Indian Geotechnical Journal, 31 (4), 2001

Ring Footings on Clay

Swami Saran* and M.M.A Al-Smadi†

Introduction

Ring foundation (circular in plan with an annuli) are usually provided for tall circular structures like smoke stacks, cement silos, water towers etc. For such structure s, the ring foundations are preferred because of full utilisation of soil capacity and less or no tension condition under the foundation. These foundations generally are subjected to vertical load due to the superstructure and the horizontal load due to wind pressure acting on the structure.

Ring foundations have not attracted much attention of research investigators. In the past, attempts to develop analytical solution for estimating settlements, under reactive pressures using elastic theory (Egorov, 1965; Brodocheva, 1968) and also finite element technique (Milovic, 1973; Bowles, 1975) have been made. However, these studies pertain to foundations under axial vertical loads only. Experimental studies under both axial and eccentric vertical loads have also been reported (Saha, 1978; Haroon, 1980; Chaturvedi, 1982; Kakroo, 1985). Dimensional analysis approach has generally been adopted to analyse the experimental results (Kakroo, 1985). Attempts to measure contact pressure distribution below the base of ring footings have also been made by conducting model tests on instrumented foundations (Kakroo, 1985). However, there is no study reported in the literature where studies on ring footings subjected to a combination of axi-symmetrical vertical and lateral loading have been carried out. This is the problem of great practical relevance because the tall structures for which such foundations are provided are subjected to large lateral load due to wind. Thus, in addition to the vertical load, the base of these foundations is subjected to large

---

* Professor of Civil Engineering, Indian Institute of Technology, Roorkee – 247 667. India.

† Ministry of Public Works & Housing, Laboratory & Research Directorate, P.O. Box 1220, AMMAN (Jordan).

moment and horizontal load. In other words, the ring foundations are in general subjected to eccentric-inclined load.

Bearing capacity, settlement and tilt are required for proportioning ring foundations subjected to eccentric inclined loads. The current practice of designing such foundations is based on many assumptions and, therefore, is very approximate.

Constitutive laws of soils define the mechanical behaviour of soils and are of prime importance for analysing almost all applied non-linear problems of soil mechanics. The most popular mathematical model for describing a constitutive law is a two constant hyperbola suggested by Kondner et al. (1963). The two constants of this model can be obtained by performing triaxial test on the pertaining soil simulating field condition. In nature, it is quite often to have over consolidated clays having saturation more than 95%. It is well established that the stress-strain characteristics of such soils is independent to confining pressure (Kondner, 1963).

In this paper, a new method has been suggested to obtain the pressure-settlement and pressure-tilt characteristics of an actual ring footing on clay subjected to eccentric inclined load using non-linear constitutive law of soil. From these characteristics, the ultimate bearing capacity of the footing can be obtained using inter-section tangent method, and the values of settlement and tilt may be read directly for the given pressure intensity. Therefore, the ring footings can be proportioned satisfying shear failure, settlement and tilt criterion.

The method proposed herein has already been used earlier for getting pressure-settlement and pressure-tilt characteristics of actual strip and square footings (Prakash et al., 1984; Agarwal, 1986). They have analysed six cases namely (i) smooth flexible strip footing, (ii) smooth rigid strip footing, (iii) rough flexible strip footing, (iv) rough rigid strip footing, (v) smooth flexible square footing and (vi) smooth rigid square footing. They concluded that the effects of roughness and rigidity of footing are very small on the average pressure-settlement and average pressure-tilt curves. Advantage of these conclusions has been taken in developing the proposed analysis for ring footings.

In general, ring footings have rough base and partial rigidity depending on the ring thickness. However as the effect of roughness and rigidity on the pressure versus average settlement, and pressure versus tilt curves are very small, for simplification of analysis, firstly the method is developed for smooth-flexible ring footing. In the end, an empirical but logical procedure has been suggested to obtain the settlement and tilt characteristics of a corresponding rigid footing subjected to eccentric inclined load. The results may be slightly on the safer side and there for can be used for actual rigid rough base ring footings.

Analysis

The analysis has been developed for studying the behaviour of a ring footings subjected to eccentric-inclined load (Fig.1). The analysis is based on the following assumptions :

1. The soil mass is a semi-infinite and isotropic medium.

2. The footing base is smooth and flexible.

3. The whole area of the footing has been divided into m × n small parts (Fig.2). Each part is assumed to carry a concentrated load acting at its centre for determining stresses in the soil mass. The concentrated load is obtained by multiplying the average pressure intensity obtained from the contact pressure distribution shown in Fig.2 acting on the element by its area.

4. The eccentric-inclined load P acting on the footing is resolved into vertical and horizontal components (i.e. $P\cos\alpha$ and $P\sin\alpha$, Fig.1). The contact pressure distribution diagrams for both $P\cos\alpha$ and $P\sin\alpha$

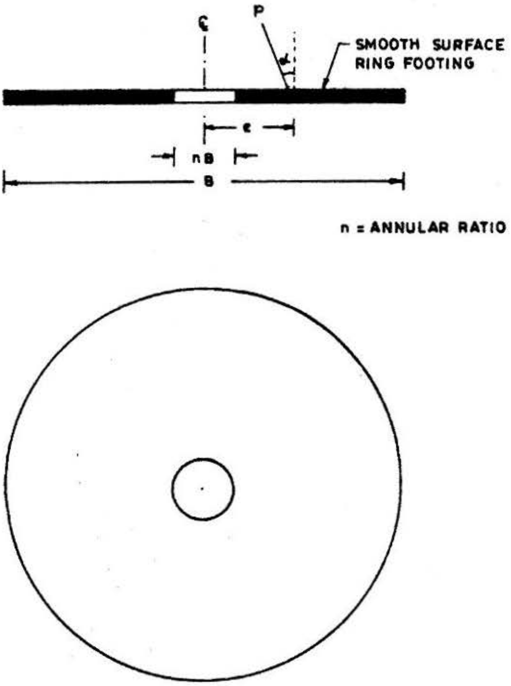


FIGURE 1 : Ring Footing Subjected to Eccentric Inclined Load

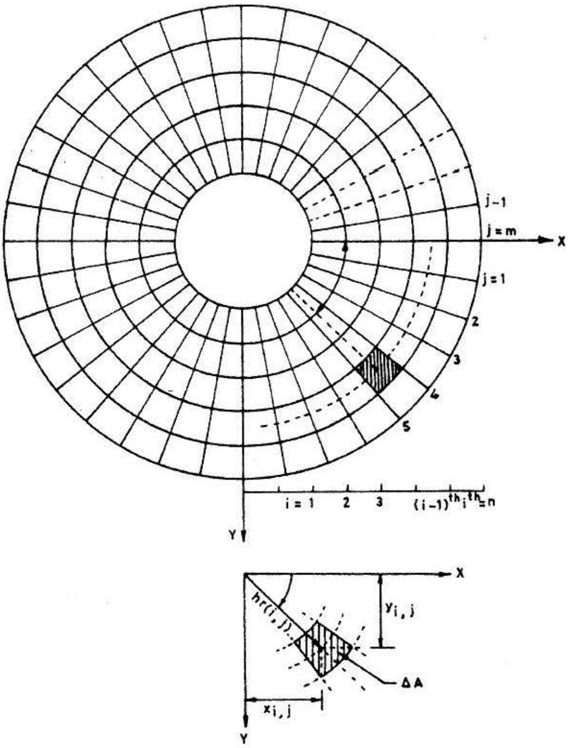


FIGURE 2 : Footing Area Divided into *n* Equal Rings and *m* Equal Sectors

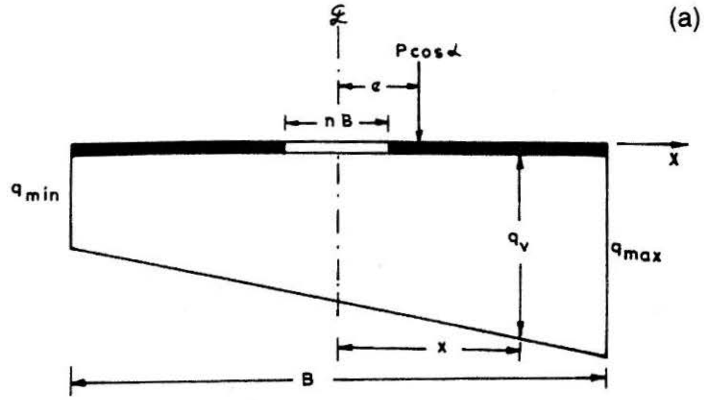
components have been assumed linear (Figs.3 and 4). Following two cases have been considered, depending on the amount of the eccentricity :

Case 1

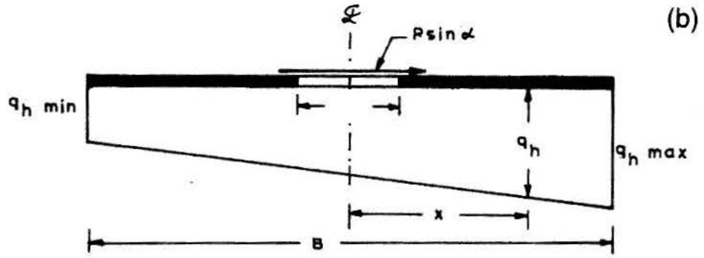
$$\frac{e}{B} \leq \frac{\left(1+n^2\right)}{8} \qquad \text{(Fig.3)}$$

The contact pressure due to load components $P\cos\alpha$ (Fig.3a) and $P\sin\alpha$ (Fig.3b) at distance x from the centre are given by (Roark, 1954; Young, 1989) :

$$q_v = \frac{P}{A}\cos\alpha\left[1+\left(\frac{8}{1+n^2}\right)\left(\frac{e}{B}\right)\left(\frac{x}{R}\right)\right] \tag{1}$$



CONTACT PRESSURE DIAGRAM DUE TO VERTICAL COMPONENT OF ECCENTRIC-INCLINED LOAD



CONTACT PRESSURE DIAGRAM DUE TO HORIZONTAL COMPONENT OF ECCENTRIC-INCLINED LOAD

FIGURE 3 : Assumed Contact Pressure Distributions for $e/B < \left[(1+n^2)/8\right]$

$$\Delta P_v = q_v \cdot \Delta A \tag{2}$$

$$q_h = \frac{P}{A}\sin\alpha\left[1+\left(\frac{8}{1+n^2}\right)\left(\frac{e}{B}\right)\left(\frac{x}{R}\right)\right] \tag{3}$$

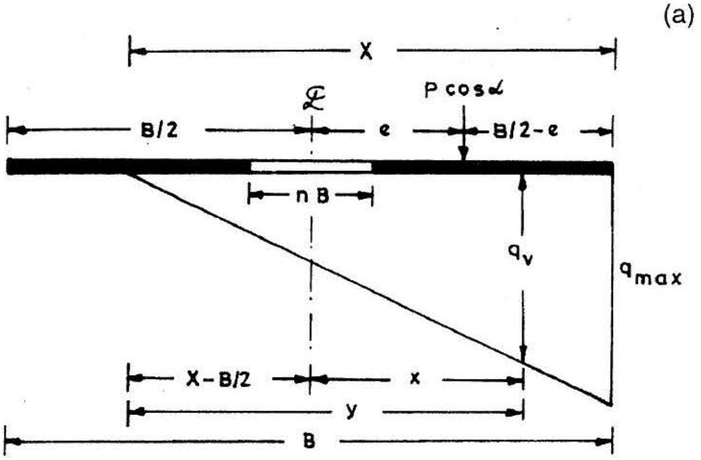
$$\Delta P_h = q_h \cdot \Delta A \tag{4}$$

where P = Applied eccentric-inclined load

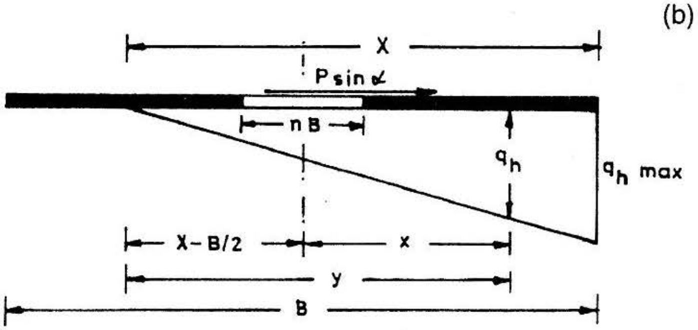
E = Eccentricity of the load

B = Diameter of the footing (= 2R)

α = Inclination of the applied load



CONTACT PRESSURE DIAGRAM DUE TO VERTICAL COMPONENT OF ECCENTRIC-INCLINED LOAD



CONTACT PRESSURE DIAGRAM DUE TO HORIZONTAL COMPONENT OF ECCENTRIC-INCLINED LOAD

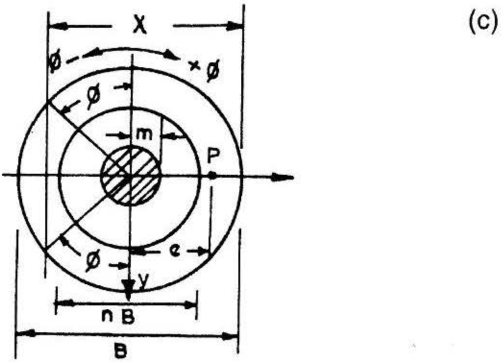


FIGURE 4 : Assumed Contact Pressure Distributions for $e/B > \left[(1+n^2)/8\right]$

A = Net area of the footing

n = Annular ratio

q_v = Contact pressure due to load component $P\cos\alpha$ at a point x

x = Coordinate of the point at which the contact pressure is desired

ΔA = Area of elementary strip

ΔP_v = Vertical point load acting in ΔA

q_h = Contact pressure due to load component $P\sin\alpha$ at a point x

ΔP_h = Horizontal point load acting in ΔA

Case 2

$$\frac{e}{B} > \frac{\left(1+n^2\right)}{8} \qquad \text{(Fig.4)}$$

The maximum pressure due the vertical component of the eccentric-inclined load (Roark, 1954; Teng, 1977; Young, 1989) is given by

$$q_{\max} = k\left(P/A\right)\cos\alpha \tag{5}$$

As shown in Fig.4a, the contact pressure, due to the vertical component of the eccentric-inclined load, at a point x from the centre of the footing is given by :

$$q_v = \left(y/X\right)q_{\max} \tag{6}$$

$$\Delta P_v = q_v \cdot \Delta A \tag{7}$$

$$X = R\left(1-\sin\phi\right) \tag{8}$$

where; X and ϕ are as shown in Fig.4c and ϕ satisfies Eqn.9.

$$\frac{e}{B} = \frac{\left(\pi/8\right)-2.5\phi-\left(5/12\right)\sin\phi\cos\phi+\left(1/6\right)\sin^3\phi\cos\phi}{\cos\phi-\left(1/3\right)\cos^3\phi-\left(\pi/2\right)\sin\phi+\phi\sin\phi} \tag{9}$$

$$y = x+\left[X-\left(B/2\right)\right] \tag{10}$$

However, values of k and X/R have been calculated by Young (1989) for different n and e/R, and presented in Table 1.

The maximum contact pressure, due to the horizontal component of the applied eccentric-inclined load, as seen in Fig.4b taken as :

$$q_{h\max} = k(P/A)\sin\alpha \tag{11}$$

The contact pressure due to the horizontal component at a point x is

$$q_h = (y/X)q_{h\max} \tag{12}$$

5. The soil mass supporting the footing is divided into a large number of thin horizontal strips (Fig.5).

6. The stresses in each layer are computed using theory of elasticity. The strains are computed for the known stress condition using constitutive law.

7. There is no slippage at the interface of layers of the soil mass.

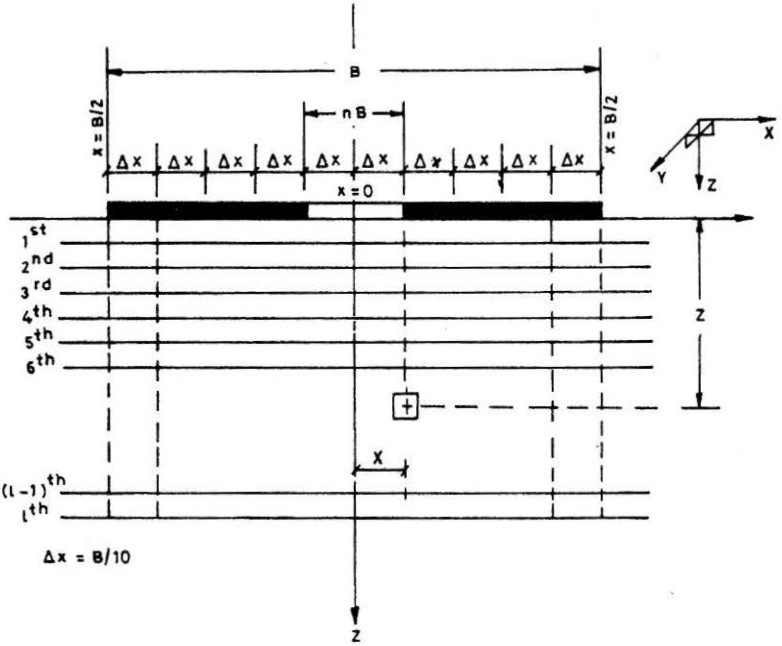


FIGURE 5 : Soil Mass Supporting a Footing Divided Equally into l Horizontal Layers

General Procedure

1. For a given load, P on the ring footing, the contact pressure distribution at the interface footing base and supporting soil is taken as shown in Figs.3 and 4, depending on the amount of eccentricity The contact pressure distribution is the loading pattern for the soil at the surface below the footing, and the stresses in the soil mass is induced according to this loading pattern.

2. The soil mass supporting the footing is divided into n layers as shown in Fig.5.

3. Taking any vertical section, the normal and shear stresses at the center of a layer at depth z below the footing are computed using the theory of elasticity (Poulos and Davis, 1973).

4. The principal stresses at a point in the soil mass and their directions with respect to the vertical z-axis are also computed using the theory of elasticity. (Poulos and Davis, 1973).

5. The constitutive equation for the supporting soil is obtained from triaxial compression test results using Kondner's two-constant hyperbola.

6. Evaluation of Strains : The strain ε_1 in the direction of major principal stress, σ_1, is determined from constitutive relationship given by Eqn.13.

$$\varepsilon_1 = \frac{a(\sigma_1 - \sigma_3)}{1 - b(\sigma_1 - \sigma_3)} \tag{13}$$

in which a and b = the constants of hyperbola; and σ_1 and σ_3 = the major and minor principal stresses. The strains ε_2 and ε_3 are computed from the following relationships, which are based on theory of elasticity:

$$\varepsilon_2 = \frac{\sigma_2 - \mu(\sigma_1 + \sigma_3)}{\sigma_1 - \mu(\sigma_2 + \sigma_3)} \varepsilon_1 \tag{14}$$

$$\varepsilon_3 = \frac{\sigma_3 - \mu(\sigma_1 + \sigma_2)}{\sigma_1 - \mu(\sigma_2 + \sigma_3)} \varepsilon_1 \tag{15}$$

in which σ_2 = the intermediate principal stress; E and μ = the elastic modulus and Poisson's ratio of the soil, respectively. The strain in the vertical direction for each layer is calculated from the following equation:

$$\varepsilon_x = \varepsilon_1 \cos^2\theta_1 + \varepsilon_2 \cos^2\theta_2 + \varepsilon_3 \cos^2\theta_3 \tag{16}$$

in which θ_1, θ_2 and θ_3 = directions of the principal strains with respect to the vertical axis.

7. The vertical settlement, ΔS, of any layer is computed by multiplying the strain ε_z with the thickness of each layer, δ_z, as follows:

$$\Delta S = \varepsilon_z \cdot \delta_z \tag{17}$$

8. The total settlement, S, along any vertical axis, x, is computed by numerically summing the settlements.

$$S = \sum_{0}^{n} \varepsilon_z \cdot dz \tag{18}$$

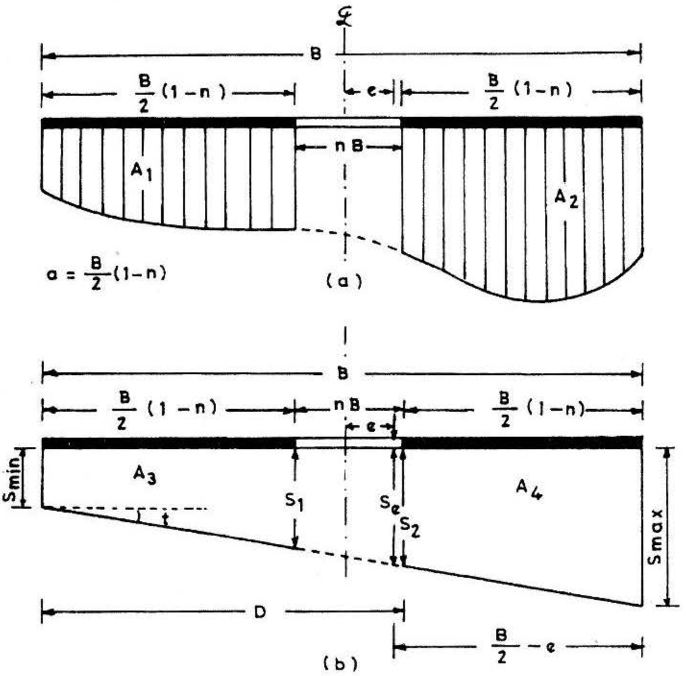


FIGURE 6 : Settlement Diagrams Under Ring Footing, when $e/B < \left[(1+n^2)/8\right]$

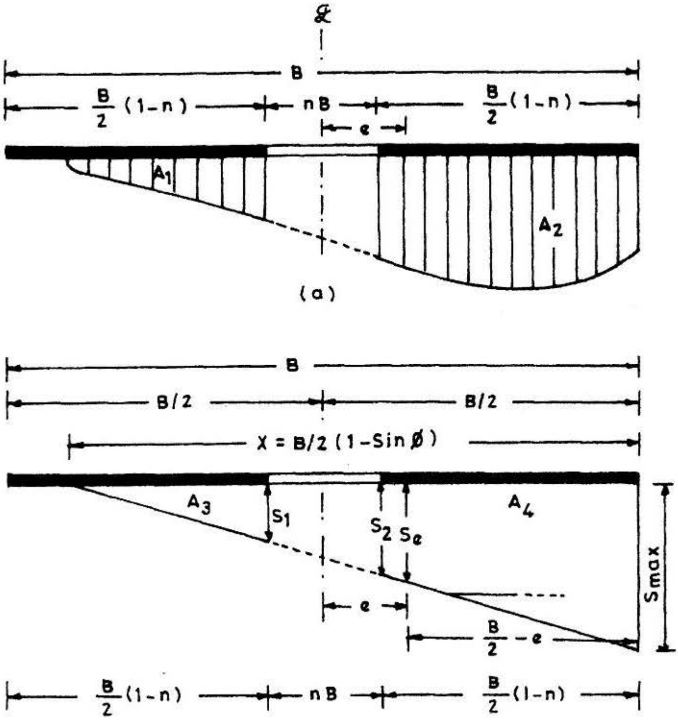


FIGURE 7 : Settlement Diagrams Under Ring Footing, when $e/B > \left[(1+n^2)/8\right]$

9. The total settlements have been computed along various vertical sections, namely, $x = -B/2$, $-4B/10$, $-3B/10$, $-B/10$, 0, $B/10$, $2B/10$, $3B/10$, $4B/10$, $B/2$ for the given pressure intensity, eccentricity, angle of inclination and the annular ratio.

The typical settlement pattern obtained by the above procedure will be as shown in Figs.6a and 7a respectively for $e/B < \left[(1+n^2)/8\right]$ and $e/B > \left[(1+n^2)/8\right]$.

10. A rigid ring footing will settle as shown in Fig.6b and 7b.

For evaluating the settlement pattern of rigid ring footings, following procedure is adopted :

Case 1

When $e/B < \left[(1+n^2)/8\right]$

The values of $S_{\max}$ and $S_{\min}$ of a rigid ring footing (Fig.6b) are obtained by equating :

i) The area of settlement diagram of Fig.6a to the area of settlement diagram of Fig.6b and

ii) The distance of the centre of settlement diagram of Fig.6a from the left edge of the footing, i.e. point 'A', to the distance of centre of settlement diagram of Fig.6b from the left edge of footing, point 'A'.

i.e. $$A_1 + A_2 = A_3 + A_4 \tag{19}$$

or

$$A_s = (a/2)\left[\left(S_{\max} + S_2\right) + \left(S_1 + S_{\min}\right)\right] \tag{20}$$

and

$$C_g = \frac{\left[A_3\left\{\dfrac{\left(2S_1 + S_{\min}\right)a}{\left(S_1 + S_{\min}\right)3}\right\} + A_4\left\{\dfrac{\left(2S_{\max} + S_2\right)a}{\left(S_{\max} + S_2\right)3} + D\right\}\right]}{\left(A_3 + A_4\right)} \tag{21}$$

Solving Eqns.20 and 21, therefore;

$$S_{\min} = A_s \frac{\left[B(4+n)(1+n) - 6C_g\right]}{\left[B^2(1-n)\left(1+n+n^2\right)\right]} \tag{22}$$

and

$$S_{\max} = A_s \frac{\left[6C_g - B(1+n)(2+n)\right]}{\left[B^2(1-n)\left(1+n+n^2\right)\right]} \tag{23}$$

where A_s = area of settlement diagram (Fig.5a) = $A_1 + A_2$

$a = B/2(1-n)$

$S_{\min}$ = minimum settlement of footing

$S_{\max}$ = maximum settlement of footing

B = width (= external diameter) of footing

C_g = centre of gravity of the settlement diagram of Fig.5a from point 'A'

n = annular ratio

= Internal Diameter/External Diameter

$D = B/2(1+n)$

S_1 and S_2 are as defined in Fig.(5b)

It may be noted that the settlement pattern (Fig.6a) is obtained by the procedure described in this paper. Therefore the values of A_s and C_g can be conveniently determined. Knowing these values, $S_{\max}$ and $S_{\min}$ may be obtained using Eqns.22 and 23.

Therefore, knowing the values of $S_{\max}$ and $S_{\min}$ the tilt (t) of the rigid footing (Fig.6b) may be computed using the following equation:

$$t = \left[\left(S_{\max} - S_{\min}\right)/B\right] \tag{24}$$

Case 2

When $e/B > \left[(1+n^2)/8\right]$

In this case the area of settlement diagram of Fig.7a is equated to the area of settlement diagram of Fig.7b. It gives

$$S_{\max} = A_s \frac{2X}{X\left(X-2Bn\right)+B^2 n} \tag{25}$$

where X is defined in Eqn.8. Values of X/R have been given in Table 1 in terms of e/B and n.

Knowing the value of $S_{\max}$, the tilt (t) for the rigid ring footing is given by:

$$t = \left(S_{\max}\right)/X \tag{26}$$

11. Settlement $S_{\max}$ and tilt (t) for a given set of e/B, a, n and B are computed for different applied load intensities by repeating Step 1 through Step 10. Consequently, the pressure versus maximum settlement ($S_{\max}$) and pressure versus tilt (t) curves can then be obtained for the rigid ring footing subjected to eccentric-inclined load.

Table 1 : Values of K and X/R for Contact Pressure and Width Calculation for Different Values of n and e/R

n		e/R															
		0.25	0.29	0.3	0.34	0.35	0.4	0.41	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85
0.0	X/R	2.00		1.82		1.66	1.51		1.37	1.23	1.10	0.97	0.84	0.72	0.60	0.47	0.35
	k	2.00		2.21		2.46	2.75		3.11	3.56	4.14	4.9	5.94	7.43	9.69	13.4	20.5
0.4	X/R		2.00	1.97		1.81	1.67		1.53	1.38	1.22	1.05	0.88	0.73	0.60	0.48	0.35
	k		2.00	2.03		2.22	2.43		2.68	2.99	3.42	4.03	4.9	6.19	8.14	11.3	17.3
0.6	X/R				2.00	1.97	1.84		1.71	1.56	1.39	1.21	1.02	0.82	0.64	0.48	0.35
	k				2.00	2.03	2.18		2.36	2.58	2.86	3.24	3.79	4.64	6.04	8.54	13.2
0.8	X/R							2.00	1.91	1.78	1.62	1.45	1.26	1.05	0.84	0.6	0.41
	k							2.00	2.10	2.24	2.42	2.65	2.94	3.34	3.95	4.98	7.16

12. Pressure-maximum settlement (S_m), and pressure-tilt (t) curves for other sets of e/B, a, n and B are obtained by repeating Step 1 through Step 11.

The details of derivations of equations used in this paper are given elsewhere (Al-Smadi, 1998)

Interpretation

Justification of Assumptions

Out of the seven assumptions, assumptions No.2 and 6 need justification.

As already mentioned, that the effect of roughness of the footing is very small and can be neglected. The results will be slightly on the safer side (Sharan, 1977; Prakash, et al. 1984).

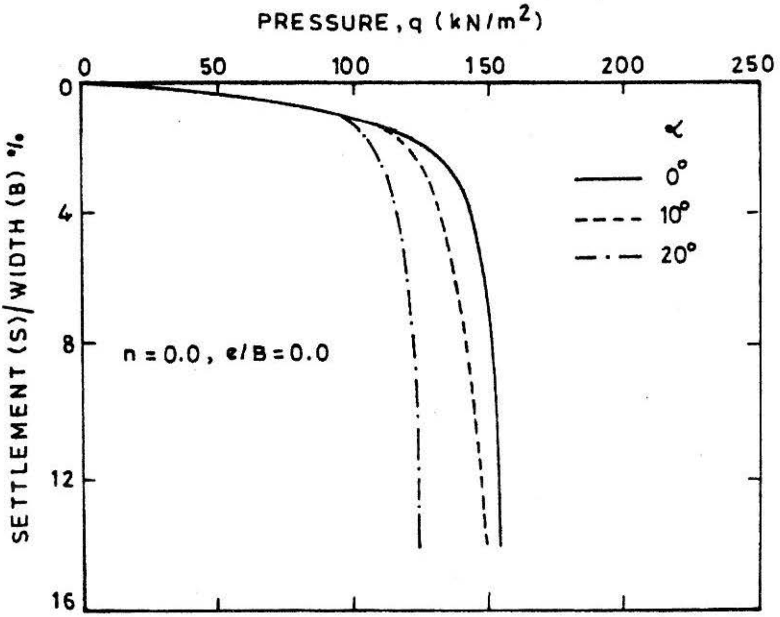
A procedure has been suggested to determine the settlement and tilt of the corresponding rigid footing from the settlement pattern of the flexible footing. Amir (1992) has validated this procedure for strip, square and rectangular footings.

In pre-consolidated saturated clays, effect of confining pressure is negligible on their stress-strain behaviour. It is very well documented that theory of elasticity can be used for determining stresses in such supporting soils (Sharan, 1977; Prakash et al., 1984; Sud, 1984; Agarwal, 1986; Amir, 1992).

Results and Discussion

Pressure-settlement and pressure-tilt characteristics of ring footing subjected to eccentric inclined loads have been obtained for footings resting on Buckshot clay (LL = 65%, PL = 20%, w_n = 33.2%, D_R = 90%) (Carrol, 1963). The average values of the hyperbola constants a and b are 95×10^{-6} m^2/kN and 11.49×10^{-3} m^2/kN respectively.

To examine the effect of size of footing, the pressure-settlement and pressure-tilt characteristics, have been obtained for ring footing having width, B = 200 mm, 300 mm, 600 mm, 1000 mm and 2000 mm for annular ratio n = 0.0, 0.2, 0.4, α = 0°, 10°, 20° and e/B = 0.0, 0.1, 0.2. It has been found that for the same pressure intensity, the settlement of the footing increases in direct proportion with the width of the footing. It means that the pressure versus settlement/width ratio plots, and pressure versus tilt curves are independent to the size of the footings. Therefore, ultimate bearing pressure is also independent to the size of the footing.



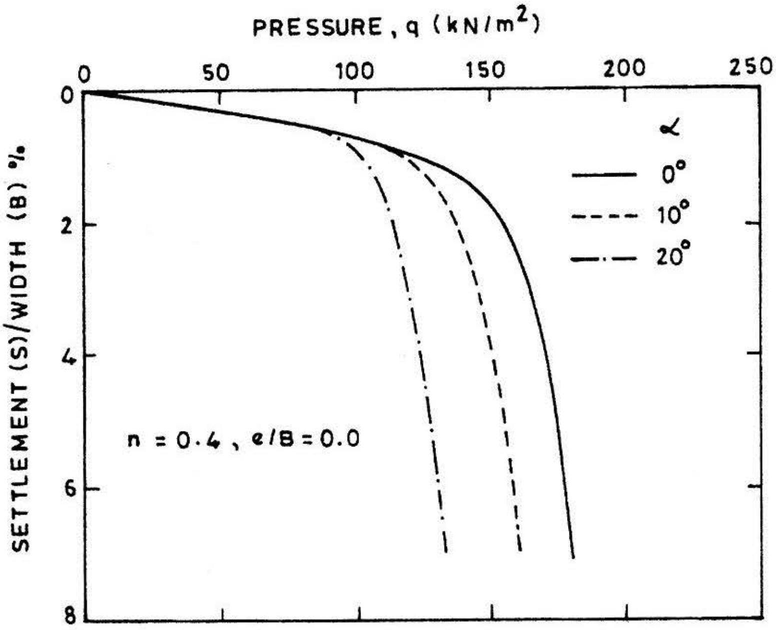
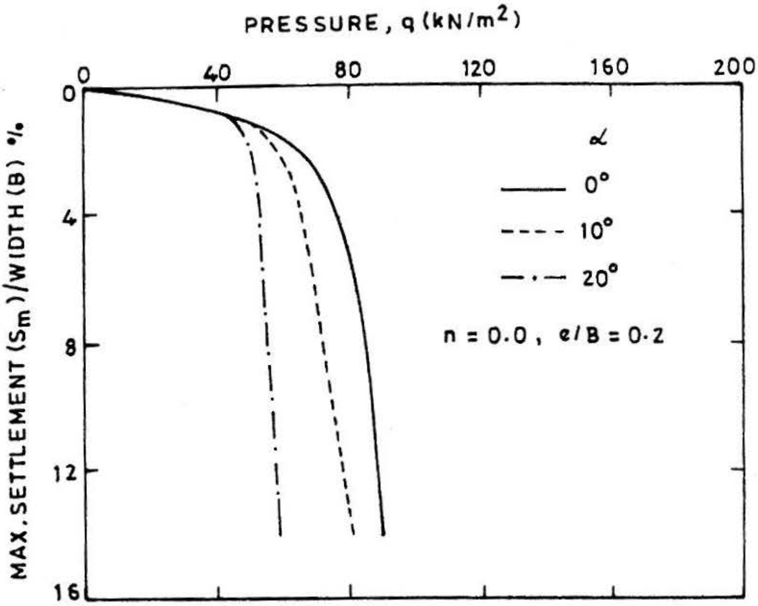


FIGURE 8 : Pressure Versus Settlement (S_{max}) Curves for n = 0 and 0.4 for e/B = 0



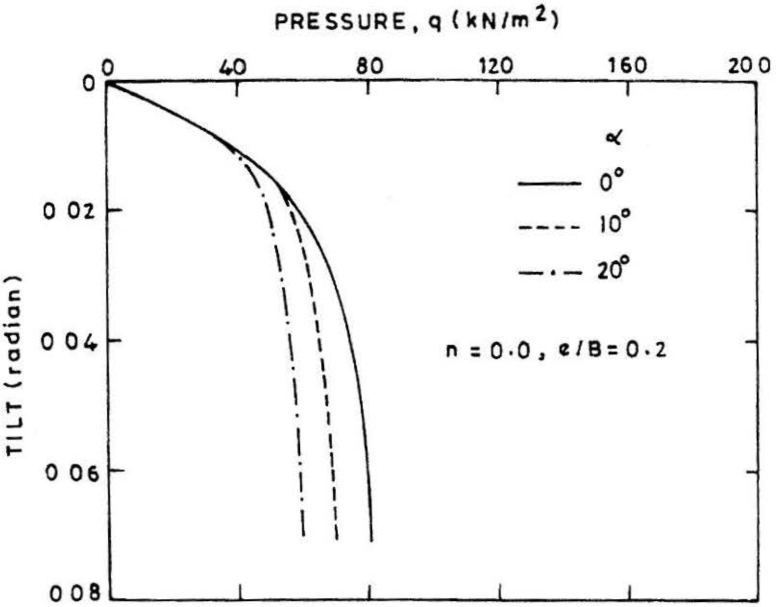
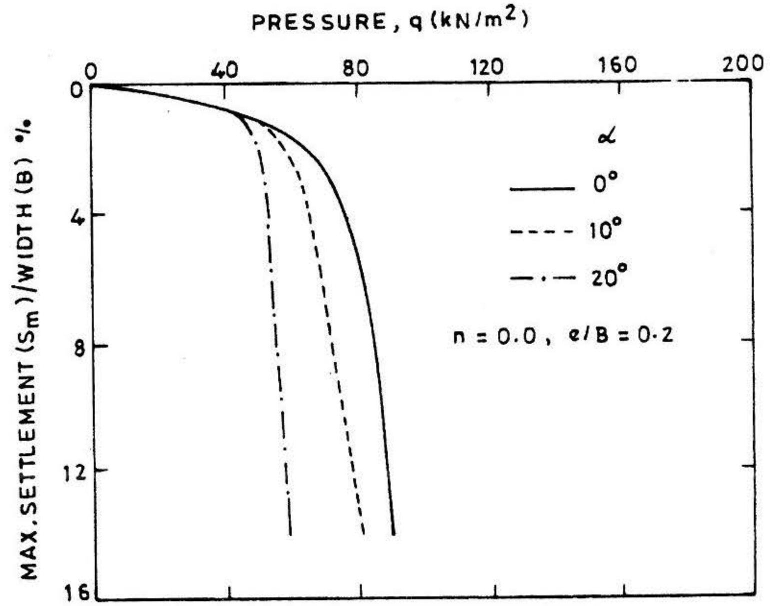


FIGURE 9 : Pressure Versus Settlement (S_{max}) and Pressure Versus Tilt Curves for n = 0 and e/B = 0.1



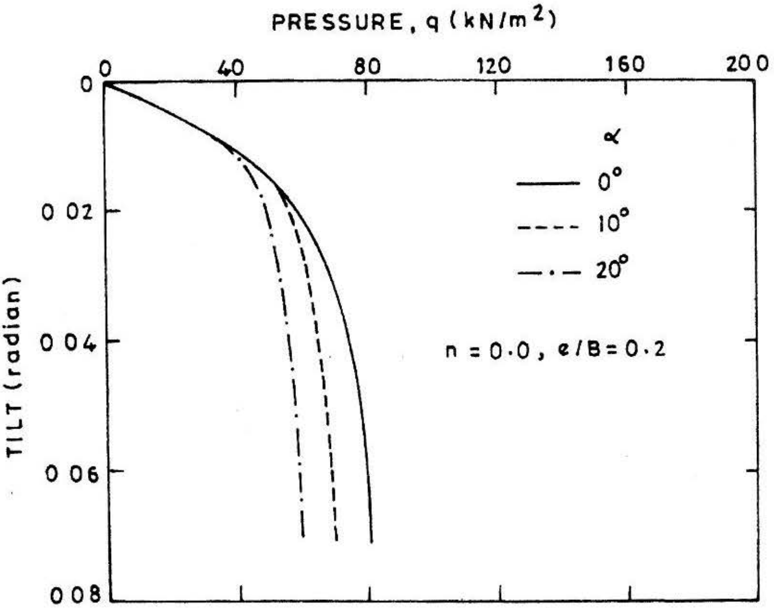
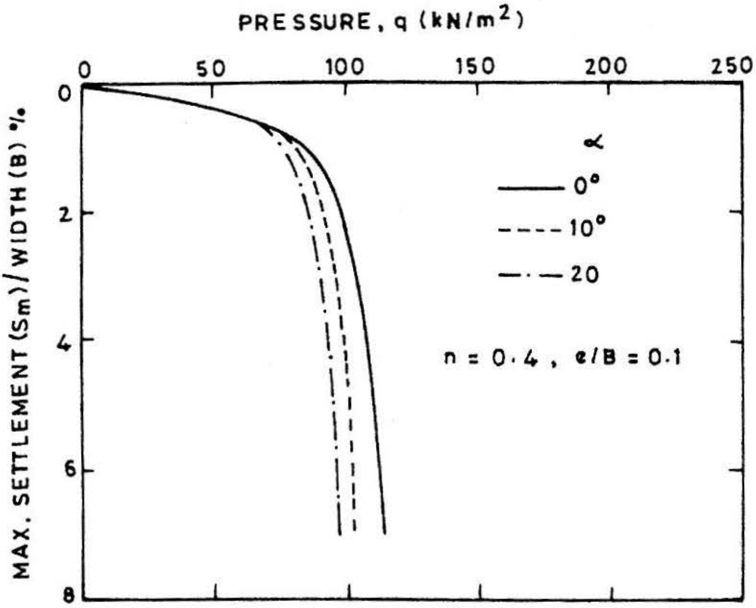


FIGURE 10 : Pressure Versus Settlement (S_{max}) and Pressure Versus Tilt Curves for n = 0 and e/B = 0.2



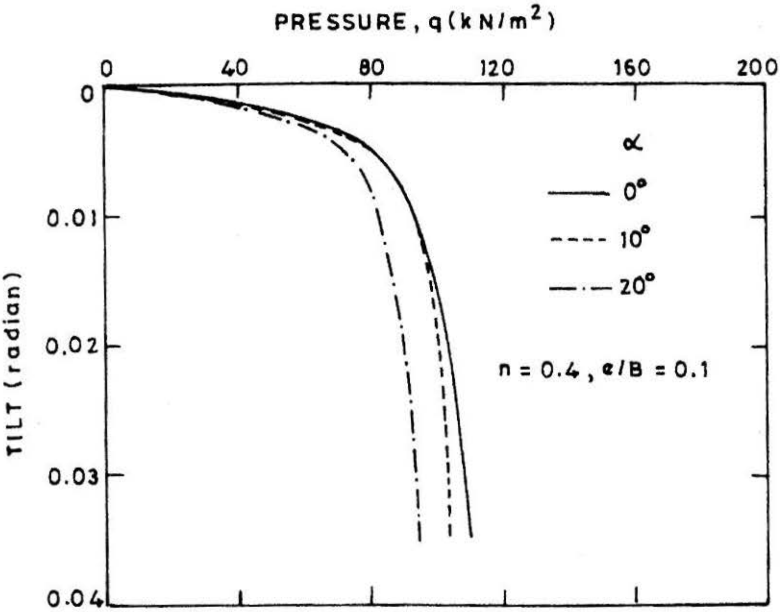


FIGURE 11 : Pressure Versus Settlement (S_{max}) and Pressure Versus Tilt Curves for n = 0.4 and e/B = 0.1

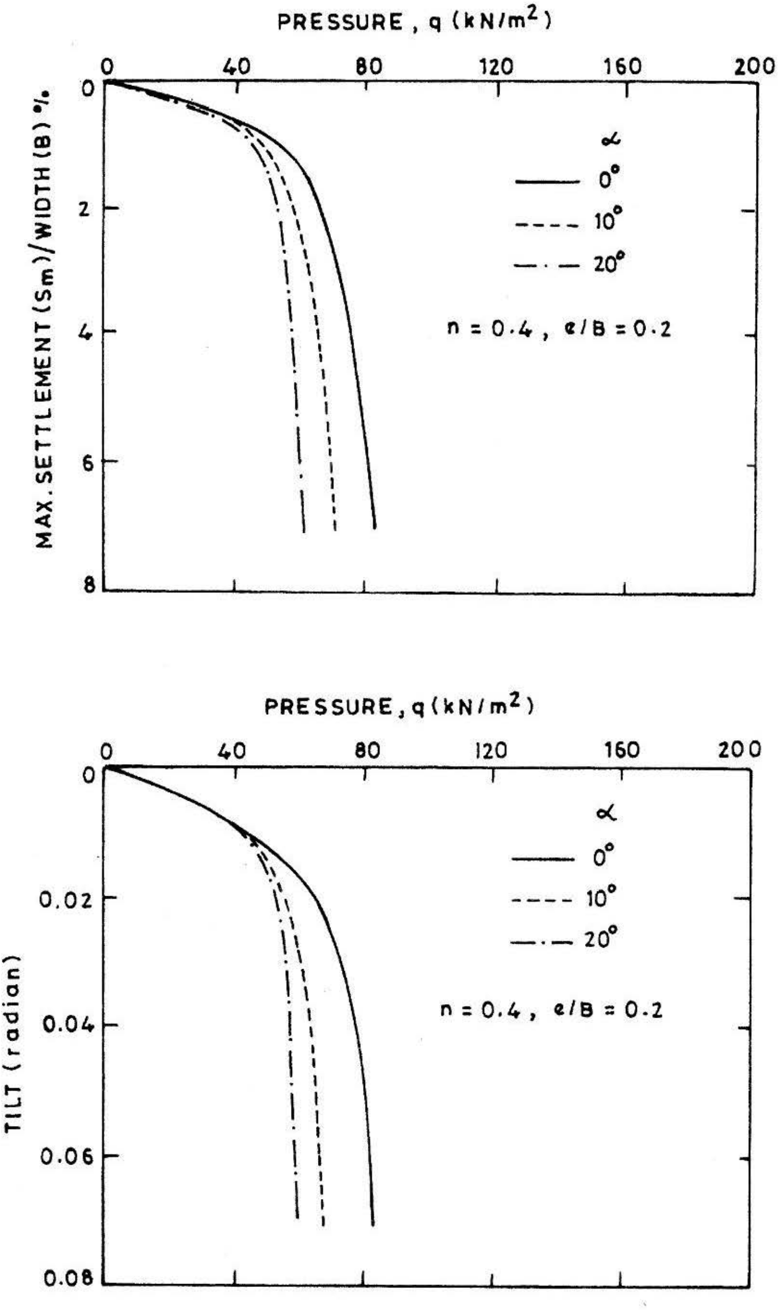


FIGURE 12 : Pressure Versus Settlement (S_{max}) and Pressure Versus Tilt Curves for n = 0.4 and e/B = 0.2

For further interpretation, typical pressure versus maximum settlement width ratio $\left(S_{\max}/B\right)$ and pressure versus tilt plots are shown in Figs.8 to 12 for $n = 0$, 0.4, a = 0.0°, 10°, 20° and $e/B = 0$, 0.1, 0.2. A careful study of these figure indicated that for the same pressure intensity, settlement and tilt of the footings increase with the increase in e/B ratio and angle of inclination of load α, but decrease with the increase in annular ratio.

No experimental data is available on the ring footings resting on clay and subjected to eccentric-inclined loads, and therefore the approach presented herein could not be validated. However, the same approach had already been used successfully for strip, square and rectangular footings (Prakash et al., 1984; Sud, 1984; Agarwal, 1986 and Amir, 1992).

Conclusions

1. A novel approach is developed to get the pressure-settlement and pressure-tilt characteristics of the ring footings subjected to eccentric-inclined load using non-linear stress-strain characteristics of the soil.

2. For the same pressure intensity, settlement and tilt of the ring footing increase with the increase in e/B ratio and angle of load inclination a. As annular ratio n increases, the settlement and tilt decreases.

References

AGARWAL, R.K. (1986) : "Behaviour of Shallow Foundations Subjected to Eccentric-Inclined Loads", *Ph.D. Thesis*, University of Roorkee, Roorkee, U.P., India.

AL-SMADI, M.M. (1998) : "Behaviour of Ring Foundations on Reinforced Soil", *Ph.D. Thesis*, University of Roorkee, Roorkee, U.P., India.

AMIR, A.A.A. (1992) : "Interference Effect on the Behaviour of Footings", *Ph.D. Thesis*, University of Roorkee, Roorkee, U.P., India.

BORODACHEVA, F.N. (1968) : "Action of a Vertical Eccentric Force on an Annular Foundation Situated on a Compressible Base", *Soil Mechanics and Foundation Engineering*, No.1, pp.34-37.

BOWELS, J.E. (1975) : *Analytical and Computer Methods in Foundation Engineering*, McGraw-Hill Kogakusha Limited, Tokyo, p.196-199.

CARROL, W.F. (1963) : "Dynamic Bearing Capacity of Soil", *Technical Report No. 3-599*, U.S. Army Engineers, Water Ways Experiment Stations, Corps of Engineers, Vicksburg, Mississippi.

CHATURVEDI, A. (1982) : "Behaviour of Eccentrically Loaded Ring Footings on Sand", *M.E. Thesis*, University of Roorkee, Roorkee, U.P., India.

EGOROV, K.E. (1965) : "Calculation of Bed for Foundation with Ring Footing", *Proc. of the* 6^{th} *International Conference on SMFE*, Montreal, Vol.II, pp.41-45.

HAROON, M. and MISRA, S.K. (1980) : "A Study on the Behaviour of Annular Footings on Sand", *Indian Geotechnical Society*, IGC-80, Bombay, Vol.1, pp.87-91.

KAKROO, A.K. (1985) : "Bearing Capacity of Rigid Annular Foundations under Vertical Loads", *Ph.D. Thesis*, University of Roorkee, Roorkee, U.P., India.

KONDNER, R.L. (1963) : "Hyperbolic Stress-Strain Response : Cohesive Soils", *Journal of the Soil Mechanics and Foundations Division*, Proc. ASCE, Vol.89, No.SM1, pp.115-143.

MILOVIC, D.M. (1973) : "Stresses and Displacements Produced by a Ring Foundation", *Proc. of 8th International Conference on Soil Mechanics and Foundation Engineering*, Vol.III, pp.167-171.

POULOS, H.G. and DAVIS, E.H. (1974) : *Elastic Solutions for Soil and Rock Mechanics*, John Wiley and Sons, Inc., New York.

PRAKASH, S., SARAN, S. and SHARAN, U.N. (1984) : "Footings and Constitutive Laws", *Journal of the Geotechnical Engineering Division*, ASCE, Vol.110, No.10, pp.1473-1488.

ROARK, R.J. (1954) : *Formulas for Stress and Strain*, McGraw Hill Book Company Inc., New York.

SAHA, M.C. (1978) : "Ultimate Bearing Capacity of Ring Footings on Sand", *M.E. Thesis*, University of Roorkee, Roorkee, U.P., India.

SHARAN, U.N. (1977) : "Pressure Settlement Characteristics of Surface Footings from Constitutive Laws", *Ph.D. Thesis*, University of Roorkee, Roorkee, U.P., India.

SUD, V.K. (1984) : "Behaviour of Shallow Foundations Adjacent to Slopes", *Ph.D. Thesis*, University of Roorkee, Roorkee, U.P., India.

TENG, W.C. (1977) : *Foundation Design*, Prentice Hall of India Private Ltd., New Delhi.

YOUNG, W.C. (1989) : *Roark's Formulas for Stress & Strain*, McGraw-Hill Book Company, New York.