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Stability Computations in Steep Cohesive Slopes: Difficulties and Their Remedies

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Introduction

Among the numerous slope stability computation methods based on the limit equilibrium principles and slice discretisation, only a few methods are categorised as "rigorous" in the sense that they satisfy all conditions of equilibrium and are valid for general shear surfaces (e.g., Spencer method, 1973; Morgenstern and Price method, 1965; GLE method- Fredlund et al., 1981). All these methods use a model of formulation which leads to a pair of simultaneous nonlinear equations. The solution of this pair of slope stability equations yields the factor of safety associated with a potential shear surface.

The nonlinear nature of these equations necessitates the use an elaborate technique to solve for the factor of safety. A number of solution techniques are available in the literature, e.g., a numerical-graphical procedure (Spencer, 1967), a two-variable Newton-Raphson technique (Morgenstern and Price, 1967; Wright, 1969), a method of successive approximation (Spencer, 1973), the best-fit regression technique (Fredlund, 1974), the rapid solver technique (Fredlund, 1981). Excellent reviews of these methosds are available in the literature (Fredund, 1984; Bhattacharya and Basudhar, 1999). According to Soriano (1976), elaborate iterative schemes such as those based on Newton-Raphson approach (Morgenstern and Price, 1967; Wright, 1969) are not stable or do not converge to a proper solution. Bhattacharya and Basudhar (1999) have presented a new solution technique, with reference to the Spencer method (Spencer, 1973), wherein the problem of finding the two unknowns

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(the factor of safety, F and an indicator of the interslice force function, θ) from the two nonlinear equations has been formulated as a nonlinear programming problem which is then solved by using the sequential unconstrained minimisation technique (SUMT). It has been reported that the proposed technique successfully overcomes some of the deficiencies of the earlier methods.

Critical slip surfaces are now determined using optimization techniques e.g., the sequential unconstrained minimization technique (SUMT) (Basudhar, 1976; Bhattacharya, 1990), dynamic programming technique (Baker, 1980); simplex reflection technique (Nguyen, 1985), alternate variable technique (Celestino and Duncan, 1981), variational calculus (Baker and Garber, 1977; Castillo and Revilla, 1977), random search technique (Greco, 1996) etc.

In the conventional procedure of determination of critical shear surface, it is required to solve the pair of nonlinear equations every time a trial slip surface is generated by the auto search technique employed in the minimization scheme. Bhattacharya and Basudhar (2001) have proposed a new procedure called the direct procedure in which the factor of safety, F as well as the characteristic interslice force inclination θ are included in the design vector, along with the co-ordinates defining the slip surface, while putting the force equilibrium and the moment equilibrium equations as equality constraints in the general mathematical programming formulation of the problem which is then solved using the well known sequential unconstrained minimisation technique (SUMT) of nonlinear programming.

In determination of critical slip surfaces, or, for that matter, in evaluation of factors of safety for arbitrary slip surfaces, problems of non-convergence may often arise (Ching and Fredlund, 1983). In rigorous slope stability computations, besides convergence, the obtained solutions have to satisfy some conditions of acceptability such that the internal forces obtained from the solution do not violate the Mohr-Coulomb failure criteria anywhere within the sliding body, no tension is implied and that the directions of forces are kinematically admissible (Morgenstern and Price, 1965; Sarma, 1979).

It has been observed that the assumption regarding the interslice force function has a marked influence on achieving convergence as well as acceptable solutions. Spencer (1973, 1981) has demonstrated that in many cases the assumption of parallel interslice forces together with introduction of a vertical tension crack running parallel to the crest results in a convergent solution. The acceptability of the solution is checked by the reasonableness of the position of the line of thrust that is obtained as a part of the solution. It has been generally observed (Spencer, 1981; Bhattacharya, 1990) that in those cases in which the line of thrust (for effective stress) is within the middle third or thereabout, the conditions of acceptability are satisfied, as can always be verified from the detailed output data. There are situations, however, where the assumption of parallel interslice forces are not compatible with the equilibrium conditions of the problem, such as a steep slope with high cohesion value (Soriano, 1976) in which case one needs to try with other interslice force functions in order to obtain an acceptable line of thrust. It has been suggested (Ching and Fredlund, 1983) that the interslice force assumption used should be consistent with the geometry of the slope and the stress distribution within the soil mass. Based on a large number of finite element based stress analysis, Fan et al. (1986) have proposed a general empirical interslice force function which can be utilized in such situations.

In view of the above mentioned difficulties in the stability computations especially those concerning steep and cohesive slopes, in this paper an attempt has been made to extend the original version of the equation solver developed earlier by the authors by incorporating the interslice force function values at the interslice boundaries in the design vector originally consisting of only two variables namely, F and θ. Appropriate acceptability criteria can be put as constraints in the nonlinear programming formulation. Starting with an assumed interslice force function, solution can be obtained which would yield, in addition to the values of of F and θ, the optimal values of the interslice force function associated with an acceptable solution. The extended version can be used to obtain an acceptable solution for a given shear surface as well as for a critical slip surface which do not satisfy the prescribed acceptability criteria.

Extension to the Earlier Solution Procedure

As mentioned earlier, the authors have proposed (Bhattacharya and Basudhar, 1999) an efficient equation solver in which the problem of finding the two unknowns (F, θ) from two nonlinear equilibrium equations

$$Z_n(F,\theta) = 0 \tag{1a}$$

and

$$M_n(F,\theta) = 0 \tag{1b}$$

has been formulated as a nonlinear programming problem as follows:

Find $\mathbf{D} = [F,\theta]^T$

Such that $f(\mathbf{D}) = Z_n^2 + M_n^2 \rightarrow \text{Min.}$

Subject to some bounds on F and θ

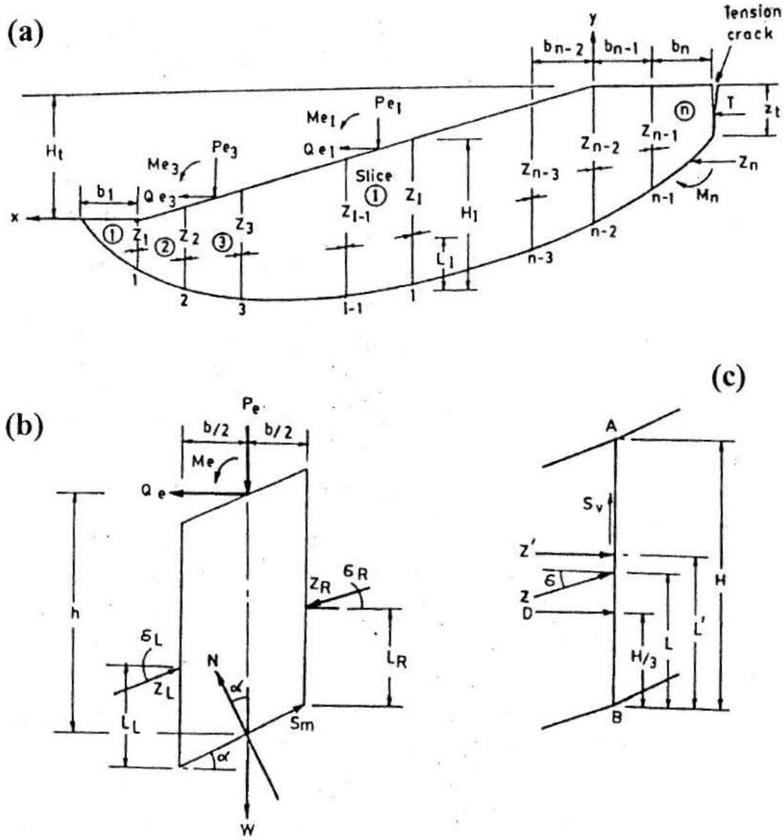

FIGURE 1 : (a) Defination and Notations; (b) Forces on a Typical Slice; (c) Forces on an Interslice Boundary

where, referring to Fig.1, Z_n and M_n are the external balancing force and moment respectively (Spencer, 1973; Bhattacharya, 1990). Both Z_n and M_n are functions of the variables (F, θ). The first variable is the value of overall factor of safety F; the second is the interslice force characteristic angle θ which, together with the coefficients k_i, determines the slopes, δ_i, of the interslice forces (Fig.1) in accordance with the following expression

$$\tan\delta_i = k_i \tan\theta \tag{3}$$

where the suffix i denotes the ith interslice boundary and increases from the toe end towards the scarp end. The coefficient k in the Spencer method is equivalent to the interslice force function f(x) in the Morgenstern and Price method. If n is the number of slices, (n−1) values are chosen or prescribed by the user for k; e.g., if k is taken to be unity throughout, then the interslice forces will be all parallel and their slopes δ_i relative to the horizontal will be each equal to θ. For further details regarding the solution technique, reference may be made to Bhattacharya and Basudhar (1999).

In those situations in which the assumption of parallel interslice forces (i.e., k = 1 throughout) does not result in an acceptable line of thrust, it is necessary to try with other k distributions. In order to allow a systematic variation of the set of k-values, it is proposed to include the same in the design vector. In accordance with the suggestions made earlier, in this case the formulation has been further extended to include the constraints on the line of thrust such that the ratios L/H lie within the middle third. In many cases, however, this might prove to be too stringent for the smooth progress of the numerical scheme and, therefore, a slightly more liberal bounds (0.25 to 0.65) for L/H, as suggested by Hamel (1968) has been adopted. And to add more flexibility the depth of tension crack (z_t) has also been included in the design vector together with an upper limit z_0 for z_t where z_0 is the depth of zero active earth pressure. The extended version of the problem formulation, in its most generalised form, is stated as follows.

$$\text{Find } D = \left[F, \theta, z_t, \mathbf{k}\right]^T \tag{4a}$$

$$\text{Such that } f(D) = Z_n^2 + M_n^2 \rightarrow \text{Min.} \tag{4b}$$

$$\text{Subject to } 0.25 \le \frac{L}{H} \le 0.65 \text{ and} \tag{4c}$$

$$z_t \le z_0 \tag{4d}$$

where L/H = the ratio denoting the position of the line of thrust,

L = height of the point of application of the interslice force, and

H = height of the corresponding interslice boundary (Fig.1c)

z_0 = the depth of zero active earth pressure.

$$k = [k_1, k_2, \ldots, k_i, \ldots\ldots, k_{n-1}]^T \tag{5}$$

where k_i is the value of the k-distribution at the ith interslice boundary.

Like the original version of the equation solver (Bhattacharya and Basudhar, 1999), the constrained minimisation problem stated above is solved by the sequential unconstrained minimisation technique in conjunction with the Powell method of multidimensional search and the quadratic interpolation technique for unidimensional search.

General Empirical Interslice Force Function

Fredlund (1984) and Fan et al. (1986) have reported about a detailed study on interslice force functions computed from finite element analysis. Based on a large number of analyses, a general empirical interslice force function has been proposed which is reproduced below for the sake of ready reference.

$$f(x) = K \exp\left[\frac{-C^N \omega^N}{2}\right] \tag{6}$$

where, K = magnitude of the interslice force function at mid slope

C = variable to define the inflection points

N = variable to specify the flatness or sharpness of curvature

ω = dimensionless x-position relative to the midpoint of the slope

The variable K is a function of the slope inclination and the depth factor. The constants C and N are related to slope inclination. These parameters were computed for a circular slip surface. Analyses for composite or noncircular slip surfaces showed variations in magnitude for some of the constants in the above expression but the general shape of the function remained the same. The constants K, C and N may be obtained from the charts prepared for them.

Illustrative Examples

Two specific instances of steep cohesive slopes are cited in the literature (Soriano, 1976; Castillo and Luceno, 1982), which have been reported to pose difficulties in arriving at a converging solution. One of these cases (Example Problem 1) (Soriano, 1976) concerns with the evaluation of factor of safety of a given shear surface while the other (Example Problem 2) (Castillo and Luceno, 1982) concerns with the determination of the critical slip surface.

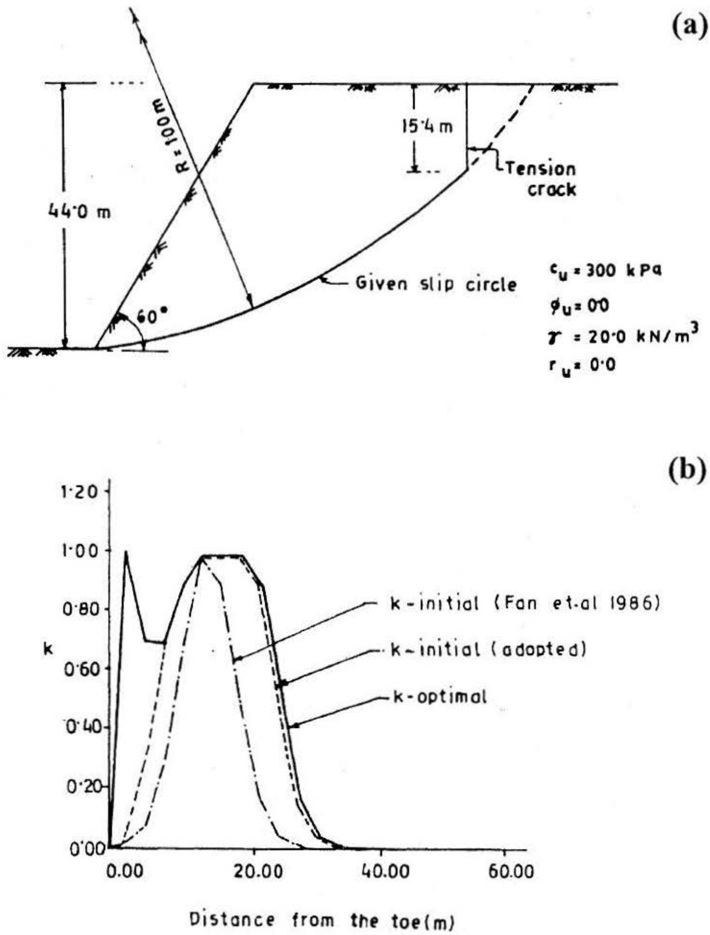

FIGURE 2 : Example Problem 1 – (a) Slip Circle Modified due to Tension Crack; (b) Optimal Interslice Force Function

Example Problem 1

Figure 2a presents a section of a steep and highly cohesive slope together with a given slip circle. Soriano (1976) has cited this as one simple case which is not solvable by the iterative schemes suggested by Spencer (1973), Morgenstern and Price (1967) etc. when $\phi_u = 0$. He has further reported that if $c_u = 300$ kPa, the factor of safety by the force equilibrium method would be larger than F = 1.89 for any value of θ, whereas by the moment equilibrium method the factor of safety is independent of θ and has the value of F = 1.86. He has further observed that even though the factor of safety is quite well defined for this slope, any iterative scheme trying to satisfy both moment and force equilibrium will fail to converge and that this would very often be the case for steep and highly cohesive slopes for which the assumption of parallel side forces (used in the Spencer method) is not compatible with equilibrium conditions.

From the above observations it can be surmised that the non-convergence of the problem at hand may be due to one or both of the following:

(i) deficiency in the iterative scheme adopted for solving the pair of nonlinear equations

(ii) incompatibility of the assumption of parallel interslice forces.

With this in view, the present study has been undertaken in the following two phases:

(i) to investigate whether, by using the proposed equation solver, it is possible to achieve convergence while still assuming parallel interslice forces i.e., k = 1 throughout.

(ii) in case the above attempt fails, to make a fresh attempt to solve the problem making use of other interslice force functions i.e., k-distributions based on a modified formulation treating the k-values as design variables.

Results and Discussion

Trial Solution Assuming Parallel Interslice Forces

As a first trial, solution was attempted using the original version of the proposed equation solver (Bhattacharya and Basudhar, 1999) considering parallel interslice forces (k = 1 throughout) and no tension crack. A total of 20 slices was considered for the analysis. However, in agreement with Soriano's observations, convergence could not be achieved

In highly cohesive slopes, problems of convergence may arise as a result of development of tension and as a remedial measure it has been suggested (Ching and Fredlund, 1983; Spencer, 1973; Spencer, 1968) to introduce tension crack in the analysis. Accordingly, in the next trial, presence has been assumed of a vertical tension crack running parallel to the crest of the slope with water pressure acting in it and the given slip circle was terminated at its bottom. The depth of the tension crack has been arbitrarily chosen as 10.0 m ($\cong 0.25\ H_t$) and, true to expectations, this has resulted in convergence. From an arbitrarily chosen initial design vector F = 2.0 and

$\theta = 0.1$ (with the corresponding $Z_n = -0.3539E{+}03$ and $M_n = -0.3115E{+}04$), the solution has converged to $F = 1.61$ and $\theta = 0.382$ (with the corresponding $Z_n = 0.8836E{-}03$ and $M_n = 0.2087E{+}02$).

Two observations can be made from this success in achieving convergence in the case of a steep slope with high cohesion value : (i) that the iterative scheme based on Powell's method as used in the proposed equation solver is quite effective in handling such cases, and (ii) that the consideration of the presence of a tension zone below the crest can greatly help overcome the problem of occasional non-convergence in such situations.

An examination of the detailed results in the above case, however, reveals that the position of the line of thrust associated with the solution is highly unsatisfactory as a part of it is located below the sliding mass (as indicated by the negative L/H values). Besides, some of the resultant interslice forces (Z') are also found to be negative. Thus, even though convergence has been achieved, the solution is not an acceptable one. In order to obtain an acceptable solution, the following measures have been adopted :

(i) The depth of tension crack (z_t) has been increased from 10.0 m to 15.0 m while still considering parallel interslice forces. This has resulted in a line of thrust which is located entirely within the sliding mass and, is, therefore, much improved compared to the previous solution with $z_t = 10.0$ m. Further, the resultant interslice forces are all positive. The corresponding solution vector is obtained as: $F = 1.51$ and $\theta = 0.481$ which are appreciably different from their previous values for $z_t = 10.0$ m. However, the line of thrust, though lying within the sliding mass is not positioned entirely within the middle third of the interslice heights.

(ii) As a next trial, solution has been obtained taking $z_t = 20.0$ m (considering that the calculated value of the depth of zero active earth pressure is nearly 20.0 m); however, there has been no further improvement on the position of the line of thrust. From the above it appears that for this slope the assumption of parallel interslice forces does not hold good, as also observed by Soriano (1976).

Trial Solution with General Interslice Force Function

As it seemed possible that more acceptable solutions can be obtained by considering other interslice force functions i.e., k-distributions, it is required to try different k-distributions in a systematic manner. For this purpose the extended formulation treating the k-values as design variables has been utilized. In accordance with the suggestions made earlier, in this case the formulation has been further extended to include constraints on the line of thrust such that the ratios L/H lie within the middle third. And to add more flexibility the depth of tension crack (z_t) has also been included in the design vector. With this extended formulation the following trials have been made:

(i) As already stated, Fredlund (1984), Fan et al. (1986) have reported about the development of a general empirical interslice force function (Eqn.4) computed from finite element analyses of a large number of problems. Charts are available which provide the values of the empirical constants to be used for a given problem. Using these charts the values of the

Table 1 : Calculated Responses Associated with the Solution to the Example Problem 1

Slice No.	σ' kPa	τ kPa	L/H	L'/H	$Z'/\Upsilon bH_t$	F_v
1.	21.42	19.98				
			0.33	0.33	0.19	2.46
2.	27.54	19.98				
			0.33	0.33	0.37	2.61
3.	45.78	19.98				
			0.37	0.37	0.49	2.24
4.	48.84	19.98				
			0.42	0.42	0.59	2.20
5.	47.72	19.98				
			0.47	0.47	0.68	2.38
6.	51.36	19.98				
			0.50	0.50	0.73	2.62
7.	47.08	19.98				
			0.52	0.52	0.79	3:14
8.	24.39	19.98				
			0.45	0.45	0.91	5.38
9.	21.92	19.98				
			0.38	0.38	1.04	High
10.	45.24	19.98				
			0.37	0.37	1:04	High
11.	55.92	19.98				
			0.38	0.38	0.97	High
12.	56.40	19.98				
			0.40	0.40	0.87	High
13.	52.90	19.98				
			0.42	0.42	0.76	High
14	48.61	19.98				
			0.45	0.45	0.65	High
15	43.95	19.98				
			0.49	0.49	0.55	High
16	38.94	19.98				
			0.54	0.54	0.47	High
17	33.52	19.98				
			0.58	0.58	0.41	High
18	27.67	19.98				
			0.58	0.58	0.38	High
19	21.32	19.98				
			0.50	0.50	0.39	High
20	14.41	19.98				

constants have been obtained to develop the interslice force function (k-distribution) for the present problem. The k-distribution thus developed (Fig.2b) has been fed as input for the next trial while keeping the initial value for the depth of tension crack z_t (now a design variable) as 15.0 m. This has yielded a solution which is associated with a line of thrust which is far more improved compared to that obtained in the first trial but yet not entirely within the middle third.

(ii) Regarding a reasonable interslice force function it has been suggested qualitatively that when the ground surface is horizontal the functional direction should approach zero and it is anticipated to approach the gradient of the ground surface near the middle of the steepest part of the slope (Ching and Fredlund, 1983). In view of the fact that the given slip surface passes through the toe and, therefore, the slope of the ground surface remains unchanged for a major portion of its length (except for a small portion near the upper end which terminates below the tension crack), it seems reasonable to flatten the initial k-distribution used in the first trial as shown in Fig.2b. Accordingly, a revised k-distribution (Fig.2b) has been considered as initial guess for the next trial while retaining the other initial values as in earlier trials. This has finally led to the desired acceptable solution in which the line of thrust is found to lie entirely within the middle third. The final k-distribution is also shown in Fig.2b and the associated F and θ values are 1.50 and 1.026 respectively. The final depth of tension crack is 15.40 m. The position of the line of thrust together with the calculated values of the internal forces corresponding to the optimal solution are presented in Table 1. The position of the associated line of thrust for total stress as indicated by the ratios L/H, where L denotes height of the point of application of the interslice force and H is the height of the corresponding interslice boundary. The ratio L'/H indicates the corresponding ratio for effective stress (Fig.1c). Since the value of r_u is zero, both these ratios have the same values. σ' and τ denote the effective normal stress (whose value in this case is same as that of the total normal stress, σ, as r_u is zero) and the shear stress at the base of a slice respectively (Fig.1b). The ratio $Z'/\Upsilon bH_t$ indicates non-dimensional value of the normal component of the resultant interslice force due to effective stress at an interslice boundary (Fig.1c). For the expressions for each of the above, reference may be made to Spencer (1973) and Bhattacharya (1990). It can be seen from the Table that the line of thrust lies entirely within the middle third. The values of $Z'/\Upsilon bH_t$, σ and τ are seen to be all positive, as they should be. Further, from the Table, it is also seen that the values of F_v, the factors of safety along vertical interfaces are all greater than the value of F, the factor of safety, as they should be. Thus, the obtained solution satisfies all conditions of acceptability.

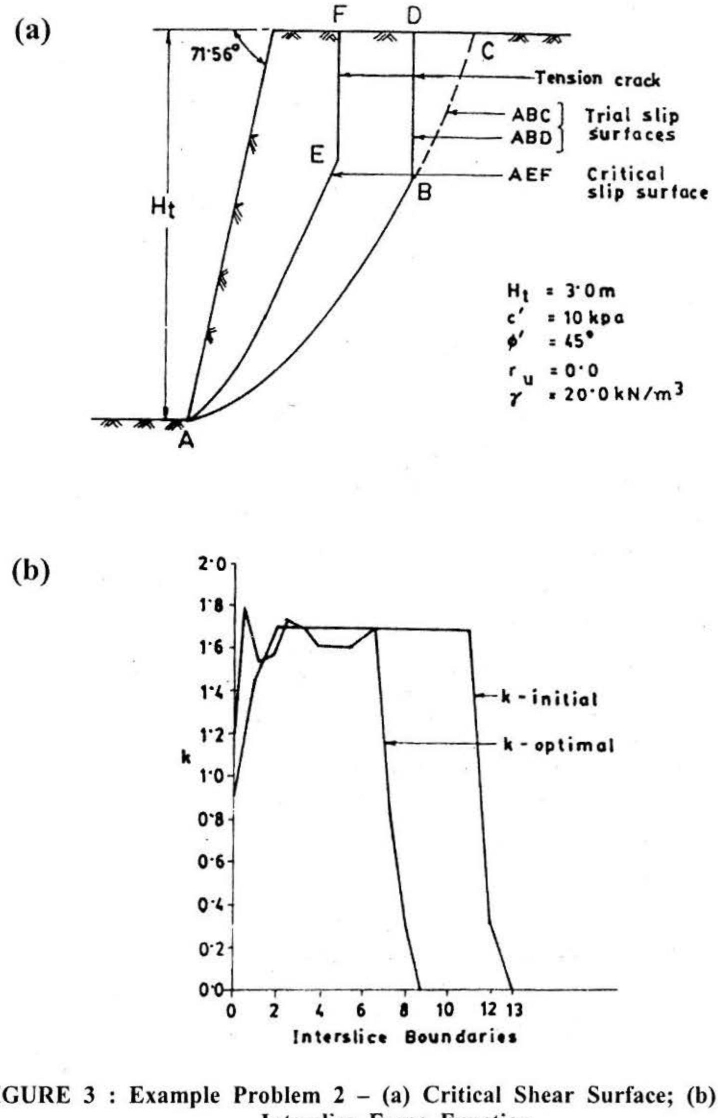

FIGURE 3 : Example Problem 2 – (a) Critical Shear Surface; (b) Optimal Interslice Force Function

Example Problem 2

Figure 3a presents another section of a steep and cohesive slope along with the soil properties. It has been reported (Castillo and Luceno, 1982) that the same slope was previously analysed by Baker and Garbar (1977) and by Castillo and Revilla (1977), using two different variational techniques and significantly different results were obtained; the values of the minimum factor of safety (F_{min}) associated with the critical slip surfaces being 1.80 and 1.60

respectively. It appears that in these solutions the acceptability criteria were not checked (Castillo, 1989). Further, the shape and location of the critical slip surface was not reported. The Direct Procedure of determination of critical slip surface developed by the authors (Bhattacharya and Basudhar, 2001) has the built-in provision for inclusion of acceptability criteria. The present analysis attempts to use the developed procedure to arrive at an acceptable solution for the problem and then compare the results with those obtained from other existing techniques based on variational methods, dynamic programming, etc.

Results and Discussion

In Fig.3a, the slip surface marked ABC has been arbitrarily chosen as the initial slip surface. A total of 13 slices has been considered for the analysis. To start with, no tension crack has been assumed, and, as is usual with the Spencer method, the interslice forces have been assumed to be all parallel i.e., k equals to unity throughout. However, the solution did not converge. It was observed that a number of constraints on the line of thrust which were initially among the violated constraints, remained so even after several cycles of minimization. This is rather unusual because, one of the features of the extended penalty function method (SUMT) of minimization adopted in the developed Direct Procedure is that, as the minimization progresses, it tends to bring the solution back to the feasible region by successively reducing the number of violated constraints. Such an occurrence, therefore, is indicative of a kind of "ill-conditioning" of the problem in the sense that under the given conditions the position of the line of thrust cannot be improved and, as a result, a solution in the feasible region cannot be obtained.

Violation of the constraints on the line of thrust indicates development of tension in the sliding mass and, therefore, in the next trial, tension crack has been introduced. The trial depth of the tension crack z_t has been taken as 1.08 m, (based on the depth of zero active earth pressure), thereby changing the initial slip surface from ABC to ABD. To render more flexibility to the computational scheme the depth of tension crack z_t has been included in the design vector. The solution has now converged to yield the final surface marked AEF (Fig.3a) with a F_{min} of 1.48 and a final z_t value of 1.0 m. The constraints on the line of thrust are all satisfied, which means the line of thrust is well within the sliding mass, though all L/H values do not lie within the middle third. The solution is, however, unacceptable as the detailed results show the value of the calculated normal stress at the base of the last slice near the scarp end of the slip surface to be negative.

Thus, clearly, the introduction of tension crack and its inclusion in the design vector have contributed to the convergence of the solution. The

Table 2 : Calculated Responses Associated with the Solution to the Example Problem 2

Slice No.	σ' kPa	τ kPa	L/H	L'/H	$Z'/\Upsilon bH_t$	F_v
1.	16.49	17.34				
			0.31	0.31	0.18	1.57
2.	11.04	13.77				
			0.29	0.29	0.30	1.60
3.	15.35	16.59				
			0.27	0.27	0.40	1.71
4.	19.27	19.16				
			0.28	0.28	0.47	1.71
5.	15.95	16.98				
			0.29	0.29	0.51	1.95
6.	14.09	15.77				
			0.30	0.30	0.52	2.35
7.	14.56	16.08				
			0.31	0.31	0.49	2.82
8.	13.85	15.61				
			0.37	0.37	0.44	3.17
9.	12.99	15.05				
			0.49	0.49	0.39	3.15
10.	11.49	14.06				
			0.64	0.64	0.35	3.02
11.	2.58	8.23				
			0.59	0.59	0.43	4.57
12.	2.40	8.12				
			0.49	0.49	0.51	8.89
13.	1.63	7.62				

unacceptability of the solution, however, may be attributed to an improper choice of the k distribution. As it seemed possible that more acceptable results might be obtained using other k distributions, the surface marked AEF has been re-solved using the original version equation solver developed by the authors and taking the k distribution marked 'k-initial' in Fig.3b. Such a k-distribution has been selected based on guidelines for the choice of interslice force functions (Fan et al., 1986). However, even with this k-distribution, the concerned normal stress still remained negative, though its value is now closer to zero. Similar attempts can be made using a number of different k-distributions till a more acceptable solution is obtained.

Obviously, a systematic search for a suitable k-distribution is desirable. Now, with the kind of formulation used in the extended version of the equation solver developed by the authors (Eqn.4) it is possible to do so by treating the set of k-values (one for each of the interslice boundaries) as design variables. This has now resulted in an acceptable solution as is evident from the calculated stresses and other internal forces presented in Table 2. It can be seen from the Table that the obtained line of thrust (indicated by the ratios L/H and L'/H , whose values are the same in this case also, as r_u is zero) is reasonable as only a small portion of it near the toe lies marginally outside the middle-third of the respective interslice heights. It is further seen that the stresses and internal forces as indicated by σ', τ and $Z'/\Upsilon bH_t$ (as explained in the case of Example Problem 1) are all positive, as they should be. The values of F_v, the factors of safety along vertical interfaces are seen to be all greater than the value of F, the factor of safety, as they should be. Thus, the obtained solution satisfies all conditions of acceptability. The final k-distribution obtained as part of the solution is marked 'k-optimal' in Fig.3b. The factor of safety has been obtained as 1.53 which is a little higher than the previous value of 1.48 corresponding to the unacceptable solution.

It appears that a more direct approach to obtain acceptable solution would be followed if during the search for the critical slip surface the set of k-values defining the interslice force function be included in the design vector. But in that case the number of design variables will be nearly doubled, which might cause computational difficulties by increasing the interdependence among the design variables. However, such a study has not been undertaken here.

Comparison with Solutions Obtained using Other Techniques

As already stated, attempts were made earlier to solve the present problem using variational technique. As reported by Castillo and Luceno (1982), two different variational formulations were used by Baker and Garbar (1977) and by Castillo and Revilla (1977) and significantly different results were obtained. The corresponding F_{min} values were reported as 1.80 and 1.60 respectively. Such a wide variation in the value of factor of safety may be explained by examining whether the acceptability criteria were taken care of in these solutions. While in the solution obtained by Castillo and Revilla the acceptability criteria were not checked (Castillo, 1989), it is not known whether the other solution was checked for acceptability.

For the sake of comparison with the dynamic programming technique, the program SSOPT (Baker, 1979) has been utilized to obtain a solution to this example problem. Dynamic programming technique is widely known to be independent of initial design points. However, in the present case it has been observed that the program SSOPT based on the same technique is markedly sensitive to certain input data. For quite a few apparently reasonable set of these input data, convergence could not be achieved. After several attempts the solution has converged to yield a F_{min} of 1.75; but detailed output show that the solution is unacceptable as it is associated with a line of thrust which almost coincided with the critical slip surface. Furthermore, negative values have been obtained for a resultant interslice force and the normal stress at a slice base (near the upper intersection point of the slip surface with the ground).

General Discussion and Conclusions

On the basis of the studies undertaken in this paper, the following concluding remarks can be made:

1. In those situations in which slope stability analysis based on rigorous limit equilibrium methods (e.g., Spencer method, Morgenstern and Price method, GLE method) faces difficulties either in convergence or in convergence to an acceptable solution (viz., steep and cohesive slopes) introduction of tension crack often helps in achieving convergence. In such situations, however, to arrive at an acceptable solution it is often required to try different k-distributions. The extended version of the equation solver proposed in this paper, allows a systematic trial with different k-distributions as well as varying depth of tension crack by treating z_t and the k-values as design variables, while including appropriate acceptability constraints in the formulation.

2. The general empirical interslice force function proposed by Fan et al. (1986) may not result in acceptable solution especially in the type of problems treated in this paper; nevertheless, it can serve as a good initial guess for the determination of optimal interslice force function i.e., k-distribution leading to acceptable solutions.

3. With reference to the Example Problem 1, it is observed from Fig.2b that the final k-distribution obtained as a part of the solution is of irregular shape with a discontinuity in the lower half of the slope. Again, with reference to the Example Problem 2, Fig.3b shows that the final k-distribution is rather irregular in shape. The reasonableness of such interslice force functions may be judged on the basis of stress analyses using finite element method. However, the value of the present investigation lies in the findings that the proposed numerical procedure can be utilised successfully in obtaining an acceptable solution by iterating over the k-distribution and eventually finding out a distribution that is compatible with the conditions of the problem at hand. A reasonable shape of the interslice force function can be ensured by inserting appropriate side constraints in the formulation. However, further work is necessary on this aspect.

4. In steep and cohesive slopes, it has been observed that problems of non-convergence arise in the determination of critical slip surfaces even wen well established computer programs based on rigorous methods of analysis coupled with optimization techniques are used, e.g. the program SSOPT based on the Spencer method and dynamic programming technique. Using the procedures outlined in this paper, it is possible to obtain a convergent as well as acceptable solution in two phases. In the

phase I, the Direct Procedure in which there is provision for introduction of tension crack with its depth as a design variable, is used which yields a critical slip surface, which may not satisfy the acceptability criteria.

Whenever the procedure converges to unacceptable solution, it indicates that the assumption of parallel interslice force is not compatible with the conditions of the problem and that other k-distributions should be tried in order to obtain an acceptable solution. In phase II, the critical slip surface obtained in phase I is re-analysed using the proposed extended version of the equation solver, in which the k-values are treated as design variables while acceptability criteria are included as constraints. This, then, results in an acceptable solution and the associated k-distribution is also obtained as a part of the solution.

5. It appears that a more direct approach to obtain acceptable solution would be followed if during the search for the critical slip surface the set of k-values defining the interslice force function be included in the design vector. But in that case the number of design variables will be nearly doubled, which might cause computational difficulties by increasing the interdependence among the design variables. However, further research is required in this direction.

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