

## **Technical Note**

### **Analysis of Block Foundations for Variable Speed Machines**

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#### **Introduction**

**M**achine foundations supported on rigid blocks may be analysed using either numerical or analytical methods. Numerical methods include Finite element method, Boundary element method etc. Further, as an alternative to rigorous boundary element solutions, cone models may also be adopted for such problems (Wolf, 1994). Analytical methods consist of linear elastic weightless spring approach, often referred to as Barkan's method (Barkan, 1962) and lumped parameter model based on elastic half-space theory (Richart et al., 1970). Sankaran et al. (1977) comprehensively reviewed literature pertaining to the different methods of analysing dynamic problems of footings partially or fully embedded into the soil. Extensive review of literature on foundation dynamics also has been carried out by Richart, Jr. (1989).

Finite element method based on discretisation of foundation-soil system is one of the most popular numerical techniques among dynamic analysts. The method has the advantage in handling spatial variation of mechanical properties of the domain and complex geometries effectively. Solution to problems of axisymmetric foundations subjected to symmetrical and asymmetrical dynamic loading in time domain using finite element approach has been dealt with by Shridhar (1995). Though, this method is very effective for dynamic problems of embedded foundations, it is not very popular among practising engineers. This is because of the complications involved in modelling the boundaries of soil domain, which often results in high cost,

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run time and memory requirements of computers. Therefore, this method of approach is presently confined to research environment.

Barkan's method, one of the analytical approaches, is quite popular among the practising engineers simply because the solutions to the complex problem of coupled vibrations of the block are readily available (Barkan, 1962; Srinivasulu and Vaidyanathan, 1976). However, this method ignores the effect of soil damping and also the soil mass participation in vibrations. It has been observed that the damped natural frequencies of foundation block do not vary significantly from undamped natural frequencies. Further, if the natural frequencies of the block are sufficiently detuned with respect to the operating frequency of the machine, the computed amplitudes will be finite and therefore meaningful. IS 2974 (Part IV)-1979 specifies the magnitude of detuning to be of the order of  $\pm 20\%$ .

In engineering practice rigid blocks are mostly provided as foundation for low frequency rotary machines such as Primary air fan (PA Fan), Secondary air fan (SA Fan), Induced draught air fan (ID Fan) etc., as they are located at ground level. Quite often the requirements of operation of the machine and site constraints pose great deal of difficulties in sizing of the block to achieve the specified detuning satisfying the frequency criterion. It is suggested that the design of block foundations will have to be based on the amplitude criterion only so long as the allowable amplitudes are kept small to ensure that the performance of the machine is smooth and satisfactory. The calculated amplitudes of the block supporting machines operating at a constant speed are either over conservative or become infinite at resonance when Barkan's approach is adopted since the method ignores damping of the soil medium. On the other hand, for block foundations supporting variable speed machines, the computed amplitudes become infinite most of the time as one or the other mode of vibration approaches the excitation frequency of machine resulting in resonance. Therefore, in such situations, consideration of soil damping in the analysis is a necessity in order to arrive at finite and meaningful amplitudes while using lumped parameter approach.

This paper presents dynamic analysis of block foundation supporting a variable speed machine using lumped parameter approach based on elastic half-space theory. The method considers the geometrical damping of the soil medium, whereas the material damping of soil medium is neglected. Formulations of equations of motion and their solutions are presented. Further the application of the said theory to solve a practical problem of a block foundation supporting I.D. Fan with a variable operating speed from 150 to 725 rpm is presented. This type of block foundation is frequently encountered in power plants. The extent of inaccuracy involved in the computed amplitudes of displacements by using Barkan's method is demonstrated.

## Formulation of Equations of Motion and Solution Technique

In general, a block foundation is subjected to three translational and three rotational modes of vibration. A general formulation of equations of motion with all six degrees of freedom being coupled is possible. However, because of the complexity involved, the results of block vibration analysis with such a formulation are difficult to interpret and evaluate.

Consider a block foundation subjected to dynamic forces such as Vertical ( $V$ ), Horizontal ( $H$ ), Moment ( $M$ ) at an instant as shown in the Fig.1. All these forces are assumed to act at centre of gravity of the machine-foundation system. The centre of gravity of the machine-foundation system is at a height ' $h$ ' from the base. The soil reaction ' $R$ ' is acting at a distance ' $s$ ' from the vertical line passing through the centre of gravity of the foundation-soil system on account of non-uniform pressure distribution at the base due to the applied moment. The weight of the foundation block including the weight of the machine is ' $W$ ' and it's mass is  $m = W/g$ , where  $g$  is acceleration due to gravity. The mass moment of inertia of the foundation block about the axis about which it rotates is  $I_m$ . From Fig.2 by grouping the forces in vertical, horizontal and moments, the equations of motion may be written as,

Vertical Motion:

$$m\ddot{z} + c_z\dot{z} + k_z z + s(c_2\dot{\theta} + k_2\theta) = V \quad (1)$$

Horizontal Motion:

$$m\ddot{x} + c_x\dot{x} + k_x x + h(c_x\dot{\theta} + k_x\theta) = H \quad (2)$$

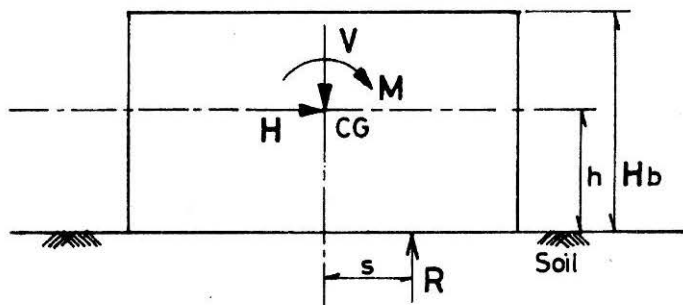
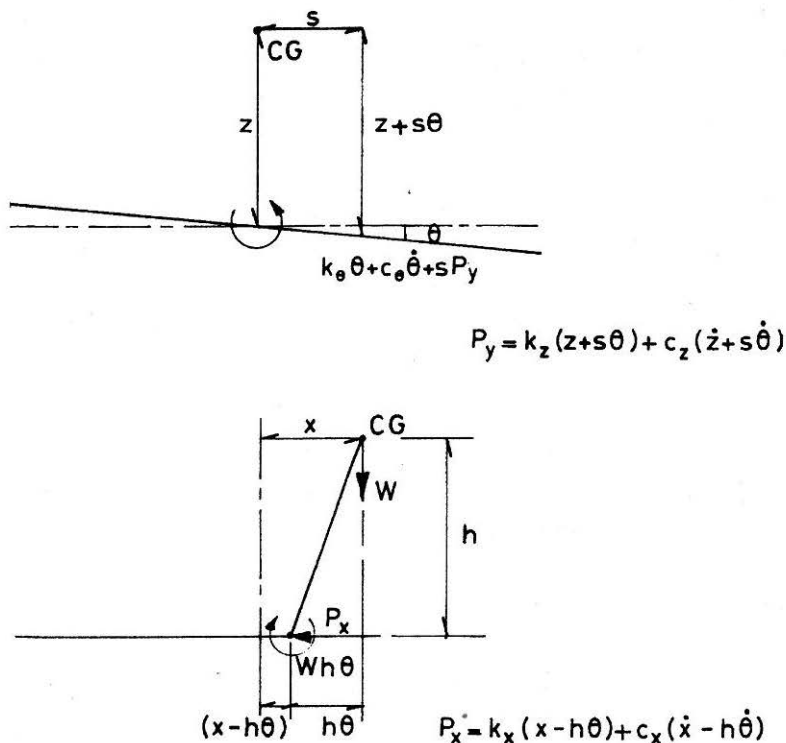


FIGURE 1 : Block Foundation Subjected to Exciting Forces



**FIGURE 2 : Motions and Forces for the Block a) Vertical Motion and Rotation b) Horizontal Motion and Rotation**

Rocking Motion:

$$I_m \ddot{\theta} + c_{\theta} \dot{\theta} + k_{\theta} \theta + sP_y - hP_x - Wh\theta = M \quad (3)$$

The resulting equations of motion are coupled and involve six soil parameters. The six soil parameters associated with vertical, horizontal and rocking motion are the three spring constants  $k_z$ ,  $k_x$  and  $k_{\theta}$  and the corresponding damping constants  $c_z$ ,  $c_x$  and  $c_{\theta}$  respectively. These parameters are to be evaluated from the properties of soil-foundation system as given in Table 1 (Richart et al., 1970)

The equations of motion, Eqns.1, 2 and 3, are to be solved simultaneously. The solution is obtained for general type of dynamic forces of the form (combination of sine and cosine functions),

$$V = V_0 e^{i\omega t}, \quad H = H_0 e^{i\omega t} \quad \text{and} \quad M = M_0 e^{i\omega t} \quad (4)$$

TABLE 1 : Spring and Damping Constants

Type of motion	Spring Constant	Damping Constant
Vertical	$k_z = 4Gr_o/(1-\nu)$	$c_z = 3.4r_o^2\sqrt{\rho G}/(1-\nu)$
Horizontal	$k_x = 32(1-\nu)Gr_o/(7-8\nu)$	$c_x = 18.4(1-\nu)r_o^2\sqrt{\rho G}/(7-8\nu)$
Rocking	$k_\theta = 8Gr_o^3/3(1-\nu)$	$c_\theta = 0.8r_o^4\sqrt{\rho G}/(1-\nu)(1+B_\psi)$ where $B_\psi = 3(1-\nu)I_m/8\rho r_o^5$

where  $r_o$  = equivalent radius of circular footing, and  
 $\rho$  = mass density of soil medium.

The equivalent radius for a rectangular block of width  $B$  and length  $L$  for different modes of vibration are:  $(A/\pi)^{0.50}$  for both vertical and horizontal modes of vibration and  $(BL^3/3\pi)^{0.25}$  for rocking mode,

where  $A$  = base area of the foundation,  
 $G$  = dynamic shear modulus, and  
 $\nu$  = Poisson's ratio of the soil medium respectively.

and the amplitudes of displacements of the form,

$$z = Z_1 e^{j\omega t}, \quad x = X_1 e^{j\omega t} \quad \text{and} \quad \theta = \theta_1 e^{j\omega t} \quad (5)$$

where

$\omega$  = frequency of excitation,

$Z_1, X_1$  = amplitudes of displacements in vertical and horizontal direction respectively, and

$\theta_1$  = amplitude of rotation at centre of gravity of machine foundation system.

Substituting Eqns.4 and 5 in Eqns.1, 2 and 3, the equations may be written in the matrix form as,

$$\begin{bmatrix} \begin{pmatrix} -m\omega^2 \\ +ic_z\omega + k_z \end{pmatrix} & 0 & s(k_z + ic_z\omega) \\ 0 & \begin{pmatrix} -m\omega^2 \\ +ic_x\omega + k_x \end{pmatrix} & -h(k_x + ic_x\omega) \\ s(ic_z\omega + k_z) & -h(k_x + ic_x\omega) & \begin{pmatrix} -I_m\omega^2 \\ +ia\omega + b \end{pmatrix} \end{bmatrix} \begin{bmatrix} Z_1 \\ X_1 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} V_o \\ H_o \\ M_o \end{bmatrix} \quad (6)$$

where

$$a = c_\theta + s^2 c_z + h^2 c_x \quad \text{and} \quad (7)$$

$$b = k_\theta + s^2 k_z + h^2 k_x - Wh$$

The solutions are obtained as,

$$Z_1 = \Delta_1/\Delta, \quad X_1 = \Delta_2/\Delta \quad \text{and} \quad \theta_1 = \Delta_3/\Delta \quad (8)$$

where,

$$\Delta = \begin{vmatrix} \begin{pmatrix} -m\omega^2 \\ +ic_z\omega + k_z \end{pmatrix} & 0 & s(k_z + ic_z\omega) \\ 0 & \begin{pmatrix} -m\omega^2 \\ +ic_x\omega + k_x \end{pmatrix} & -h(k_x + ic_x\omega) \\ s(ic_z\omega + k_z) & -h(k_x + ic_x\omega) & \begin{pmatrix} -I_m\omega^2 \\ +ia\omega + b \end{pmatrix} \end{vmatrix} \quad (9)$$

Expanding the determinant  $\Delta$ , Eqn.9 may be expressed in the form of  $A + iB$  by collecting the real and imaginary terms separately, where

$$A = (-m^2 I_m) \omega^6 + \omega^4 \{ b m^2 + m(ac + I_m d - h^2 c_x^2 - s^2 c_z^2) + I_m e \}$$

$$- \omega^2 \{ m b d - m(h^2 k_x^2 + s^2 k_z^2) + c_\theta g^* + f I_m + e(k_\theta - Wh) \}$$

$$+ f(k_\theta - Wh)$$

and

$$B = m\omega^5(am + I_m c) - \omega^3 \left\{ \begin{matrix} m(bc + ad) + I_m g^* + ec_\theta \\ -2m(h^2 k_x c_x + s^2 k_z c_z) \end{matrix} \right\} \quad (10)$$

$$+ \omega \{ g^*(k_\theta - Wh) + f c_\theta \}$$

where

$$c = c_x + c_z$$

$$d = k_x + k_z$$

$$e = c_x c_z$$

$$f = k_x k_z$$

$$g^* = c_z k_x + k_z c_x$$

Similarly,

$$\Delta_1 = \begin{vmatrix} V_o & 0 & s(k_z + ic_z\omega) \\ H_o & (-m\omega^2 + ic_x\omega + k_x) & -h(k_x + ic_x\omega) \\ M_o & -h(k_x + ic_x\omega) & (-I_m\omega^2 + ia\omega + b) \end{vmatrix} \quad (11)$$

Determinant  $\Delta_1$  in Eqn.11 may be expressed in the form of  $C + iD$  by collecting the real and imaginary terms separately, where

$$C = V_o m I_m \omega^4 - \omega^2 \left\{ V_o (bm + c_x a^* + k_x I_m) \right. \\ \left. - M_o s k_z m - s e (H_o h + M_o) \right\} \\ + V_o k_x b^* - s f (H_o h + M_o)$$

and

$$D = -\omega^3 \{ V_o (am + I_m c_x) - m M_o s c_z \} \\ + \omega \{ V_o (k_x a^* + c_x b^*) - s g^* (H_o h + M_o) \} \quad (12)$$

where

$$a^* = a - h^2 c_x = c_\theta + s^2 c_z$$

$$b^* = b - h^2 k_x = k_\theta + s^2 k_z - Wh$$

and

$$\Delta_2 = \begin{vmatrix} (-m\omega^2 + ic_z\omega + k_z) & V_o & s(k_z + ic_z\omega) \\ 0 & H_o & -h(k_x + ic_x\omega) \\ s(ic_z\omega + k_z) & M_o & (-I_m\omega^2 + ia\omega + b) \end{vmatrix} \quad (13)$$

Determinant  $\Delta_2$  in Eqn.13 may be expressed in the form of  $E + iF$  by grouping the real and imaginary parts separately, where

$$E = m H_o I_m \omega^4 - \omega^2 \left\{ H_o (bm + a c_z + k_z I_m) - H_o s^2 c_z^2 \right. \\ \left. + M_o h (m k_x + e) - V_o h s e \right\} \\ + k_z (b H_o + h M_o k_x) - V_o h s f - H_o s^2 k_z^2$$

and

$$F = -\omega^3 \{H_o(am + c_z I_m) + mM_o hc_x\} + \omega \{H_o(ak_z + bc_z) + g^*h(M_o - V_o s) - 2s^2 k_z c_z H_o\} \quad (14)$$

and finally,

$$\Delta_3 = \begin{vmatrix} (-m\omega^2 + ic_z\omega + k_z) & 0 & V_o \\ 0 & (-m\omega^2 + ic_x\omega + k_x) & H_o \\ s(ic_z\omega + k_z) & -h(k_x + ic_x\omega) & M_o \end{vmatrix} \quad (15)$$

Expanding the determinant  $\Delta_3$  in Eqn.15 in terms of real and imaginary parts separately as  $G + iH$ , where

$$G = m^2 M_o \omega^4 - \omega^2 \left\{ mk_x (M_o + hH_o) + e(M_o - hH_o - V_o s) - mk_z (M_o - V_o s) \right\} + f(M_o + H_o h - V_o s)$$

and

$$H = -\omega^3 \{mc_x (M_o + hH_o) + mc_z (M_o - V_o s)\} + \omega g^* (M_o + hH_o - V_o s) \quad (16)$$

Equation 8 may now be expressed as,

$$\begin{aligned} Z_1 &= (C + iD)/(A + iB) = K + iL \\ X_1 &= (E + iF)/(A + iB) = M + iN \\ \theta_1 &= (G + iH)/(A + iB) = O + iP \end{aligned} \quad (17)$$

Modulus values of the Eqn.17 may be worked out to evaluate the vertical, horizontal and rotational amplitudes  $Z_1$ ,  $X_1$  and  $\theta_1$  respectively. The net horizontal amplitude ( $X_{top}$ ) at the top of the block may then be computed as,

$$X_{top} = X_1 + \theta_1 (H_b - h) \quad (18)$$

where  $H_b$  is the total height of the block. A simple computer program has



been developed in FORTRAN language, which computes net amplitudes of both horizontal and vertical displacement at top of the block for a given force input.

It is seen that vertical motion is independent of the other two motions and the horizontal and rocking motions are always coupled with value of  $s = 0$  in Eqns.1, 2 and 3. Kuppusamy (1977) reported that the eccentricity of soil reactions has no significant influence on peak amplitude of horizontal vibrations as well as resonant frequency. However, marginal increase in peak amplitude of vertical vibrations has been observed due to eccentricity without significant increase in the resonant frequency. As far as possible, the block foundations are proportioned in such a way that the resultant force due to mass of the machine and that of the foundation passes through the centre of gravity of base area of contact. In situations where eccentricity is unavoidable, it is ensured that the same is limited to 3 to 5% of plan dimensions. Satisfying this criterion will reduce the coupling effect of 's' to a minimum. In the present example  $s = 0$  condition is considered in view of the above facts.

Effect of embedment has not been considered for spring and damping constants in Table 1. It has been shown by Lysmer and Kuhlemeyer (1969) theoretically that vibration amplitudes decrease with depth of embedment. Therefore, the computed amplitudes of vibration are on conservative side when embedment effects are not considered. Moreover, the space between the excavated portion of the pit and the foundation block is generally backfilled with soil and compacted. The uncertainties involved in evaluating the properties of backfilled soil necessitates ignoring the embedment effects. The spring and damping parameters involve estimation of dynamic shear modulus  $G$  and Poisson's ratio  $\nu$  which may be evaluated by conducting field tests. Laboratory resonant column tests may also be performed simulating actual field conditions. Designers face utmost difficulty in choosing the appropriate values of soil parameters with confidence. However, by performing number of field tests, a close range of values can be arrived at and recommended for the site. Although, spring and damping constants are frequency dependent (Table 1), it has been found that accurate solutions may be obtained by assuming them to be frequency independent (Hall and Kissenpfennig, 1976).

Based on cyclic plate load test, the coefficient of uniform compression of the soil medium may be evaluated. Elastic modulus of soil medium may be computed using the following equation (IS 5249-1992).

$$E = \frac{C_u (1 - \nu^2) \sqrt{A}}{1.13} \quad (19)$$

**TABLE 2 : Recommended Value of  $C_u$  for 10 m<sup>2</sup> Area  
(IS 2974-Part I, 1969)**

Permissible SBC – kN/m <sup>2</sup>	$C_u$ – kN/cu.m
< 150	$3 \times 10^4$
150 – 350	$3 \times 10^4 - 5 \times 10^4$
350 – 500	$5 \times 10^4 - 10 \times 10^4$
> 500	$> 10 \times 10^4$

The dynamic shear modulus ( $G$ ) of soil medium may then be computed by the relationship,

$$G = E/2(1+\nu) \quad (20)$$

If the data from the cyclic plate load tests are not available, then coefficient of uniform compression ( $C_u$ ) may be evaluated from known allowable safe bearing capacity (SBC) of the soil medium as given in Table 2. A range of values of coefficient of uniform compression are chosen for the particular soil type in order to assess the corresponding variation of maximum vibration amplitudes. This will eliminate the uncertainties involved in case the exact values of soil parameters are not available.

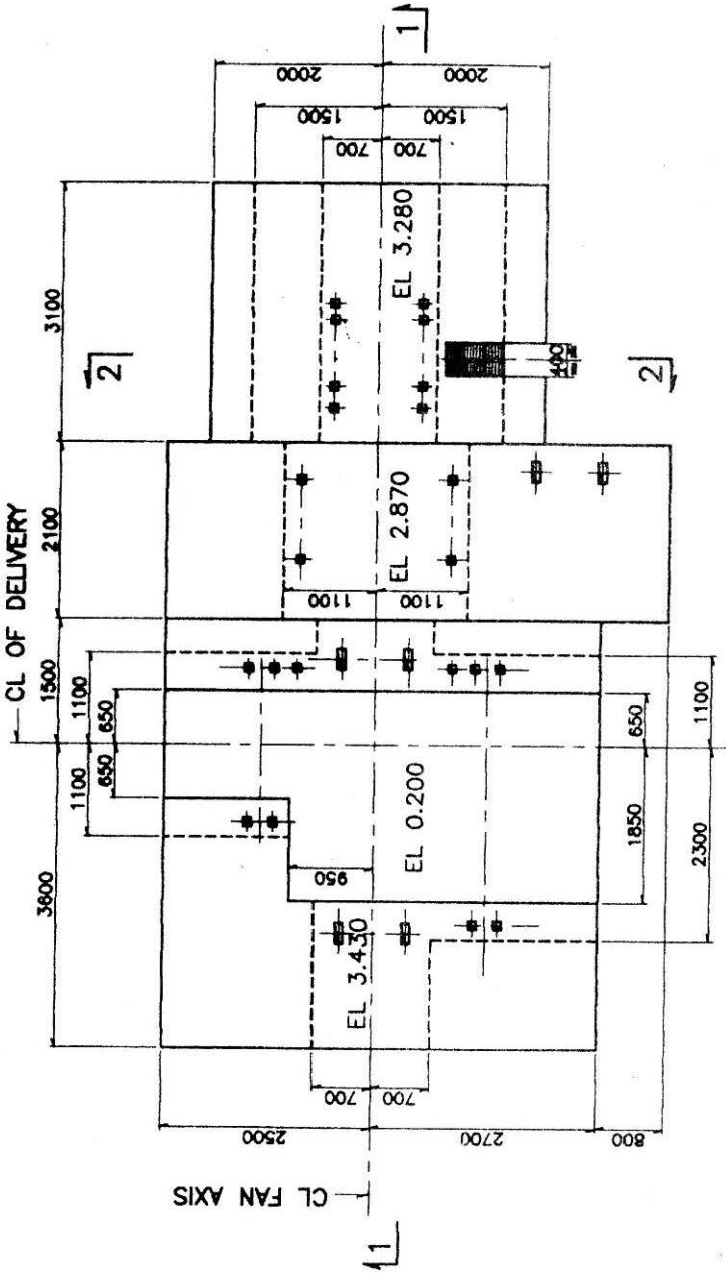
In the following section, a practical problem is chosen to investigate the behaviour of the block foundation by using lumped parameter model based on elastic half-space theory as described in the above sections. The results obtained are compared with Barkan's approach.

### Worked-out Example

The details of the I.D. fan block foundation is shown in Figs.3, 4 and 5. This is a typical type of block foundation often encountered in Power plants and subjected to varying dynamic forces depending upon the excitation frequency. The example presented in the following sections has been engineered by the authors for The Mysore paper mills modernisation project (1 × 16 MW) located at Bhadravathi, Karnataka.

#### *Data Obtained from the Machine Manufacturer*

Operating speed or frequency of excitation  $\omega$  = 150 to 725 rpm  
 Weight of the machine = 260.5 kN



PLAN AT TOP

FIGURE 3 : Plan of L.D. Fan Foundation

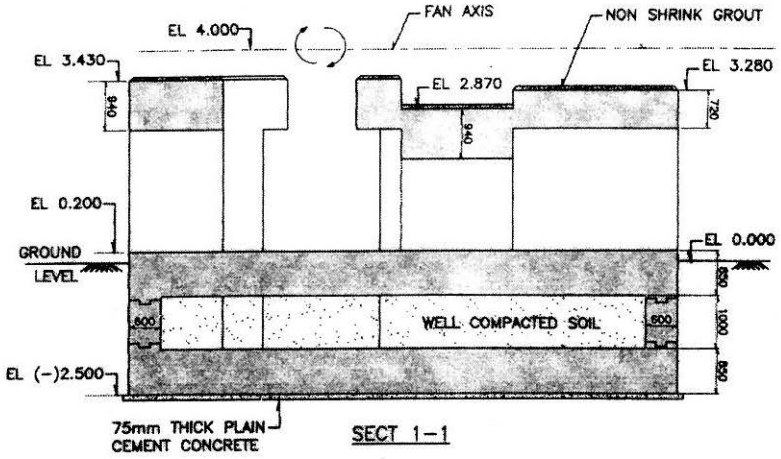


FIGURE 4 : Longitudinal Section of I.D. Fan Foundation

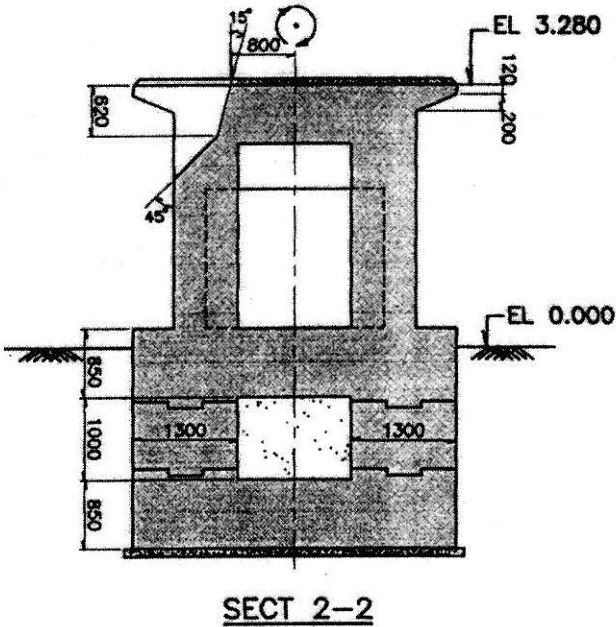


FIGURE 5 : Transverse Section of I.D. Fan Foundation

Vertical dynamic force $V_o$	= 52.5 kN
Horizontal dynamic force $H_o$	= 52.5 kN
Dynamic moment $M_o$	= 173 kNm

The dynamic forces indicated above are normal unbalance forces at 725 rpm

### *Data of the Block Foundation and from Geotechnical Investigations*

Weight of foundation including machine weight $\omega$	= 5560.9 kN
Mass moment of inertia $I_m$	= 7161.6 kN m sec <sup>2</sup>
Base area of foundation $A$	= 51.52 m <sup>2</sup>
Coefficient of uniform compression of soil medium $C_u$	= $3 \times 10^4$ kN/m <sup>3</sup>
Poisson's ratio of soil medium $\nu$	= 0.15
Safe bearing capacity of soil medium at founding level	= 200 kN/m <sup>2</sup>

### **Natural Frequencies of the Block**

Natural frequencies of the block in vertical, horizontal, coupled rocking and torsional modes of vibration using Barkan's approach is tabulated in Table 3, for  $C_u = 1.5 \times 10^4$  kN/m<sup>3</sup>,  $3.0 \times 10^4$  kN/m<sup>3</sup> and  $5.0 \times 10^4$  kN/m<sup>3</sup> respectively. It is quite evident from Table 3 that the operating frequency of the machine varying from 150 to 725 rpm coincides with one or the other modes of vibration resulting in resonance.

**TABLE 3 : Natural Frequencies of the Block (Barkan's Method)**

Mode of vibration	$C_u = 1.5 \times 10^4$ kN/m <sup>3</sup>	$C_u = 3.0 \times 10^4$ kN/m <sup>3</sup>	$C_u = 5.0 \times 10^4$ kN/m <sup>3</sup>
Vertical (rpm)	352	498	643
Horizontal (rpm)	249	352	455
Rocking (rpm)	471	667	861
	219	309	399
Torsion (rpm)	297	420	542

## Amplitudes of Displacements for the Block Foundation

The amplitude calculations are carried out for the block foundation subjected to dynamic forces arising out of normal unbalance at different operating frequencies of the machine by using Barkan's approach and elastic half-space theory. The following sections describe the behaviour of both vertical and horizontal amplitudes of the block with various  $C_u$  values using elastic half-space theory. Also, the results obtained are compared with those obtained by Barkan's approach. The computed amplitudes are checked against the stipulations by Indian standard practices.

### Comparison of Amplitudes of Displacements using Barkan's Approach

Amplitudes of both vertical and horizontal displacements at top of the block and rotation at centre of gravity of the block have been worked out by using Eqns.19 and 20 over a range of excitation frequencies i.e., 150 to 725 rpm. Variation of vertical amplitude of displacements with operating frequency of the machine by using lumped parameter model based on elastic half-space theory and by Barkan's approach for  $C_u = 3.0 \times 10^4 \text{ kN/m}^3$  is shown in Fig.6. It is quite evident that consideration of geometrical damping of soil medium in half-space theory reduces the amplitudes significantly in comparison to the amplitudes computed by Barkan's approach. Also, it is

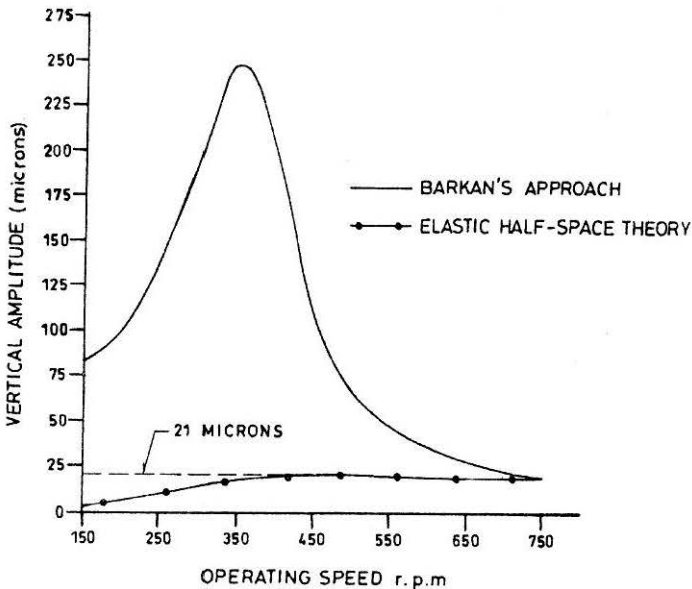
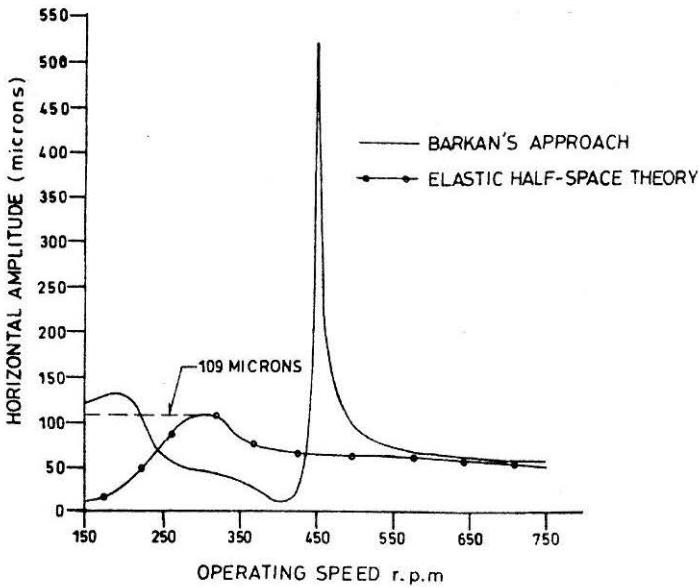


FIGURE 6 : Comparison of Vertical Amplitude of Displacement between Elastic Half-Space Theory and Barkan's Method with Operating Speed



**FIGURE 7 : Comparison of Horizontal Amplitude of Displacement between Elastic Half-Space Theory and Barkan's Method with Operating Speed**

noticed that the calculated vertical amplitudes are unrealistic at or near resonance by Barkan's method, whereas the same is significantly lower and realistic if obtained from half-space theory.

Similarly, variation of horizontal amplitudes of displacements at the top of the block with operating frequencies of the machine for  $C_u = 3.0 \times 10^4$  kN/m<sup>3</sup> is shown in Fig.7. In this case also, lower values of amplitudes are obtained in comparison to those calculated by using Barkan's method. The advantage of the proposed approach becomes evident in the computation of amplitudes at or near resonance condition.

### ***Behaviour of Amplitude of Displacement of the Block***

#### **Vertical Amplitude of Displacement**

The variation of vertical amplitude ( $Z_1$ ) at top of concrete block in microns with frequency of excitation ( $\omega$ ) for three different values of coefficient of uniform compression  $C_u = 1.5 \times 10^4$ ,  $3.0 \times 10^4$  and  $5.0 \times 10^4$  kN/m<sup>3</sup> of soil medium is shown in Fig.8. It can be observed from Fig.8 that the amplitudes of vertical displacement progressively increase with increase in frequency of excitation for  $C_u = 1.5 \times 10^4$ ,  $3.0 \times 10^4$  and  $5.0 \times 10^4$  kN/m<sup>3</sup>. The vertical amplitudes of displacements attain peak at a certain frequency of excitation for

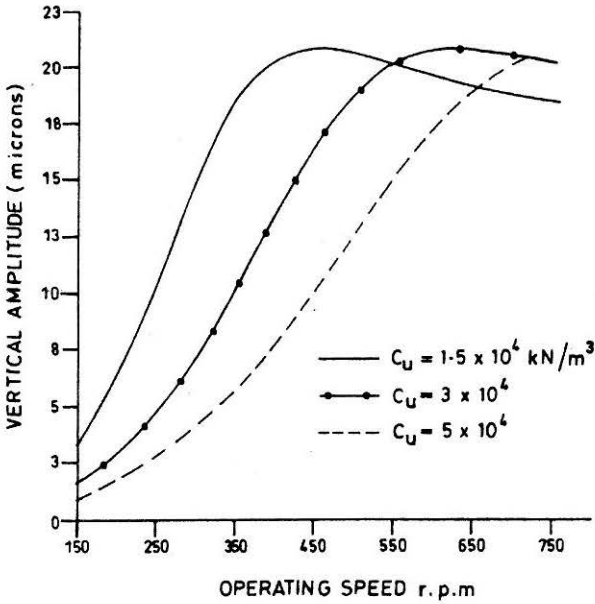


FIGURE 8 : Variation of Vertical Amplitude of Displacement with Operating Speed for Various Values of  $C_u$

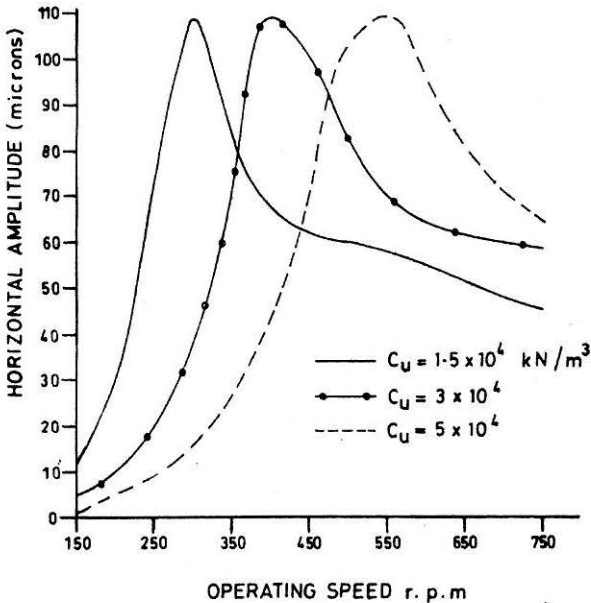


FIGURE 9 : Variation of Horizontal Amplitude of Displacement with Operating Speed for Various Values of  $C_u$



$C_u = 1.5 \times 10^4$  and  $3.0 \times 10^4$  kN/m<sup>3</sup>. Whereas, for  $C_u = 5 \times 10^4$  kN/m<sup>3</sup>, the peak amplitude of vertical displacement seems to occur at frequency of excitation  $\omega > 725$  rpm, which is beyond the range of operating speed of the machine. Within the range of parameters considered, the maximum peak amplitude of vertical displacement for the block is about 21 microns.

### Horizontal Amplitude of Displacement

Variation of horizontal amplitude of displacement at the top of the block ( $X_{top}$ ) with different values of excitation frequencies for  $C_u = 1.5 \times 10^4$ ,  $3.0 \times 10^4$  and  $5.0 \times 10^4$  kN/m<sup>3</sup> is shown in Fig.9. Here also it is observed that the amplitude of horizontal displacements progressively increase with increase in frequency for all the three values of  $C_u$  considered in the analysis (Fig.9). The horizontal amplitudes of displacements attain peak at a certain frequency of excitation for  $C_u = 1.5 \times 10^4$ ,  $3.0 \times 10^4$  and  $5.0 \times 10^4$  kN/m<sup>3</sup>. Thus the nature of behaviour of horizontal amplitude of displacement with frequency of excitation is similar for all the three values of  $C_u$  considered in the analysis. The maximum peak amplitude of horizontal displacement is of the order of 109 microns.

Permissible amplitudes of displacements as per IS 2974(Part IV)-1979 is 200 microns in both vertical and horizontal directions for block foundations. It is seen that the computed amplitudes of both vertical and horizontal displacement at the top of the block are well within the limiting amplitudes as per IS code.

### Conclusions

Analysis of block foundation supporting a variable speed rotary equipment using lumped parameter approach based on elastic half-space theory is presented. As the geometrical damping of soil medium is considered in the present approach, the computed amplitudes of displacements are finite and more realistic when compared with the amplitudes obtained by Barkan's approach. For under-tuned block foundations supporting a machine operating at a constant rated speed, invariably the speed coincides with sub-synchronous natural frequencies of the block resulting in transient resonance condition. The presented approach has distinct advantage in computation of amplitudes of displacements at transient resonance condition, although such a resonance lasts for a short duration of time. The usefulness of the procedure is quite evident in arriving at realistic and meaningful amplitudes for all practical problems of block foundations supporting rotary equipments operating over a wide range of frequencies. It is recommended that a block foundation analysed using lumped parameter model based on elastic half-space theory may be considered safe from the point of view of machine performance, when the amplitudes of vibration are within the permissible limits, even at resonance condition. The present BIS limitations with regard to frequency and amplitudes of displacements need to be reviewed.

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## References

- BARKAN, D.D. (1962) : *Dynamics of Bases and Foundations*, McGraw Hill, Inc., New York.
- HALL Jr., J.R. and KISSENFENNIG, J.F. (1976) : "Special Topics on Soil-Structure Interaction", *Nuclear Engineering Design*, 38, pp.273-287.
- IS 2974 (Part IV)-1979 : "Code of Practice for Design and Construction of Machine Foundations - Foundations for Rotary type Machines of Low Frequency", *Bureau of Indian Standards*, New Delhi, India.
- IS 5249-1992 : "Determination of Dynamic Properties of Soil-Method of Test", *Bureau of Indian Standards*, New Delhi, India.
- KUPPUSAMY, T. (1977) : "Block Foundation Subjected to Coupled Modes of Vibration", *International Symposium on Soil-Structure Interaction*, Univ. of Roorkee, Roorkee, India, pp.429-436.
- LYSMER, J. and KUHLEMEYER, R.L. (1969) : "Finite Dynamic Model for Infinite Media", *Journal of Engineering Mechanics Division*, ASCE, Vol.95, pp.859-872.
- RICHART, F.E., WOODS, R.D. and HALL, J.R (1970) : *Vibrations of Soils and Foundations*, Prentice-Hall, Inc., New Jersey.
- RICHART Jr., F.E. (1989) : *The Art and Science of Geotechnical Engineering: At the Dawn of Twenty First Century: A Vol. Honouring Ralph. B. Peck*, Prentice-Hall, Englewood Cliffs, New Jersey, pp.31-54.
- SANKARAN, K.S., SUBRAHMANYAM, M.S. and KRISHNASWAMY, N.R. (1977) : "Dynamics of Embedded Foundations - A Reappraisal", *International Symposium on Soil-Structure Interaction*, Univ. of Roorkee, Roorkee, India, pp.413-419.
- SRINIVASULU, P. and VAIDYANATHAN, C.V. (1976) : *Hand Book of Machine Foundations*, Tata McGraw-Hill, New Delhi.
- SHRIDHAR, D.S. (1995) : "Studies on Dynamic Soil-Foundation Interaction in Time Domain", *Ph.D Thesis*, Submitted to Indian Institute of Technology, Bombay.
- WOLF, J.P. (1994) : "Cone Models as a Strength of Materials Approach to Foundation Vibration", *Proc., 10<sup>th</sup> European Conference on Earthquake Engineering*, Vienna, Austria, pp.1-10.