Indian Geotechnical Journal, 32 (2), 2002

Uplift Response of Buried Pipes in Sands Using FEM

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Introduction

Pipes laid underground get subjected to considerable uplift thrust when the water table exists at a level much higher than the ground surface. Such situation often arises in the case of pipes employed for the transportation of gas and oil under water. In order to maintain the integrity of such pipes, it is necessary that the pipe foundation system has an adequate uplift resistance. In case if the pipe itself is not in a position to counteract the prevailing uplift pressures, anchors are then provided to impart additional uplift support to such pipes. Considerable investigations, both theoretical and experimental, are available on the estimation of the uplift resistance of anchors (Meyerhof and Adams, 1968; Vesic, 1971; Murray and Geddes, 1987; Rowe and Davis, 1982a and 1982b; Koutsabeloulis and Griffiths, 1989; Subba Rao and Kumar, 1994; Basudhar and Singh, 1994; and Kumar, 1999). However, not much information is reported on the determination of the uplift resistance of pipes. The uplift resistance of buried pipes is normally estimated from the available uplift results on the strip anchors (Trautmann et al., 1985; and Matyas and Davis, 1983a and 1983b). In the present paper, a computational study on the basis of elastoplastic finite element method has been carried out to obtain the load deformation response of buried pipes till the occurrence of complete shear failure in the soil mass. The development of plastic zones within the soil mass is also examined. The effect of embedment ratios of the pipes and the friction angle of the soil mass on the results has been studied in detail. Comparisons of the computed failure loads have been made with the different available theories often used for finding the uplift resistance of strip anchors.

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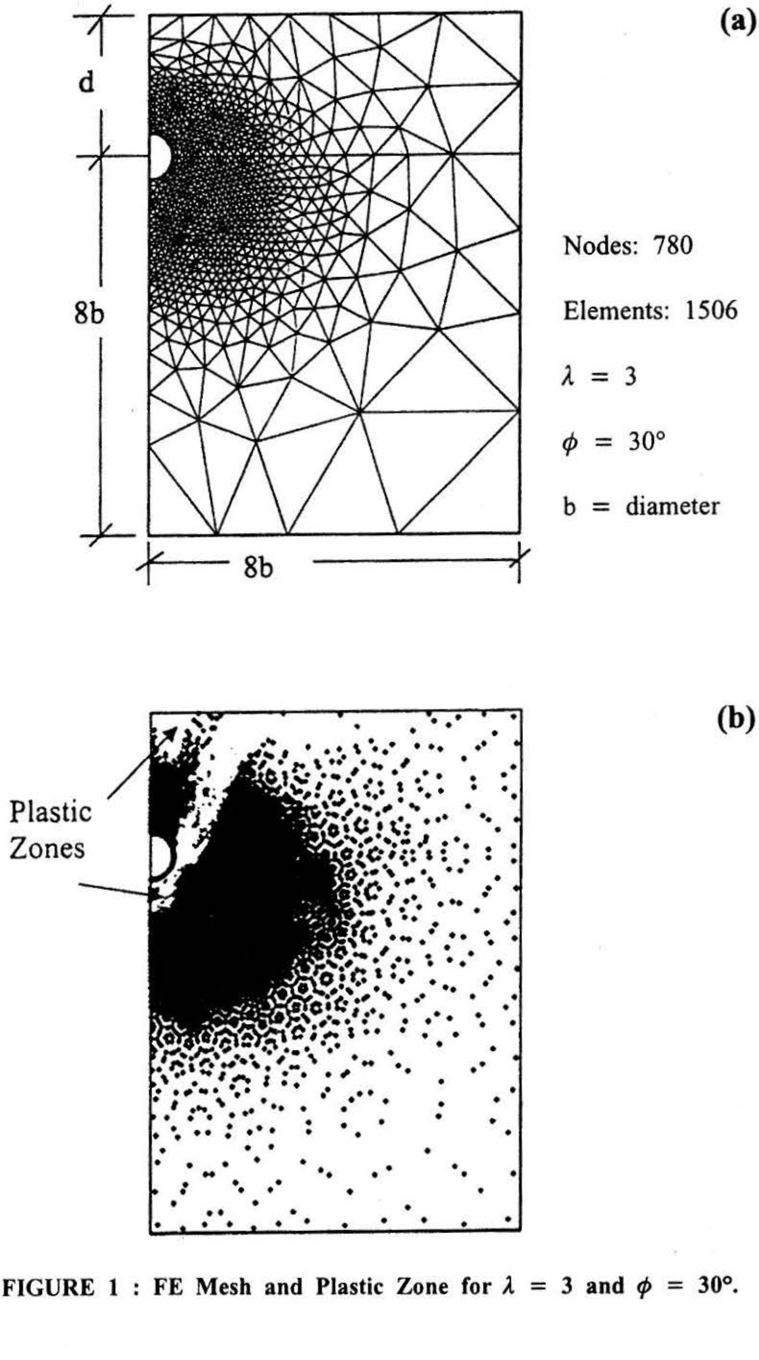

FIGURE 1 : FE Mesh and Plastic Zone for $\lambda = 3$ and $\phi = 30°$.

Definition of the Problem

A long pipe of diameter b is embedded at a depth d from the ground surface as shown in the Fig.1(a). The pipe is assumed to be perfectly rigid, rough and is buried in a cohesionless soil medium. It is required to assess the load deformation response of the pipe till the ultimate pullout failure, and then to establish the magnitudes of failure load. In the present analysis, the value of the b was kept equal to 0.5 m, and the embedment ratio $(\lambda = d/b)$ was varied in between 1 and 7. The values of the elastic modulus (E) and the Poisson's ratio (ν) of soil were taken equal to 20,000 kPa and 0.3, respectively. The friction angle (ϕ) of the soil was varied between 30 and 50 degrees. The soil unit weight (γ) was kept equal to 20 kN/m^3.

Constitutive Model

It was assumed that the soil medium is linearly elastic perfect plastic material following Mohr-Coulomb failure criterion and an associated flow rule. The incremental stresses $\{d\sigma\}$ were related to the total incremental strains $\{d\varepsilon\}$ via elasto-plastic stiffness matrix $[D^{ep}]$ (Chen and Mizuno, 1990).

For the present plane strain problem,

$$\left[D^{ep}\right]_{4\times4} = \left[D^{e}\right]_{4\times4} - \frac{\left[D^{e}\right]_{4\times4}\left[\frac{\partial F}{\partial \sigma}\right]_{4\times1}\left[\left(\frac{\partial F}{\partial \sigma}\right)^{T}\right]_{1\times4}\left[D^{e}\right]_{4\times4}}{\left[\left(\frac{\partial F}{\partial \sigma}\right)^{T}\right]_{1\times4}\left[D^{e}\right]_{4\times4}\left[\frac{\partial F}{\partial \sigma}\right]_{4\times1}} \tag{1}$$

where, F is the yield function

$$F = \frac{I_1}{3}\sin\phi + \sqrt{J_2}\left(\cos\alpha - \frac{\sin\alpha\sin\phi}{\sqrt{3}}\right) - c\cos\phi \tag{2}$$

$$\left[\left(\frac{\partial F}{\partial \sigma}\right)^{T}\right]_{1\times4} = \left[\frac{\partial F}{\partial \sigma_x}\ \frac{\partial F}{\partial \sigma_y}\ \frac{\partial F}{\partial \sigma_z}\ \frac{\partial F}{\partial \tau_{xy}}\right] \tag{3a}$$

$$J_2 = \left[\frac{\left(s_x^2 + s_y^2 + s_z^2\right)}{2} + \tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right] \tag{3b}$$

$$J_3 = s_x s_y s_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - s_x\tau_{yz}^2 - s_y\tau_{xz}^2 - s_z\tau_{xy}^2 \tag{3c}$$

$$\alpha = \frac{1}{3}\sin^{-1}\left(-\frac{3J_3\sqrt{3}}{2J_2^{3/2}}\right) \tag{3d}$$

where

$$I_1 = \sigma_x + \sigma_y + \sigma_z;$$

$$s_y = \sigma_x - I_1/3;$$

$$s_y = \sigma_y - I_1/3;$$

$$s_z = \sigma_z - I_1/3; \text{ and}$$

$\left[D^e\right]_{4\times 4}$ = elastic stiffness matrix for the plane strain case.

The yield function F can also be related with the major and minor principal stresses (σ_1 and σ_3) with the Eqn.(4)

$$F = \frac{(\sigma_1 - \sigma_3)}{2} - \frac{(\sigma_1 + \sigma_3)}{2}\sin\phi - c\cos\phi \tag{4}$$

FE Mesh and Boundary Conditions

In accordance with the FE analysis of Rowe and Davis (1982b), in which the vertical uplift response of horizontal rigid strip anchors was examined, it was assumed that the effect of the loading of the pipe is negligible at horizontal and vertical distances of magnitudes 8b from the center of the pipe respectively. On account of the symmetry of the pipe about its vertical central axis, only one half of the soil domain enclosed within the extreme vertical boundaries was considered. For the chosen soil domain, along both the vertical boundaries (one of which forms the central vertical axis of the pipe) displacement constraint only in the horizontal direction was provided. Along the horizontal boundary line below the pipe, displacement constraints both in horizontal and vertical directions were imposed. No separation of the soil material from the periphery of the pipe was considered during the course of the analysis. The soil mass as enclosed within the defined boundaries of the domain was discretized into a mesh of six noded linear strain triangular elements. The finite element mesh was generated in such a fashion that the elements approaching the periphery of the pipe become gradually smaller in sizes. Typical generated FE meshes for the embedment ratios equal to 3 and 5 are shown in Figs.1(a) and 2(a). The number of elements vary in between 1390 and 1506, and the number of nodes were in between 719 and 780.

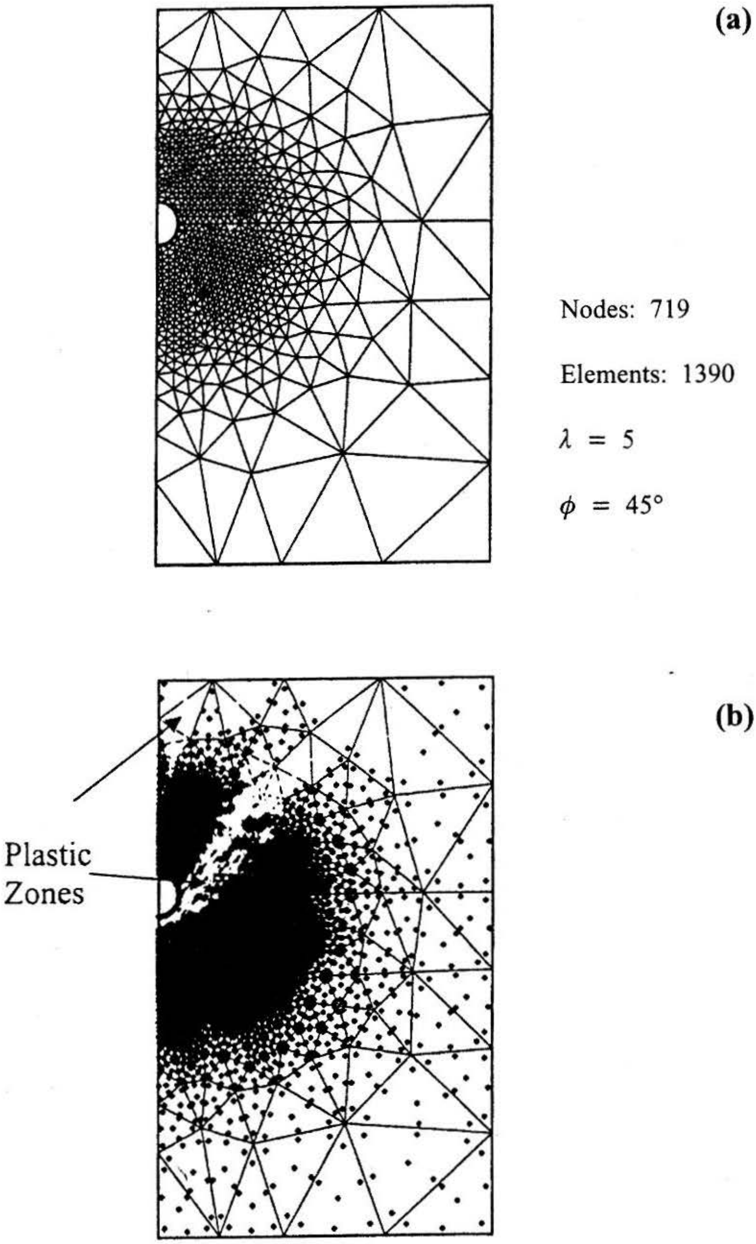

FIGURE 2 : FE Mesh and Plastic Zone for λ = 5 and ϕ = 45°.

Analysis

The finite element code SAGE-CRISP version 4.0 was used for carrying out the analysis. Before launching the elastoplastic FE analysis, in situ stresses everywhere were determined by assuming K_0 condition in the soil mass, and the magnitude of earth pressure coefficient K_0 was kept equal to 1.0. Since it has already been demonstrated by Rowe and Davis(1982b) that the variation in K_0 does not bring any significant change in the uplift resistance of anchors, its effect on the pullout resistance of pipes was not, therefore, investigated in the present paper; in fact a theorem in plasticity also states that the variations in initial stresses have no influence on the magnitudes of materials following associated flow rule (Chen, 1975). After establishing the in-situ stresses from the K_0 condition, the finite element analysis was then started by imposing equal vertical upward displacement increments everywhere along the periphery of the pipe; displacement constraint in the horizontal direction everywhere along the periphery of the pipe was employed to simulate the rough pipe surface. Each increment of the displacement was further subdivided into 50-100 small increments. During each small displacement increment, modified Newton-Raphson iterative technique was used to account for the non-linearity due to the elastoplastic stiffness matrix. It was seen that 8-12 iterations were sufficient enough to achieve the convergence. A displacement convergence criterion was used to check for the convergence. The convergence was said to be achieved when the square root of the sum of displacements in the current iteration over the square root of the sum of displacements up to the current iteration becomes equal to or less than 1%. Since greater vertical stresses exist at the bottom of the pipe than at its top, a certain amount of initial upward displacement of the pipe was necessary in all the cases to achieve its vertical force equilibrium. After achieving the initial force equilibrium of the pipe, displacement increments of the pipe were continued till the complete failure of the pipe was noticed.

The uplift force P_u per unit length of the pipe for any displacement can be determined by integrating either the nodal vertical reactions or the vertical components of the stresses along the surface of the pipe. In the present paper P_u was obtained by numerically integrating the vertical component of the stresses everywhere at the first row of Gauss integration points along the periphery of the pipe. It has been done following the findings of Griffiths (1982) and Woodward and Griffiths (1998, 2000), for computing the bearing capacity of foundations using finite elements, in which case it was shown that the integration of stresses provided slightly a better match with the slip line solution than the summation of vertical reactions along the foundation surface.

The average vertical uplift pressure p_u was defined as P_u/b. Using the

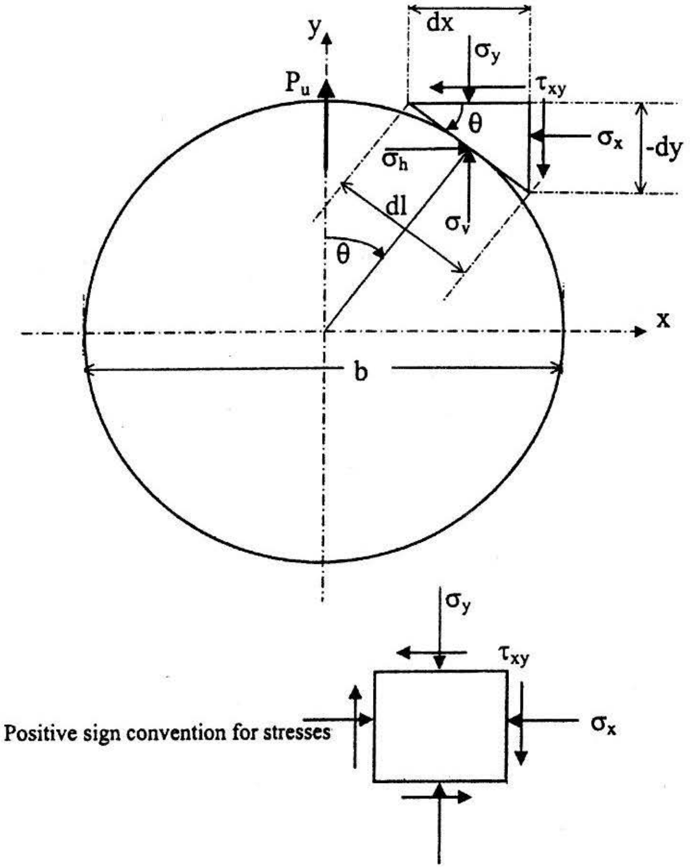

FIGURE 3 : Equilibrium of a Soil Element along the Pipe Periphery

Fig.3, with the positive sign conventions of stresses as indicated, it can be shown from the equilibrium of forces for an element along the pipe that

$$P_u = 2\int_{\theta=0}^{\pi} \sigma_v \mathrm{dl} = 2\int_{\theta=0}^{\pi} \frac{\left\lfloor \sigma_y \mathrm{dx} + \tau_{xy}(-\mathrm{dy}) \right\rfloor}{\mathrm{dl}} \mathrm{dl} \tag{5a}$$

$$p_u = \frac{2}{b}\int_{\theta=0}^{\pi} \left(\sigma_y \cos\theta + \tau_{xy} \sin\theta\right) \mathrm{dl} \tag{5b}$$

The magnitude of the p_u at failure ($p_{u,\,ult}$) was expressed in the form of uplift factor F_γ as earlier defined by Rowe and Davis (1982b).

$$p_{u,\,ult} = \gamma\, \mathrm{d}F_y \tag{6}$$

Results

The obtained uplift pressure displacement response was related in a non dimensional way in the same fashion as was earlier presented by Rowe and Davis (1982). The non dimensional uplift pressure was expressed as $p_u/(\gamma d)$, and the uplift displacement (δ) of the pipe was presented in the form of parameter $E\delta/(\gamma bd)$. The non dimensional uplift pressure-displacement relationships in all the cases are shown in Figs.4 to 8. It can be seen that in all the cases, initially, the pressure displacement response remains linear, and then the slope (stiffness) of the plotted curves decreases continuously till the complete failure occurs. The magnitudes of the failure load, the failure displacement and the initial stiffness of the curves become larger for higher values of λ and ϕ. From the obtained pressure versus displacement relationship, the failure loads were determined in all the cases. From the known magnitude of failure loads, the uplift factor F_γ was then determined using Eqn.(6). The variation of the F_γ with λ and ϕ is shown in Fig.9. It can be seen that the uplift factor increases continuously with the increases in ϕ and λ. The variation of the F_γ with λ is found almost linear. In addition to examining the pressure displacement relationships of the pipes, the failure status of all the elements at their integrating points was also noticed. It was seen that even at complete collapse, the soil mass lying just above the pipe remains mostly non-plastic. The collapse of the pipes in all the cases occurs on account of the development of a thin curved plastic (shear) zone which starts from the bottom of the pipe and then extending up to the ground

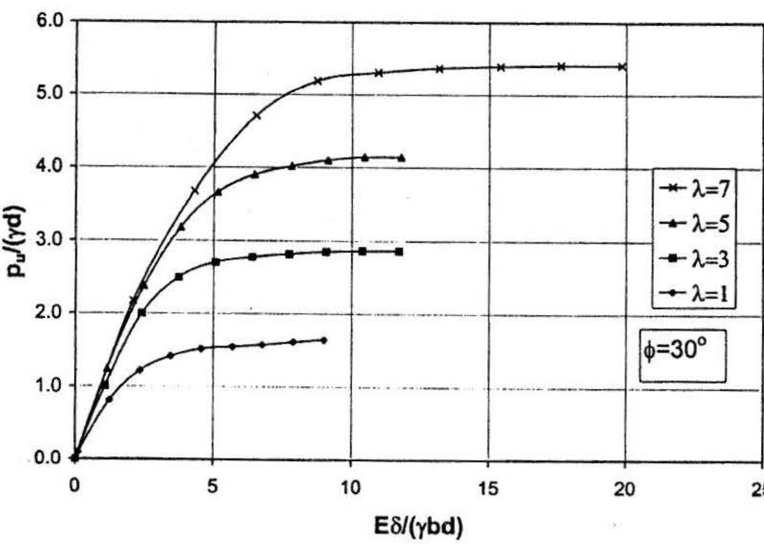

FIGURE 4 : Non-dimensional Uplift Pressure-Displacement Response for ϕ = 30°

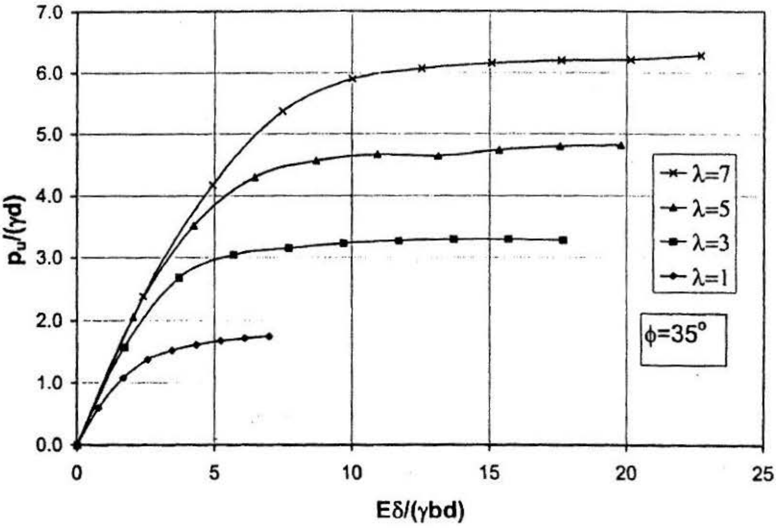

FIGURE 5 : Non-dimensional Uplift Pressure-Displacement Response for ϕ = 35°

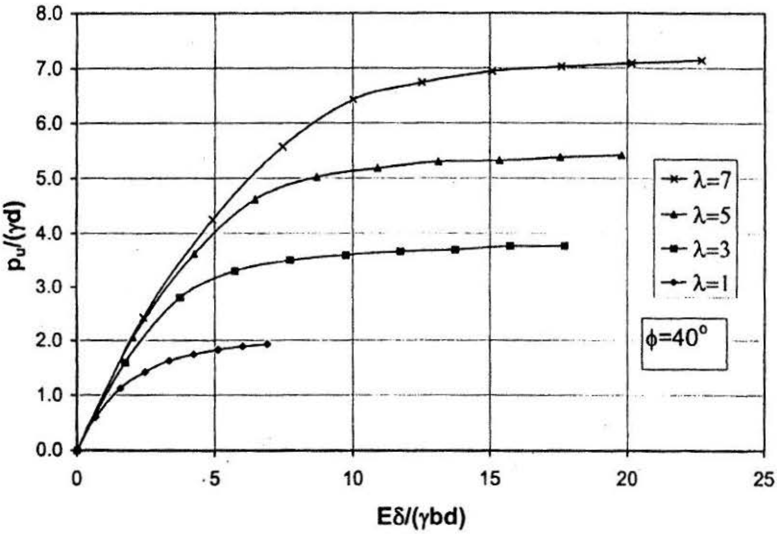

FIGURE 6 : Non-dimensional Uplift Pressure-Displacement Response for ϕ = 40°

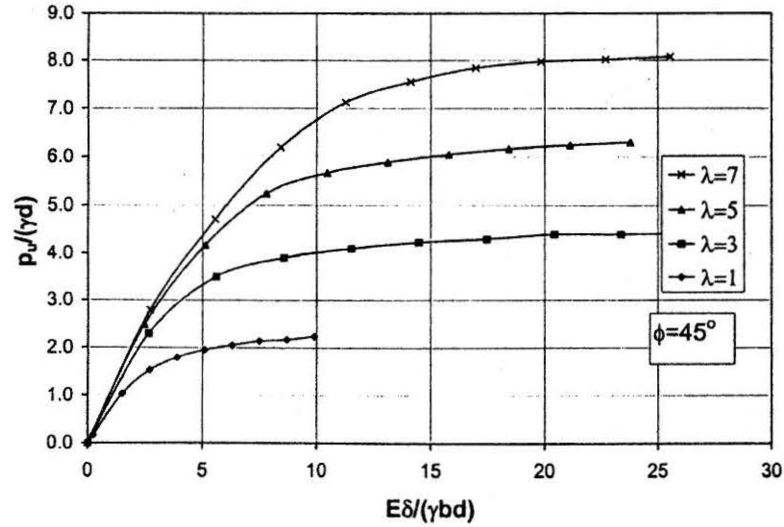

FIGURE 7 : Non-dimensional Uplift Pressure-Displacement Response for $\phi = 45°$

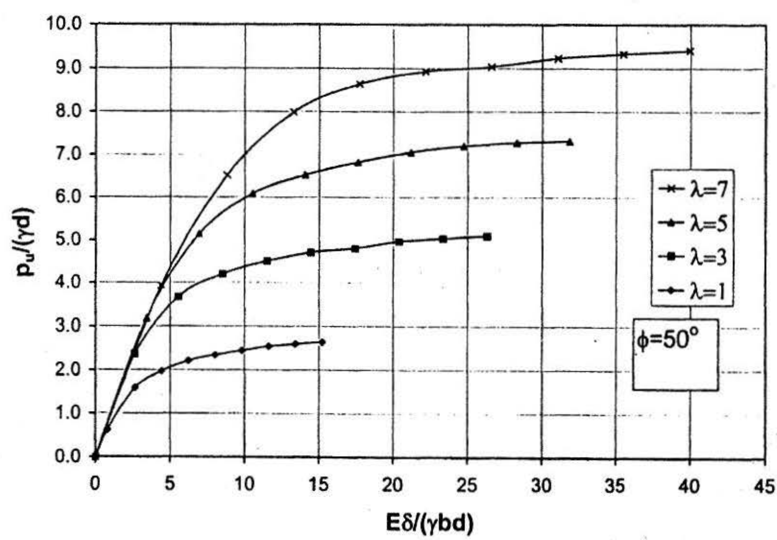

FIGURE 8 : Non-dimensional Uplift Pressure-Displacement Response for $\phi = 50°$

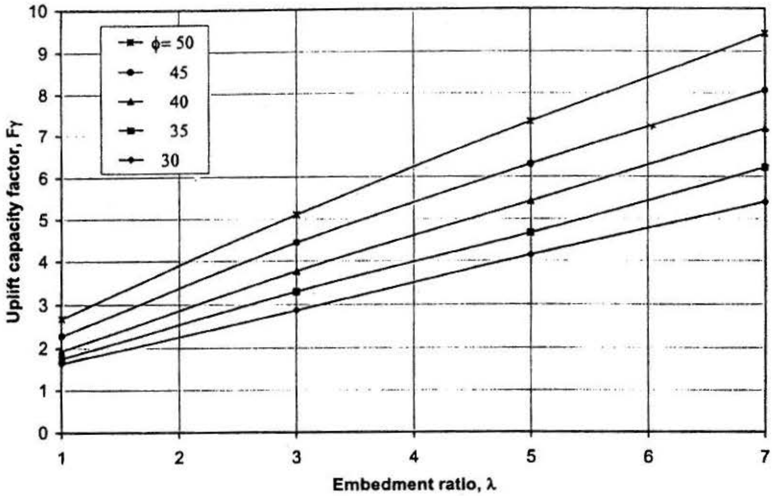

FIGURE 9 : The Variation of the Uplift Factor F_γ with λ and ϕ

surface. The obtained pattern of the plastic zones for two different cases viz. case (i) $\phi = 30^\circ$ and $\lambda = 3$; and case (ii) $\phi = 45^\circ$ and $\lambda = 5$, is indicated in Figs.1(b) and 2(b). In these figures, black dots represent the non-plastic Gauss integration points and the color of the integration points was indicated white if the failure was noticed at that point. White patches near to the pipe surface and those in a small region close the ground around the vertical axis of symmetry represent plastic zone.

Comparisons

The results of the strip anchors subjected to uplift pressure are often utilized in predicting the pullout response of buried pipes. The obtained uplift factor, F_γ, from the present FE analysis was compared with the FE analysis of Rowe and Davis (1982) for associated flow rule material (dilatancy angle, $\psi = \phi$), the limit equilibrium study of Meyerhof and Adams (1968), the method of stress characteristic solution of Subba Rao and Kumar (1994) and the upper bound limit analysis solution of Murray and Geddes (1987). The obtained comparison is shown in Figs.10 and 11. It can be seen that the present FE results on the pipes compare most favorably with the upper bound limit analysis solution for anchors on the basis of linear rupture surfaces. The method of the stress characteristic solution provides the lowest uplift factors. It should be noted that in all the cases the earlier FE study of Rowe and Davis provides highest values of the uplift factors as compare to any other theory.

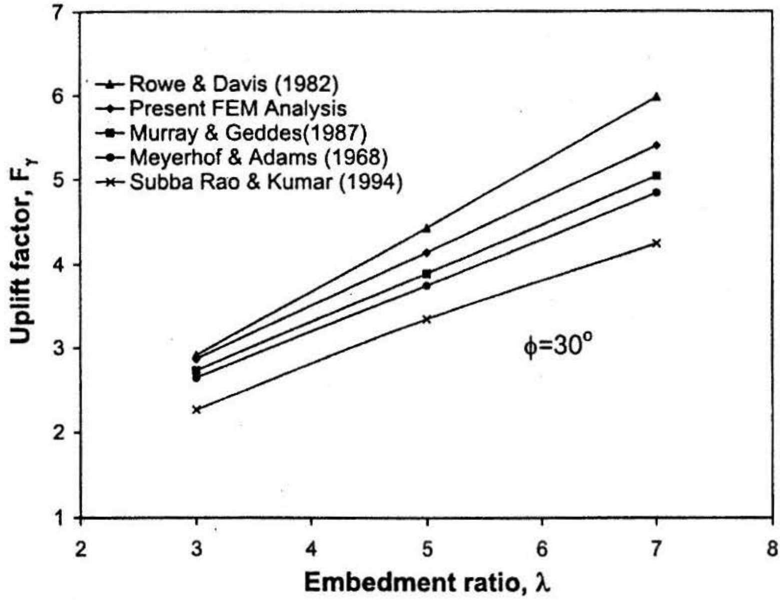

FIGURE 10 : Comparison of the Uplift Factor F_γ for $\phi = 30°$

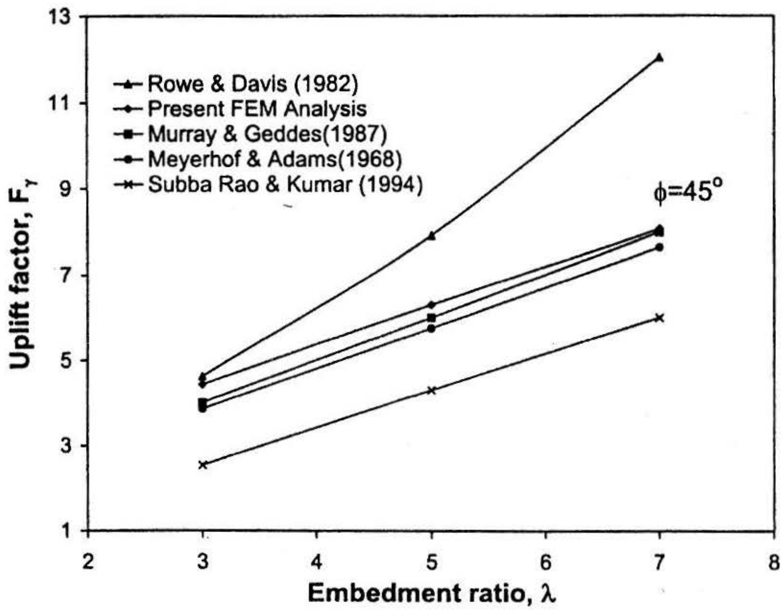

FIGURE 11 : Comparison of the Uplift Factor F_γ for $\phi = 45°$.

Comments

1. The analysis has been carried out using the assumption of the associated flow rule. This was done to avoid the computational complexities involved on account of asymmetric nature of the stiffness matrix $[D^{ep}]$ in dealing with the non-associated material. However, it is well known that the assumption of the associated flow rule results in much higher dilation than is observed for most of soils. Therefore, the solution given in this article will overestimate the uplift resistance of pipes buried in non-associated flow rule material. Based on the upper bound theorem of limit analysis, it has recently been demonstrated by Drescher and Detournay (1993) that depending on the dilatancy angle ψ an approximate magnitude of the collapse load for the non-associated soil can be obtained simply by replacing the c and ϕ with c_m and ϕ_m as given below.

$$\phi_m = \tan^{-1}(\eta \tan\phi); \qquad c_m = \eta c$$

where
$$\eta = \frac{\cos\psi \cos\phi}{1-\sin\psi \sin\phi}$$

2. The material has been assumed to be perfectly plastic. No hardening/ softening behavior of the sand was incorporated in the investigation.

3. It should be mentioned that all the results have been presented in non-dimensional form, and the selected values of b, E and γ do not affect the final results. The results can therefore be utilized to obtain the uplift response of buried pipes in the field for any given input parameters for sand material and pipe diameter. The effect of the Poisson ratio of the sand was not explored. However, its affect on the uplift response of anchors has been shown very marginal by Rowe and Davis (1982a and b).

4. The pipe has been assumed to perfectly rigid. The relative rigidity/ flexibility of the pipe-soil system, and the separation of the soil from the pipe has not been simulated in the analysis.

Conclusions

Based on elastoplastic finite element study, the uplift force displacement response of rigid pipes buried in sands at shallow depths is examined in detail. The failure loads have been obtained in the form of uplift factor F_γ. The magnitude of the uplift factor increases with the increases in the values of the embedment ratio (λ) of the anchor and the friction angle (ϕ) of the soil. The variation of the F_γ with λ is found almost linear. The obtained magnitudes of failure loads compare quite favorably particularly with the existing upper bound limit analysis solution of strip anchors subjected to uplift pressure. In all the cases, it has been noticed that even at complete collapse, the soil mass lying above the pipe remains more or less non-plastic. The collapse of the pipes occurs on account of the development of a thin curved plastic shear zone starting from the bottom of the pipe and then terminating at the ground surface.

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