles causes a change in the factor of safety only in its second decimal place. Presumably, the factor of safety will have no relation to the directional angles and Fellenius point, provided that enough trial slip surfaces are considered in the analysis from a wide range of trial centers.

Effect of Number of Slices Used in the Analysis on Factor of Safety

The effect of number of slices, into which the wedge of soil above the trial slip surface is divided, on the factor of safety is investigated for the slope data shown in Table 1 and the results are presented in Table 8.

From Table 8, it is apparent that the factor of safety is not significantly affected by the variation in number of slices, the maximum variation being 0.5% when 7 slices have been used in the analysis relative to that when the reasonably large numbers of 60 slices are used. The area of slices in the program is computed analytically including the area of the arc portion at the

Table 8 : Influence of Number of Slices Used on Factor of Safety

Number of Slices		Factor of Safety	% Variation from FoS with 50+10 Slices
Slope Portion	Crest Portion		
50	10	1.6003	0
10	2	1.602	0.125
5	2	1.608	0.5

bottom of the slice and it therefore represents the true area of the slice. Further, the change of inclination of N_i and T_i is uniform over the entire slip surface because the shape of the slip surface is circular. This may be the possible reason why the number of slices has no influence on the factor of safety. The results are however limited to a single problem given in Table 1. The influence of number of slices reported in this section is in line with that reported earlier by Spencer (1967).

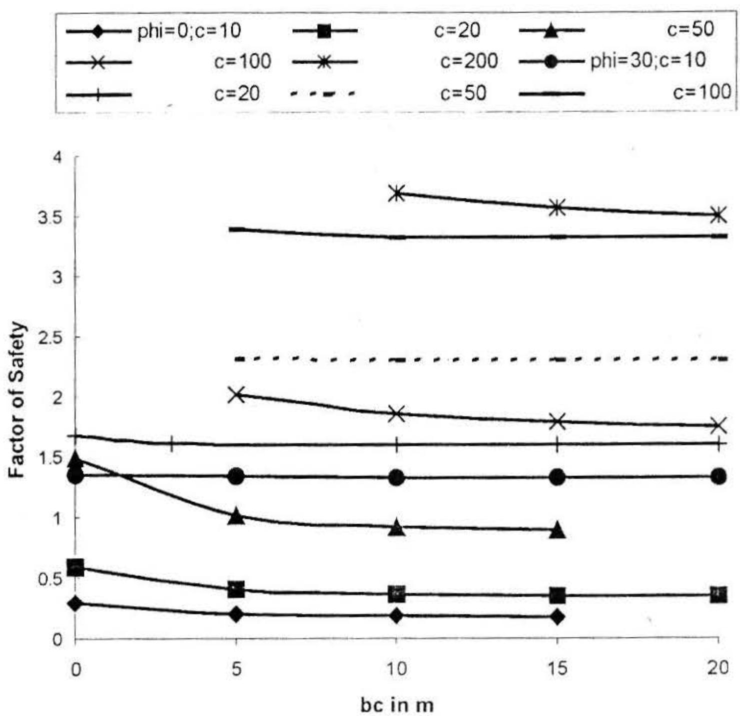


FIGURE 12 : Influence of bc on Factor of Safety

Type of Failure and Factor of Safety

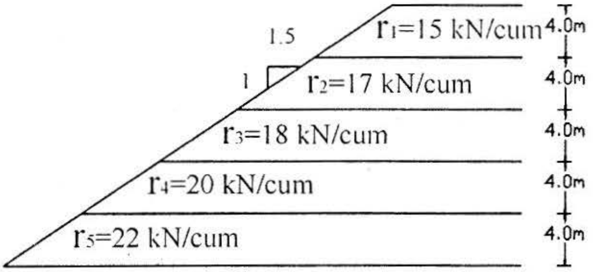
The program provides option to the user to choose the desired type of failure, either toe or base failure. In slope failure, the value of bc (Fig.6a) limits the number of trial slip surfaces at a center and hence may affect the factor of safety. Fig.12 presents the factor of safety as a function of bc. From Fig.12, it is apparent that the factor of safety decreases with increase in bc up to a certain value of bc and thereafter remains constant with further increase in bc. It therefore becomes necessary to use maximum possible value for bc in order to find minimum factor of safety subject to field/design considerations. Similar is the case with bt in base failure.

Layered Soil Slope: Equivalent and Average Unit Weight

Figures 13a and 13b show two typical layered soil slopes, which are analyzed for factor of safety using the AutoLISP program. It has been already stated that the program considers the equivalent unit weight for each slice, and hence the method may be considered exact. An approximate method may also be considered, in which instead of computing and using equivalent unit weight separately for each slice, an average unit weight for the entire slope common to all the slices, may be used in computing the weight of the slice (W_i). The average unit weight may be defined, for a 5-layered soil slope as:

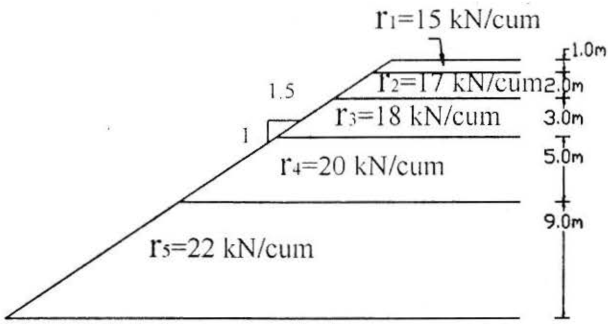
$$\gamma_e = \frac{(\gamma_1 h_1 + \gamma_2 h_2 + \gamma_3 h_3 + \gamma_4 h_4 + \gamma_5 h_5)}{(h_1 + h_2 + h_3 + h_4 + h_5)} \tag{5}$$

Similar expressions may be written for soil slope with less or more number of layers. For the problems shown in Figs.13a and 13b, the average



c=20kPa; ϕ=30; ru=0; bc=10m; bt=0

FIGURE 13a : A Typical Layered Slope



c=20kPa; ϕ=30; ru=0; bc=10m; bt=0

FIGURE 13b : A Typical Layered Slope

unit weight defined by Eqn.5 works out to be 18.4 kN/m^3 and 20.05 kN/m^3 respectively. The two problems are solved by the Exact and Approximate methods and the results are presented in Table 9.

From Table 9, it is apparent that the error due to the use of average unit weight in a layered soil slope is significant but it appears to be on safer side. The magnitude of the error is also variable depending on the problem under consideration.

Effect of Location and Type of Tension crack on Factor of Safety

The problem shown in Table 1 is solved by considering a tension crack at a distance dc from the crest of the slope. Table 10 presents the factor of safety as a function of dc for the slope with a tension crack, in dry condition as well as when the crack is filled with water.

From Table 10, it is apparent that the factor of safety initially decreases and then increases with increase in the distance of the crack from the crest of the slope for both the types of tension crack. Also, the tension crack at

Table 9 : Factor of Safety by Exact and Approximate Methods

Sl. No.	Problem	Factor of Safety		% Error
		Exact Method	Approximate Method	
1.	Fig. 13a	1.665 (-3, 18)	1.590 (-3, 16)	-4.5
2.	Fig. 13b	1.591 (-4, 17)	1.550 (-3, 16)	-2.6

Table 10 : Effect of Type and Location of Tension Crack

Sl. No.	dc (m)	Factor of Safety	
		Dry Crack	Crack Filled with Water
1.	0	1.617 (-4, 17)	1.559 (-3, 19)
2.	1	1.605 (-2, 19)	1.552 (-2, 19)
3.	2	1.603 (-1, 19)	1.554 (-1, 19)
4.	5	1.639 (2, 19)	1.595 (2, 19)
5.	10	1.773 (7, 19)	1.730 (7, 19)

the same location gives a lower factor of safety, when it is filled with water compared to that of a dry crack. Presumably, the difference in factor of safety between these two cases is due to the moment of additional hydrostatic pressure in the crack. This hydrostatic pressure depends on the depth of crack, which, in turn, is a function of the shear parameters and unit weight of the soil.

Effect of Hard Stratum and its Depth below base on Factor of Safety

When the slope consists of a relatively stronger soil at top and weaker soil at bottom and underlain by a hard stratum at a depth, dhs, below the base, the critical failure surface is likely to give a base failure. The problem shown in Fig.14, related to this case, is solved by using the AutoLISP program. Table 11 presents the factor of safety as a function of dhs. From Table 11, it is apparent that increase in the depth of hard stratum below the base decreases the factor of safety. However, when hard stratum is at a depth

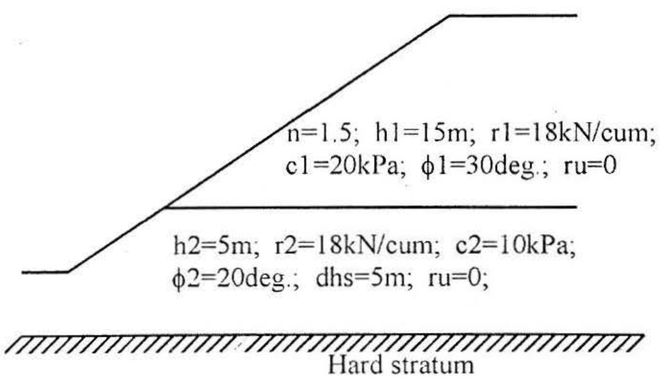


FIGURE 14 : Typical Slope with a Hard Stratum at 5 m Below Base

Table 11 : Effect of Depth of Hard Stratum on Factor of Safety

Sl. No.	dhs (m)	Factor of Safety	Critical Center
1.	0	1.202	(–8, 10)
2.	2	1.174	(–6, 11)
3.	5	1.174	(–6, 11)
4.	10	1.174	(–6, 11)

greater than certain critical value, it does not have any influence on factor of safety.

Stability Number and Factor of Safety

The use of stability charts has become a common practice nowadays to determine the factor of safety at least in the preliminary stability analysis. Considering the fundamental Taylor's stability number, it is defined as

$$Sn = \frac{c}{F\gamma H} \tag{6}$$

The basic assumption in the method is that the Stability number has a unique value for a given slope angle, β, and the friction angle, ϕ.

In this study, factor of safety has been determined for several soil (c = 10, 20, 50, 100 and 200 kPa; ϕ = 30° and 40°) and slope properties (H = 5, 20 and 50 m) with γ = 18 kN/m^3 and n = 1.5 (slope is 1V to nH), using the program and the Stability number has been determined for all these cases. Figure 15 presents the results in the form of a graph between the Stability number and unit cohesion for different heights of the slope. From Fig.15, it is apparent that the Stability number is not a unique number, but it increases considerably with increase in unit cohesion of the soil for the same friction angle, height and inclination of slope and density of soil. It also increases with decrease in height of the slope, for the same slope, friction angle and cohesion. Though this observation is limited to the problems considered in this study and needs further confirmation, it appears to question the use of Stability number for determination of factor of safety.

Comparison of Results of the program with standard problems

The results of the program are compared with those available in standard literature.

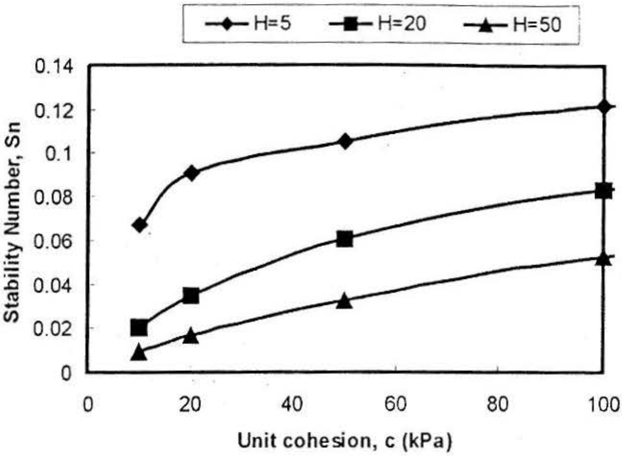


FIGURE 15a : Variation of Stability Number with Soil Properties (ϕ = 30°)

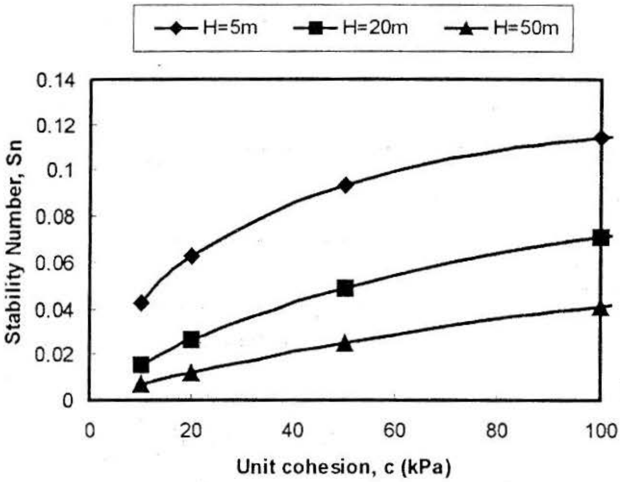


FIGURE 15b : Variation of Stability Number with Soil Properties (ϕ = 40°)

Example 16-1 (page 549; Bowles, 1984)

Given a homogeneous slope with a slope of 1V on 1.5H has H = 12 m, γ = 18.0 kN/m^3 and s = c_u = 30 kPa, r_u is assumed to be 0.0.

Required: What is F if (a) ϕ = 0° and (b) ϕ = 15° and show location of critical circle ?

Table 12 : Comparison of Results of the Program with Standard Problems

Ex. No.	Factor of Safety as per	
	Bowles (1984)	AutoLISP program
16-1(a)	0.90	0.873
16-1(b)	1.56	1.604

Solution of the above problem is given in Table 12.

Table 12 indicates that the factor of safety determined by the AutoLISP program compares closely with that by the method used in Bowles (1984).

Conclusions

The discussion in the preceding section draws the following conclusions.

1. A computer program in AutoLISP has been presented for slope stability analysis. The program has the ability to consider typical conditions in slope stability analysis, such as a) Non-homogeneous soil slope, b) Tension crack in both dry and filled with water condition c) Hard stratum at any depth d) pore pressure ratio.

2. Size of grid (Lx and Ly), increment to the coordinates of trial center (ΔX_t and ΔY_t), and radius increment (Rand) are found to be the helpful search parameters that enable reliable location of critical slip surface.

3. The magnitude of search parameters required for locating the critical slip surface have been determined in terms of the height of the slope for a wide range of heights.

4. Where, suitable search parameters are used, the number of slices and directional angles do not significantly influence the resulting factor of safety.

5. It is preferable to use maximum possible values of bc and bt, subject to field/design considerations, in the stability analysis for locating the critical slip surface.

6. The use of average unit weight in the stability analysis of a non-homogeneous soil slope introduces significant error in the factor of

safety, although this error is on the safer side. An exact method of considering the unit weight for each slice by weighted average has been used in the program that ensures accuracy of the resulting factor of safety.

7. The factor of safety initially decreases and then increases with the increase in the distance of the crack from the crest of the slope.

8. Deeper the hard stratum below the base of the soil slope, lesser will be the factor of safety of the slope up to a certain maximum depth after which the hard stratum has no influence.

9. For a given slope, friction angle and unit weight, the Stability number is not unique, but increases considerably with increase in unit cohesion and decreases with increase in height, other parameters remaining the same.

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Notations

c_i'	=	Effective average cohesion at the base of i^{th} slice
b_i	=	Width of ith slice
W_i	=	Weight of soil in the i^{th} slice
u_i	=	Average pore pressure at the base of the i^{th} slice
ϕ_i'	=	Effective average friction angle of soil at the base of the ith slice
α_i	=	Slope of the tangent to the trial slip surface at the mid point of the base of the ith slice
n	=	number of slices
hi	=	height of the i^{th} slice
h_{wi}	=	piezometric head midway along slip surface of the i^{th} slice.
γ	=	Saturated unit weight of the soil
γ_w	=	Unit weight of water
h_{e1}	=	height of the slice in the layer 1
h_{e2}	=	height of the slice in the layer 2 = h_2
h_{e3}	=	height of the slice in the layer 3
h_{et}	=	Total height of the slice = c_y-d_y
H	=	Height of slope
dhs	=	Depth of hard stratum below base

Lx and Ly	=	Size of grid in X and Y directions
ΔX_t and ΔY_t	=	Increment to the X and Y Coordinates of the Trial center
Rand	=	Increment to the radius of slip surface at each trial center
by, cy, dy	=	y – coordinates of the points b, c and d respectively with respect to point p as origin