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Physical and Numerical Modelling of Heat Flow through Landfills

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Introduction

Flow of heat through geomaterials is of practical interest in many areas of environmental geomechnics. The problem of heat and moisture migration in soils is an important research topic related to the storage and disposal of municipal and radioactive wastes, high-temperature discharges from power plants and industrial processing units (Hensley and Savvidou, 1993). The potential impact of waste disposal on groundwater quality is a major consideration in the design, evaluation and certification of waste disposal facilities. The impact of a waste disposal facility on groundwater quality will depend on the nature of the site, the type of waste, the local hydrogeology, the presence of a dominant path and, perhaps most importantly, the nature of the barrier that is intended to limit and control contaminant migration. Analysis of heat transfer has application in the design of waste storage facilities such as landfills. Municipal waste landfill basal liners and subsoil are subject to long-term thermal loadings due to exothermic degradation processes in the landfill and these loadings may result in the liner heating up to between 40 and 60^0C (Holzlohner, 1995; Doll, 1997). This heating may under certain conditions (in a composite liner with a watertight upper geomembrane or in segments of a simple clay liner with no access to leachate from above) cause desiccation in the clay liner.

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At present, there exists a critical need for physical observations of heat flow phenomena in landfills and subsoil. These observations are required to evaluate the current predictive methods for such phenomena. The flow of heat through soils is often simulated with the help of theoretical modelling approaches, which are fully dependent on complete understanding of the fundamental mechanisms and processes involved. The controlled field experiments of heat flow simulation have the advantage of modelling the total complexity of the full-scale problem. However, these tests are difficult to perform and often offer little control over the boundary conditions. In this regard, a controlled laboratory experiment of heat flow simulation through landfill liners will provide a valuable information in connection with their thermal stability. The purpose of investigating the heat flow in the landfill liner system and the subsoil is to develop a predictive capability of the experimental as well as numerical methods. Physical modelling in line with the numerical modelling enhances the predictive capability of the numerical methods.

The temperature at the base of the landfill is particularly sensitive to the type of waste, rate of landfilling and the saturated thickness of waste (Rowe, 1998). Temperature has implications with respect to potential fluid movement and moisture content decrease in the compacted clay liner (CCL). The excess heat will alter the moisture content of the clay liner, thereby causing the desiccation cracks to occur on the liner surface. Desiccation cracks create zones of weakness in a liner and cause reduction in the overall strength and stability, as well as increase the compressibility of the liner. Cracks also create pathways for transport of contaminants towards groundwater system. In the present study, an effort has been made to model the heat flow through landfill liners and subsoil using the experimental set-up fabricated in the laboratory in order to understand the desiccation cracking of the CCL. The finite element method (FEM) has also been used to simulate the heat flow through liners. The experimental results are compared with that of FEM results. The experimental and FEM results have clearly indicated the usefulness of carrying out heat flow simulation through the liners in order to know their potential risk of desiccation when subjected to temperature variations in the landfill.

Thermal Properties

The analysis of heat flow through landfills indicates the importance of thermal properties of wastes, liners and subsoil. The other applications where the thermal properties of soils have more importance are ground freezing, soil shrinkage, nuclear waste disposal, the underground soil heating and design of buried electrical cables. A study carried out by Thomas and Ferguson (1999) has revealed that the heat flow through the municipal solid waste landfills basically depends on the specific heat (C_{ps}), thermal conductivity (k) and thermal diffusivity (α) of wastes and the compaction state of the liner including the subsoil. Heat can be transferred to the soil by pure conduction alone in which case the heat passes through the static pore fluid and through the soil particles themselves, or it can be dominated by convection in which case the transfer of heat is due to the motion of the pore fluid (Savvidou, 1988). The different mechanisms tend to reflect the grain size of the soil, conduction featuring in fine-grained soils and convection in coarser materials. Conductive heat flow is primarily through the solid phase of a soil mass.

Heat Capacity

Heat capacity is the amount of heat absorbed or released by a unit volume for a corresponding rise or fall of one degree Celsius. Specific heat is the heat capacity of a unit mass of a substance or heat needed to raise the temperature of one gram of a substance by one degree Celsius. The soil-heat flux is the rate of flow of heat energy into, from, or through the soil. The temperature of the soil is determined to a considerable extent by its own properties. The temperature of soils depends upon the factors responsible for: (a) differences in the intensity of absorption of heat, (b) variations in the specific heat of the soil, and (c) differences in the heat conductivity.

The conduction heat flow process in soils is quantified by Fourier's law. In a thermally isotropic medium, Fourier's law for two dimensional heat flow (Incropera and DeWitt, 1996) is given by

$$q_x = -k\frac{\partial T}{\partial x} \quad \text{and} \quad q_y = -k\frac{\partial T}{\partial y} \tag{1}$$

where $T = T(x, y)$ is a temperature field in the medium, q_x and q_y are the components of the heat flux (W/m^2) in x- and y- directions respectively, k is the thermal conductivity (W/m °C) and $\partial T/\partial x$ and $\partial T/\partial y$ are the temperature gradients along x- and y- directions respectively. Fourier's law is the generalisation based on experimental evidence. It is also an expression, defines an important material property, the thermal conductivity. Fourier's law applies for all matter regardless of its state: solid, liquid or gas.

Thermal Conductivity

Thermal conductivity is one of the most important thermal properties of the clay liner and the landfill wastes, which controls the heat flow. In general, the thermal conductivity of a soil is larger than that of a liquid, which is larger than that of a gas. Thermal conductivity (k) is defined as the quantity of heat flowing per unit time through unit area of material when the difference of temperature between the faces is unity. The thermal conductivity is independent of the temperature gradient, but not necessarily of temperature itself. Experimental observations indicate that the thermal conductivity is dependent upon the soil water content, the soil texture, the mineralogical composition, the soil compaction and the organic matter content of soil and, therefore varies from point to point in the soil profile.

The thermal conductivity of soils is a key parameter governing the behaviour of heat exchange facilities utilising the ground for heat transfer and storage. In designing new facilities or evaluating existing facilities, the thermal conductivity of soils is a determining factor. In spite of this dependence, it appears that the current engineering practice of designing facilities such as design of landfills and ground-source heat pump installations, this critical soil parameter is simply estimated from some list of known values. The thermal conductivity of a typical soil is likely to be in the range of 0.3 - 3.0 W/m °C. The thermal conductivity can be determined using a relatively simple transient heat flow method in which a line heat source, called a thermal needle, is inserted into the soil ((Mitchell, 1993). The needle contains both a heating wire and a temperature sensor. When heat is introduced into the needle at a constant rate, the temperatures T_2 and T_1 at times t_2 and t_1 are related to the thermal conductivity k according to the following equation (Mitchell and Kao, 1978):

$$k = \frac{4\pi}{Q}\left[\frac{\ln(t_2)-\ln(t_1)}{T_2-T_1}\right] \tag{2}$$

where Q is the heat input between t_1 and t_2. A range of thermal properties for the typical soil and landfill waste is given in Table 1 (Mitchell, 2002). All these properties are strongly affected by bulk density, water content, and internal sources and sinks of heat.

Landfill Temperature: A Previous Study

The literature contains a wide range of reported temperatures in landfills. The temperature within the heart of a landfill is influenced mainly

TABLE 1 : Thermal Properties of Soil and Waste

Material	k (W/m °C)	α [$\times 10^{-7}$] (m^2/s)	C_{ps} (J/kg °C)
Soil	0.30 – 3.0	2.0 – 3.0	450 – 1000
Waste	0.10 – 1.0	1.0 – 2.5	300 – 1700

by the degree and type of microbial activity. The main source of heat during the anaerobic methanogenic phase is the decomposition of carbohydrates, fats and proteins. Temperatures of 50°C are not uncommon within landfills while peak temperatures approaching 70°C have been recorded. An estimate of the heat generation (Rees, 1980) is given by:

$$\begin{aligned} &1\,\text{kg}\ C_6H_{12}O_6 \Rightarrow 0.25\,\text{kg}\ CH_4 + 0.69\,\text{kg}\ CO_2 + 0.056\,\text{kg Biomass} \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad + 632\,\text{kJ Heat} \\ &(\text{Carbohydrate}) \quad (\text{Methane}) \quad (\text{Carbon dioxide}) \end{aligned}$$

It is reported that the temperature measured at Tokyo Port landfill, Japan with a value of about 60°C remained constant within ± 2°C between 1985 and 1988 (Rowe, 1998). In such situations, the analysis of heat flow through landfills can be considered as steady state, which will give practically acceptable values of temperature distribution in the landfill liners and the subsoil.

High temperatures (50°C to 70°C) have been reported in a number of European landfills. For example, Brune et al. (1991) reported temperatures ranging from 24°C to 38°C in a leachate drain beneath 4 - 6 year old waste at the Altwarmbuchen Landfill. This landfill had been rapidly filled (10 - 20 m/annum) and although it was in a methane forming phase after 4 - 6 years, the leachate was still acidic and very strong. In contrast, at the Venneberg Landfill (Rowe, 1998) the temperature in the drain only ranged from 14°C to 20°C. This landfill has been filled much slower (2 m/annum) than the Altwarmbuchen Landfill and the leachate was only lightly loaded with both organic and inorganic contaminants. In both cases, the temperature was measured about 4 years after the landfill was completed.

The service life of compacted clay liners below geomembranes is usually affected by the temperature rise in the landfill. For a clayey liner, the service life is the period of time during which the bulk hydraulic conductivity of the liner may be expected to fall within the design range. Provided that a clayey liner has been properly designed and constructed, and appropriate attention has been given to clay-leachate compatibility, there is no reason to believe that it would not perform within the design range of hydraulic conductivity, for thousands of years, provided that it is not allowed to desiccate after placement. Desiccation can be related to a change in water content in the clay that could occur: (a) after construction of the clay liner and before placing the geomembrane; (b) after placing the geomembrane and before covering with waste; and (c) after placement of waste.

The behaviour of composite liners under the real conditions was investigated by Gottheil and Brauns (1995), by conducting large scale laboratory experiments. Their results have clearly indicated that the mineral component of a composite liner is exposed to the risk of desiccation if the liner is underlain by a permeable subgrade. Meibner and Wendling (1998) have investigated the formation of cracks in clayey soils, which take place when the water content is decreasing due to moisture and temperature gradient. A laboratory model was designed and used to determine the tensile strength, in relation to different parameters, such as temperature, air humidity, water content and void ratio.

Physical Modelling of Heat Flow

In the section, the details of experimentation and the results of comprehensive laboratory testing programme conducted on heat flow simulation through experimental landfills using the specially fabricated experimental set-up are presented.

Locally available Powai soil (silty soil) and sand are used in the experimental studies on heat flow through CCL and composite liner (CCL + Geomembrane) consisting of 50 mm and 100 mm thick compacted clay and 0.5 mm thick High Density Polyethylene (HDPE) geomembrane. The silty soil is used to represent the landfill base and the sand is used to represent the landfill waste. Marine clay is used for making the compacted clay liner. The properties of the soils used in the experiments are listed in Table 2.

TABLE 2 : Soil Properties

Property	Silty soil	Marine clay
Specific gravity	2.76	2.68
Liquid limit (%)	48	79
Plastic limit (%)	33	40
Shrinkage limit (%)	–	11
Texture		
Sand (%)	39	5
Silt (%)	58	62
Clay (%)	3	33
Standard Proctor test		
MDD (g/cc)	1.54	1.31
OMC (%)	26.0	31.0
Classification (USCS)	ML	CH
Hydraulic conductivity (cm/sec)	$3.62*10^{-6}$	$1.89*10^{-8}$

Experimental Set-up and Accessories

The experimental set-up along with the accessories needed to measure the temperature is fabricated specially for the study of heat flow through the liners and the subsoil. A general experimental set-up for temperature measurement through the liner and subsoil is shown in Fig.1. This set-up is employed for conducting 1-g heat flow simulation through the liners. The overall experimental set-up consists of the following components:

- Steel tank : 300 mm × 300 mm × 450 mm high (10 mm thick steel plate)
- Temperature indicator : Cr-Al type with 12 sensors, (Range −10 to 199.9°C)
- Thermocouples : K-type (Cr-Al), 3 mm diameter
- Temperature controller : Range (0 to 300°C)
- Heater : 1000 W, 25 mm diameter

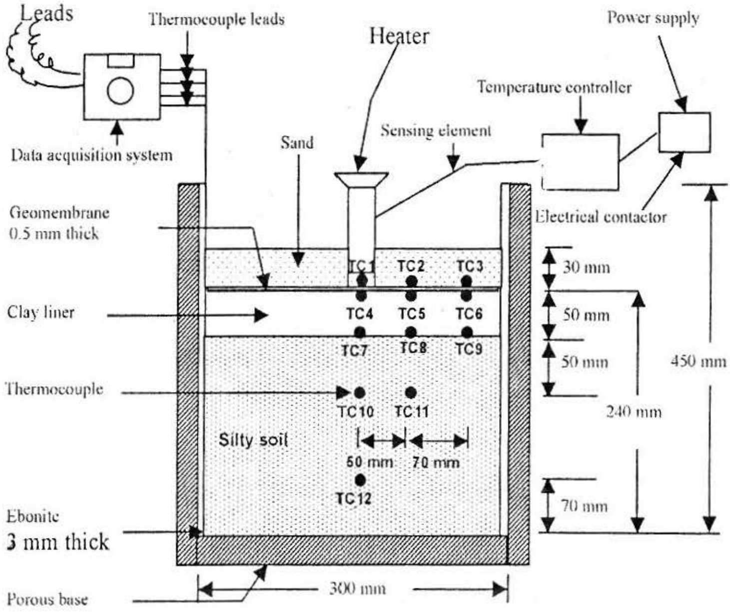

FIGURE 1 : Laboratory Model for Temperature Measurement through Liner

Experimental Programme

Four experiments were carried out: (1) with 50 mm thick compacted clay liner (Experiment No. 1), (2) 50 mm thick compacted clay liner with geomembrane (Experiment No. 2), (3) with 100 mm thick compacted clay liner (Experiment No. 3), and (4) 100 mm thick compacted clay liner with geomembrane (Experiment No. 4). Silty soil and marine clay are mixed with water (water content being ± 3% of OMC). The silty soil is placed in layers to allow for the placement of thermocouples in the required position and compacted gently. The marine clay of 50 mm thickness is placed above the silty soil to form a CCL in Experiment No.1. The fine sand is placed above the CCL to represent the waste as well as the surcharge. In Experiment Nos.2 and 4, geomembrane is laid on the CCL before placing the sand. The 50 mm thick CCL is subjected to a surcharge load of 0.25 kN/m^2 and 100 mm thick CCL is subjected to a surcharge of 1.0 kN/m^2.

Heater is placed 20 mm above the liner top as shown in Fig.1 for all the experiments. Electrical contactor feeds 230 volts to the heater (1000 W) through a regulation with the help of temperature controller. This temperature controller controls the heater temperature at desired setting of temperature (say 60°C etc.). The sensing element of the temperature controller is a thermocouple (TC) attached to the heater.

All the four experiments are carried out with two values of temperature setting. Initially, a constant heater temperature of 60°C was maintained for 48 hours and subsequently increased to 75°C, to simulate the field conditions. In the case of CCL alone, the temperature was maintained for 48 hours at 75°C only. When the composite liner was used, heating at 75°C was continued till the desiccation crack was observed. In Experiment Nos. 1 and 2, the surcharge on the liner was 0.25 kN/m^2 where as in the case of Experiment Nos.3 and 4, the surcharge on the liner was 1.0 kN/m^2. The results and discussion of heat flow simulation experiments conducted on the liners are presented in the following sections.

Experimental Results

For the heat flow simulation through landfill liners, 1-g experiments are conducted to study the desiccation cracking of the CCL. It has been observed that the identical temperatures were recorded for symmetrically placed thermocouples on either side of the heat source. The temperature distributions in the CCL (50 mm and 100 mm thick) and the subsoil are given in Tables 3 and 4 for Experiments Nos.2 and 4 respectively. Figures 2 and 3 show the variation of temperature with depth for the cases of CCL with 50 mm thick alone and composite liner respectively. The variation of temperature with depth for the cases of CCL with 100 mm thick alone and composite liner is shown in Figs.4 and 5 respectively. It is seen from the figures that the temperature decreases with the increase in depth. From Figs.3 and 5, it is observed that the drop in temperature across the geomembrane is about 8°C. The variation of normalised temperature (T/T_h) with time for the Experiment Nos.1 and 2 are shown in Figs.6 and 7 respectively, where T is the temperature at any time and depth and T_h is the heater temperature equal to 75°C. Heating was continued for twenty two days (528 hours). It was observed that 50 mm thick clay liner with a surcharge of 0.25 kN/m^2 developed cracks at the end of 528 hours. However no cracks were observed in the liner subjected to a surcharge load of 1.0 kN/m^2, even after continuously heating for 528 hours. It is observed that the normalised temperature increased upto about 24 hours and thereafter it decreased with increasing time, both in horizontal and vertical directions.

TABLE 3 : Observed Temperatures in Composite Liner System

(Heater Temperature = 75°C, Surcharge = 0.25 kN/m^2, CCL Thickness = 50 mm)

Time Elapsed (hour)	TC-1	TC-2	TC-3	TC-4	TC-5	TC-6	TC-7	TC-8	TC-10	TC-12
0.0	38.3	34.1	27.3	35.4	32.6	27.8	30.6	28.8	28.0	26.4
0.5	42.2	35.8	27.5	36.5	33.4	27.9	30.8	29.2	28.0	26.4
1.0	43.6	36.6	27.5	38.4	33.4	27.9	31.1	29.2	27.9	26.3
1.5	44.2	37.3	27.8	38.5	33.6	28.2	31.4	29.7	27.9	26.3
2.0	45.0	37.7	27.9	39.4	33.8	28.2	31.7	29.7	28.0	26.3
3.0	45.9	37.7	28.0	39.5	34.0	28.4	32.0	30.2	28.1	26.3
4.5	46.6	38.4	28.5	39.8	34.7	28.8	32.4	30.7	28.5	26.5
7.0	47.6	39.1	29.3	40.5	35.5	29.7	33.0	31.2	28.9	27.9
12.0	48.6	39.3	29.3	40.8	35.7	29.8	33.4	31.7	29.3	27.3
24.0	48.4	39.4	29.4	40.4	37.0	29.8	33.7	31.5	29.7	27.5
48.0	47.9	39.0	29.5	40.3	36.8	29.8	33.3	31.4	29.4	27.3
72.0	46.8	37.8	29.0	39.7	36.1	28.9	32.5	30.6	28.8	26.7
120.0	45.2	35.4	27.1	36.5	33.7	27.7	30.5	28.4	27.6	25.8
168.0	44.0	32.2	25.2	33.7	31.1	25.8	28.5	27.3	25.8	24.4
288.0	42.3	31.1	24.3	30.0	28.8	24.8	27.1	25.8	24.8	23.4
384.0	40.1	30.3	24.0	29.2	28.1	24.3	26.7	25.4	24.5	23.3
528.0*	37.5	28.7	23.7	28.3	27.6	24.1	26.3	24.9	24.1	23.2

* Development of Cracks Observed

TABLE 4 : Observed Temperatures in Composite Liner System

(Heater Temperature = 75°C, Surcharge = 1.0 kN/m^2, CCL Thickness = 100 mm)

Time Elapsed (hour)	TC-1	TC-2	TC-3	TC-4	TC-5	TC-6	TC-7	TC-8	TC-10	TC-12
0.0	38.3	28.8	23.1	33.7	29.8	24.0	23.4	23.3	22.7	22.0
0.5	43.4	29.8	23.2	36.5	30.7	24.1	23.5	23.3	22.7	22.0
1.0	43.8	30.5	23.4	37.1	31.3	24.3	23.4	23.3	22.7	22.0
1.5	44.1	30.9	23.5	37.6	31.7	24.6	23.4	23.2	22.6	22.0
2.0	44.8	31.5	23.8	38.0	32.3	24.9	23.6	23.2	22.6	22.0
3.0	45.3	31.9	24.1	38.5	32.7	25.1	23.7	23.6	22.7	22.0
4.0	45.5	32.1	24.3	38.6	32.8	26.3	23.7	23.6	22.6	21.8
11.0	46.9	34.2	26.3	40.1	33.9	26.1	25.1	24.7	23.1	22.0
24.0	46.4	34.1	26.1	40.4	33.7	26.2	25.1	24.8	23.1	21.9
48.0	46.2	33.4	25.6	39.7	33.2	26.3	24.9	24.4	23.0	21.8
96.0	46.2	33.5	26.2	39.5	33.2	26.6	25.4	25.2	23.7	22.6
123.0	46.6	33.7	26.0	40.0	34.1	26.7	25.6	25.2	24.2	23.1
154.0	47.5	34.5	26.7	40.7	34.1	25.7	25.6	25.4	23.7	22.7
216.0	46.4	33.2	25.7	39.5	32.8	24.5	25.0	24.8	23.2	22.1
288.0	44.7	31.3	24.5	37.4	31.0	22.5	23.9	23.6	22.2	21.3
360.0	41.7	29.0	22.5	34.6	28.6	22.4	22.0	21.8	20.4	19.6
408.0	39.8	28.2	22.2	32.2	28.0	22.2	21.6	21.3	19.1	18.6
528.0	36.5	26.7	21.9	30.8	26.2	20.8	20.3	19.9	19.0	18.5

Discussion

The experimental set-up designed and fabricated for conducting laboratory tests on heat flow through landfills including clay liners is found to be quite successful. Based on the laboratory investigations, it has been observed that the temperatures are higher in the horizontal direction, i.e., the thermal conductivity in horizontal direction (k_x) is greater than the thermal conductivity in vertical direction (k_y). It is found that the desiccation cracking will occur when the clay liner is subjected to a temperature of 40°C or more. The risk of desiccation cracking in the CCL is governed by the moisture migration and water-vapour transport due to thermal gradients. However, the extent of desiccation and shrinkage cracks in the compacted clay liners can be reliably clarified in long-term field experiments.

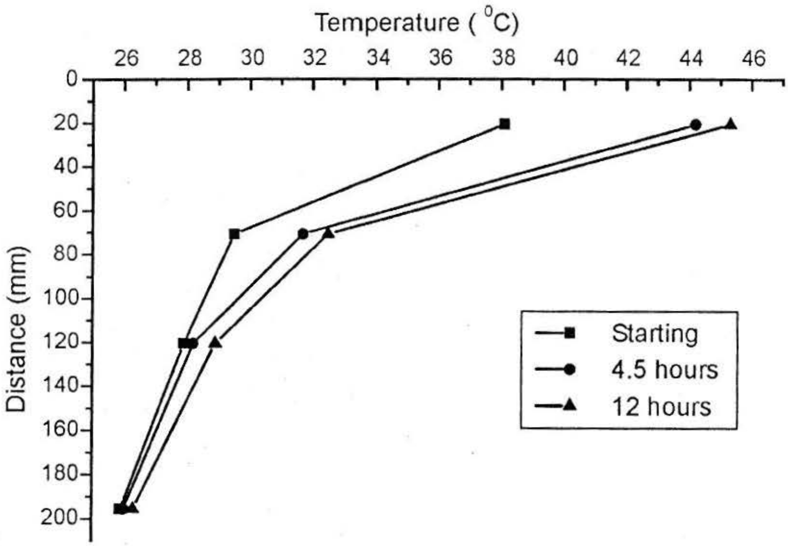

FIGURE 2 : Variation of Liner and Subsoil Temperature with Depth (50 mm Thick CCL only)

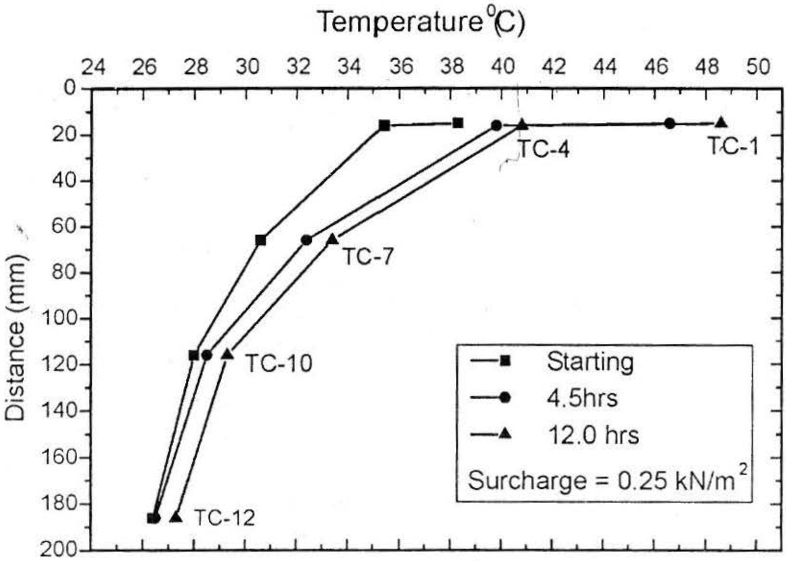

FIGURE 3 : Variation of Liner and Subsoil Temperature with Depth (Composite Liner with 50 mm Thick CCL)

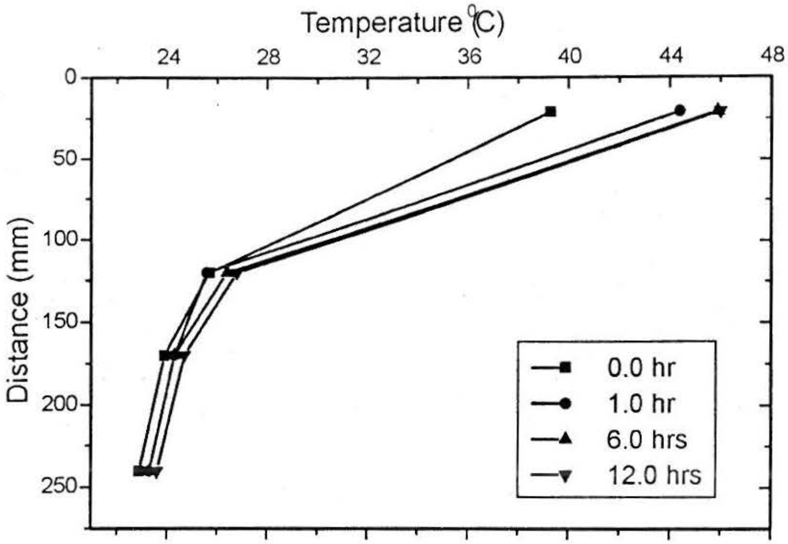

FIGURE 4 : Variation of Liner and Subsoil Temperature with Depth (100 mm Thick CCL only)

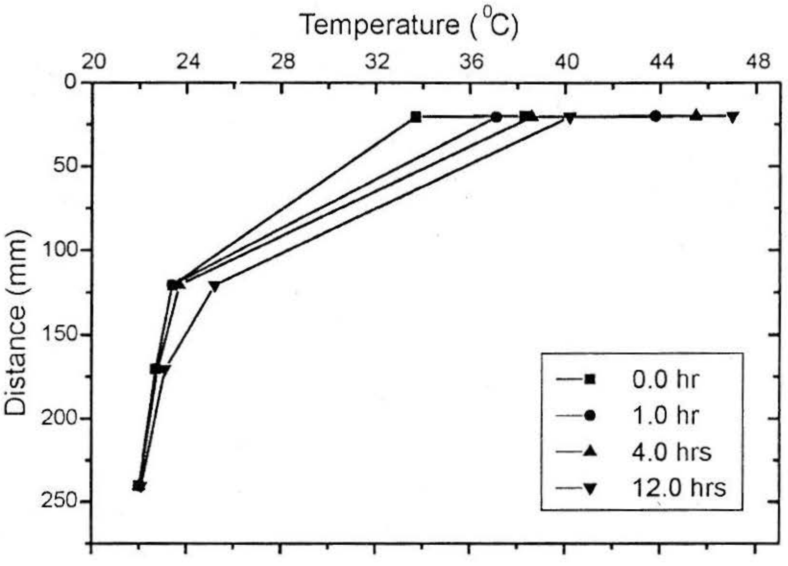

FIGURE 5 : Variation of Liner and Subsoil Temperature with Depth (Composite Liner with 100 mm Thick CCL)

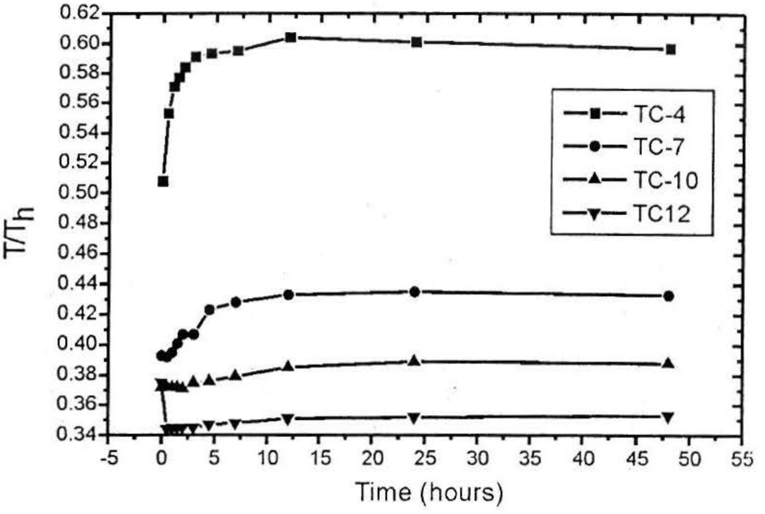

FIGURE 6 : Comparison of Temperature in Vertical Direction (50 mm Thick CCL only)

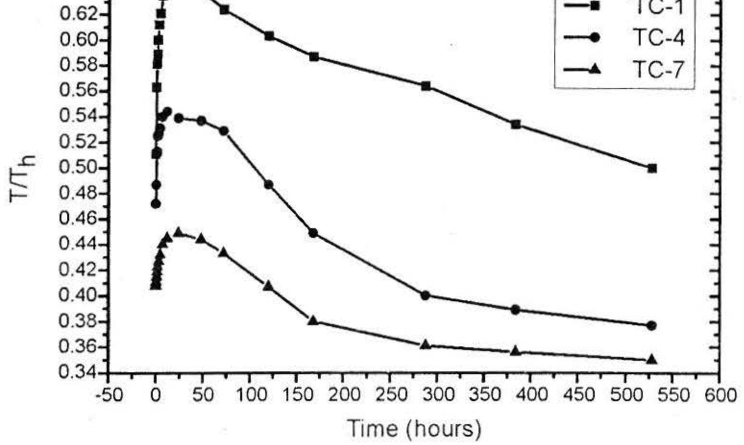

FIGURE 7 : Comparison of Temperature in Vertical Direction (Composite Liner with 50 mm Thick CCL))

Finite Element Modelling of Heat Flow

The finite element method (FEM) is a widely accepted numerical procedure for solving the differential equations of engineering and physics and is the computational basis of many computer-aided design systems. The finite element formulation is given for the two-dimensional steady state heat flow (Eqn.3) including source term (Q) using linear triangular elements. Equation 3 can be derived by imposing the principle of conservation of heat energy over an arbitrary fixed volume V of the medium, which is bounded by a closed surface S. The two-dimensional steady state heat flow equation can be written as (Lewis et al., 1996)

$$k_x \frac{\partial^2 T}{\partial x} + k_y \frac{\partial^2 T}{\partial y} + Q = 0 \tag{3}$$

where T is the temperature, k_x and k_y are the thermal conductivities in x- and y-directions respectively, and Q is an internal heat source or sink. The internal source or sink must exist along the entire z-direction for the heat transfer to be two-dimensional.

Triangular Elements

The linear triangular element shown in Fig.8 has straight sides and three nodes i, j and k. A consistent labelling of the nodes is a necessity and

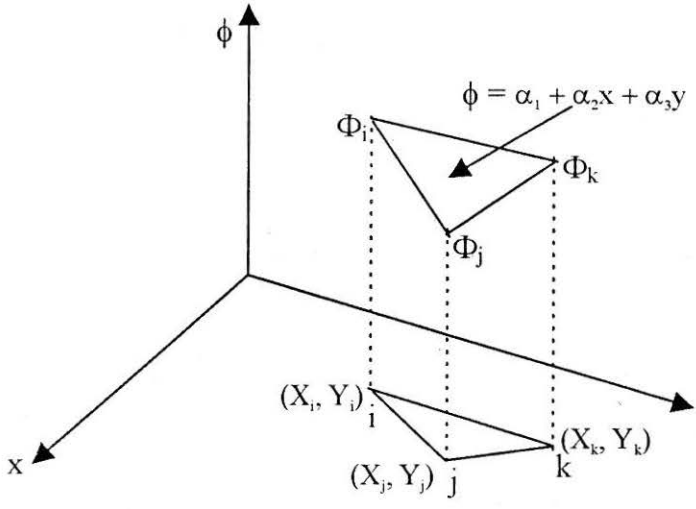

FIGURE 8 : Linear Triangular Element

the labelling here proceeds counterclockwise from node i, which is arbitrarily specified. The nodal values of ϕ are Φ_i, Φ_j and Φ_k whereas the nodal coordinates are (X_i, Y_i), (X_j, Y_j) and (X_k, Y_k). The value ϕ here represents the scalar quantity, i.e., temperature (T).

The interpolation polynomial is

$$\phi = \alpha_1 + \alpha_2 x + \alpha_3 y \tag{4}$$

with the nodal conditions

$$\begin{aligned} \phi &= \Phi_i && \text{at } x = X_i,\ y = Y_i \\ \phi &= \Phi_j && \text{at } x = X_j,\ y = Y_j \\ \phi &= \Phi_k && \text{at } x = X_k,\ y = Y_k \end{aligned}$$

substitution of these conditions into Eqn.4 produces the system of equations

$$\begin{aligned} \Phi_i &= \alpha_1 + \alpha_2 X_i + \alpha_3 Y_i \\ \Phi_j &= \alpha_1 + \alpha_2 X_j + \alpha_3 Y_j \\ \Phi_k &= \alpha_1 + \alpha_2 X_k + \alpha_3 Y_k \end{aligned} \tag{5}$$

which yields

$$\begin{aligned} \alpha_i &= \frac{1}{2A}\left[\left(X_j Y_k - X_k Y_j\right)\Phi_i + \left(X_k Y_i - X_i Y_k\right)\Phi_j + \left(X_i Y_j - X_j Y_i\right)\Phi_k\right] \\ \alpha_2 &= \frac{1}{2A}\left[\left(Y_j - Y_k\right)\Phi_i + \left(Y_k - Y_i\right)\Phi_j + \left(Y_i - Y_j\right)\Phi_k\right] \\ \alpha_3 &= \frac{1}{2A}\left[\left(X_k - X_j\right)\Phi_i + \left(X_i - X_k\right)\Phi_j + \left(X_j - X_i\right)\Phi_k\right] \end{aligned}$$

where the determinant

$$\begin{vmatrix} 1 & X_i & Y_i \\ 1 & X_j & Y_j \\ 1 & X_k & Y_k \end{vmatrix} = 2A \tag{6}$$

and A is the area of the triangle.

Substituting for α_1, α_2 and α_3 in Eqn.4 and rearranging produces an equation for ϕ in terms of three shape functions Φ_i, Φ_j and Φ_k that is

$$\phi = N_i\Phi_i + N_j\Phi_j + N_k\Phi_k \tag{7}$$

where

$$\begin{aligned} N_i &= \left[a_i + b_i x + c_i y\right] \\ N_2 &= \left[a_j + b_j x + c_j y\right] \\ N_3 &= \left[a_k + b_k x + c_k y\right] \end{aligned} \tag{8}$$

and

$$\begin{aligned} a_i &= X_j Y_k - X_k Y_j, \quad b_i = Y_j - Y_k \quad \text{and} \quad c_i = X_k - X_j \\ a_j &= X_k Y_i - X_i Y_k, \quad b_j = Y_k - Y_i \quad \text{and} \quad c_j = X_i - X_k \\ a_k &= X_i Y_j - X_k Y_i, \quad b_k = Y_i - Y_j \quad \text{and} \quad c_k = X_j - X_I \end{aligned}$$

The scalar quantity ϕ is related to the nodal values by a set of shape functions that are linear in x and y. This means that the gradients $\partial\phi/\partial x$ and $\partial\phi/\partial y$ are constant within the element. For example,

$$\frac{\partial\phi}{\partial x} = \frac{\partial N_i}{\partial x}\Phi_i + \frac{\partial N_j}{\partial x}\Phi_j + \frac{\partial N_k}{\partial x}\Phi_k \tag{9}$$

but

$$\frac{\partial N_\beta}{\partial x} = \frac{b_\beta}{2A} \qquad \beta = i, j, k$$

Therefore

$$\frac{\partial\phi}{\partial x} = \frac{1}{2A}\left[b_i\Phi_i + b_j\Phi_j + b_k\Phi_k\right] \tag{10}$$

since b_i, b_j and b_k are constants, Φ_i, Φ_j, Φ_k are independent of the space coordinates and the derivative has a constant value. A constant gradient within any element means that many small elements have to be used to accurately approximate a rapid change in ϕ.

The element contribution to the system of equations $\{R^{(e)}\}$ using Eqn.3, considering the thermal conductivities in x and y direction, is given by

$$\left\{R^{(e)}\right\} = \int_A [N]^T \left(k_x \frac{\partial^2 T}{\partial x^2} + k_y \frac{\partial^2 T}{\partial y^2} + Q \right) dA \tag{11}$$

where [N] is the row vector containing the element shape functions. Since the interpolation function T(x, y), does not have continuous derivatives between the elements, the second-derivative terms in Eqn.11 must be replaced by first-derivative terms.

The second derivative terms in Eqn.11 can be replaced by applying the product rule for differentiation. Consider the quantity

$$\frac{\partial}{\partial x}\left\{[N]^T \frac{\partial T}{\partial x}\right\} \tag{12}$$

differentiation gives

$$\frac{\partial}{\partial x}\left\{[N]^T\right\} = [N]^T \frac{\partial^2 T}{\partial x^2} + \frac{\partial [N]^T}{\partial x}\frac{\partial T}{\partial x} \tag{13}$$

Rearranging and substituting for $[N]^T \partial^2 T/\partial x^2$ in Eqn.11 produces

$$-\int_A \left\{[N]^T k_x \frac{\partial^2 T}{\partial x^2}\right\} dA = -\int_A \left\{k_x \frac{\partial}{\partial x}\left([N]^T \frac{\partial T}{\partial x}\right)\right\} dA + \int_A \left\{k_x \frac{\partial [N]^T}{\partial x}\frac{\partial T}{\partial x}\right\} dA \tag{14}$$

The first integral on the right-hand side of Eqn.14 can be replaced by an integral around the boundary using Green's theorem. Application of the theorem and making substitution for second order terms in Eqn.11 along with $T^{(e)} = [N]\left\{T^e\right\}$ yields

$$\left\{R^{(e)}\right\} = -\int_\Gamma \left\{[N]^T \left(k_x \frac{\partial T}{\partial x}\cos\theta + k_y \frac{\partial T}{\partial y}\sin\theta\right)\right\} d\Gamma + \left\{T^e\right\}\int_A \left(k_x \frac{\partial [N]^T}{\partial x}\frac{\partial N}{\partial x} + k_y \frac{\partial [N]^T}{\partial y}\frac{\partial N}{\partial y}\right) dA - \int_A Q[N]^T dA \tag{15}$$

where θ is the angle to the outward normal and Γ is the element boundary. Equation (15) is close to the desired form and has the general form

$$\left\{R^{(e)}\right\} = \left\{I^{(e)}\right\}+\left[k^{(e)}\right]\left\{T^{e}\right\}-\left\{f^{(e)}\right\} \tag{16}$$

where
$$\left\{I^{(e)}\right\} = -\int_{\Gamma}\left\{[N]^{T}\left(k_x\frac{\partial T}{\partial x}\cos\theta+k_y\frac{\partial T}{\partial y}\sin\theta\right)\right\}d\Gamma \tag{17}$$

$$\left[k^{(e)}\right] = +\int_{A}\left(k_x\frac{\partial[N]^{T}}{\partial x}\frac{\partial N}{\partial x}+k_y\frac{\partial[N]^{T}}{\partial y}\frac{\partial N}{\partial y}\right)dA \tag{18}$$

$$\left\{f^{(e)}\right\} = -\int_{A}Q[N]^{T}\,dA \tag{19}$$

The finite element formulation presented above has been used to simulate the heat flow through experimental landfills, which were considered for the physical modelling of heat flow in the previous section. The results and discussion of the finite element analysis of heat flow simulation are presented in the subsequent sections.

FEM Analysis of Model Tests

In this section, two experimental landfills with CCL of 50 mm and 100 mm thickness are considered to study the heat flow through liners using the finite element formulation given in the above section. A computer code HEAT2D in FORTRAN 77 was developed and is used to simulate the heat flow through the liners. A flow chart of the two-dimensional heat flow simulation process is given in Fig.9.

Results

The two cases of 1-g experimental results obtained on landfills tested in the laboratory are considered. In the first case, a 50 mm thick compacted clay liner underlain by the silty soil and in the second case, a 100 mm thick compacted clay liner underlain by the silty soil are considered. In both the cases, the heater was placed 20 mm above the liner and heater temperature was maintained at 75°C. The finite element discretisation for the first case is shown in Fig.10, where element numbers are circled and node numbers are shown with bullet marks. Only half the portion of the experimental landfill is considered because of the line of symmetry (boundary along nodes 1–66). Zero flux boundaries are used along the nodes 1–66, bottom boundary (nodes 66–70) and side boundary (nodes 70–5). Along the ground surface

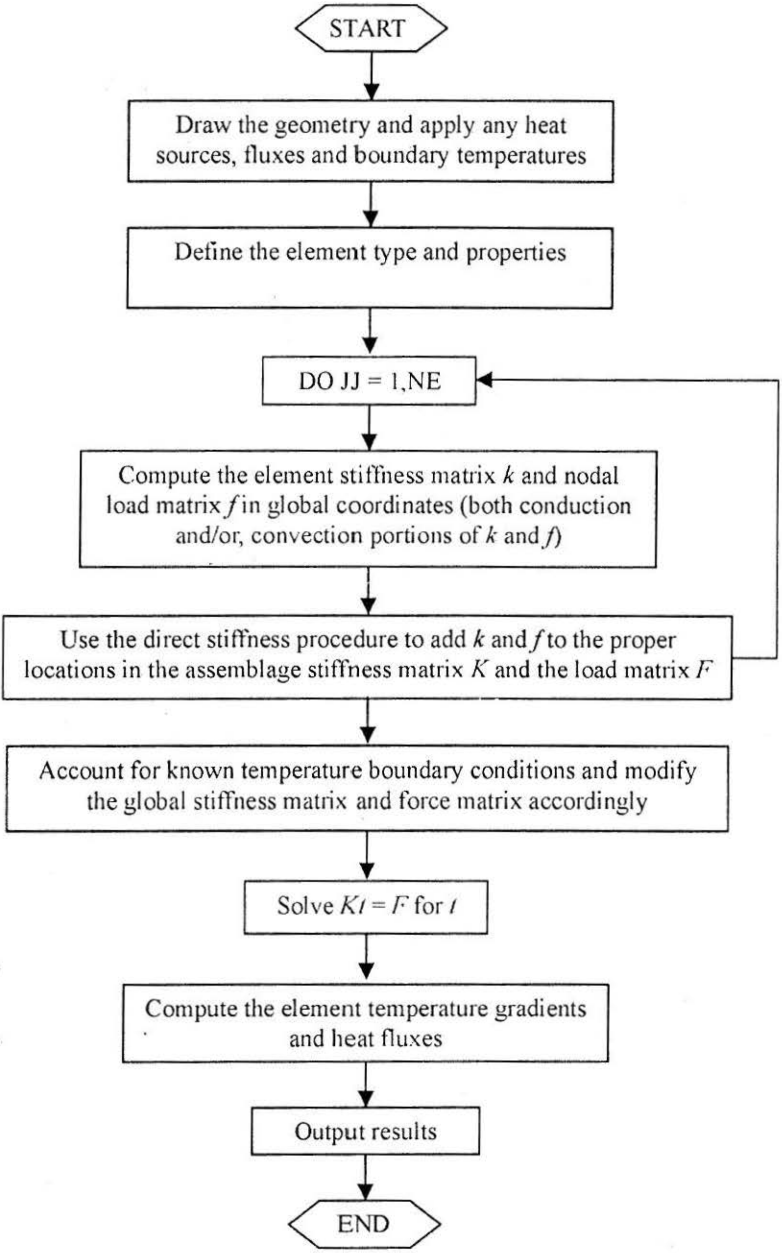

FIGURE 9 : Flowchart for Two-dimensional Heat Transfer Process

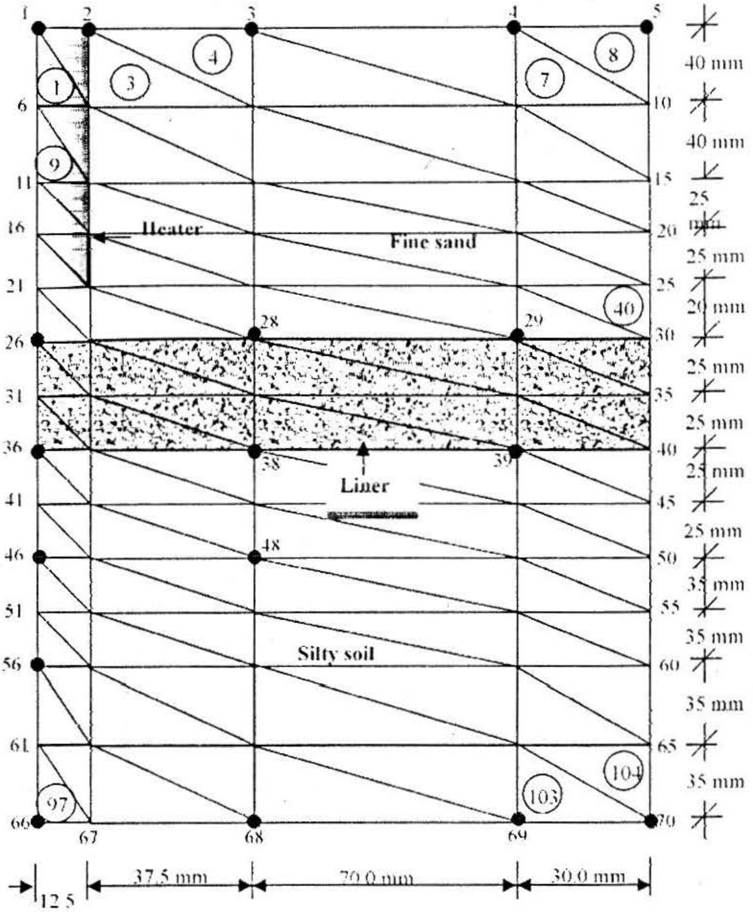

FIGURE 10 : F E Discretisation: CCL with Thickness 50 mm (Nodes 26, 28, 29, 36, 38, 39, 46, 48 and 56 are the location of the Thermocouples)

(nodes 1–5) a specified room temperature of 25°C is used as a boundary condition in the finite element analysis.

The experimental values of temperature distribution are compared with the results of finite element analysis and are presented in Table 5 for 50 mm thick CCL. The experimental values of the second case are also compared with the results of finite element analysis and are presented in Table 6. The thermal conductivities of heater material, fine sand, liner and silty soil are respectively, 0.015, 0.00836, 0.00548 and 0.00703 W/cm °C. The thermal conductivities of fine sand, liner and silty soil are obtained by the known values of their parameters. In both the cases, finite element analysis

Table 5 : Comparison of Experimental and F E Results

(Heater Temperature = 75°C, CCL Thickness = 50 mm)

Node No. / TC	Temperature (°C)	
	Experimental	FEM
26 (TC-4)	46.8	53.257
28 (TC-5)	42.5	43.911
29 (TC-6)	30.7	30.307
36 (TC-7)	33.2	38.734
38 (TC-8)	32.8	36.058
39 (TC-9)	29.3	28.419
46 (TC-10)	29.3	32.682
48 (TC-11)	28.9	31.522
56 (TC-12)	26.6	28.124

Table 6 : Comparison of Experimental and F E Results

(Heater Temperature = 75°C, CCL Thickness = 100 mm)

Node No. / TC	Temperature (°C)	
	Experimental	FEM
26 (TC-4)	45.9	52.754
28 (TC-5)	37.7	38.438
29 (TC-6)	28.0	27.941
36 (TC-7)	26.8	27.416
38 (TC-8)	26.6	27.358
39 (TC-9)	25.6	25.210
46 (TC-10)	25.6	26.943
48 (TC-11)	24.7	26.108
56 (TC-12)	23.6	25.869

overestimates the temperature values by 1 to 12%. At boundary nodes, it underestimates the temperature values by about 2%. Figures 11 and 12 show the comparison of finite element and experimental results for 50 mm and 100 mm thick CCL respectively. The observed experimental temperatures are considered at the end of 24 hours of continuous heating.

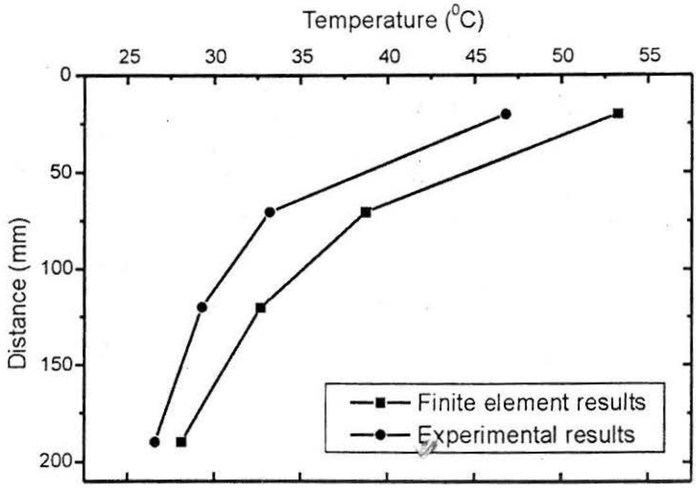

FIGURE 11 : Comparison of Experimental and FEM Results (50 mm Thick CCL only)

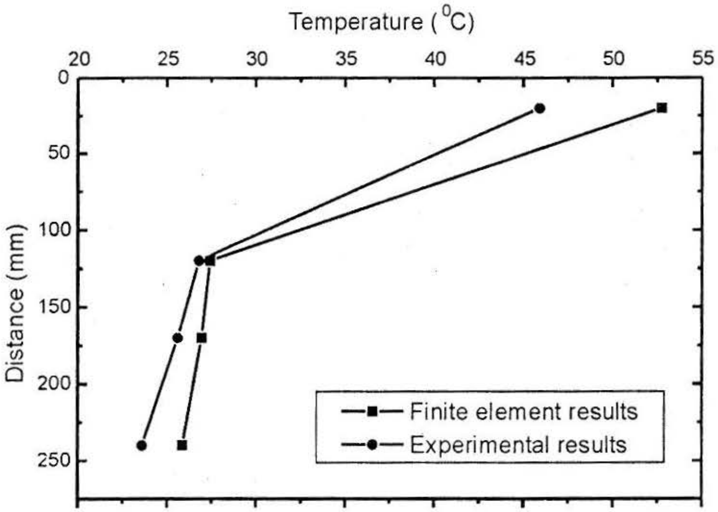

FIGURE 12 : Comparison of Experimental and FEM Results (100 mm Thick CCL only)

Discussion

For both cases of experimental landfills, the temperature values estimated by FEM are presented at the liner top and bottom surface. Knowing the temperatures at nodal points, the temperature contours through the landfill waste, liner and the subsoil can be constructed. The theoretical drop in temperature across the 1 m thick liner is about 4°C for the input temperature of 50°C when the no flux boundary is considered at the bottom and sides. A good agreement of results is obtained between FEM and experiments.

A quantitative exact prediction of desiccation is not possible. This is due to the uncertainties with respect to the thermal vapour diffusion coefficients, hydraulic conductivity to field conditions and the unknown effects of consolidation and shrinkage. In view of this, the finite element method can give an idea of how the temperatures vary across the liner thickness. By knowing the temperatures at the liner top and bottom, a rough estimate can be made for the possible location of potential desiccation cracking along the liners.

Concluding Remarks

The heat flow through landfill basically depends on the thermal conductivity (k), thermal diffusivity (α) and specific heat (C_{ps}) of wastes and the compaction state of the liner including the subsoil. The study presented in the paper has been dealt with the development of a methodology for studying heat flow through CCL and composite liners. The desiccation cracking of the CCL will occur at a temperature of 40°C and above. This cracking of the clay liner is mainly attributed to the moisture migration away from the liner due to temperature gradients. The results of the study indicated that the CCL of a composite liner system is exposed to risk of desiccation when the clay liner is underlain by a permeable subgrade. The desiccation cracking increases both the diffusive and advective contaminant transport through the liner system and can reduce the service life of both geomembrane and underlying CCL. It is concluded that the care must be given to minimising or preventing desiccation both in the short-term and long-term monitoring of landfills.

The most important advantage of the finite element method is that it enables us to approximate, with high confidence, more complicated problems, such as those with more than one thermal conductivity, for which closed form solutions are difficult to obtain. The automation of the FEM through general computer programs makes the method extremely powerful. In practice, it would be anticipated that variations throughout the landfill system would develop with time owing to moisture migrations, local soil variability or other effects. The finite element analysis can be repeated for any assumed variation in the thermal conductivity.

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