Characterisation of Soil Spatial Variability and Its Influence on Slope Stability

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Introduction

The stochastic description of the heterogeneous soil media gives a rational way of dealing with the variability of soil properties, which is not possible by deterministic approaches. Such an approach takes into account the uncertainties involved in various stages of soil profiling. The point to point variations of soil properties though expressed through the statistical parameters such as mean and standard deviation, the correlation of the property values with their separating distance expresses both qualitatively and quantitatively the nature, strength and extent of this correlation. Figure 1 shows the parameters necessary for characterisation. The terms in Fig.1(a), $\bar{u}$, $\tilde{u}$, δ_u represent the average value, the standard deviation of fluctuation of u from the average and δ_u, the scale of fluctuation of u, which measures the distance within which the soil property u(z) shows strong correlation in the vertical direction. Figure 1(b) shows the characterisation in two dimensions. In this case, $\bar{h}$ is the average depth, $\tilde{h}$ characterizes the magnitude of difference and δ_u is the characteristic or the correlation distance. Such stochastic characterization of soil profiles/properties is very important in geotechnical engineering, especially in stability problems, where the risk of failure is a function of spatial variability of natural soils (Vanmarcke, 1977a). Vanmarcke had pointed out that such probabilistic characterisation would essentially provide a basic format for quantifying geotechnical engineering information regarding the subsurface conditions at a particular site and provide a basis for predicting the performance of a geotechnical engineering structure and for quantifying the probability failure.

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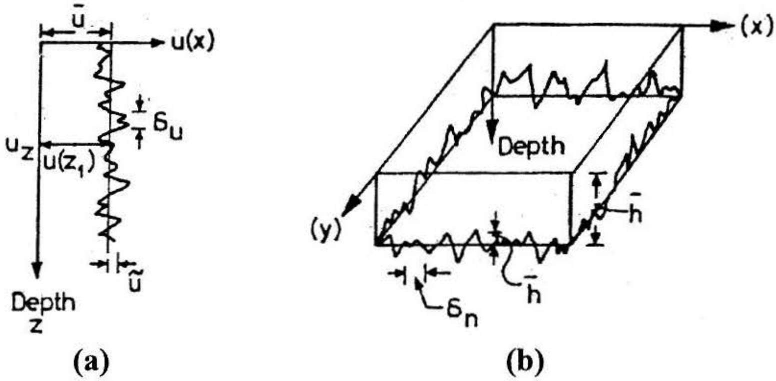

FIGURE 1 : Parameters of (a) Homogeneous Randomly Varying Soil Property; (b) Randomly Varying Depth of Rock

Further it enables a geotechnical engineer to assess critically and compare various site investigation and testing programs and also evaluate their effectiveness (Kulatilake and Varatharajah, 1986). Literature pertaining to geological and geotechnical investigations shows that spatial variability modelling had been attempted with two different approaches, using techniques based on analysis of random fields and geostatistics. Both the techniques have been separately dealt with by geotechnical and mining engineers and engineering geologists (Journel and Hujibregts,1978; De Groot and Baecher, 1993). However, in literature, there is no attempt to examine them collectively, and to draw useful information with regard to exploration programs. It is also necessary to examine the significance of the information obtained with reference to the stability of geotechnical structures. The paper focuses on these concepts in characterization of variability of soils and their application to a typical soil data pertaining to soft soil deposits, and addresses these aspects in two sections. The paper presents two examples demonstrating the influence of spatial variability on the stability of an embankment and a cut slope on soft soil.

Random Field Theory

Stationarity

The random field theory implicitly assumes that data are stationary, i.e., the statistical properties of the time series are unaffected by any shift of the spatial origin or statistically it means, the first two moments (the mean and the covariance) are required to be constant. Any kind of data are considered to be stationary if it strictly adheres to the following conditions: (1) the mean, is constant with distance or no trend/drift exists in the data, (2) the

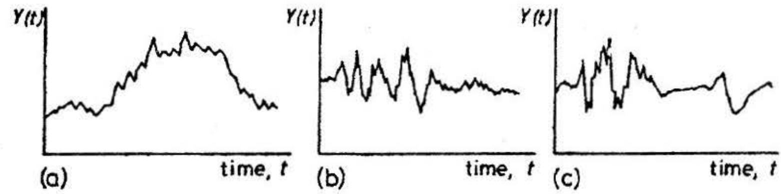

FIGURE 2 : Forms of Non-Stationaity (a) Trend; (b) Variance; (c) Relationship (After Bennett, 1979)

variance is constant with distance, (3) there are no seasonal variations and (4) there are no irregular fluctuations. Non stationarity may however result from the trend, variance and relationship as demonstrated by Bennett (1979) and shown in Fig.2. As applications of both random field theory and geostatistics are facilitated by stationary data, treatment must be first given to data transformation, by which a non-stationary data set is transformed to a stationary one. The standard methods to process the data transformation are (1) classical decomposition; (2) differencing; (3) variance transformation (Jaksa, 1995).

The decomposition method has been widely used in geotechnical literature with reference to data transformation than other methods. This method aim at estimating and removing the deterministic component called trend, and keeps the residual random component stationary. As suggested by Lumb (1974) and Brockwell & Davis (1987), a non-stationary data set is transformed to a stationary data set, by removing a low-order polynomial trend (upto second order, Journel and Huijbergts, 1978), which is usually estimated by Ordinary Least Squares (OLS). The de-trended data set is checked for stationarity by means of nonparametric tests such as Kendall's τ test (Daniel, 1990). For details, refer to Appendix I.

Autocorrelation Functions (ACF)

Lumb (1966) gave the initial treatment of soil variability and suggested that spatial variability models represented by mean linear trends provide relatively the basic models of the geotechnical properties, which are considered as random variables. Since the introduction of the concept of autocorrelation by Agterberg (1970), many works have been directed at evaluating spatial autocorrelation for different geotechnical properties accounting for the vertical and horizontal variations. For engineering purposes, spatial variability is separated into two parts: (i) a deterministic trend, (ii) residual variability about the trend, $x_i = t_i + u_i$. The trend is characterised by a line, curve or surface, and the residuals are characterised statistically, by a mean, standard deviation and autocorrelation function.

Rather than characterize soil properties at every point, data are used to estimate a smooth trend and remaining variations are described statistically. The residual variations are seldom independent from one place to another. Usually they display waviness about the trend, rather than being completely erratic. This waviness actually reflects spatial structure ignored by the trend, creating autocorrelation among the residuals (De Groot and Baecher, 1993). The decrease in the standard deviation of soil properties averaged over a volume of soil or over a linear distance of soil is reflected in the mathematical expressions, called autocorrelation functions (ACFs) that are often used to model this decay of auto correlation with separation distance. The estimation of autocorrelation of a given property and the correlation distance, within which the property is said to be strongly correlated, is the primary task in spatial variability modelling. The basic statistical properties routinely used are the autocovariance, C_k, and autocorrelation ρk, at lag k.

$$C_k = \mathrm{Cov}(X_i, X_{i+k}) = E\left[(X_i - \bar{X})(X_{i+k} - \bar{X})\right] \tag{1}$$

$$\rho_k = C_k / C_0 \tag{2}$$

The sample autocorrelation function $|r_k|$ at lag k is defined as:

$$r_k = \frac{\sum_{i=1}^{N-k}(X_i - \bar{X})(X_{i+k} - \bar{X})}{\sum_{i=1}^{N}(X_i - \bar{X})^2} \tag{3}$$

The ACF is used to determine the distance over which a property exhibits strong correlation. The sample autocorrelation function (ACF) or correlogram is the graph of r_k for lags, k = 0, 1, 2, 3 ... K, where K is the maximum number of lags allowable. Several stipulations have been formulated regarding the maximum number of lags, out of which, $K = N/4$, (Box and Jenkins, 1970) where N is the total number of data, is generally used. Also the accuracy of the sample autocorrelation function is directly related to the number of observations/data. Though for full three dimensional analysis, the minimum number of test results needed for precise estimates of ACF is of the order of 10^4, Lumb (1974) has stated that for one-dimensional study, the value of N in the range of 20 to 100 would suffice for good results.

Models for Autocorrelation Functions

The work by Vanmarcke (1977a, 1977b and 1983) on the application

Table 1 : Theoretical Autocorrelation Functions (Vanmarcke, 1993)

No.	Model Type	Autocorrelation Function
1	Simple Exponential	$e^{-2\lvert h\rvert/\delta}$
2	Squared Exponential (Gaussian)	$e^{-\left[\pi(\lvert h\rvert/\delta)^2\right]}$
3	Regression (2 Order)	$\left[1+4(\lvert h\rvert/\delta)\right]e^{-4\lvert h\rvert/\delta}$
4	Cosine Exponential	$\cos(\lvert h\rvert/\delta)e^{-\lvert h\rvert/\delta}$
5.	Triangular	$\{1-(\lvert h\rvert/\delta)\}$ $\lvert h\rvert \le \delta$ 0 $\lvert h\rvert \ge \delta$

of random field theory to soil variability modelling contributed significantly to these concepts and its application to geotechnical problems. Vanmarcke (1977a) proposed a new parameter δ_v, called scale of fluctuation, which accounts for the distance within which the soil property shows relatively strong correlation. The various models of theoretical autocorrelation functions used to determine the scale of fluctuation are shown in Table 1. He derived the variance function (Γ_v^2) using spatial averaging principle and showed the fluctuations are smoothened in the averaging process through a parameter called, standard reduction factor, Γ_v. However, geotechnical literature, treating soil variability, has given more attention to the ACF method rather than variance function method.

Geostatistics

Geostatistics is based on the regionalised variable, represented by random functions, unlike the classical approach, which treats samples as independent realisations of a random function (Armstrong, 1998). Regionalized variables are those that fall between random variables and completely deterministic variables. These variables could be used to describe any phenomenon with spatial distribution such as elevation, any ground property, etc., This mathematical technique widely used in mining applications was originally developed by Krige and Matheron (1965), with a view to assist in the estimation of changes in ore grade within a mine. Since the property exhibits spatial continuity throughout and it is not always possible to sample every location, the immediate practical need arises for estimating unknown values from data taken at specific locations that can be sampled. The sampling and estimating of regionalized variables are done so that, later, a pattern of variation of the property under investigation can be mapped for

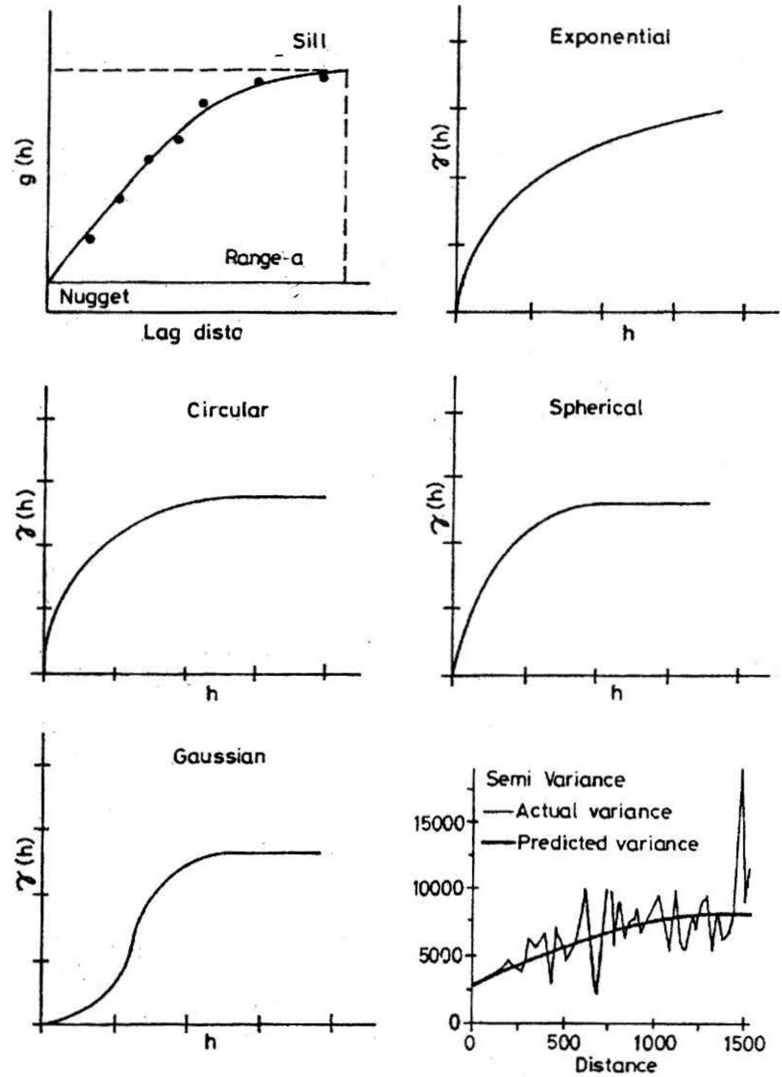

FIGURE 3 : Typical Semivariogram and Few Types of Semivariogram Model

a specific location, using kriging operation. Kriging is an interpolation method, which uses semivariogram in calculating estimates of surface at specific locations. Owing to such successful application of geostatistics principles in mines, later it pervaded its applications in diversified engineering disciplines. Soulie et al. (1990) presented a geostatistical analysis of the spatial variability, both vertical and horizontal, of vane measurements of the undrained shear strength, s_u, of a soft clay deposit in the valley of James

Bay area of Quebec and estimated the value of s_u at points of the deposit where no measurements were made, by using kriging method.

Semi Variograms

The spatial variation is quantified by the semivariogram, where the semi-variance measures the degree of spatial dependence between samples. The variogram is a function, which characterizes the dependence existing between variables at different points in space. This dependence is assumed to be a function of the distance that separates values of variables rather than the values themselves. The magnitude of the semivariance between points depends on the distance between the points. The value of the sample-semivariogram for a separation distance h (lag) is the average squared difference in z values between pairs of input sample points separated by h, given by

$$\gamma(h) = \frac{1}{2h}\left[\sum_{i=1}^{n}\left\{z(x_i)-z(x_{i+h})\right\}^2\right] \tag{4}$$

where x_i are the locations of the samples, $z(x_i)$ are their values and n is the number of pairs (x_i, x_{i+h}) separated by a distance h.

The geostatistics also assumes the data to be stationary, i.e., the semivariograms depends only on the separation distance and not on the locality of the pairs. It has been shown by Davis (1986), that if the trend is subtracted from the regionalised variable, the residuals will, lend themselves be regionalised variables and will have local mean value zero. If the regionalised variable is stationary and normalised to have a mean of zero and a variance of 1.0, the semivariogram will be the mirror image of the autocorrelation function. The essential components of semivariogram are the sill, range, and nugget as shown in Fig.3 (Armstrong, 1998).

1. Sill - The semivariance increases as the distance increases until at a certain distance away from a point the semivariance, then will no longer increase, causing a flat region to occur on the semivariogram called a sill.

2. Range - From the point of interest to the distance where the flat region begins is termed the range or span of the regionalized variable. Within this range, locations are related to each other, and all known samples contained in this region. This region is also referred to as the neighborhood, and must be considered when estimating the unknown point of interest. A typical semivariogram reaches a limit called its sill at a distance called range.

Table 2 : Typical Semivariogram Models (Armstrong, 1998)

No	Model Type	Analytical Semivariogram
1	Simple Exponential	$\gamma(h) = C\left[1-\exp(-\lvert h\rvert/a)\right]$
2	Squared Exponential (Gaussian)	$\gamma(h) = C\left[1-\exp\left(-\frac{\lvert h\rvert^2}{a^2}\right)\right]$
3	Spherical	$\gamma(h) = C\left[\frac{3\lvert h\rvert}{2a}-\frac{1}{2}\left(\frac{\lvert h^3\rvert}{a^3}\right)\right]$ $\lvert h\rvert < a$ C $\lvert h\rvert \geq a$
4	Power	$\gamma(h) = C\lvert h\rvert^{\alpha}$ with $0 < \alpha \leq 2$
5	Nugget	$\gamma(h) = 0$ $h = 0$ $= C$ $\lvert h\rvert > 0$
6	Cubic	$\gamma(h) = C\left(7r^2-8.75r^3+3.5r^5-0.7r^7\right)$ if $r < 1$ $= C$ otherwise, where $r = h/a$

3. Nugget - The initial value of semivariance may not start from the origin, but intersects the semivariance axis at a distance, called nugget.

The variogram consists of the following features: (1) it always starts at 0 [for $h = 0$, $z(x+h) = z(x)$], it could be also discontinuous just after the origin; (2) it generally increases with h; (3) it rises upto a certain level called the Sill and flattens out, or alternatively, it may just go on rising.

Models for Variograms

Clark (1979) provided a number of semi-variogram models, and described the process of fitting a model to an experimental semi-variogram using a trial and error approach. The experimental variogram is obtained from the Eqn.(4) for one, two or three dimensions depending upon the problem at hand. The experimental semivariogram is usually determined upto half of the total sampled extent (Journel and Huijbregts, 1978; Clark, 1979; Brooker, 1989). The minimum number of pairs needed for a reliable estimate of semivariance is between 30 and 50 (Journel and Huijbregts, 1978; Brooker, 1989). The experimental variograms obtained above have to be fitted to the standard analytic variogram models. Some of the common semivariogram models are given in Table 2 and various types of semi variograms are shown in Fig.3. The process of fitting or determining the 'appropriate' model for the given data set, for the semi variograms are concerned, needs an exercise of judgement coupled with experience. In order of importance (Armstrong, 1998), the following features are to be regarded properly while fitting the models to the semivariance data: the nugget effect, the slope at the origin, the range, the sill and the anisotropies.

The nugget could be determined by extrapolating the semivariance back to the origin and the slope can be assessed with the initial values; the range can be fixed visually and the sill is the value where the variogram reaches a constant semivariance value. Depending on the behaviour of the slope and nugget, corresponding models may be chosen. The range is the equivalent of correlation distance within which the property shows strong correlation. It is reported (Soulie et al., 1990) and elsewhere that it is difficult to say which model best fits the data, or represents the soil variability for the particular site, however, a model should reflect the main characteristic of the spatial variability.

Issues in Correlogram and Semivariogram

Nugget Effect

The nugget effect, which is common both in autocorrelation function and variogram, is attributed to the combination of the three separate phenomena namely: Microstructures within the geological material, sampling or statistical errors, and measurement errors (Rendu, 1981). In fact, the nugget effect that is obtained from the experimental semivariogram depends greatly on the physical distance between the individual samples that form the data set. As the sampling distance increases, it is possible to obtain a better estimate of C_0. However, while one is able to reduce the sampling interval to a very small distance, the cost of the exploration program increases dramatically. As a result, it is often unreasonable and in fact unnecessary to reduce the sampling spacing below some nominal value. Unfortunately, this minimum sampling distance is dependent on the geological material being examined and cannot be known prior to investigation. Common practice is to begin sampling with a relatively coarse grid, and then to fill with a repeatedly finer grid, until the sample spacing no longer influences the resultant experimental semivariogram (Jaksa, 1995).

Nested Structures

Nested structures are sources of variability, which come into play simultaneously for all distances and are influenced by the scale of observation. They indicate the presence of processes operating at different scales and are reported both in the random field theory and geostatistics literature (Journel and Huijbregts, 1978; Vanmarcke, 1978). Both a shorter range and a longer range will be apparent, wherein the shorter range represents the micro-variability of the property and the longer range is attributed to other influences, such as by depositional features or jointing within the soil mass (Marsland and Quarterman, 1982). In an investigation by Jaksa et al. (1994), it was indicated that the apparent variability of horizontal correlation distance, from two different analysis in keswick clay, which varied from 1 to 2 m to 0.15 m, could be explained by nested structures. Vanmarcke (1978) reasoned this to the fact that geotechnical properties may exhibit two or more superimposed scales of fluctuation, depending on the modeling scale.

Model Fitting

The three main approaches in the literature for estimating the parameters of the semivariogram model are: inspection, weighted least squares, and likelihood methods (Alexander et al., 2001). It is apparent that before the wide use of computers, semivariograms were fitted based on visual inspection and presently, statisticians commonly use variants of maximum likelihood estimation to fit semivariograms while researchers from physical sciences use variants of least squares approach. However, Journel and Huijberts (1978) suggested that automatic fitting of models to experimental semivariograms such as least squares methods should be avoided, since least squares assumes that sample points are independent observations which is not true of the experimental variogram and the behaviour of the variogram very close to the origin is not known. And yet it is vital and least squares does not take account of this anomaly (Armstrong, 1998). Though it is necessary to obtain best fit by trial and error approach, certain guidelines are given in the literature. Clark (1979) and Brooker (1991) recommend guidelines for the fitting the appropriate model to the experimental variograms. For example, the tangent at the origin intersects the sill at a point with an abscissa $2a\sqrt{3}$, a, 1.73a respectively for spherical, exponential and gaussian models, where 'a' represents the range. It is to be noted that experimental semivariogram is usually determined only up to half of the total sampled extent (Journel and Huijberts, 1978; Clark, 1979; Brooker, 1989). With respect to model fitting of ACFs, the method of Ordinary Least Square is adopted, and tried with the recommended models.

Correlation Distance

The common technique in time series analysis (Box and Jenkins, 1970) to determine the correlation distance is by Barlett's approximation (Eqn.5). The following equation gives the Barlett's limit corresponding to two standard errors of estimates (Jaksa, 1995)

$$\left|r_k\right| = \pm\frac{1.96}{\sqrt{N}} \tag{5}$$

where N is the total number of observations/data. The $r_k|$ denotes the Barlett's limit and the correlation distance corresponding to this limit is equal to the scale of fluctuation δ. The correlation distance corresponds to the ordinate of the intersection of Barlett's limit and sample ACF.

The correlation distance (h), or the scale of fluctuation (δ) (determined from the sample ACF), the Barlett's distance (r_B) (obtained from the general Barlett's limit condition) and the range, (determined in the semivariogram), in essence, represents the same distance of strong correlation for any property, while the property variation satisfies all the necessary assumptions of stationarity. The detailed investigation in the relation between, δ, $r_{B,}$ and a, revealed that the exact equality between these parameter values does not occur at all instances, unless the sampling program has been done perfectly with minimum measurement and human errors (Jaksa, 1995). Since these values are found to be heavily dependent on many factors, such as physical features of the soil, presence of any anomalous materials/layers, measurement accuracy and the related other geological phenomena, the close disagreement between these values is inevitable, unless the errors has been carefully avoided/minimised at every stage of soil profiling. Further, Jaksa et al. (1997) demonstrated that the scale of fluctuation and hence, the correlation distance, is greatly influenced by (1) the spacing of the data; (2) the stationarity of data; (3) the degree of polynomial trend removed from the data; and (4) measurement errors. Further, it is pointed out that it is paramount that data used to assess the spatial variability of geotechnical materials be obtained at sample spacing less than the correlation distance of the material. It has also been demonstrated (Jaksa, 1995) that the model parameters for the sample ACF or semivariogram will widely differ, for the data, with and without the trend removal, depending on the stationarity of the data.

Analysis of Autocorrelation Functions

The sample autocorrelation function has been obtained for cone tip resistance profile of Singapore peaty clay, to demonstrate the function, properties and utility of the ACFs in the geotechnical context. This site is reported to be located along the Singapore river bank with mainly peaty clay deposit with layers of silty clay and marine clay at its bottom (Chang, 1986). The soil profile and their statistical description are shown in Fig.4(a) and Table 3.

The 18 m profile of cone-tip resistance values (kg/cm^2) shown in Fig.4(a), appears to follow an increasing trend with depth, with a mean of

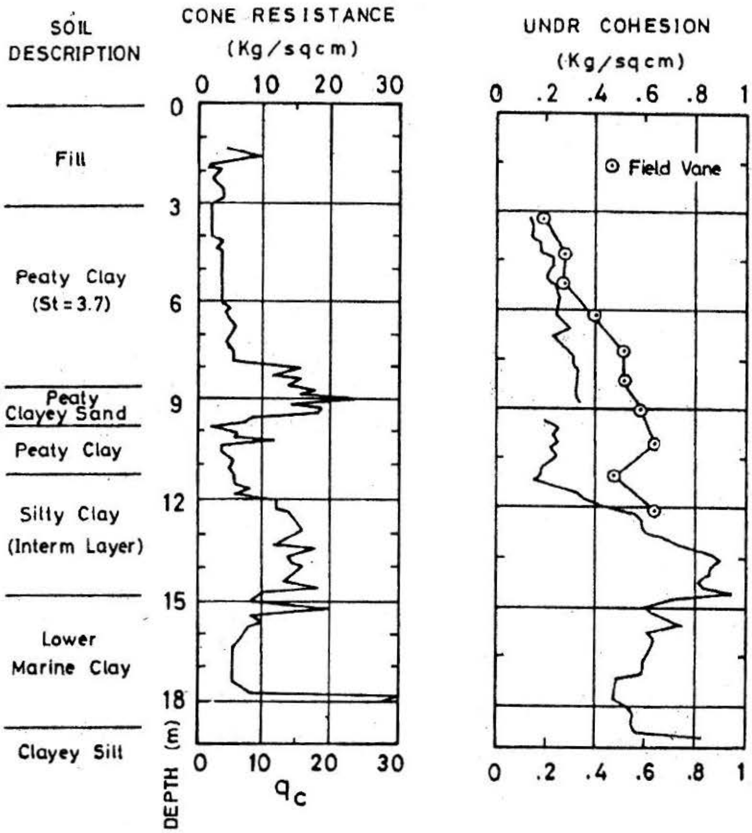

FIGURE 4 : (a) Cone Tip Resistance Profile for the Singapore Clay; (b) Cohesion Profile for the Singapore Clay

Table 3 : Statistical Description of the Geotechnical Properties of Various Soil Profiles

Statistical Values	Singapore Clay	
	Cone Tip Value	Cohesion
Mean (kg/cm^2)	7.81	0.38
Skewness	−0.299	0.69
Kurtosis	1.0144	−0.7
Coefficient of Variation	0.678	0.65

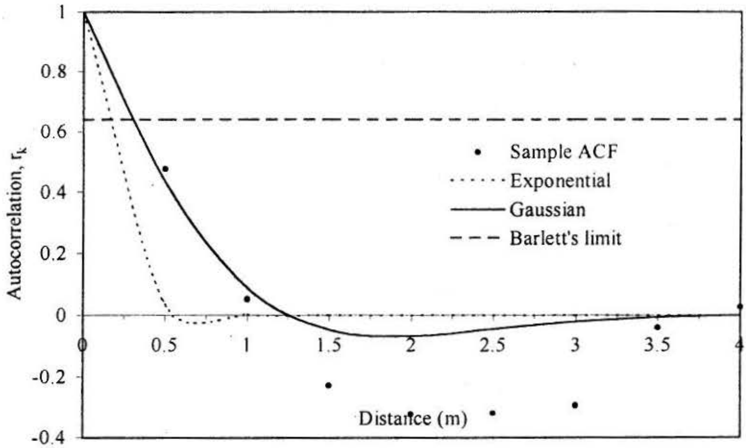

FIGURE 5 : Sample ACF with Models Fitted and Showing Barlett's Limit for Cone Tip Resistance, Singapore Clay

7.91 (kg/cm^2) and coefficient of variation of 0.678. Kendall's τ test (Daniel, 1990) confirmed the non-stationarity of the data hence the data is de-trended. The constructed sample ACF for the de-trended data, with the fitted models is shown in Fig.5. The exponential and the Gaussian models show the best fit with the model parameter values, for which correlation distances are 0.3 m and 0.52 m. The correlation distances as obtained by the sample ACF and Barlett's limit are tabulated in Table 4 for comparison. For the above data, Barlett's distance using Barlett's limit (0.64) gives correlation distances of 0.18 m and 0.3 m respectively.

Analysis of Semi Variograms

The correlation distance as discussed earlier, could also be derived using semi variograms using the geostatistical principles. In order that the efficiency or the robustness of the determination method using random filed

Table 4 : Correlation Distances by Sample ACF and Barlett's Limit for Cone Tip Resistance

Models Fitted	Barlett's Limit	Correlation Distance(m)	
		h (Sample ACF)	r_B (Barlett's limit)
Exponential Gaussian	0.64	0.30	0.18
		0.52	0.30

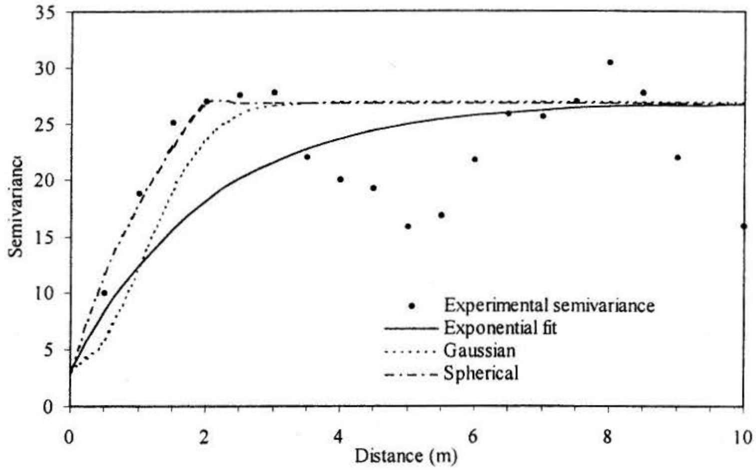

Figure 6 : Experimental Semivariogram with Models Fitted for Cone Tip Resistance Values, Singapore Clay

analysis or geostatistics be felt, semi variograms are also constructed for the same soil profile (Fig.4a). As the geostatistical analyses also requires the stationarity of the data, the cone resistance data of the Singapore clay is de-trended and the estimated semivariogram for the residuals is shown in Fig.6.

The experimental semivariogram is constructed using (Eqn.4) and the available models were used to fit these values. Though the behaviour near the origin is well defined, with a nugget of 3 $(kg/cm^2)^2$, the semivariances scatter with wide deviations at higher lags. A range of 2m corresponding a sill of 27 $(kg/cm^2)^2$ with a nugget of 3 $(kg/cm^2)^2$ characterizes for this semivariogram. Along with the usual spherical and exponential models, the Gaussian model is also fitted to this data to show the relative behaviour of these models. The gaussian model behaves closely to the spherical model reaching the sill immediately than the exponential model.

Discussions

The correlation distances estimated from sample ACF, Barlett's limit and semivariogram for the cone tip resistance data of the Singapore clay are 0.3 m (exponential model), 0.18 m, and 2 m respectively. The results reveal that the correlation distance for the cone tip resistance agrees with that obtained from the Barlett's limit (0.3 m and 0.18 m), but appreciable deviation is observed, between the result from sample ACF (0.3 m) and semivariogram (2 m). The reason for such a deviation in correlation distance values might be that the sampling distance is not close, leaving only smaller number of data. The accuracy of the estimation of sample ACF and the semivariogram is significantly influenced by the number of pairs of observations available for their determination. Notwithstanding the errors involved in the sampling process, the analysis brings out a similarity in the correlation distance, determined from two different principles.

The very nature of soil variability presents a higher level of difficulty in quantifying them in the design stage of important geotechnical structures. The lack of adaptability of the conventional design methods to take the soil variability into account makes a design engineer handicapped, though a good deal of effort in design is desired. The capability of the random field theory and the geostatistics in dealing the soil variability is being now increasingly highlighted, in literature. Since the theoretical formulations of such analyses are well supported in the geotechnical engineering context, immediate application of these analyses is called for, with the view of to perform better and rational design of geotechnical structures and assess its reliability. The outcome of such analysis always brings out a better understanding of the subsoil, the pattern of property variation, leading to an economical site exploration and interpolation of the property values, at points where sampling is not possible, through kriging.

Influence of Spatial Variability in Slope Stability Problems

The spatial variability concepts, applied to the slope stability problems have been demonstrated in literature (Alonso, 1976 and Vanmarcke, 1977a).

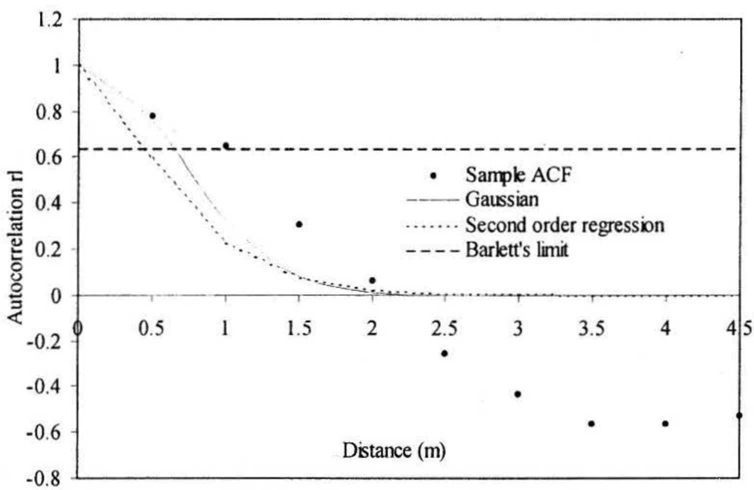

FIGURE 7 : Sample ACF with Models Fitted and Showing Barlett's Limit for Cohesion, Singapore Clay

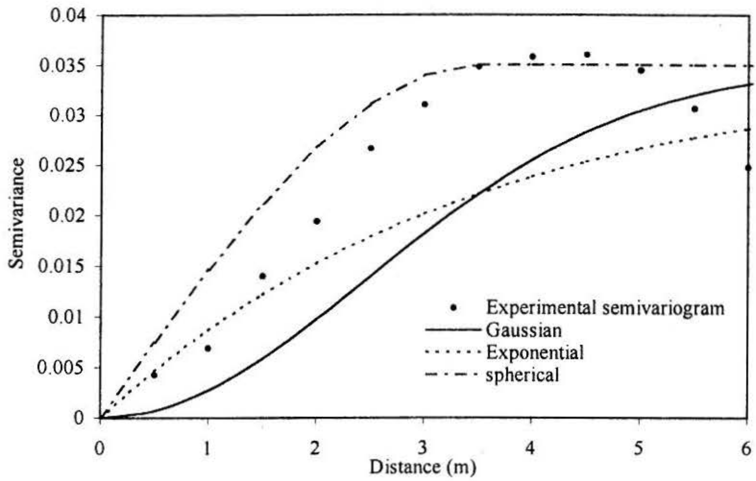

FIGURE 8 : Experimental Semivariogram with Models Fitted for Cohesion, Singapore Clay

The significance of the parameter such as correlation distance in horizontal and vertical directions, along the soil slope, could be brought out more clearly, when a complete probabilistic slope stability analysis is done. With a view to emphasize the practical necessity of incorporating soil variability in the routine slope stability analysis, the influence of the variability of the cohesion of the Singapore clay on an embankment and cut slope stability is discussed.

The variability of the cohesion is quantified by the correlation distance both from the random filed analysis and semivariogram, by the methods discussed earlier. The sample ACF, with fitted models and Barlett's limit is shown in Fig.7, wherein Gaussian and second order regression models are used. The experimental semivariogram has a sill value of 0.0351 (kg/cm^2)^2,

Table 5 : Correlation Distances by Sample ACF and Barlett's Limit for Cohesion Values

Models Fitted	Barlett's Limit	Correlation Distance(m)	
		h (Sample ACF)	r_B (Barlett's limit)
Gaussian	0.64	1.70	0.75
Second Order Regression		1.42	0.50

a range of 3.5 m with zero nugget and the exponential, spherical and gaussian models fitted to the data are shown in Fig.8. The comparative values are shown in Table 5. The result shows that the correlation distance obtained from the sample ACF, Barlett's limit and semivariogram are 1.7 m (Gaussian model), 0.75 m and 3.5 m, respectively.

Slope Stability Analysis

The influence of spatial variability of undrained shear strength with depth in terms of cohesion is considered in the analysis. Autocorrelation function which expresses the correlation between any two points as a function of distance lag is of modified Gaussian type (Calle, 1985), given by

$$\rho(\delta x, \delta y, \delta z) = e^{\frac{\delta_x^2+\delta_z^2}{D_h^2}}\left[(1-\alpha)+\alpha e^{\frac{\sigma_y^2}{D_v^2}}\right] \tag{6}$$

where D_h and D_v are autocorrelation parameters, which are related to the scales of fluctuation. Terms δ_x, δ_y and δ_z are distance lags between any two spatial points in x, y and z directions. The parameter α is the ratio of vertical variance (the variance of fluctuations relative to the mean value along a vertical line) to the total variance (the variance relative to the mean value, over the whole deposit). For $\alpha = 1$, the autocorrelation function takes on the classical Gaussian form, which is often suggested in literature. For calculation of probability of failure, First Order Second Moment (FOSM) method is used and probability of failure is defined as factor of safety being equal to or less than one.

Since the primary purpose is to demonstrate the influence of spatial variability, as quantified by theories mentioned, only vertical correlation distance is treated as variant and all other parameters are kept constant in the analysis. Herein two typical examples are discussed to illustrate the influence of correlation distance on the probabilities of failure.

Example 1

An embankment resting on the soil profile (Fig.4) is considered as shown in Fig.9. The variation of probabilities of failure for the slip circle passing through the foundation soil (FS = 2.01), with the change in vertical correlation distance is shown in Fig.11. The figure shows the influence of vertical correlation distance in the range of 0.1 m to 5 m. In this case the fill is purely a uniform cohesionless soil (ϕ = 35° and c = 0) and since the major failure surface passes through the embankment side slope, the influence of vertical correlation distance of the foundation soil is not significant in this

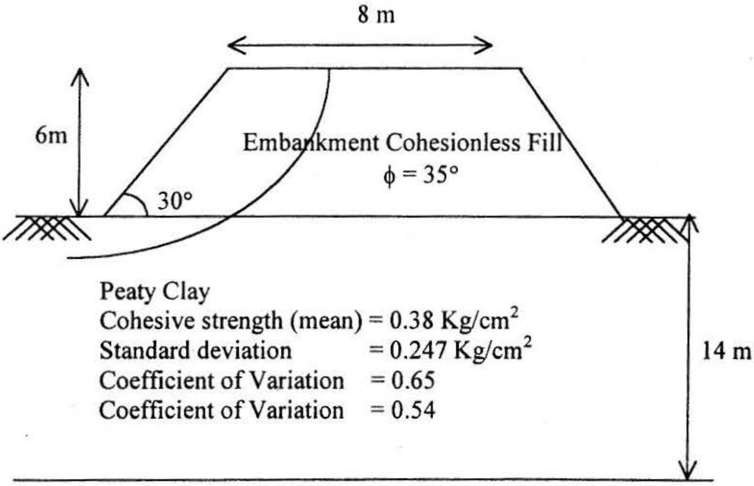

Figure 9 : Embankment on Singapore Clay

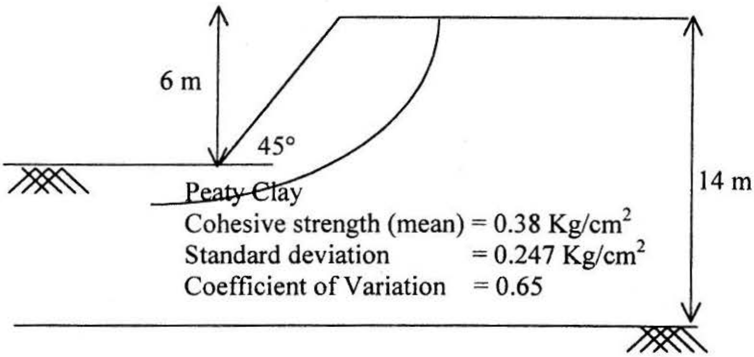

FIGURE 10 : Cut Slope on Peaty Clay

case. As a result, the probabilities of failure associated with this stability analysis are very low.

Example 2

A typical cut slope on the soil profile of Fig.4 with heights of 6 m and 10 m is considered (Fig.10). Figure 12 shows the influence of vertical correlation distance in the range of 0.1 m to 10 m. It could be observed that the increase in vertical correlation distance increases the probabilities of failure of the slope. Vertical correlation distances for shear strength

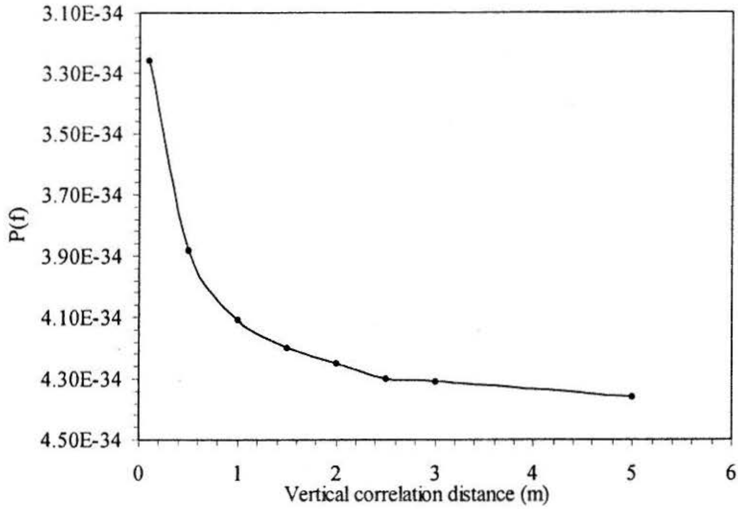

FIGURE 11 : Influence of Vertical Correlation Distance on Probability of Failure of the Embankment Slope on Soft Clay

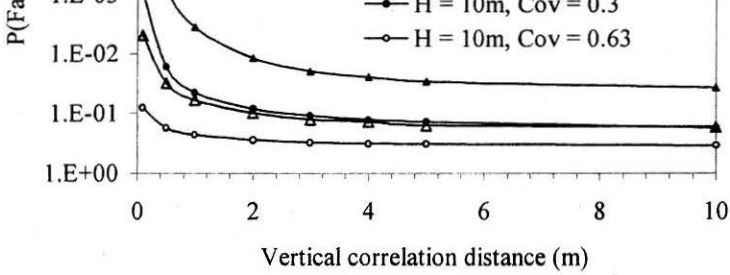

FIGURE 12 : Influence of Vertical Correlation Distance on Probability of Failure of a Cut Slope on Soft Clay

parameters, normally lies in the range of 0.1 m to 5 m. It is apparent from Fig.12 that the variation of probabilities of failure is very significant for this range of vertical correlation distance. In this case, the probabilities of failure (corresponding to factor of safety of 2) is influenced significantly, say for H = 6m and C.O.V = 0.3, it increases to four orders of magnitude. Conservative estimates of probabilities of failures are likely, if the correlation distance is not considered in the probabilistic analysis.

Concluding Remarks

The application of random field theory and geostatistics as an efficient descriptor of soil variability is demonstrated. The correlation distance which measures the distance of strong correlation exhibited by soils is treated both from the random field theory and geostatistic principles. The estimation of correlation distance helps the soil exploration program to be economical and rational and in the interpolation of the property values at the unsampled points. Case studies were presented to detail the methods involved and the stochastic characterisation of the soil variability is highlighted with different soil properties. The study clearly points out the similarity of the correlation distances, as evaluated from the two different principles. The significance of incorporating the correlation distances in the probabilistic analysis of soil slopes is shown through embankment and cut slope stability problems.

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Notation

X_i, X_{i+k}	=	random variables at ith and i+k points
C_k	=	autocovariance
ρ_k	=	autocorrelation
r_k	=	Sample ACF
$\gamma(h)$	=	semivariance
h	=	correlation distance
a	=	range
C	=	sill
C_o	=	nugget
δ	=	scale of fluctuation

Appendix I

Kendall's τ test (Daniel, 1990)

Kendall's τ is a measure of correlation of say two individual properties or data X and Y, based on the ranks of observations, wherein the objective is to test the null hypothesis that X and Y are independent (which implies $\tau = 0$) against one of the following alternatives: $t \neq 0$, $t > 0$, $t < 0$. The alternative $t \neq 0$ is interpreted to mean that there is an association between X and Y, $t > 0$, implying a direct association between X and Y and $t < 0$, means that X and Y are inversely associated. The test statistic is given by

$$\tau = \frac{S}{n(n-1)/2}$$

$$S = P - Q$$

where n = number of (X, Y) observations,

P = number of pairs in natural order,

Q = number of pairs in reverse natural order.

The obtained τ is compared with τ^* tabulated and accordingly the correlation between X and Y could be decided.