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Technical Note

Earthquake Response of Soil-Structure System

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Introduction

This paper deals with a technique to obtain the time period of vibration of a structure considering the effect of underlying soil medium. The present state of the art uses extensive modelling of soil media based on Finite element technique or Boundary element method and couples this with the super-structure to find out the natural frequency of the coupled soil-structure system. The analysis often becomes too expensive and computationally tedious for the enormous amount of data one needs to generate for a detailed analysis.

The method used herein considers a structure with large degrees of freedom and this can be effectively analysed without resorting to much elaborate soil modelling and yet arrived at a result which is reasonable and effective for practical design engineering practice.

Formulation Based on Single-Degree-of Freedom

The period of vibration of a given structure increases with decreasing soil stiffness. This logical conclusion has been widely noted in the field. As the earthquake response of a structure is dependent on the time period, it is now realised that for calculating the response of a structure considering a fixed base and ignoring the effect of soil can lead to serious errors.

The structure is taken as multi-mode responding in a single-degree-of freedom in its fixed base condition. An idealised single-degree-of freedom system founded on soil medium is shown in Fig.1.

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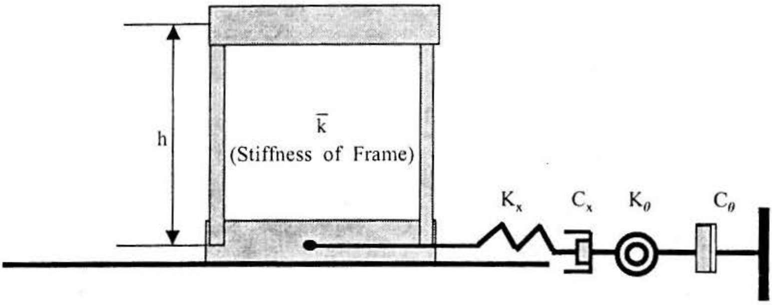


FIGURE 1 : Soil-Structure Interaction Model for System with Single-Degree-of Freedom.

The base excitation of a structure is specified by the free-field motion of the ground surface. Under the influence of such an excitation, the base of the structure will displace x horizontally and, in addition will rotate θ around horizontal axis. The configuration of the coupled system may then be specified by the displacement x(t) and θ(t) and by the inter-floor deformation, u(t). For rigidly mounted structure, x(t) = y(t), the ground displacement, and rotation $\theta(t) = 0$. The height h must then be interpreted as the distance from the base to the centroid of the inertia forces for the assumed mode; and m, k and c must be interpreted as the associated generalised mass, generalised stiffness and generalised damping coefficient

Based on the above information recent version of many international codes (ATC, UBC, FEMA etc.) are all taking this criterion into cognisance and proposing various expressions to predict a modified response of the system. However most of the existing solutions restrict the analysis to a single-degree-of freedom system to predict the modification to fundamental time period. Both ATC and FEMA have proposed an expression (Veletsos and Meek, 1974) for effective building period

$$\bar{T} = T\sqrt{1+\frac{\bar{k}}{K_x}\left(1+\frac{K_x h^2}{K_\theta}\right)} \tag{1}$$

where, $\bar{T}$ = modified time period of the structure due to presence of soil;

T = Fundamental time period of the fixed base structure;

$\bar{k}$ = stiffness of the fixed base structure

$$= \frac{4\pi^2 W}{gT^2}$$

K_x = horizontal spring constant of the soil given by (Richart et al. 1970)

$$= \frac{32(1-\upsilon)Gr_x}{(7-8\upsilon)}$$

K_θ = rotational spring constant of the soil given by (Richart et al. 1970)

$$= \frac{8Gr_\theta^3}{3(1-\upsilon)}$$

h = effective height or inertial centroid of the system,

W = total weight of the structure,

G = Dynamic Shear Modulus of the soil and

r_x, r_θ = Equivalent radius of a circular footing in horizontal and rocking mode.

Equation 1 is a standard formula used extensively to assess the effect of dynamic soil-structure interaction of a structure foundation system under earthquake and has been found to predict reasonable results and are quite close to the field observations. However, one of the major limitations of the above formulation is that it is restricted to a mathematical model having a single degree of freedom. The expression checks for the effect of soil on structure for the fundamental mode only. It can neither predict the modified time response for higher modes nor the eigen vectors for the complete soil-structure system. While it is a fact that in majority of cases the fundamental mode governs the response of the structure but there are cases where even the higher modes have significant contribution (depending on the modal mass distribution factor) and cannot be ignored in the analysis.

Squaring both sides and considering the expression $T = 2\pi/\omega$, Eqn.1 can be written as

$$\frac{1}{\overline{\omega}^2} = \frac{1}{\omega^2} + \frac{m}{K_x} + \frac{m\overline{h}^2}{K_\theta} \tag{2}$$

which can be further reduced to

$$\frac{1}{\overline{\omega}^2} = \frac{1}{\omega^2} + \frac{1}{\omega_x^2} + \frac{1}{\omega_\theta^2} \tag{3}$$

Equation 3 gives a modified natural frequency relation for the system with single-degree-of freedom.Using $\omega = \sqrt{k/m}$, Eqn.3 can be rewritten as

$$\frac{1}{k_e} = \frac{1}{k} + \frac{1}{K_x} + \frac{h^2}{K_\theta} \tag{4}$$

in which, k_e = equivalent stiffness of the soil-structure system.

The theory, mentioned below, is based on a single degree of freedom and extended to systems having multi-degrees-of freedom and tried to overcome the limitations discussed in the earlier section.

System with Multi-Degrees-of Freedom

The 3-D frame shown in Fig.2 is considered for the presentation of the proposed method. The frame structure has n degrees-of freedom and subjected to soil reactions in the form of translational and rotational springs.

For a system having n degrees of freedom, Eqn.4 can be extended to matrix form and multiplying by the mass matrix $[\mathbf{M}]_{n\times n}$ on both sides we have

$$\frac{[\mathbf{M}]_{n\times n}}{[K_e]_{n\times n}} = \frac{[\mathbf{M}]_{n\times n}}{[K]_{n\times n}} + \frac{[\mathbf{M}]_{n\times n}}{K_x} + \frac{[\mathbf{M}]_{n\times n}[h^2]_{n\times n}}{K_\theta} \tag{5}$$

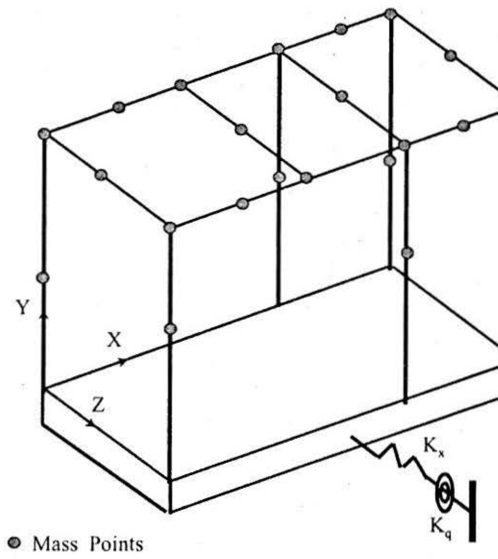


FIGURE 2 : A 3-D Frame having Multi-Degree-of Freedom with Representative Foundation Spring

where $\left[K_e\right]$ = equivalent stiffness matrix of the soil structure system of order (n × n);

$[M]$ = diagonal mass matrix of order (n × n) having masses lumped at the element diagonals;

$[h]$ = radius vectors of the lumped masses to the centre of the foundation springs of order (n × n) and is again a diagonal matrix;

K_x, K_θ = the translational and rotational spring stiffness, respectively, of the total foundation system represented by respective unique values as mentioned earlier for a single-degree case.

For structures considered in 3D, $h = \sqrt{x^2 + y^2 + z^2}$ where x, y, z is the position vectors of the lumped masses with respect to the centre of stiffness.

Multiplying both sides of Eqn.5 by $[M]^{-1}$, one can write

$$\frac{[I]}{\left[K_e\right]} = \frac{[I]}{[K]} + \frac{[I]}{K_x} + \frac{\left[h^2\right]}{K_\theta} \tag{6}$$

where $[I]$ = Identity matrix of order (n × n).

Equation 6 can be rewritten as

$$[I]\left[K_e\right]^{-1} = [I][K]^{-1} + \left[I/K_x\right] + \left[h^2/K_\theta\right]$$

or

$$\left[F_e\right] = [F] + \left[F_x\right] + \left[F_\theta\right] \tag{7}$$

where $[F]$ = Flexibility matrix of the soil structure system with suffixes as mentioned earlier for stiffness matrices.

Once the flexibility matrix of the equivalent soil structure system is known the stiffness matrix may be obtained from the expression $\left[\mathbf{K}_e\right] = \left[\mathbf{F}_e\right]^{-1}$. Now, knowing the modified stiffness matrix, the eigen solution may be carried out using the usual procedure e.g.

$$\left[\mathbf{K}_e\right][\phi] = \left[\lambda_e\right][\mathbf{M}][\varphi]$$

Estimation of Damping Ratio for the Soil-Structure System

The normal procedure for calculating the damping ratio is to guess a value, say 2 to 5% for the structure, and assume the same value for all the modes. Next, obtain the value of S_a/g for a particular structure per mode corresponding to the time period based on the curve given in IS-1893. The basis of assuming this damping ratio is purely judgmental and is dependent on either the experience of the engineer, recommendation of codes, or based on field observations on the performance of similar structures under previous earthquakes.

If the effect of soil is neglected, it is possible to obtain the material damping of the structure depending on what constitutes the structural material (e.g. steel, RCC etc.). However, when the whole system is resting on soil, an analyst is usually faced with the following stumbling blocks for which a solution is still eluding, especially for modal analysis in the time domain.

The difficulties encountered can be summarised as follows:

1. The damping matrix of the coupled soil-structure system becomes non-proportional for which the damping matrix does not de-couple using orthogonal transformation.

2. As the damping ratio of the structure and the soil can be widely varying it becomes difficult to assess a common damping ratio, which will effect the soil as well as structure.

3. Even after elaborate FEM modelling of the soil the damping ratio contribution per mode still remains at best a guess-estimation.

A method presented hereafter to estimate the contribution of soil medium to the combined soil-structure system under earthquakes in various modes and does not resort to an elaborate modelling of the soil. The contribution of soil damping to the structural system is estimated and the estimation herein is approximate. None the less, it gives a reasonable basis to arrive at some realistic damping value rather than to guess a damping value at the outset presuming that it will be same for all the modes. This is true, especially for a coupled soil-structure system, where a widely varying damping for the soil and structure makes it difficult for the analyst to arrive at a unified rational value applicable to the system.

Formulation of Damping for Single-Degree-of Freedom

Neglecting the higher order, the material-damping ratio for a soil-structure system having single degree of freedom is given by (Kramer,1995)

$$\frac{\bar{\xi}}{\bar{\omega}^2} = \frac{\zeta}{\omega^2} + \frac{\zeta_x}{\omega_x^2} + \frac{\zeta_\theta}{\omega_\theta^2} \tag{8}$$

where $\bar{\xi}$ = damping ratio of the equivalent soil structure system;

ζ = damping ratio of the fixed base structure;

ζ_x = horizontal damping ratio of the soil

$= 0.288/\sqrt{B_x}$

$$B_x = \frac{(7-8v)mg}{32(1-v)\rho_s r_s^3}$$

m = total mass of the structure and foundation;

g = acceleration due to gravity;

v = Poisson's ratio of the soil;

ρ_s = mass density of the soil;

ζ_θ = damping ratio of the soil in rocking mode;

$\zeta_{\theta x} = 0.15/\left\{(1+B_\theta)\sqrt{B_\theta}\right\}$

$$B_\theta = \frac{0.375(1-v)J_\theta g}{\rho_s r_\theta^5} \text{ and}$$

J_θ = mass moment of inertia of the foundation and the structure.

Converting the damping ratio equation to stiffness-mass basis

$$\frac{m\bar{\xi}}{k_e} = \frac{m\zeta}{k} + \frac{m\zeta_x}{K_x} + \frac{mh^2\zeta_\theta}{K_\theta} \tag{9}$$

that is

$$\bar{\xi} = k_e\left[\frac{\zeta}{k} + \frac{\zeta_x}{K_x} + \frac{\zeta_\theta}{K_\theta}\right] \tag{10}$$

For very high values of K_x and K_θ; $k_e \rightarrow k$ when $\bar{\xi} \rightarrow \zeta$.

Extension to Systems with Multi-Degree-of Freedom

On extending the Eqn.10 to multi-degree-of freedom of order n and writing it in matrix form we have

$$\frac{[\bar{\zeta}][M]_{n\times n}}{[K_e]_{n\times n}} = \frac{[\zeta][M]_{n\times n}}{[K]_{n\times n}} + \frac{[\zeta_x][M]_{n\times n}}{K_x} + \frac{[\zeta_\theta][M]_{n\times n}[h^2]_{n\times n}}{K_\theta} \tag{11}$$

Eqn.11 can be reduced to

$$\frac{[\bar{\zeta}]}{[K_e]} = \frac{[\zeta]}{[K]} + \frac{[\zeta_x]}{K_x} + \frac{[\zeta_\theta][h^2]}{K_\theta} \tag{12}$$

that is

$$[\bar{\zeta}] = [K_e]\{[\zeta][F] + [\zeta_x][F_x] + [\zeta_\theta][F_\theta]\} \tag{13}$$

where $[\bar{\zeta}]$ = damping ratio matrix of the combined soil structure system having n number of modes

It is to be noted that $[\bar{\zeta}]$ is non-proportional and not a diagonal matrix, and based on the matrix operation as shown above has off-diagonal terms.

A study on the parametric effect shows that $[\bar{\zeta}]$ becomes nearly a diagonal matrix (i.e. the off-diagonal terms vanish or approach to zero) when damping ratio of the structure and the soil foundation systems are nearly equal. However when the damping ratios are widely varying, the off-diagonal terms do not vanish and their magnitudes are relatively smaller than the diagonal terms, ζ_{ii}, which has the most dominant effect on the system. Thus, if it is possible to arrive at a foundation layout where the damping ratio of the structure and foundation are very close to one another and the assumption of the diagonal terms as modal damping ratio per mode is quite correct.

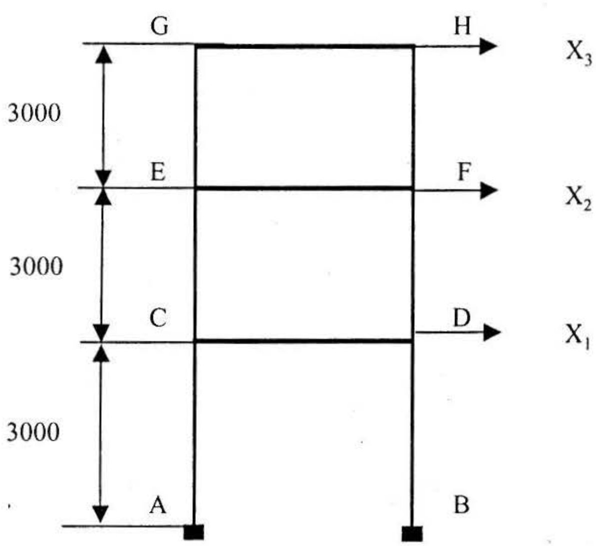
Even when the off-diagonal terms exist for widely varying values for practical design engineering purpose, consideration of the term, ζ_{ii} of damping ratio matrix is realistic for it gives a reasonably rational basis of the estimation of damping ratio per mode rather than guessing a value based on gut feeling.

The above theory based on suitable example is explained hereunder.

Example

Shown in Fig.3 is a three-storied steel frame subjected to dynamic forces as shown. The damping ratio for steel is found to vary between 2 to 5%. Determine:

- The fixed base natural frequencies of the structure.
- The fixed base eigen vectors.
- Modified natural frequency with foundation stiffness.
- Modified eigen vectors.
- Take K_x = 35000 KN/m and K = 50000 KN/m for the Soil-Foundation.
- Consider $\zeta_x = 0.10$ and $\zeta_\theta = 0.15$ respectively
- Analyse the floor shears for earthquake based on IS-1893 Zone III for
  - Fixed base
  - Considering the soil effect



(All Dimensions are in mm)

FIGURE 3 : Sketch Diagram of Three-Storied Space Frame

Let

$$K_{AC} = K_{DB} = 1.5 \times 10^3 \text{ kN/m}$$
$$K_{CE} = K_{DF} = 1.0 \times 10^3 \text{ kN/m}$$
$$K_{EG} = K_{FH} = 0.75 \times 10^3 \text{ kN/m}$$
$$M_{GH} = 200 \text{ kN sec}^2/\text{m}$$
$$M_{EF} = 400 \text{ kN sec}^2/\text{m}$$
$$M_{CD} = 400 \text{ kN sec}^2/\text{m}$$

Solution

The stiffness and mass matrix is given by

$$[\mathbf{K}] = \begin{bmatrix} 5000 & -2000 & 0 \\ -2000 & 3500 & -1500 \\ 0 & -1500 & 1500 \end{bmatrix}$$

and

$$[\mathbf{M}] = \begin{bmatrix} 400 & & \\ & 400 & \\ & & 200 \end{bmatrix}$$

Based on the above we have found earlier that

$$\omega_1 = 1.281 \text{ rad/sec;}$$
$$\omega_2 = 3.162 \text{ rad/sec;}$$
$$\omega_3 = 4.135 \text{ rad/sec.}$$

Thus the time periods for the fixed base structure is given by

$$T_1 = 4.97 \text{ sec}$$
$$T_2 = 1.987 \text{ sec}$$
$$T_3 = 1.52 \text{ sec}$$

The mode shapes or the eigen vectors and normalised eigen vectors are

$$[\phi] = \begin{bmatrix} 0.000333 & 0.000333 & 0.000333 \\ 0.000333 & 0.000833 & 0.000833 \\ 0.000333 & 0.000833 & 0.003145 \end{bmatrix}$$

$$[\phi] = \begin{bmatrix} 0.01615 & 0.03244 & 0.0344512 \\ 0.0350718 & 0.01622 & -0.03172 \\ 0.04493 & -0.02433 & 0.02477 \end{bmatrix}$$

Calculation for the Combined Soil-Structure System

Here stiffness matrix of the fixed base structure with $[K]$ on inversion gives

$$[F] = \begin{bmatrix} 0.000333 & 0.000333 & 0.000333 \\ 0.000333 & 0.000833 & 0.000833 \\ 0.000333 & 0.000833 & 0.003145 \end{bmatrix}$$

$$[F_x] = \begin{bmatrix} 1/35000 & 0 & 0 \\ 0 & 1/35000 & 0 \\ 0 & 0 & 1/35000 \end{bmatrix}$$

$$[h^2] = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 36 & 0 \\ 0 & 0 & 81 \end{bmatrix} \quad \text{and}$$

$$[F_\theta] = \begin{bmatrix} 9/50000 & 0 & 0 \\ 0 & 36/50000 & 0 \\ 0 & 0 & 81/50000 \end{bmatrix}$$

As $[F_e] = [F]+[F_x]+[F_\theta]$, $[F_e]$ can be written as

$$[F_e] = \begin{bmatrix} 0.000542 & 0.000333 & 0.000333 \\ 0.000333 & 0.001581905 & 0.000833 \\ 0.000333 & 0.0008333 & 0.001982 \end{bmatrix}$$

which is combined flexibility matrix of the soil structure system.

Inversion of the above flexibility matrix gives

$$[K_e] = \begin{bmatrix} 2195.19 & -344.34 & -224.4212 \\ -344.34 & 804.662 & -306.223 \\ -224.42 & -306.223 & 671.0682 \end{bmatrix}$$

The matrix $[K_e]$ gives a combined stiffness matrix for structural system considering the soil compliance. It is symmetric and is completely different from the one which adds up the springs directly to the diagonal. This matrix has no rigid body mode and can be used directly for static analysis too. Moreover if we take K_x and K_θ very high the $[K_e]$ converges to the fixed base matrix $[K]$.

Thus based on the above modified stiffness matrix and mass matrix $[M]$, one can write the eigen values as

$$\omega_1 = 1.10286 \text{ rad/sec;}$$

$$\omega_2 = 1.9916 \text{ rad/sec;}$$

$$\omega_3 = 2.4136 \text{ rad/sec.}$$

Thus the time periods for the combined soil-structure system is given by

$$T_1 = 5.5697 \text{ sec}$$

$$T_2 = 3.154 \text{ sec}$$

$$T_3 = 2.603 \text{ sec}$$

Normalised modified eigen vectors considering soil stiffness is given by

$$[\phi_i] = \begin{bmatrix} -0.013 & 0.0479 & 0.00589 \\ -0.0409 & -0.00772 & -0.0276 \\ -0.0360 & -0.0169 & 0.05835 \end{bmatrix}$$

Calculations of Modal Damping

Assuming

ζ = 5% for the structure;

ζ_x = 10% for the soil in translation mode;

ζ_θ = 15% for the soil in rocking mode, Eqn.12 can be written as

$$\left[\bar{\xi}\right] = \begin{bmatrix} \mathbf{0.092} & -0.028 & -0.020 \\ -0.007 & \mathbf{0.10908} & -0.028 \\ -0.002 & -0.0126 & \mathbf{0.1115} \end{bmatrix}$$

It will be seen that the main diagonal terms are dominant and can be considered as the modal damping ratio contribution for each mode.

Suppose a closely spaced damping data are given as

ζ = 5% for the structure;

ζ_x = 6% for the soil in translation mode;

ζ_θ = 5.5% for the soil in rocking mode,

The modal damping matrix reduces to

$$\left[\bar{\xi}\right] = \begin{bmatrix} \mathbf{0.0525} & -0.0015 & -0.001016 \\ -0.0004 & \mathbf{0.05312} & -0.00144 \\ -0.00014 & -0.00066 & \mathbf{0.05315} \end{bmatrix}$$

Here, the matrix becomes practically diagonal with off-diagonal terms having very low values.

Thus for the present problem ζ may be considered as

ζ_1 = 9.2% for the first mode;

ζ_2 = 10.9% for the second mode;

ζ_3 = 11.1% for the third mode,

Calculation of Earthquake Force for Fixed Base Structure

Modal Mass Participation Ffactor

m	ϕ_1	$m\phi_1$	$m\phi_1^2$
400	0.01615	6.46	0.104329
400	0.03507	14.028	0.491962
200	0.04493	8.986	0.403741
		29.474	**1.000032**

m	ϕ_2	$m\phi_2$	$m\phi_2^2$
400	0.03244	12.976	0.420941
400	0.01622	6.488	0.105235
200	−0.02433	−4.866	0.118389
		14.598	**0.644565**

m	ϕ_3	$m\phi_3$	$m\phi_3^2$
400	0.03445	13.7804	0.47475407
400	−0.01372	−5.488	0.07529536
200	0.02477	4.954	0.12271058
		13.2464	**0.67276001**

For first mode

$$\kappa_1 = \frac{29.474}{1.000032} = 29.47306$$

For second mode

$$\kappa_2 = \frac{14.598}{0.644565} = 22.64777$$

For third mode

$$\kappa_3 = \frac{13.2464}{0.67276} = 19.689$$

Assuming 5% damping for the structure we have

Mode	Time period (secs)	Sa (m/sec^2)	Remarks
1	4.9	0.4905	Sa value obtained from the chart given in IS-1893 for 5% damping
2	1.98	0.6867	– Do –
3	1.52	0.7848	– Do –

For Zone III

$K = 1.0,\ \beta = 1.0,\ I = 1.2,\ F_0 = 0.2$ as per the code

Thus base shear is given by

$$V = \sum_{i=1}^{3} K \cdot I \cdot \beta \cdot F_0 \cdot \kappa_1 \cdot S_a m_i \phi_i$$

Substituting data on the above formula we have

Mode	Base Shear V	Remarks
1	102	Fixed Base Case
2	5.45	
3	4.91	

Calculation for Coupled Soil-Structure Interaction

m	ϕ_1	$m\phi_1$	$m\phi_1^2$
400	−0.013	−5.2	0.0676
400	−0.041	−16.4	0.6724
200	−0.036	−7.2	0.2592
		−28.8	**0.9992**

m	ϕ_2	$m\phi_2$	$m\phi_2^2$
400	0.0479	19.16	0.917764
400	−0.0077	−3.08	0.023716
200	−0.0169	−3.38	0.057122
		12.7	**0.998602**

m	ϕ_3	$m\phi_3$	$m\phi_3^2$
400	0.006	2.4	0.0144
400	−0.0276	−11.04	0.304704
200	0.0583	11.66	0.679778
		3.02	**0.998882**

Modal Mass Participation Factor

For the first mode

$$\kappa_1 = \frac{-28.8}{0.9992} = -28.8231$$

For the second mode

$$\kappa_2 = \frac{12.7}{0.998602} = 12.717$$

For the third mode

$$\kappa_3 = \frac{3.02}{0.9988} = 3.0233$$

Modal Damping for Each Mode

As calculated earlier

Mode	Damping	Time (s)	Sa (m/s^2)	Remarks
1	9.2%	5.7	0.343	Calculated from curve based on interpolation corresponding to 9.2% damping
2	10.9%	3.2	0.294	Calculated from curve based on interpolation corresponding to 10.9% damping
3	11.15%	2.6	0.245	Calculated from curve based on interpolation corresponding to 11.15% damping

Calculation for Base Shear

Base shear for the frame with coupled soil-structure interaction is given by

Mode	Base Shear V	Remarks
1	68.4	Couple Soil-Foundation System
2	11.4	
3	0.537	

Calculation of Storey Forces

The storey forces for the two case are calculated hereafter

Storey		1st	2nd	top
m		400	400	200
h		3	6	9
mh^2		3600	14400	16200
$\frac{mh^2}{\sum_{i=1}^{3} mh^2}$		0.10526	0.42105	0.47368
Coupled Soil-Structure System				
	Base Shear Mode 1	7.20E+00	2.88E+01	3.24E+01
	Base Shear Mode 2	1.20E+00	4.80E+00	5.40E+00
	Base Shear Mode 3	5.66E−02	2.26E−01	2.55E−01
Fixed Base				
	Base Shear Mode 1	1.08E+01	4.31E+01	4.84E+01
	Base Shear Mode 2	5.74E+00	2.29E+01	2.58E+01
	Base Shear Mode 3	5.17E+00	2.07E+01	2.33E+01

Comparison of Results

Time Period

Structure type	T1	T2	T3
Fixed Base Structure	4.9	1.987	1.52
Soil –structure interaction	5.697	3.154	2.603

The time periods are increasing with introduction of soil springs as predicted at the outset.

Acceleration

Structure type	Mode-1	Mode2	Mode3
Fixed Base Structure	0.4905	0.6867	0.7848
Soil –structure interaction	0.34335	0.2943	0.245

The acceleration decreases with soil-structure effect in this case.

Damping

Structure type	Mode-1	Mode2	Mode3
Fixed Base Structure	5%	5%	5%
Soil –structure interaction	9.2%	10.9%	11.15%

Damping constant for all mode for fixed base case varies with mode for coupled analysis but is neither 5% min. nor 15% maximum but somewhere in-between which is quite logical.

Base Shear (kN)

Structure type	Mode-1	Mode2	Mode3
Fixed Base Structure	102	5.45	4.91
Soil –structure interaction	68.4	11.4	0.537

Significant reduction in base shear considering the soil effect though conceptually it can be predicted that amplitude of vibration will increase.

Shear Force per Floor

Storey	Modes 1		Mode 2		Mode 3	
	Fixed Base	Coupled with soil	Fixed Base	Coupled with soil	Fixed Base	Coupled with soil
1	10.8	7.2	5.74	1.2	5.17	0.0056
2	43.1	28.8	22.9	4.8	20.7	0.226
Top	48.4	32.4	25.8	5.4	23.3	0.255

A significant variation in floor shears per mode is indicated in the above.

Conclusions and Remarks

1. The major advantage of the present technique is the calculation of the time period without resorting to an elaborate modelling of the soil. Two representative spring values for the foundation is capable of modifying the stiffness of the super-structure having any conceivable degree of freedom.

2. This cuts down significantly the modelling as well as the cost of computation.

3. No rigid body motion exists.

4. Stiffness matrix of the soil-structure system is symmetric and real.

5. Beam, plates, shell etc. can be used to model the super structure and this does not generically violate the procedures followed for FEM analysis of the superstructure.

6. Since the matrix has no rigid body mode, the procedure may be also used directly for calculating the static response. No additional computational effort is required.

7. Though approximate, the procedure outlined furnishes a rational basis for estimating the modal-damping ratio per mode for the coupled soil-structure system.

8. The results are logical and, in general, satisfies the trend as observed in more rigorous analysis based on complex damping and eigen value problem (where a matrix of order $n \times n$ gets inflated to the order $2n \times 2n$, thus adding to the cost of computation).

9. If a very high value of K_x and K_θ is given, the stiffness matrix converges to the fixed base matrix.

10. No necessity for elaborate model of soil just one spring stiffness value or the boundary value of a super-element would suffice.

11. Though the example above is based on spring-dashpot model, the stiffness matrix of the structure can be based on continuum approach having as many degrees of freedom as one likes to select.

12. The modified stiffness matrix technique can well be used for both modal and time-history response.

13. The analyst need not bother about the appropriate soil model to be conceived for the coupled soil structure interaction analysis, a unique value of the soil spring is all he needs to consider for his input.

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