Indian Geotechnical Journal, 32 (2), 2002

Foundation Vibration on Layered Soil System

D.K. Baidya* and A. Sridharan†

Introduction

The design of foundation for machine involves a systematic application of the principles of soil engineering, soil dynamics and theory of vibration. Since the classical work by Lamb (1904) and subsequently by Reissner (1936), Bycroft (1956), Lysmer and Richart (1966) and Richart et al (1970), the subject of vibratory response of foundations has attracted the attention of several researchers. The state of the art on the subject has since then made significant strides [Gazetas (1983, 1991), Wolf (1994) and Kameswara Rao (1998)]. Methods are now available not only for computing the response of machine foundations resting on the surface of the elastic half space but also for embedded foundations and foundations on piles.

Amongst different methods developed in the past, Lysmer analog model is popular because of its simplicity. This methodology has been proved to be quite accurate for the analysis of foundation in low to medium frequency range. Important steps in the analysis by this method are prediction of natural frequency (resonant frequency) and resonant amplitude. Foundations for machines are generally designed keeping natural frequency of the system well away from the operating frequency to avoid resonance. It is observed from typical frequency-amplitude response curve that amplitude of vibration is insignificant at an operating frequency, $f_{or} < 0.5f_n$ or $> 1.5f_n$. Hence, avoidance of resonance become the primary objective in the design of machine foundations.

Natural frequency of the foundation soil system depends on several factors namely, shape and size of the foundation, depth of embedment,

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* Associate Professor, Civil Engineering Department, I.I.T., Kharagpur – 721302, India. E-mail: baidya@civil.iitkgp.ernet.in

† Honorary Professor, Department of Civil Engineering, I.I.Sc., Bangalore – 560012, India.

dynamic soil properties, nonhomogeneities in the soil, frequency of vibration etc. Most of the above factors studied by the past investigators [Barkan (1962), Anadakrishna and Krishnaswamy (1973), Dasgupta and Rao (1978), Kameswara Rao (1998), Johnson et al. (1975), Nagendra and Sridharan (1982), Sridharan et al (1981) to name a few]. However, nonhomogeneities in soil system have not been addressed adequately in the past for the analysis of foundations subjected to dynamic loads. A few investigators namely, Warburton (1957), Novak and Beredugo (1972), Kagawa and Kraft (1981), Gazetas (1983), Wolf (1994) and Baidya and Murali Krishna (1996, 2000) have considered the nonhomogeneity of soil in studying the dynamic response of foundations. There is still ample scope for further investigation in this area to improve the understanding of the dynamic response of foundation resting on nonhomogeneous soil system.

In this paper an attempt is made to study the effect of layering (both position and thickness) on dynamic response of foundation soil system. The objective of the present study includes:

(i) proposing a suitable method for the analysis of vibrating footing resting on layered soil system,
(ii) identifying the model parameters and proposing suitable method for estimation,
(iii) an experimental investigation on two-layered soil system under two different series namely, a soft layer over a stiff layer and vice versa,
(iv) investigation of the effect of layering on the dynamic response of footings, and
(v) verifying the adequacy of the proposed model using experimental results.

Proposed Model

Lysmer's analog model for the analysis of foundation for dynamic loading on homogeneous soil as shown in Fig.1(a) can be extended for the nonhomogeneous soil system with modification as shown in Fig.1(b). The equivalent stiffness and damping as shown in Fig.1(b) can be assumed constant over low to medium frequency range as it has been used in original analog model. Hence, dynamic analysis of foundation on layered soil system by the proposed method requires appropriate estimation of equivalent stiffness and damping of the soil foundation system. Methods for estimating the above parameters are presented below:

Stiffness

Baidya (1992) studied the dynamic response of foundations resting on different layered soil systems using equivalent lumped parameter model and

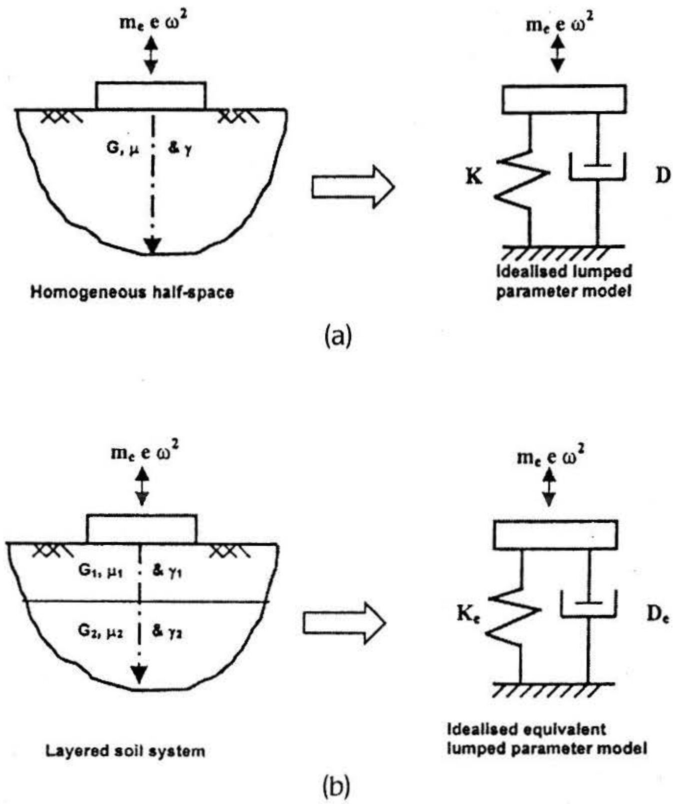


FIGURE 1 : Proposed Model for Dynamic Analysis of Foundation Resting on (a) Homogeneous Half-Space and (b) Layered Soil System

suggested a simple method for estimating the equivalent lumped parameters i.e. equivalent stiffness and equivalent damping. The expressions for equivalent stiffness for a multi-layered soil system are reproduced below:

$$K_i = \frac{\pi G_i r}{F_i - F_{i-1}} \tag{1}$$

$$K_e = \frac{1}{\sum_{0}^{i} 1/K_i} \tag{2}$$

where K_e = equivalent stiffness of the system,

K_i = stiffness of the i-th layer,

r = radius of the circular footing,

G_i = shear modulus of soil in i-th layer,

F_i = depth function which is as given below:

$$F_i = \frac{1-\mu}{2}\tan^{-1}\left(h_i/r\right)+\frac{1}{4}\left\{\frac{\left(h_i/r\right)^2}{1+\left(h_i/r\right)^2}\right\} \tag{3}$$

Murali Krishna et al. (1997) investigated the performance of the above method for foundation resting on stratum by model block vibration tests in a tank using Lazan oscillator. They have compared the stiffness obtained from the above method with that obtained from the experimental results and have shown results were in good agreement. Hence, the same method is suggested for the estimation of equivalent stiffness of the two layered soil system based on the satisfactory performance of the method on stratum (single layer over a rigid boundary). Using Eqns.1 to 3, equivalent stiffness and natural frequency of two-layered soil system can be expressed as,

$$K_e = k_e G_1 r \tag{4a}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K_e}{m}} \tag{4b}$$

where $k_e = 1\bigg/\left(\frac{1}{k_1}+\frac{1}{nk_1}\right)$

$$k_1 = \frac{\pi}{[F]_0^{h_1/r}}$$

$$k_2 = \frac{\pi}{[F]_{h_1/r}^{\infty}}$$ and

$$n = G_2/G_1$$

Using Eqn.4, equivalent stiffness of two-layered soil system for different values of shear modulus ratio, n is presented in Fig.2. This figure presents equivalent stiffness for two- layered soil under two categories, i. e., (i) a soft layer over a stiff layer ($n > 1$) and (ii) a stiff layer over a soft layer ($n < 1.0$). It can be seen from the Fig.2 that with the increase of thickness of soft upper layer, equivalent stiffness reduces and it attains almost the half space value when thickness of the upper layer is approximately six times the radius

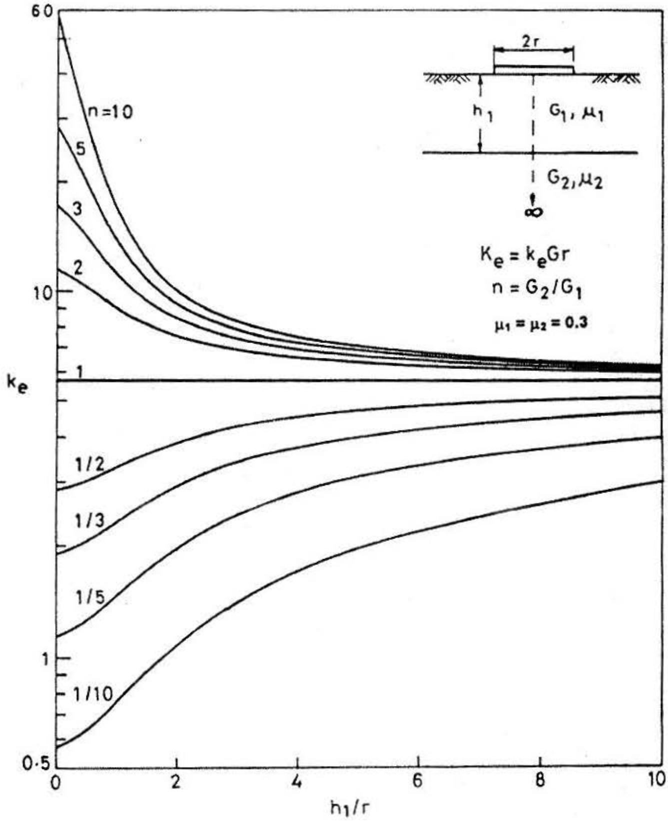


FIGURE 2 : Equivalent Stiffness Factor, k_e, versus Non-Dimensional Depth of First Layer, h_1/r for Two Layered Soil System

of the footing. On the other hand, with the increase of upper stiff layer, equivalent stiffness increases but it does not attain the half space value at depth equal to six times the radius of the footing. This indicates presence of a soft layer even at a great depth has significant influence on equivalent stiffness. Fig.2 can be used directly to estimate equivalent stiffness of two-layered soil system. Using Eqns.1 to 3, equivalent stiffness of any multi-layered soil system can be estimated.

Damping

Damping of any vibrating system is a very complicated term and it is particularly so for foundation vibration resting on soil. Damping of vibrating foundation consists of two parts, i.e., material damping due to hysteresis effect and radiation damping due to dissipation of energy within unbounded soil medium. Material damping of soil can be approximated between 5% and

10% of the critical damping whereas, radiation damping depends on several factors and its value can be as high as 50% of the critical damping. From the elastic half space theory (Richart et al. 1970), it can be estimated approximately for vertical vibration as $0.425/\sqrt{B_z}$, where B_z is the modified mass ratio. This method, however, sometimes gives misleading results. Complexity in damping increased further for layered soil. Baidya and Sridharan (1994), Baidya and Murali Krishna (1996), Baidya and Rathi (1999) and Baidya and Muralikrishna (2000) suggested a method for estimating equivalent damping of foundation resting on a stratum. No simple method for estimating the equivalent damping of layered soil systems is developed yet. This aspect is discussed based on the observations on the experimental results later in this paper.

Experimental Program

Tests were conducted in a pit of size 1.2 m × 1.2 m × 0.91 m dug at field. The pit was filled up by two different materials namely, sand and sawdust to form layered system. The investigation was carried out in two series: (i) saw dust over sand (a soft layer over a stiff layer) and top saw dust layer was varied in steps of $\cong B/2$, where, B is the width of the footing (0.30 m), and (ii) sand over saw dust (a stiff layer over a soft layer) and top sand layer depth was varied as in series (i). In total, six layered beds in each series were prepared and tests were conducted with three static weights (3.5, 4.1 and 4.7 kN) and three eccentric angles (θ = 5, 10 and 15°), where θ is the eccentric angle to be set to vary the magnitude of dynamic loads). Eccentric moment, $w_e e$ is 0.1, 0.2 and 0.3 N-m respectively, for θ = 5, 10 and 15°. Detailed experimental program is also shown in Fig.3. Lazan type mechanical oscillator was used for inducing vibration and a steel footing of size 300 × 300 × 50 mm was used as model footing in the investigation.

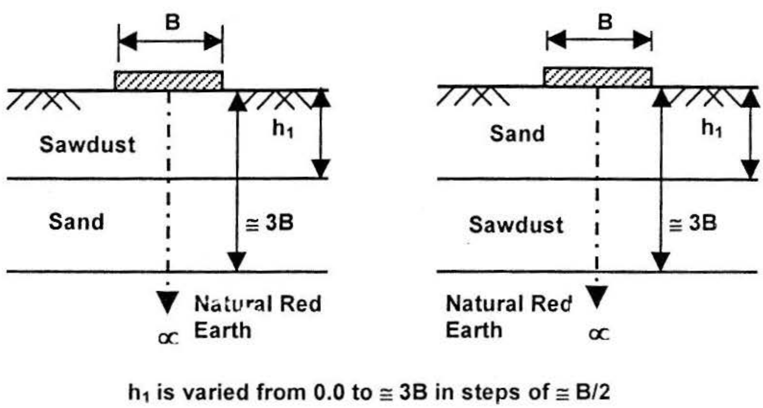


FIGURE 3 : Experimental Program.

Test Set-Up and Procedure

A square pit of size 1.2 m × 1.2 m and depth 0.91 m was excavated in the field to carry out the tests for the present investigations. Using well graded local river sand (ϕ = 36° and γ = 17 kN/m^3) and sawdust (γ = 2.3 kN/m^3) required layered test beds were prepared. Sand and sawdust were chosen as the test materials with an aim to investigate over a wide range of shear modulus ratio (ratio of shear modulus of sand and sawdust is approximately 8) of soil. Further, these materials were chosen because of the fact that it was easy to work with and maintain uniformity while preparing the layered beds.

To maintain the uniform condition throughout the test program, the pit was filled by sand in steps of 150 mm thick layer and each layer was compacted using a compactor and at a fixed compacting effort to achieve density approximately 17 kN/m^3. Sawdust was a very difficult material and compaction by dropping weight is ineffective. Sawdust layer was compacted by walking over it by several passes and putting static weight for some time. Density of sawdust achieved by this way was approximately 2.3 kN/m^3 and this was maintained fairly in the entire investigation. Sand was placed in the pit in steps of 150 mm thick layer. In two steps a 300 mm thick sand layer at the bottom was prepared first. Remaining depth of the pit was filled up with sawdust in steps of 150 mm thick layer following the procedure described before. This is one of the twelve layered beds planned in the investigation. After completing the test on this bed, sawdust layer was removed completely. Thickness of the sand layer at the bottom was then increased by 150 mm and remaining depth was filled up again with sawdust. Thus, in total six layered beds were prepared in series (i). Similarly another six layered beds were prepared in series (ii) just reversing the position of the layer. Although it was planed to have thickness of each layer in terms of footing dimension, it is difficult to achieve exactly while preparing the layered bed in the field. Hence, actual depth achieved in each bed of each series is presented in Table 1.

After preparing a desired layered bed, the surface of the top layer was leveled and a steel footing (300 × 300 × 50 mm) was placed centrally on the layered bed. Oscillator was then placed over the plate and a number of steel ingots were placed on the top of the oscillator to provide required static mass. Whole set-up was then tightened by nuts and bolts so that the whole system acted as a single mass during vibration. Proper care was taken to maintain C.G. of the whole system and the C.G. of footing in same vertical line. The oscillator was then connected through a flexible shaft to a variable speed DC motor. A Bruel and Kajaer vibration pick up was placed on the top of the footing to measure vibration amplitudes with B&K vibration meter.

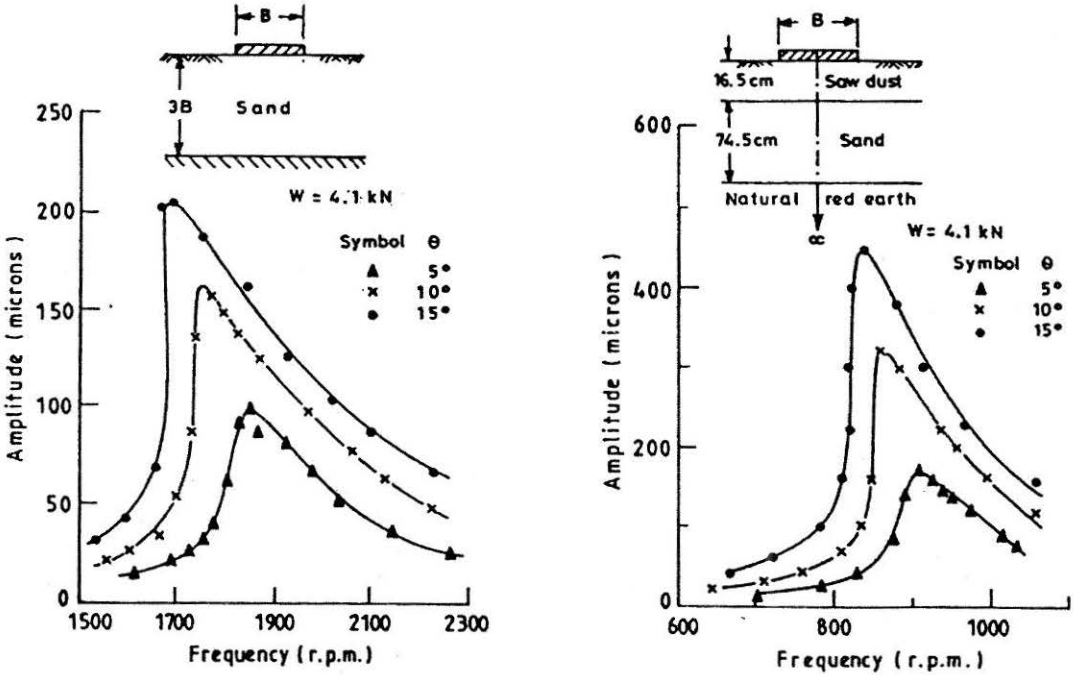
TABLE 1 : Actual Thickness of the Layers Achieved During Tests

Series (i)		Series (ii)	
Thickness of Sawdust Layer at Top (mm)	Thickness of Sand Layer at Bottom (mm)	Thickness of Sand Layer at Bottom (mm)	Thickness of Sawdust Layer at Top (mm)
0.00	910	0.00	910
165	745	160	750
310	600	335	575
460	450	490	420
610	300	645	265
910	0.00	910	0.00

The oscillator was then run slowly through a motor using speed control unit. The foundation was thus subjected to vertical vibration. Frequency and corresponding amplitude of vibration were recorded by means of photo tachometer and vibration meter respectively. The amplitudes were noted at a frequency interval of 25 to 50 rpm to obtain a complete frequency amplitude response and to locate the resonant peak accurately. Frequency corresponding to maximum displacement amplitude from the response curve is considered as resonant frequency.

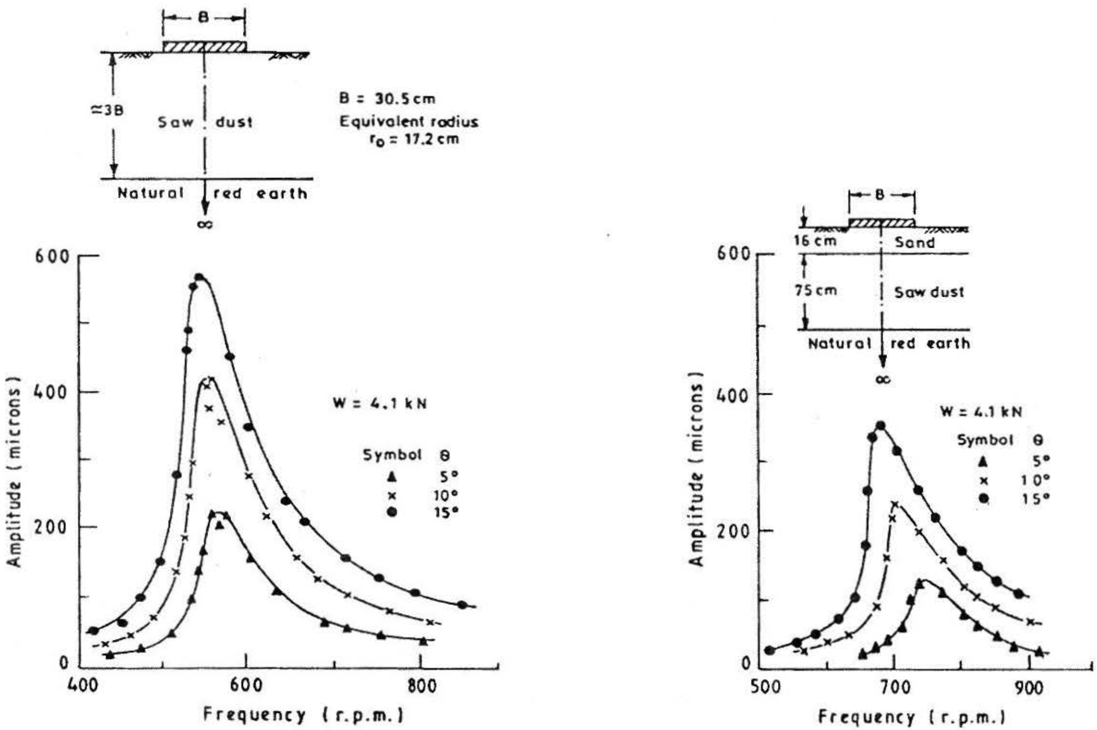
Results and Discussion

Obtaining frequency and corresponding displacement amplitude of vibration from each layered bed, frequency versus amplitude curves are plotted to locate resonant frequency and amplitude. Figs.4(a) and (b) present the frequency-amplitude response curves obtained from the two layered beds of series (i). Figs.5(a) and (b) present the frequency-amplitude response curves obtained from the two layered beds of series (ii). It can be seen from Figs.4(a) and (b) that resonant frequencies reduced significantly (1860 rpm to 925 rpm for $\theta = 5°$) due to replacement of sand layer of depth B/2 at top by a sawdust layer. On the other hand, it can be seen from the Figs.5(a) and (b) that resonant frequency increased (from 600 to 750 rpm for $\theta = 5°$) due to replacement of sawdust layer of depth B/2 at top by a sand layer. From each layered bed a total of nine response curves (three static weights and three θ values for each weight) are obtained. Resonant frequencies of each layered bed obtained from response curves are presented as f_0 in Tables 2 and 3 respectively, for series (i) and (ii). Similarly, resonant amplitudes obtained from the response curves are presented as Z_m in Tables 4 and 5 for series (i) and (ii) respectively. It is observed from Table 2 that with further increase of sawdust layer thickness at top, resonant frequencies decrease



(a) (b)

FIGURE 4 : Frequency versus Displacement Amplitude Curves for Series (i): (a) Pit Filled up with Sand and (b) A 165 mm Thick Sawdust Layer over a 745 mm Thick Sand Layer



(a) (b)

FIGURE 5 : Frequency versus Displacement Amplitude Curves for Series (ii): (a) Pit Filled up with Sawdust and (b) A 160 mm Thick Sand Layer over a 750 mm Sand Layer

TABLE 2 : Observed and Predicted Resonant Frequencies of Layered Soil Systems in Series (i)
[A Soft (Sawdust) Layer over a Stiff (Sand) Layer]

W (kN)	$w_e e$ (N-m)	h/r = 0.0		h/r = 0.96		h/r = 1.8		h/r = 2.67		h/r = 3.54	
		$f_o^{\$}$	$f_p^{\$}$	f_o	f_p	f_o	f_p	f_o	f_p	f_o	f_p
	0.1	31.3	31.2	15.9	14.4	13.6	11.8	11.5	10.9	11.0	10.4
3.5	0.2	29.8	29.1	14.9	13.9	13.0	11.4	10.9	10.5	10.6	10.0
	0.3	28.8	37.2	14.3	13.4	12.6	11.0	10.7	10.2	10.3	9.7
	0.1	31.0	30.7	15.4	13.9	13.1	11.3	10.9	10.5	10.5	10.0
4.1	0.2	29.2	28.7	14.7	13.6	12.6	11.1	10.7	10.3	10.2	9.8
	0.3	28.3	26.9	14.1	13.1	12.2	10.8	10.4	10.0	10.0	9.6
	0.1	30.0	29.5	15.0	13.5	12.7	11.0	10.7	10.2	10.3	9.7
4.7	0.2	28.3	27.6	14.5	13.3	12.4	10.9	10.4	10.1	10.0	9.6
	0.3	27.8	26.6	13.9	13.0	12.1	10.7	10.3	9.9	9.9	9.5

$\$$ in cps.

TABLE 3 : Observed and Predicted Resonant Frequencies of Layered Soil Systems in Series (ii) [A Stiff (Sand) Layer over a Soft (Sawdust) Layer]

W (kN)	$w_e e$ (N-m)	h/r = 0.0		h/r = 0.93		h/r = 1.95		h/r = 2.85		h/r = 3.75	
		$f_o^{\$}$	$f_p^{\$}$	f_o	f_p	f_o	f_p	f_o	f_p	f_o	f_p
	0.1	10.0	9.9	12.5	12.2	19.0	16.0	24.3	19.6	28.1	23.4
3.5	0.2	9.5	9.5	11.9	12.0	17.8	15.4	22.5	18.7	26.8	22.2
	0.3	9.3	9.3	11.7	11.4	17.3	14.8	21.7	18.7	25.8	21.1
	0.1	9.5	9.4	12.4	11.8	18.7	15.4	23.8	19.0	27.8	22.7
4.1	0.2	9.3	9.3	11.8	11.6	17.5	15.1	22.3	18.2	26.2	21.8
	0.3	9.2	9.1	11.3	11.3	16.7	14.6	21.5	17.7	24.8	20.8
	0.1	9.2	9.3	12.2	11.5	18.0	15.0	23.3	18.4	26.7	22.0
4.7	0.2	9.0	9.2	11.6	11.4	17.0	14.8	21.7	18.0	25.2	21.2
	0.3	8.9	9.0	11.1	11.1	16.2	14.5	20.8	17.5	24.3	20.6

TABLE 4 : Observed Resonant Amplitudes and Equivalent Damping for Layered Soil Systems of Series (i) [A Soft (Sawdust) Layer over a Stiff (Sand) Layer]

W (kN)	$w_e e$ (N-m)	h/r = 0.0		h/r = 0.96		h/r = 1.80		h/r = 2.67		h/r = 3.54	
		Z_m †	D_e @	Z_m	D_e	Z_m	D_e	Z_m	D_e	Z_m	D_e
	0.1	110	12.9	165	8.6	193	9.9	210	6.7	223	6.3
3.5	0.2	170	16.8	320	8.8	330	8.5	440	6.4	480	5.9
	0.3	215	20.4	455	9.3	483	8.7	610	6.9	605	7.0
	0.1	95	12.8	140	8.6	134	9.0	200	6.0	195	6.2
4.1	0.2	165	14.8	290	8.3	280	8.6	340	7.3	370	6.5
	0.3	210	17.4	405	8.9	480	8.4	490	7.4	525	6.9
	0.1	80	13.3	130	8.1	117	9.0	150	7.0	178	5.9
4.7	0.2	130	16.4	253	8.3	250	8.4	300	7.0	375	6.4
	0.3	175	18.8	363	8.7	390	8.4	420	7.5	460	6.8

† in microns (10^{-3} mm) @ in percent (%)

TABLE 5 : Observed Resonant Amplitudes and Equivalent Damping for Layered Soil Systems of Series (ii) [A Stiff (Sand) Layer over a Soft (Sawdust) Layer]

W (kN)	$w_e e$ (N-m)	h/r = 0.0		h/r = 0.93		h/r = 1.95		h/r = 2.85		h/r = 3.75	
		Z_m	D_e	Z_m	D_e	Z_m	D_e	Z_m	D_e	Z_m	D_e
	0.1	260	5.5	160	8.7	85	16.8	70	20.6	70	20.6
3.5	0.2	485	5.9	315	9.0	160	17.9	140	20.6	120	24.0
	0.3	650	6.6	425	10.0	235	18.3	180	24.2	163	27.0
	0.1	220	5.5	130	9.3	78	15.7	65	18.9	65	18.9
4.1	0.2	420	5.8	240	10.1	148	16.5	125	19.6	115	21.4
	0.3	580	6.3	355	10.2	220	16.6	165	22.4	150	24.8
	0.1	200	5.3	100	10.6	75	14.1	60	17.8	60	17.8
4.7	0.2	370	5.7	225	9.4	135	15.7	120	17.8	113	19.0
	0.3	500	6.4	325	9.7	200	15.9	170	18.8	150	21.5

further but at a slower rate whereas it is observed from Table 3 that with further increase of sand layer thickness at top, resonant frequency increases. It can also be seen that with the increase in dynamic force level (θ), the resonant frequency decreases (Tables 2 and 3) whereas resonant amplitude increases (Tables 4 and 5). Further, it can be seen that resonant frequency and amplitude decrease with the increase in static weight on any layered bed for a constant θ. These observations qualitatively agree with the existing findings (Richart et al., 1970). But main objective of present investigation is to find out the effect of layering on the dynamic response. Figure 6 presents the effect of layering on resonant frequency for static weight of 4.1 kN and $\theta = 10^{\circ}$. It can be seen from the Fig.6(a) that with the increase of thickness of saw dust layer at top, resonant frequency decreases and resonant amplitude increases. On the other hand it can be seen from Fig.6(b) that with the increase of thickness of sand layer at top, resonant frequency increases and resonant amplitude decreases.

The effect of layering on resonant frequency can also be seen from Table 2 and 3 for all static weights and θs. With the increase of thickness of sawdust layer at top, stiffness of the system reduces which results in reduction in resonant frequency (Eqn.4b). Similarly, with the increase of thickness of sand layer at top, stiffness of the system increases which results in increase in resonant frequency. Hence, Variation of resonant frequency is mainly due to variation of equivalent stiffness of the system with the variation of thickness of the layer. This observation qualitatively agrees well with results presented in Fig.2. It can be seen from Fig.2 that when the top layer is softer than the bottom, equivalent stiffness reduces up to a depth equal to 2r or B and beyond this depth variation is not that significant. This sharp variation of stiffness results in sharp variation of resonant frequency. On the other hand, when the top layer is stiffer than the bottom, equivalent stiffness increases gradually which results in gradual increase of resonant frequency. To study the effect of layering on the response quantitatively, experimental results are analysed and presented in the subsequent sections.

Analysis of Experimental Results and Comparison

Lysmer's 'lumped' parameter model is generally used in low to medium frequency range for the analysis of block type machine foundations because of its simplicity. Hence, proposed modified model as shown in Fig.1(b) is used for the analysis of experimental results. Expressions for resonant frequency and resonant amplitude in terms of other parameters according to the above model are as given below:

$$\frac{Z_m W}{w_e e} = \frac{1}{2D_e\sqrt{1-D_e^2}} \tag{5}$$

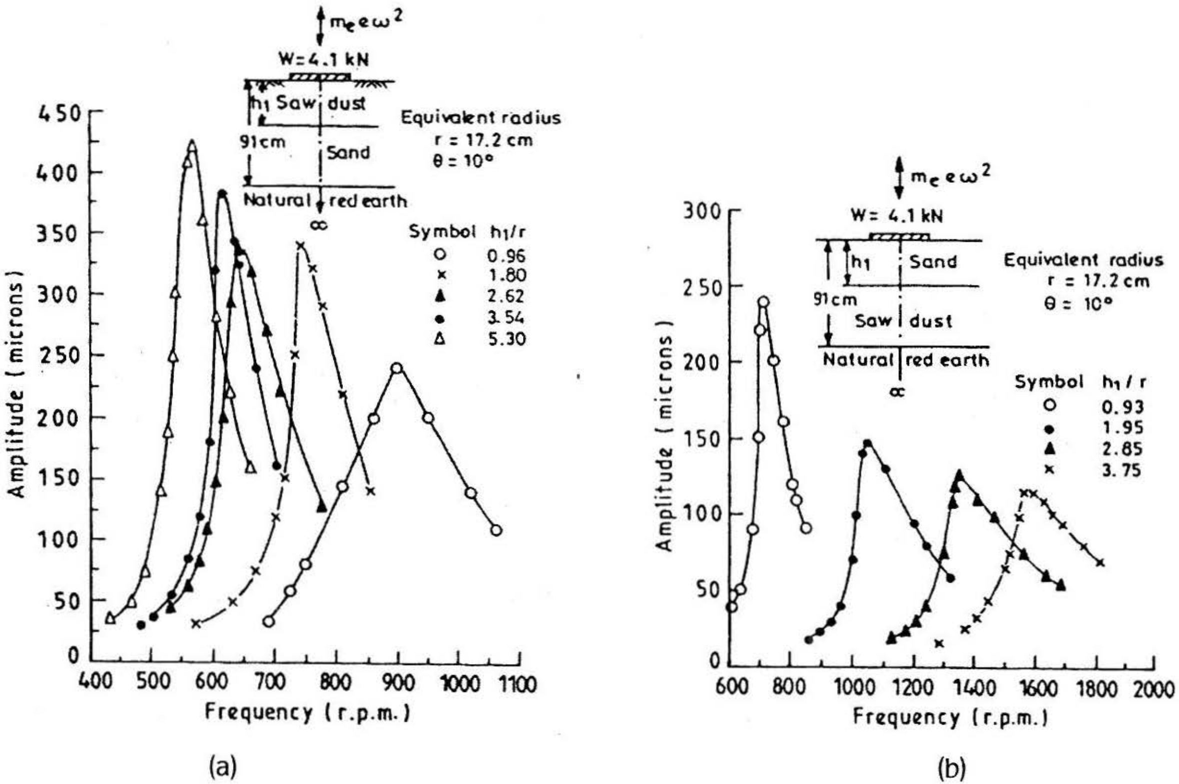


FIGURE 6 : Variation of Response with the Variation of Thickness of Top Layer: (a) A Soft Layer over a Stiff Layer and (b) A Stiff Layer over a Soft Layer

$$f_{nr} = \frac{f_n}{\sqrt{1-2D_e^2}} \tag{6}$$

$$\cong f_n \text{ where } D_e \text{ is very small (less than 0.15)}$$

where Z_m = resonant amplitude,

W = total weight of the foundation and machine,

$w_e e$ = eccentric moment,

f_n = natural frequency,

f_{nr} = resonant frequency and

D_e = equivalent damping factor.

Using recorded resonant amplitudes from the tests (Tables 4 and 5) and other input parameters in Eqn.5, equivalent damping of different layered beds is obtained and presented in Table 4 and 5 for series (i) and (ii) respectively. It is observed that damping of different layered beds obtained from Eqn.5 are less than 15% except a few cases. Further, it can be seen from Tables 4 and 5 that the top layer has maximum effect on equivalent damping of the layered system. During foundation vibration three important vibration waves generate namely, Rayleigh wave, shear wave and compression wave and carried away energy from the vibrating foundation which is called radiation damping. Out of above three waves, energy carried by Rayleigh wave is $2/3^{rd}$ of the total. Since, the Rayleigh waves travel near the surface of the ground, the radiation damping is influenced more by the surface layer. Estimation of damping of a vibrating foundation is a very critical task even for homogeneous soil and it become further complicated for layered soil. On the other hand, foundation is so designed that the operating frequency always be kept well away from the resonant frequency at which effect of damping is insignificant. Hence, in the absence of appropriate method for estimating damping of layered soil system, it can be assumed suitably based on the damping value of the surface layer.

Observed resonant frequencies presented as f_0 in Table 2 and 3 can be assumed as the natural frequencies of the system [at low damping $f_r \cong f_n$ (Eqn.6)]. The natural frequency of any layered system can be obtained using Eqns.1 to 4 if the dynamic soil properties are known. To verify the adequacy of Eqn.4 for predicting the natural frequency of any layered system, present experimental results are used. It can be seen from equations 1 to 4 that shear modulus and Poisson's ratio of the layered material are essential for the prediction of natural frequency by this method. When the pit is full either by sand or sawdust, it is considered as half space (depth is three times the width

TABLE 6 : Shear Modulus Values of Sand and Sawdust

Shear modulus* of	W = 3.5 kN			W = 4.1 kN			W = 4.7 kN		
(MN/m^2)	$w_e e$ = 0.1	$w_e e$ = 0.2	$w_e e$ = 0.3	$w_e e$ = 0.1	$w_e e$ = 0.2	$w_e e$ = 0.3	$w_e e$ = 0.1	$w_e e$ = 0.2	$w_e e$ = 0.3
Sand	13.92	12.08	10.6	15.78	13.85	12.14	16.75	14.68	13.60
Sawdust	1.68	1.57	1.48	1.76	1.76	1.68	1.98	1.96	1.89

γ_{sand} = 17.0 kN/m^3

$\gamma_{sawdust}$ = 2.3 kN/m^3

μ_{sand} = 0.3 (assumed)

$\mu_{sawdust}$ = 0.0 (assumed)

of the footing) and resonant frequency obtained corresponding to this is assumed as resonant frequency of footing resting on the half space. This is fairly true because of the fact that the variation of response is insignificant beyond 610 mm depth (Fig.6). Using the resonant frequency obtained from the response of 910 mm thick sawdust layer in Eqn.4, half space stiffness for the sawdust is obtained. Half-space stiffness obtained thus is equated with that obtained from Eqn.1 (h/r from 0 to ∞) and shear modulus value of sawdust is obtained. Similarly, half-space stiffness for sand obtained from the test result is equated with that obtained from Eqn.1 to obtain shear modulus of sand. Finally, these (shear modulus) are presented in Table 6. Bottom of the pit is not considered as rigid. Natural earth beyond 910 mm depth is assumed approximately the same as that of sand with respect to shear modulus value (this is observed in separate test on red earth alone). There are several other methods to estimate the shear modulus of the soils. Different methods, however, measure shear modulus in different strain levels. Since the soil is strongly strain dependent, different methods estimate different values even for the same soil. Hence, to eliminate strain dependency, the same model block vibration test is used as it is used in the investigation of layered soil system.

Finally, using above shear modulus values from Table 6 in Eqns.1 to 4, natural frequencies of different layered systems are obtained and presented as f_p in Tables 2 and 3 for series (i) and (ii) respectively for comparisons. All layered beds in series (i) are two-layered system (sawdust-sand) whereas it is three-layered system in series (ii) [sand-sawdust-sand (assuming red earth as sand with respect to G)]. This is considered while calculating natural frequency using Eqns.1 to 4. Square footing is converted to an equivalent circular footing with radius $= \sqrt{300^2/\pi}$ since the suggested method is applicable only for circular footing. It can be seen from Tables 2 and 3 that the predicted results agreed well with test results except for a few cases in series (ii). In series (ii), sand is placed over sawdust which is very light (approx. $1/7$ times of sand) compared to sand. The dense sand layer over the sawdust layer in series (ii), acted as surcharge on to sawdust layer below and shear modulus value of the sawdust increased due to this surcharge effect. With the increase of thickness of sand layer at top, shear modulus values of sawdust layer increases. In the analysis, however, a constant value is used and hence the differences. In actual soil, however, this situation does not occur and it is expected that natural frequency of any layered soil system can be predicted satisfactorily using the above equations.

Conclusions

Influence of layering (both position and thickness) of soil on the dynamic response of foundation soil system is investigated analytically and experimentally. From the analytical study on the equivalent stiffness of two-layered soil system following conclusions can be drawn:

- Equivalent stiffness decreases with the increase of thickness of soft layer at top, and it increases with the increase of thickness of stiff layer at top.

- When soft layer at top, effect of increase of thickness of layer on equivalent stiffness is very significant (decrease sharply at B/2 depth) up to a depth equal to B and beyond this depth effect is not that significant. On the other hand, when stiff layer at top, effect of increase of thickness of layer on equivalent stiffness is significant (gradual) up to a quite large depth.

- For soft soil, depth of influence is shallower (attain half space value before a depth of 6r or approximately 3B) compared to stiff soil (does not attain half space value even at a depth of 10r or approximately 5B).

From the model block vibration tests carried out on two-layered soil systems following important observations are made:

- Resonant frequency decreases with the increase of thickness of soft layer at top, and it increases with the increase of thickness of stiff layer at top.

- When soft layer at top, effect of increase of thickness of layer on resonant frequency is very significant (decrease sharply at B/2 depth) up to a depth equal to B and beyond this depth effect is not that significant. On the other hand when stiff layer at top, effect of increase of thickness of layer on resonant frequency is significant up to a quite large depth.

- Since the resonant frequency of layered soil depends on the equivalent stiffness (higher the stiffness higher is the resonant frequency), observations made on experimental results strongly supported the conclusions drawn from the analytical study.

Resonant frequencies of different layered soil systems obtained from the experimental study are compared with that predicted based on the equivalent stiffness of the layered soil systems. It is found that predicted resonant frequencies matched well with that obtained from the experimental results except for a few cases, which has been discussed before. Since no suitable method for estimating the equivalent damping of the layered soil system is developed yet, prediction of resonant amplitude (resonant amplitude depends only on damping) became difficult. However, equivalent damping of layered soil systems are estimated from the experimental results using lumped parameter model and observed that the damping of layered soil system is strongly influenced by the layer at top. For practical purpose damping of layered soil system can be assumed suitably based on the damping of top layer. Hence, overall conclusion can be drawn from the present study that the resonant frequency of machine foundation in low to medium frequency range resting on any layered soil system can be estimated satisfactorily from static equivalent stiffness of the system. Resonant amplitude can also be estimated assuming a suitable damping value based on the damping of surface layer in the absence of accurate method for estimating it.

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Notations

B = Width of the footing

B_z = Modified mass ratio for vertical vibration

$$= \frac{(1-\mu)}{4}\frac{W}{\gamma r^3}$$

D = Damping factor in percent

D_e = Equivalent damping factor

e = Eccentricity of rotating part

F = Non-dimensional depth factor

f = Frequency in rpm

f_n	=	Natural frequency of the vibrating system
f_{nr}	=	Resonant frequency of vibrating system
f_o and f_p	=	Observed and predicted frequency respectively in cps
G	=	Shear modulus of soils
G_i	=	Shear modulus of ith layer
G_1 and G_2	=	Shear modulus values of layer 1 and 2 respectively
h_i	=	Depth of i-th layer
h_1	=	Depth of top layer
K_i	=	Stiffness of i-th layer
K_1 and K_2	=	Stiffness of layer 1 and 2 respectively
K_e	=	Equivalent stiffness of the layered soil system
m	=	Total vibrating mass (mass of foundation plus machine)
n	=	G_2/G_1
r	=	Radius of the circular footing or radius of equivalent circle for non circular footing
w_e	=	Weight of eccentric rotating part
Z_m	=	Resonant amplitude of vibration
μ	=	Poisson's ratio of soil
γ	=	Unit weight of soil
θ	=	Angle for setting eccentricity in the oscillator