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Finite Element Analysis of Buried Flexible Pipes

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Introduction

Behavior of flexible pipes buried in soil is considerably influenced by geotechnical considerations. Pipes are considered as rigid or flexible and specified in terms of diameter to thickness ratios of the order of 100 to 250. Design of buried pipes involves use of internal pressure considerations based on which the diameter and thickness are arrived. The performance and stability evaluations are performed in terms of allowable deflection limit and buckling resistance. Backfill properties and installation conditions affect these performance limits. The most common and well-known method of designs is based on the approach as suggested by Marston over 70 years ago and involves the consideration that soil reaction can be modeled as springs represented by modulus of soil reaction. Many researchers have shown the drawbacks of this theory but still this theory is used for the design purpose because of its simplicity (Moser, 1990; Tohda and Yoshimura, 1997; Davis and Bardet, 1998 and 2000). Tohda and Yoshimura presented some case studies of failures of buried pipelines and attributed them to the fact that these were designed based on Marston-Spangler's theory and that the theory did not model actual field conditions. The buckling of a pipe occurs when the hoop stress in the pipe exceeds the yield strength of the pipe material. This is due to (a) high D/t ratios (i.e., large diameters and low thickness values), (b) very low pipe stiffness (c) excessive external loads and

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backfill (d) heavy compaction, impact loads, and earthquake loads, and (e) vacuum pressures in the pipe. The allowable buckling pressure given by international codes such as American Water Works Association (AWWA, 1996) is derived from Meyerhof and Baikey (1963) formula, which depends on the idealization that soil is represented as series of springs (Moser, 1990).

In the present context, a number of reasons for developing improved design methods and performance assessment of buried pipes can be highlighted as follows.

1. The assessment of stability and performance is normally done using conventional methods of limit equilibrium and arbitrary factors of safety. These approaches are quite old and proved to be conservative or non-conservative depending on the actual conditions (Selig and Packard, 1987; Jeyapalan et al., 1987).

2. There are considerable advances in understanding soil behaviour, which help in understanding mechanics of load transfer, extent of additional load that can be imposed on the pipe without adversely affecting the stability and performance.

3. There are changes in construction technology, which if modeled properly can give a realistic picture of safety and performance.

4. The cost of pipe installation is high and there is a need to examine the stability and performance in terms of buckling and deflection more precisely.

The objective of this paper is to present a critical appraisal of mechanical behaviour of buried flexible pipes and propose a design methodology for prediction of performance of buried flexible pipes, using finite element analysis. A design chart for the analysis of a pipe section in terms of deflection and buckling is presented. While the methodology presented here is for simple loading and boundary conditions, the approach can be extended to consider the installation processes and site conditions specific to regions or local bodies.

Background Information

Deflection

The modified Iowa formula (Moser, 1990) is the best known and simple equation for the prediction of deflection of buried pipes.

$$\Delta = D_l \frac{K_b W_c r^3}{EI + 0.061E' r^3} \tag{1}$$

where
- D = deflection in mm,
- K_b = bedding constant,
- W_c = vertical load on the pipe (kN/m),
- r = mean radius of the pipe (mm),
- E = modulus of elasticity of the pipe (MN/m^2),
- I = moment of inertia in (mm^4/mm),
- E' = modulus of soil reaction (MN/m^2) and
- D_l = Deflection lag factor.

The deflection lag factor (D_l) is the ratio of initial deflection to final deflection of the pipe. Spangler recommended a deflection lag factor of 1.25 to 1.5 to incorporate the effect of long-term deflections in flexible pipes. This relationship is developed based on the pressure distribution assumed by Spangler as shown in Fig.1a. Modulus of soil reaction (E') depends on the backfill material and degree of compaction. The values of E' are usually adopted based on the Howard (1977) results. The limitations of Iowa deflection equation are summarised as follows (Jeyapalan and Boldon, 1985; Jeyapalan et al., 1987):

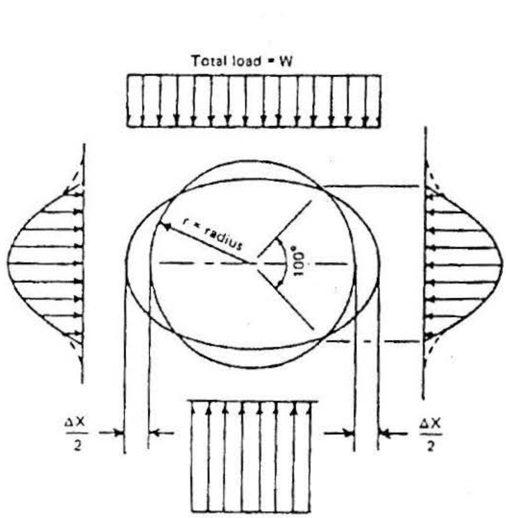

FIGURE 1a : Pressure Distribution Around the Pipe (Spangler and Handy, 1973)

- Equation was developed for corrugated metal pipes, it needs to be re-examined for other pipes.
- Vertical deflection is not equal to horizontal deflection.
- The pressure distribution on the top of the pipe is not uniform as assumed by Spangler.
- Soil-pipe interaction is not considered.
- Very flexible pipes are being manufactured for pipe stiffness values as low as 35 kPa, which is significantly lower than those pipes tested by Spangler.
- Construction induced deflection like installation and compaction efforts govern the performance of flexible pipes.

Buckling

Buckling is a general phenomenon that occurs in thin walled pipes whose D/t ratios are high. The pipe fails because of lack of stiffness. Local buckling may occur particularly if the pipe is sufficiently restrained to prevent excessive deflections. Hence, it is necessary to calculate the buckling pressure under which the pipe buckles. A number of equations are available for the calculation of critical buckling pressure. The earliest relationship is from Timoshenko and Goodier (1951), which gives critical buckling pressure as $p_{cr} = \dfrac{3EI}{\left(1-\nu_p^2\right)r^3}$ where EI is called flexural rigidity of the pipe and 'r' is the radius of the pipe. For pipes buried at cover depths greater than 1.5 m, the value of p_{cr}, is calculated from (Meyerhof and Baikey, 1963), given by the equation $p_{cr} = \sqrt{\dfrac{32E'EI}{D^3}}$, where E' is the modulus of subgrade reaction and D is the diameter of the pipe. These two relationships are well known and well accepted and are presented in codal provisions such as AWWA (1996) and the AWWA formula is given by,

$$q_a = \frac{1}{FS}\left(32R_w B'E'\frac{EI}{D^3}\right)^{1/2} \tag{2}$$

where q_a = allowable buckling pressure,

R_w = water buoyancy factor,

B′ = dimensionless empirical coefficient of elastic support, and

FS = design factor of safety (equal to 1.0).

Parameters E′, EI, and D are as described earlier. Smith and Young (1991) indicate that the assumption that the soil represented as a series of springs as Wrinkler's model in Spangler's theory of deflection and in the approach of Meyerhof and Baikey is imperfect. These methods involve the measurements of soil stiffness in terms of modulus of subgrade reaction, which is a property of pipe soil system rather than that of the soil alone. Therefore, this parameter E′ is unreliable in most of the situations. The spring model considers the pipe to be a main structural component and assumption of some form of pressure distribution around the pipe is considered. This is a major disadvantage contributing to the inaccuracy of the above methods. In addition, shear interaction between the springs is ignored. Hence, the assumption that soil acts as an isotropic elastic medium is more realistic than the Wrinkler's model based on spring analogy (Smith and Young, 1991). Katona (1978) has shown that the predicted response of a buried pipe in a non-linear backfill is of the same order as that predicted by a linear model. Chang et al. (1980) concluded that in view of the uncertainties involved in pipe soil system, a linear model is as good as any other model for the design of a buried flexible pipe to get insight into the actual stresses and deformations in soil-pipe interaction within the working stress range.

Recent Developments

Continued research and development in the pipe materials indicated that mechanical behaviour of pipe is essentially controlled by soil stiffness (E_s), pipe stiffness (E_p), depth of burial (h), unit weight of soil (γ) etc. Recognizing the role of the pipe stiffness and soil stiffness, considerable developments have been made in the design of flexible pipes. The role of the soil surrounding the pipe in contributing to the mechanical behaviour of the pipe is realized. The ability of bending of pipes expressed in terms of D/t ratios is expanding in concurrence with the development and use of suitable materials. Some of the approaches and contributions (Hoeg, 1968; Gumbel, 1983; Davis and Bardet, 1998 and 2000; Tohda and Yoshimura, 2001) that aided the understanding of behaviour of flexible pipes are reviewed in the following sections.

Hoeg (1968)

Hoeg examined the stresses in buried pipes both experimentally and analytically. Experiments were conducted on steel tube of 115 mm diameter in sand and the buried cylinders had two different D/t ratios of 40 and 80. The experiments were conducted at different depths of sand cover above the top of the crown of the cylinder. A pressure of 1000 kN is applied on the top. The deformations as well as contact pressures were measured. He presented analytical solutions for the deformation and distribution of stresses in an elastic medium. Hoeg's analysis showed good agreement with experimental results.

Gumbel (1983)

Gumbel developed charts for the calculation of deflection and buckling of the buried pipes. The basic parameters used in the design procedure are:

1) pipe properties such as flexural stiffness (S_f) and D/t ratio,

2) soil property such as elastic modulus (E_s),

3) external loads uniform and distortional components P_z and P_y respectively,

4) Performance criteria: allowable deflection, factor of safety against buckling.

Gumbel has developed charts for different values of load distribution parameter ranging from 0.05 to 0.8, for an arching coefficient (α) of 1.0. For the known value of P_y and P_z the values of E_s and S_f for both the stiffness and stability of the system are obtained by a single entry on the appropriate design chart. The design of backfill for a given pipe as well as design of pipe section for a given backfill.

Davis and Bardet (2000)

Davis and Bardet introduced a simple method of analysis of buried pipes, considering the equilibrium state of the soil around the pipe, which gives both horizontal and vertical strain. The horizontal and vertical pipe strains are determined from the Mohr's strain circle. From the vertical strain, the vertical deflection can be calculated. This analysis also enables calculation of pipe load and maximum hoop force N_{max}.

Tohda and Yoshimura (2001)

Tohda and Yoshimura proposed design charts for the design of buried pipes considering the dimensionless parameters such as stiffness ratio (k), flexural stiffness of the pipe (S_p) and deflection ratio (ω). This analysis has been carried out for rigid, medium and flexible aluminum pipes for different types of buried conditions. In Indian context, steel pipes are being increasingly used and hence the charts of Tohda and Yoshimura are not applicable. They have not considered the buckling response in terms of developed hoop stresses, which are critical in earthquake prone areas wherein the pipes often fail by buckling (Davis and Bardet, 1998)

Performance Limits

The use of empirical approaches such as Spangler's equation and the use of theoretical approaches presented earlier yields the actual stress and deformation in the pipes. The results from the above equations are compared with the performance limits expressed in terms of tolerable deflection (Moser, 1990) and critical buckling pressure (Moore, 1989). The critical buckling pressure is evaluated from Timoshenko's buckling formula if depth of burial is less than 1.5 m. If the depth is more than 1.5 m critical buckling pressure is calculated using Meyerhof's buckling formula. In the recent times use of Moore's (1989) equation (Eqn.3) based on elastic continuum approach is well established for finding the critical hoop force.

$$N_{cr} = 0.66\left(E_p I\right)^{1/3}\left\{\frac{E_s}{1-\nu_s^2}\right\}^{2/3} \tag{3}$$

To assess performance in terms of deflection permissible deflection is taken as 5% of diameter of the pipe and that of buckling by Moore's elastic continuum equation. The numerical values of allowable deflection and critical hoop force are presented in Table 1.

The above sections give a brief summary of existing methods to evaluate stresses and deformation in pipe as well as their performance limits. The methods followed in codes are Spanglers' approach for deflection and Meyerhof and Baikie formula for buckling. The other methods proposed by Hoeg, Gumbel and Davis & Bardet are derived based on analytical considerations and are not followed widely and hence their general validity

TABLE 1 : Performance Limits of Deflection and Buckling

Failure mode	Critical / Allowable limit	Reference	Value (Present Study)
Deflection	5% of diameter	Moser (1990)	60 mm
Critical buckling / hoop force	Equation (3)	Moore (1989)	Varies with pipe thickness 0.36 to 2.11 MN

is not completely established. The review presented also shows that studies based on elastic continuum approach are more accurate and valid for the analysis rather than those developed based on spring analogy for soil support as was given by Smith and Young (1991). Hence to get insight into pipe soil behaviour, numerical analysis using finite element method is conducted and the results are examined in detail.

Method of Analysis

Numerical analysis using finite element method to evaluate stresses and strain has become a powerful technique in the recent years. In the present study this technique is used to develop frame work for prediction of deformation and buckling responses covering different ranges of pipe and soil stiffness that reflects the characteristics of the steel pipes buried in soil. Standard commercial finite element program, Numerically Integrated elements for System Analysis, NISA (1998) is used for the analysis.

Framework for Analysis

The approach of Tohda and Yoshimura (2001) is possible to examine the behavior in terms of parameters that are specific to steel pipe and soil properties. The dimensionless parameters such as stiffness ratio (k), flexural stiffness of the pipe (S_p), hoop stress ratio (H_r) and deflection ratio (ω) are given by,

$$k = \frac{E_s}{S_p} \tag{4}$$

$$S_p = \frac{E_p t^3}{\left\{12\left(1-\nu_p^2\right)r^3\right\}} \tag{5}$$

$$H_r = \frac{\sigma_h}{\gamma h} \tag{6}$$

$$\omega = \frac{\delta E_s}{\gamma h} \tag{7}$$

The results of the analysis are useful in the development of design chart for an identified backfill material and pipes of different stiffness that are considered appropriate in a given locality. It is also useful to predict the deflection and hoop stress in the steel pipe.

Table 2 : Validation of the Finite Element Model (Stresses in Pipe in kPa)

Inclination, q from crown (degrees)	Finite Element Analysis of Duns and Butterfield, 1971	Savin's Stress Function Duns and Butterfield, 1971	Results from the present study
0°	490	545	490
90°	889	827	903

Result and Discussion

Validation

Initially validation of the problem is done with reference to the closed form solution for buckling provided by Duns and Butterfield (1971), for a concrete pipe (D = 150 mm, E_p = 199.48 MPa, ν_p = 0.33, D/t = 50) buried in soil (E_s = 68.95 kPa, ν_s = 0.3) buried at a depth of 150 mm from the ground surface. Plane strain analysis with a four-noded quadrilateral element is used for the analysis. Both horizontal and vertical movements are restrained at the bottom boundary and horizontal movements are restrained on the side boundaries. No external loads are applied to the pipe and in order to get critical condition the pipe is assumed to be running empty. Validation results are presented in Table 2. Once the geometry is validated, a detailed numerical analysis was carried out to examine the stresses and deformations developed in buried flexible steel pipes. Typical sketch of the buried pipe considered in the present study is given in Fig.1b. The finite element grid used for the present study is given in Fig.1c. The analysis was conducted for different

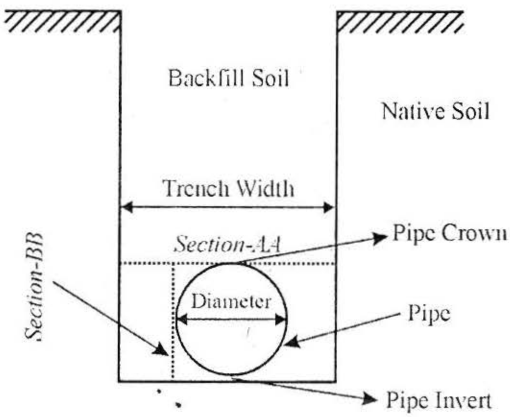

FIGURE 1b : Definition Sketch of Trench buried Pipe

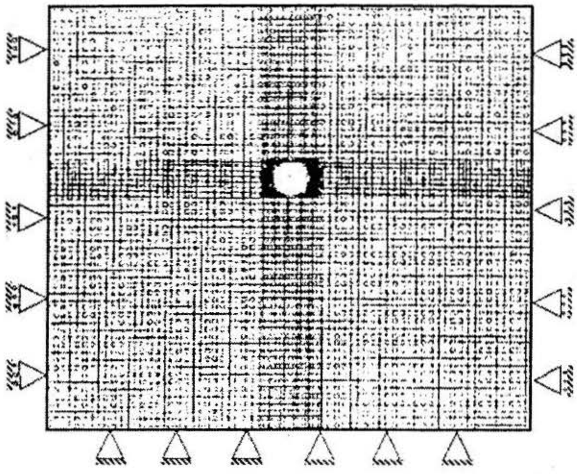

FIGURE 1c : Finite Element Grid and Boundary Conditions

h/D ratios, where h is the height of the soil above the top of the pipe, and D is the outside diameter of the pipe. Two different types of backfill corresponding to loose and dense states are considered. The properties of pipe, backfill soil and native soils are given in Table 3 and Table 4.

Comparison

The deflection values in mm, obtained from the analysis for loose sand are compared with the values obtained from different methods and are given in Table 5. It can be observed that the differences are marginal depending on the method used. The values of the present study are comparable to the values obtained from other approaches and are in the same range.

Pressure Distribution Around the Pipe

The pressure distribution all around the pipe for both loose and dense sands is shown in Fig.2. The tangential stresses around the pipe-soil interface

Table 3 : Properties of the Pipe

Diameter (mm) D	1200
Thickness (mm) t	6, 8, 10, 12
Modulus of elasticity (Pa) E_p	210×10^9
Poisson's ratio ν_p	0.30

Table 4 : Soil Properties

Property	Native Soil	Backfill Soil (Loose Sand)	Backfill Soil (Dense Sand)
Bulk density (kN/m^3) g	20.00	13.17	15.43
Soil modulus (MPa) E_s	6.77	5.98	30.00
Cohesion (kPa) c	20.00	0.00	0.00
Angle of internal friction (°) ϕ	20	32	42
Poisson's ratio ν_s	0.21	0.30	0.30

Table 5 : Deflection in mm Obtained from Different Approaches

h/D	Present study	Davis & Bardet (2000)	Gumbel (1983)	Spangler and Handy (1973)
1	6.74	3.29	3.80	3.22
2	8.51	4.65	5.40	5.87
3	10.12	6.58	11.40	8.04
4	12.07	7.78	15.27	9.82

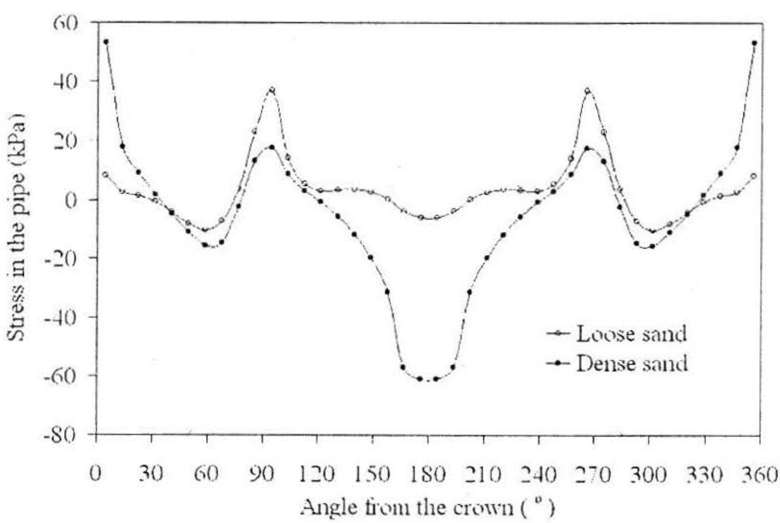

FIGURE 2 : Pressure Distribution Around the Pipe.

contribute to differences in the estimation of vertical loads and horizontal loads, which leads to difference in actual performances (Shinulevich et al., 1985). The figure shows that the pressure variation in the case of dense sand is more pronounced compared to that of loose sand. Maximum pressures are obtained at angles 0°, and 180° from the crown. The pattern of pressure variation shows a good agreement with the trend shown by Duns and Butterfield (1971) and Tohda et al. (1997).

Vertical Pressure on Top of the Pipe

Pressure distribution along the section-AA (shown in Fig.1a) on top of the pipe needs to be examined to assess the extent of load transfer on the pipe. In most of the cases, the calculated total load on the pipe was reported to be different than that of the load calculated using the method of Spangler and Handy (1973). Figures 3 and 4 show the variation of vertical pressure acting on the top of the pipe for different h/D ratios for both loose and dense sand respectively. It can be observed that the earth pressure distribution in loose and dense sand is different from the uniform pressure distribution assumed in Spangler's theory. The pressure in the case of dense sand is much higher compared to that of loose sand for all h/D ratios.

Horizontal Pressure on the Sides

Horizontal pressure distribution along the vertical section-BB as shown

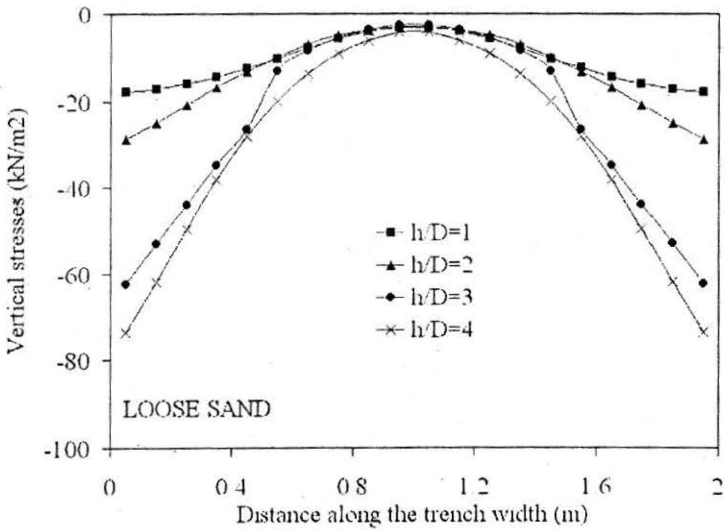

FIGURE 3 : Vertical Pressure on the Top of the Pipe from the Soil (Loose Sand)

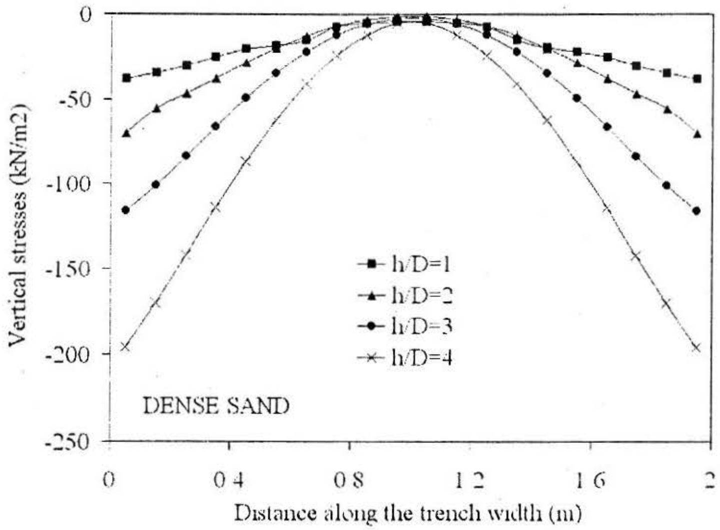

FIGURE 4 : Vertical Pressure on the Top of the Pipe from the Soil (Dense Sand)

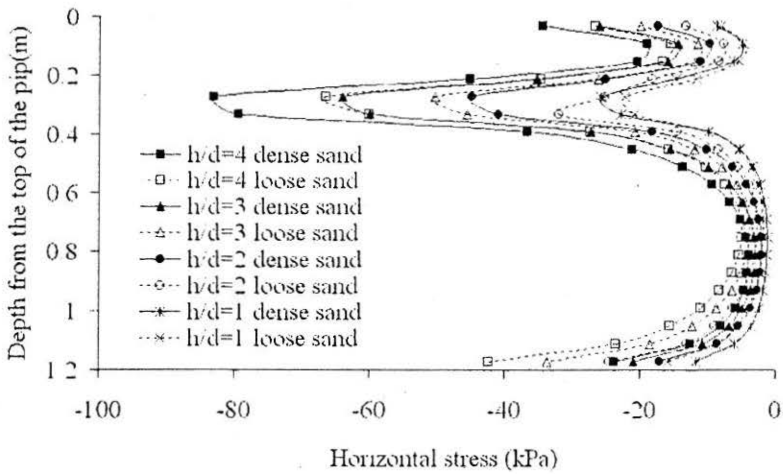

FIGURE 5 : Horizontal Pressure on the Sides of the Trench in the Pipe Zone

in Fig.1a is considered for different h/D ratios. Figure 5 shows the horizontal pressure distribution on the sides of the trench for both loose and dense back fills. The distribution shows maximum and minimum pressure values at 0.30 m from crown and 0.90 m respectively in contrast to the parabolic distribution that was assumed in the Spangler's theory.

Table 6 : Arching Coefficient for Various h/D Ratios in Loose and Dense Sand Backfill

h/D	Arching Coefficient (a)	
	Loose Sand	Dense Sand
1.00	0.62	1.76
2.00	0.43	1.50
3.00	0.36	1.42
4.00	0.30	1.34

Arching Coefficient (α)

Arching coefficient denotes the extent of load transfer on the pipe from the overburden above and is expressed as the ratio of the actual vertical force to the weight of the soil prism above. Values ranging from 0.70 to 1.30 were reported in literature depending on the trench and embankment conditions. Moser (1990) recommends a value of 1.0 for the arching coefficient. Table 6 shows that as h/D ratio increase the arching coefficient value decreases in case of both dense and loose sand. The analysis shows that in the case of loose sand the load coming on the pipe is lesser than that of actual load. This phenomenon is because of higher settlement of backfill soil compared to that of native soil. But in the case of dense sand because of the higher soil modulus of backfill soil (30 MPa) compared to native soil (6.77 MPa) settlement of backfill soil is less compared to that of native soil, which will create a downward shear on the sides. Due to this downward shear force the total load coming on the pipe is more than that of the actual load.

Figure 6 gives the variation of deflection for loose sand with stiffness ratio for different h/D ratios. It is observed that the variation in deflection with respect to increasing stiffness ratio or decrease in thickness is not significant and the deflection obtained is considerably lower than allowable limit of 5% of diameter, Moser (1990). Figure 7 shows the variation of hoop stress with stiffness ratio this shows that hoop stress increases with increase in stiffness ratio or decrease in thickness of the pipe. The results for the case of dense sand are presented in Tables 7a and 7b, which show the variation of deflection and hoop stress for different h/D ratios. It is inferred that the deflection and hoop stress values are slightly higher in comparison to the loose sand values for all the h/D ratios considered.

In order to develop design charts the pipe soil system response is expressed in terms of normalised parameters given by stiffness ratio,

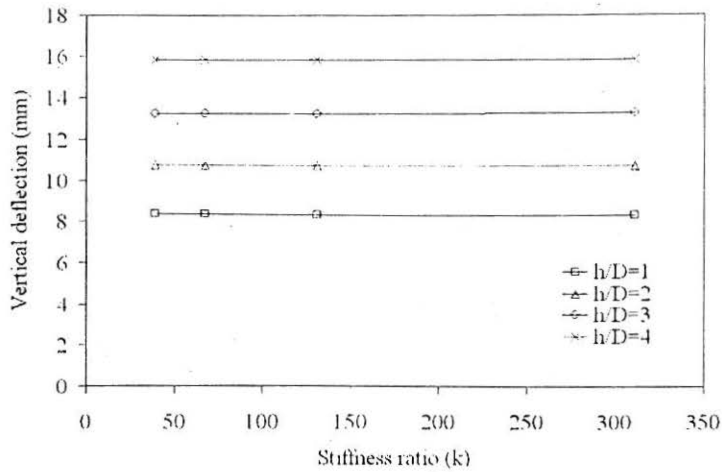

FIGURE 6 : Variation of Maximum Deflection with Stiffness Ratio

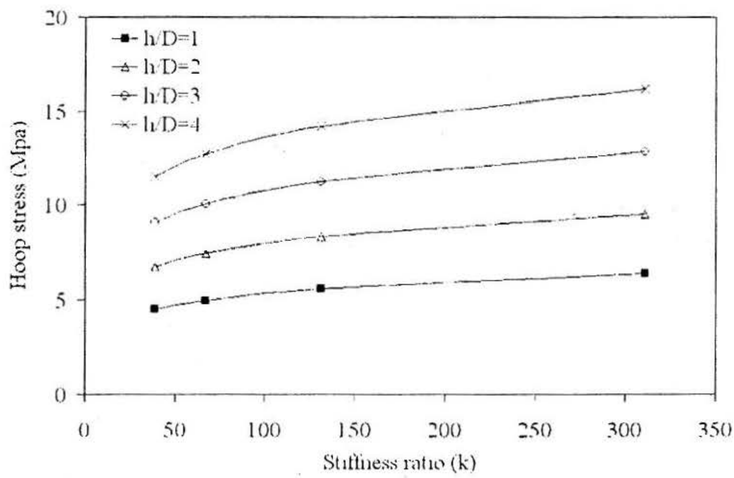

FIGURE 7 : Variation of Maximum Hoop Stress with Stiffness Ratio

Table 7a : Defection in mm (Loose Sand)

Stiffness ratio	h/D = 1.0	h/D = 2.0	h/D = 3.0	h/D = 4.0
1560	8.376	10.762	13.285	15.863
658.125	8.369	10.759	13.279	15.854
336.96	8.365	10.758	13.277	15.847
195	7.785	10.143	12.624	15.207

Table 7b : Hoop Stress in MPa (Loose Sand)

Stiffness ratio	h/D = 1.0	h/D = 2.0	h/D = 3.0	h/D = 4.0
1560	7.789	12.475	17.67	23.235
658.125	7.117	11.579	16.47	21.595
336.96	6.649	10.318	15.413	20.24
195	6.255	8.619	12.475	16.195

normalised hoop stress and deflection ratio, which are defined by Eqns.4, 6 and 7. The normalised deflection and hoop stress are provided with respect to stiffness ratio as given in Figs. 8 and 9. It is implied that h/D ratio has considerable influence on hoop stress mobilization. This trend is also reflected in the normalised plot given with deflection ratio. In the design charts of present study for both loose and dense sands are considered useful to estimate the deflection and hoop stress of steel pipe buried in soil in a specified range of stiffness ratio. The use of the proposed design charts is also illustrated with a typical example.

The following are the properties considered for pipe and backfill soil for the calculations.

Pipe properties: Steel pipe ($E_p = 210 \times 10^9$ N/m^2) Diameter of 1.6 m and thickness of 10 mm

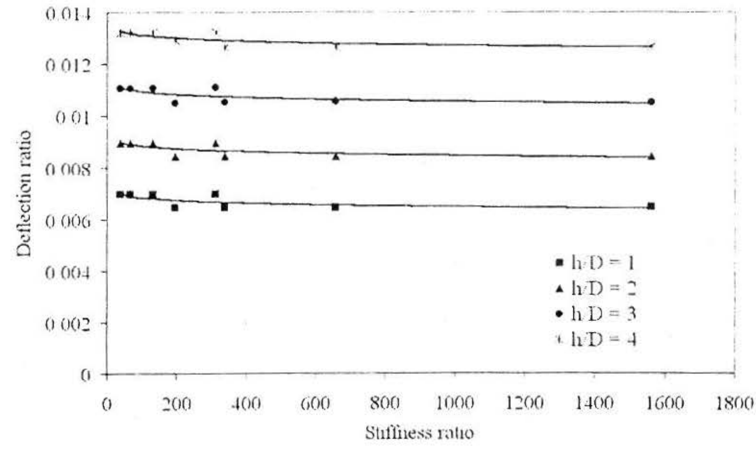

FIGURE 8 : Variation of Maximum Vertical Deflection Ratio with Stiffness Ratio (for h/D = 1, 2, 3 and 4)

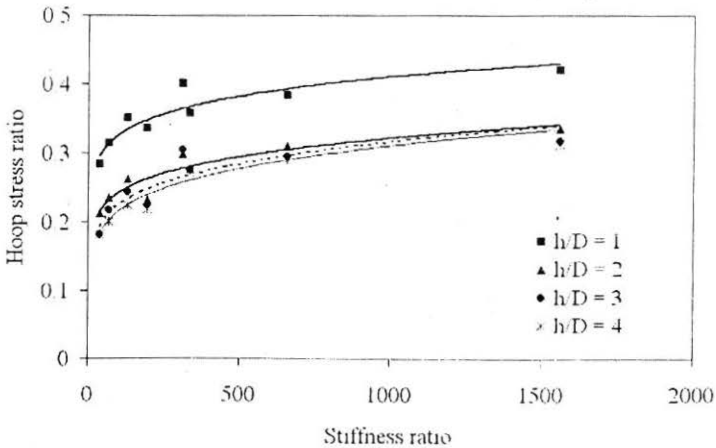

FIGURE 9 : Variation of Normalised Maximum Hoop Stress with Stiffness Ratio (h/D = 1, 2, 3 and 4)

Soil properties: Soil with bulk density $\gamma = 18$ kN/m^3 and a soil modulus of $E_s = 15 \times 10^6$ N/m^2.

The h/D ratio is taken as 1.00. Using the Eqns.4 and 5, stiffness ratio (k) is calculated as 399.36. From Figs.8 and 9 corresponding to the stiffness ratio both deflection and hoop stress are calculated. Using finite element analysis, deflection and hoop stress values are obtained. The results obtained from both the procedures are presented in Table 8.

Concluding Remarks

The paper presents a critical appraisal of methods to analyze the behavior of buried flexible steel pipes and shows that modulus of the backfill significantly affects the behavior. The study also indicates that the assumptions involved in Spangler's formula are not realistic and that predictions of deflections using this formula need to be treated with caution. This observation is in agreement with the results of previous investigators.

Table 8 : Comparison of Deflection and Hoop Stress Values Obtained from Non-Dimensional Charts and NISA

	From figures 8 and 9	Obtained from NISA
Deflection (mm)	9.75	10.12
Hoop stress ($\times 10^6$ N/m^2)	10.316	10.01

The pressure distribution around the pipe is different from that assumed by Spangler to develop Iowa formula. The horizontal pressure acting on a vertical plane is not exactly parabolic and is also not symmetric for upper and lower half of the pipe section.

Design charts have been developed to predict deflection and buckling of flexible steel pipes. Different thickness of pipes in terms of D/t ratios (ranging from 100 to 200) and different h/D ratios that cover the practical range of interest considering loose and dense sands as backfill materials are considered. Three non-dimensional parameters deflection ratio, hoop stress ratio and stiffness ratio are identified to provide design charts considering pipe-soil interaction behaviour using finite element analysis. The use of charts is illustrated with typical example.

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Notations

Δ	=	vertical deflection of the pipe
α	=	arching coefficient
δ	=	change in diameter (D/D)
γ	=	density of the soil
ν_p	=	Poisson's ratio of the pipe material
ν_s	=	Poisson's ratio of the soil
σ_h	=	hoop stress
ω	=	deflection ratio
B'	=	dimensionless empirical coefficient of elastic support

B_d	=	width of the trench
D	=	diameter of the pipe = $2r$
D_l	=	deflection lag factor
E or E_p	=	modulus of elasticity of the pipe material
E'	=	modulus of soil reaction / subgrade reaction
E_s	=	modulus of elasticity of the soil
FS	=	factor of safety
h	=	height of fill above the top of the pipe
H_r	=	hoop stress ratio
I	=	moment of inertia of pipe section ($t^3/12$)
k	=	stiffness ratio
K_b	=	bedding constant
p_{cr}	=	critical buckling pressure
q_a	=	allowable buckling pressure
r	=	radius of the pipe
R_w	=	water buoyancy factor
S_p	=	pipe stiffness
t	=	thickness of the pipe
W_c	=	load coming on to the pipe (Marston's load)