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Elastic Settlement of Footings Embedded into Elastic Stratum / Half Space

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Introduction

Two distinct kinds of action within the foundation soil may cause the subsidence or settlement of structures resting on soil. When the developed shearing stresses due to imposed load exceed the shearing strength of the material, the soil fail by sliding downward and laterally and the structure settles. In the other case, a structure settle by virtue of compressive stress and accompanying strains, which are developed in the soil as a result of the load imposed upon it. Other settlement i.e., settlement due to compressive stress and the accompanying strain mainly consists of two parts: (1) immediate or elastic settlement and (2) consolidation settlement. In sandy soil immediate or elastic settlement is the main concern whereas consolidation settlement for clayey soil. Several methods are available to estimate the both and this paper deals with the estimation of elastic settlement.

Elastic settlement is commonly computed by either using elastic formulation based on Boussinesq's equation or using empirical equations combined with graphs/charts and tables. However, elastic equations for computing elastic settlement are available for a very few specific cases like surface foundations on elastic half space. But, settlement of footings depends on several factors namely, shape and size of the footing, depth of embedment, layering and nonhomogeneities in the soil etc. Chart presented by Steinbrenner (1934) can be used to estimate settlement of rectangular footing on half space/stratum. Using the elastic solution of Timoshenko and Goodier (1970) settlement of circular and rectangular footing on half space can be

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estimated. To estimate settlement reduction due to embedment of footing into the half space, work of Fox (1948) is generally used. To estimate the settlement of footing considering the effect of embedment, stratum thickness and shape, graphical charts presented by Janbu et al (1956) and Christian and Carrier (1978) are generally used. Schmertmann (1970) presented a method to estimate the settlement of footing on sand based on cone penetration test results. Bowles (1987) presented a method to estimate settlement of footing embedded into stratum using Steibrenner's solution and Fox's chart to account for effect of stratum and embedment depth respectively. To consider the effect of layering in settlement computation work of Burmister (1943, 1945), Palmer and Barber (1940), Ueshita and Meyerhof (1967) can be referred.

It can be seen from the above review that use of graphs/charts and tables is the only option to estimate settlement taking into account the effect of different factors. Further it can be seen that different methods need to be adopted to account for the effect of different factors. Hence, in this paper an attempt is made to develop close form solutions for settlements of different shape of footings embedded into elastic stratum or elastic half space based on Mindlin's solution. This includes the following:

(1) Establishing equations for settlements of circular and rectangular footings,

(2) investigation on the effect of embedment (into half space or finite stratum) of footing on elastic settlement,

(3) investigation on applicability of equivalent circle method for other footing shapes,

(4) preparation of charts for estimating elastic settlement considering above factors, and

(5) verification of applicability of the suggested method with the help of a number of case studies.

Analysis

Mindlin (1936) presented a solution for the three-dimensional elasticity problem of homogeneous isotropic solid for the case of concentrated force acting in the interior of a semi-infinite solid. Vertical displacement, w at any point A (x, y, z) for a point load, $\underline{Q}$ acting at $(0, 0, h)$ as given by him is (Fig.1):

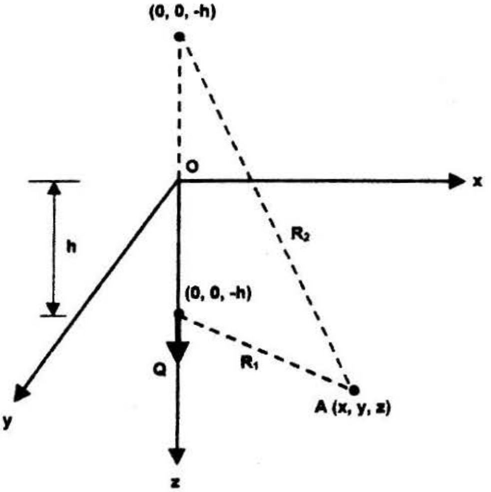

FIGURE 1 : Mindlin's Problem

$$w = \frac{(1+\mu)Q}{8\pi E(1-\mu)}\left[\frac{3-4\mu}{T_1}+\frac{8(1-\mu)^2-(3-4\mu)}{T_2}+\frac{(z-h)^2}{T_1^3}+\frac{(3-4\mu)(z+h)^2-2zh}{T_2^3}+\frac{6zh(z+h)^2}{T_2^5}\right] \quad (1)$$

or,

$$w = \frac{(1+\mu)Q}{8\pi E(1-\mu)}X \quad (2)$$

where E = Elastic modulus of the soil,

μ = Poisson's ratio of the soil

$$X = \left[\frac{3-4\mu}{T_1}+\frac{8(1-\mu)^2-(3-4\mu)}{T_2}+\frac{(z-h)^2}{T_1^3}+\frac{(3-4\mu)(z+h)^2-2zh}{T_2^3}+\frac{6zh(z+h)^2}{T_2^5}\right]$$

in polar coordinate system

$$T_1 = \left[r^2+(z-h)^2\right]^{1/2}$$

$$T_2 = \left[r^2+(z+h)^2\right]^{1/2}$$

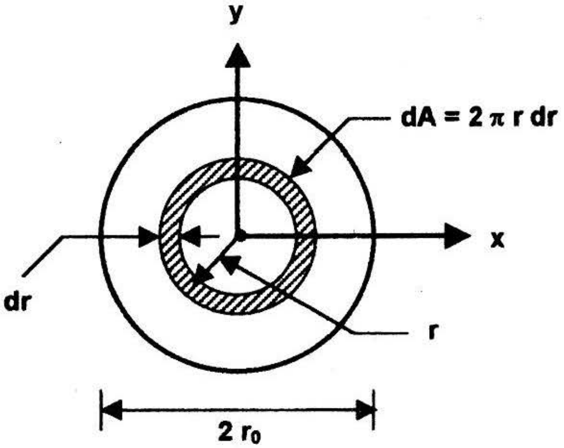

FIGURE 2 : Sketch Showing the Method for Calculating Settlement for Uniformly Loaded Circular Area

$$r^2 = \left(x^2 + y^2\right)$$

in rectangular coordinate system

$$T_1 = \left[x^2 + y^2 + (z-h)^2\right]^{1/2}$$

$$T_2 = \left[x^2 + y^2 + (z+h)^2\right]^{1/2}$$

If an area loaded with intensity, q instead of a point load, Q, the total displacement at any point can be obtained integrating over the whole area. Integration by analytical means for an arbitrary area is difficult and numerical integration techniques can be adopted for such case. For regular shapes (circular, square, rectangular, etc.), it can be integrated analytically. Expressions for settlement for different shape of footings are developed and presented in the following sub sections.

Circular Footings

Settlement at any point along the axis and at a depth, z due to circular loaded area of radius, r_0 with intensity, q at a depth, h can be obtained integrating Eqn.2. Considering a thin annular area with radius, r and thickness dr within the circular loaded area (Fig.2) situated at a depth, h, total displacement at any point along the axis and below a depth, z can be written as,

$$w = \frac{(1+\mu)Q}{4E(1-\mu)}\int_0^{r_0} X r dr \tag{3}$$

After integration, the above equation reduced to the form as,

$$w = \frac{2(1+\mu)Q}{Er_0} F(h/r_0, z/r_0) \tag{4}$$

where

$$F(h/r_0, z/r_0) = \frac{1}{8\pi(1-\mu)}\left[\begin{array}{l} (3-4\mu)\left(\sqrt{A_3}-A_1\right) \\ \quad +\left(5-12\mu+8\mu^2\right)\left(\sqrt{A_4}-A_2\right) \\ \quad +(3-4\mu)\left(A_2-\frac{A_2^2}{\sqrt{A_4}}\right)+A_1-\frac{A_1^2}{\sqrt{A_3}} \\ \quad +\frac{2(h/r_0)(z/r_0)}{\sqrt{A_4}}-\frac{2(h/r_0)(z/r_0)}{A_4^{3/2}}A_2^2 \end{array}\right] \tag{5}$$

$$\begin{aligned} A_1 &= (z-h)/r_0 \\ A_2 &= (z+h)/r_0 \\ A_3 &= 1+(z-h)^2/r_0^2, \text{ and} \\ A_4 &= 1+(z+h)^2/r_0^2 \end{aligned}$$

Eqn.4 gives settlement at any point along the axis and at a non-dimensional depth, z/r_0 when the footing is situated at a depth, h/r_0.

Rectangular Footings

The total settlement at any point along the axis and below a depth, z of the uniformly loaded [$q = Q/(4axb)$] rectangular footing of size $2ax2b$ situated at a depth h can be obtained integrating Eqn.2 as (Fig.3),

$$w = \frac{(1+\mu)Q}{8\pi E(1-\mu)ab}\int_0^a\int_0^b X dx dy \tag{6}$$

After integration twice the above equation takes the form as

$$w = \frac{8bq(1+\mu)}{E} F(h/b, z/b, a/b) \tag{7}$$

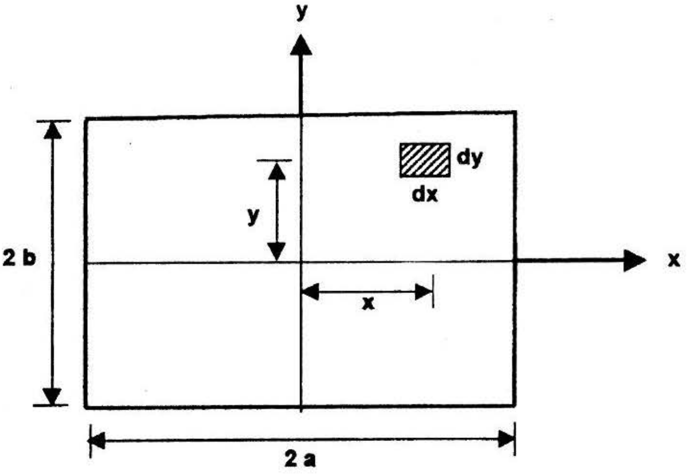

FIGURE 3 : Sketch Showing the Method for Calculating Settlement for Uniformly Loaded Rectangular Area

where

$$F(h/b, z/b, a/b)$$

$$= \frac{1}{16\pi(1-\mu)}\left[\begin{array}{l} (3-4\mu)\left[\frac{a}{b}\ln\frac{(1+B_1)B_3}{(1+B_2)B_4}+\ln\frac{(a/b+B_1)B_7}{(a/b+B_2)B_8}\right] \\ +8(1-\mu)^2\left[\frac{a}{b}\ln\frac{1+B_2}{B_3}+\ln\frac{a/b+B_2}{B_7}\right] \\ -2(1-2\mu)^2\left[(z/b+h/b)\tan^{-1}\frac{a/b}{(z/b+h/b)B_2}\right] \\ -(2-4\mu)\left[(z/b-h/b)\tan^{-1}\frac{a/b}{(z/b-h/b)B_2}\right] \\ +\frac{2(z/b)(h/b)(a/b)B_5}{B_2B_6} \end{array}\right] \quad (8)$$

$$B_1 = \mathit{sqrt}\left[1+(a/b)^2+(z/b-h/b)^2\right]$$

$$B_2 = \mathit{sqrt}\left[1+(a/b)^2+(z/b+h/b)^2\right]$$

$$B_3 = \mathit{sqrt}\left[(a/b)^2+(z/b+h/b)^2\right]$$

$$B_4 = \mathit{sqrt}\left[(a/b)^2+(z/b-h/b)^2\right]$$

$$B_5 = 1+(a/b)^2+2(z/b+h/b)^2$$
$$B_6 = (a/b)^2+(a/b)^2(z/b+h/b)^2+(z/b+h/b)^2+(z/b+h/b)^4$$
$$B_7 = sqrt\left[1+(z/b+h/b)^2\right], \text{ and}$$
$$B_8 = sqrt\left[1+(z/b+h/b)^2\right]$$

Results and Discussions

Circular Footings

Eqn.4 gives settlement at any point along the axis and at a depth, z/r_0 due to the load applied at a depth, h/r_0 (i.e. footing placed at depth, h/r_0). Settlement just below the footing can be obtained substituting $z/r_0 = h/r_0$ in Eqn.5. Similarly, settlement below the center of the surface footing can be obtained substituting $z/r_0 = h/r_0 = 0$ in Eqn.5. Obtaining F for $z/r_0 = h/r_0 = 0$ from Eqn.5 and substituting it into Eqn.4, settlement of circular footing on the half space is obtained as

$$w = \frac{2(1-\mu^2)}{E} q r_0 \tag{9}$$

The above equation is same as the Boussinesq solution for settlement of circular footing on the ground surface. Settlement of circular footing on the surface of the elastic half space or embedded into elastic half space can be obtained using suitable values of h/r_0 and z/r_0 in Eqns.4 and 5 as mentioned above. Using the same equations, settlement of circular footing resting on or embedded into an elastic stratum of finite depth can also be obtained modifying the function F suitably. Expressions for settlement of circular footing embedded into a half space, w_h can be obtained after modification as,

$$w_h = \frac{2\pi(1+\mu) q r_0}{E} F(h/r_0, h/r_0) \tag{10}$$

The above equation gives settlement of footing at just below it, which is summation of settlement between depths, h/r_0 to ∞. Eqn.5 gives settlement at depth, z/r_0 when the footing at depth, h/r_0 which is nothing but the summation of settlement between depths, z/r_0 to ∞. If the rigid boundary is present at depth, z/r_0 , it can be assumed that the settlement between depths, z/r_0 to ∞ is zero. Hence, settlement of footing embedded into a finite stratum, w_s, can be obtained subtracting settlement obtained by Eqn.5 from settlement obtained by Eqn.10 and can be expressed as,

$$
\begin{aligned}
w_h &= \frac{2\pi(1+\mu)q r_0}{E}\left[F\left(h/r_0, h/r_0\right)-F\left(h/r_0, z/r_0\right)\right] \\
&= \frac{2\pi(1+\mu)q r_0}{E} F_s
\end{aligned}
\tag{11}
$$

where $F_s = F\left(h/r_0, h/r_0\right)-F\left(h/r_0, z/r_0\right)$

Settlement of circular footing embedded into elastic half space can be obtained from Eqn.10 whereas settlement of circular footing embedded into elastic stratum can be obtained from Eqn.11. To obtain settlement of footing embedded into stratum from Eqn.11, z/r_0 always be greater than h/r_0. Settlement of circular footing embedded into half space can also be obtained from Eqn.11 substituting $z/r_0 = \infty$ into it. Following the procedures described above, values of F_s for different embedment depths, h/r_0, different stratum depths, z/r_0 and for Poisson's ratio, $\mu = 0.3$ are obtained from Eqns.5 and 11. Finally, $1/F_s$ versus h/r_0 curves are plotted for different z/r_0 and presented in Fig. 4. It can be seen from the Fig. 4 that with the increase of h/r_0, $1/F_s$ increases i.e. F_s decreases which results in decrease in

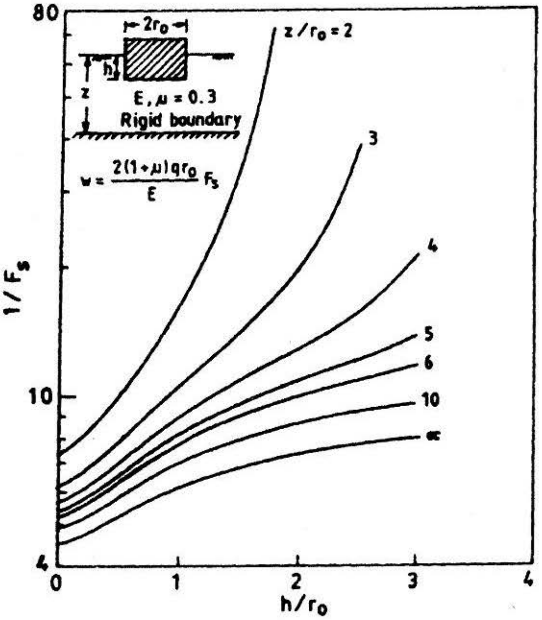

FIGURE 4 : Settlement Factor, $1/F_s$ Versus Non-Dimensional Depth of Embedment, h/r_0 for Circular Footing (μ = 0.3)

settlement for a constant stratum depth. Further, with the increase of stratum depth, z/r_0 , F_s increases which results in increase in settlement. Maximum settlement can be observed for circular footing on the surface of homogeneous half space (as in Eqn.9).

Rectangular Footings

Settlement at any point along the axis and at a depth, z/b for a constant a/b ratio and embedment depth, h/b $(h/b < z/b)$ can be obtained from the Eqn.7. Settlement below the center of the embedded rectangular footing with depth of embedment, h/b can be obtained substituting z/b by h/b in Eqns.7 and 8. Further, settlement of the rectangular footing on the surface of the elastic half space can be obtained substituting $z/b = h/b = 0$ in Eqns.7 and 8. Settlement of the square footing (i.e. $a/b = 1$) is obtained modifying Eqns.7 and 8 as,

$$w = \frac{8qb\left(1-\mu^2\right)}{\pi E}\ln\left(\sqrt{2}+1\right) \tag{12}$$

The above solution is same as can be found by Boussinesq's equation for square surface footing (Timoshenko and Goodier, 1970). Similarly, settlement equation for any rectangular footing on the surface of the elastic half space is obtained from Eqns.7 and 8 as,

$$w = \frac{4bq(1-\mu)}{\pi E}\left[\frac{a}{b}\ln\frac{1+\sqrt{1+(a/b)^2}}{a/b}+\ln\left(\frac{a}{b}+\sqrt{1+(a/b)^2}\right)\right] \tag{13}$$

Eqns.12 and 13 are for settlements at the center of the square and rectangular footings resting on the surface of the elastic half space. Settlement of any rectangular footing embedded into elastic stratum/half space can be obtained similarly as described for circular footing. Settlement of rectangular footing embedded into elastic half space, w_h and embedded into stratum, w_s is obtained modifying Eqns.7 and 8 as

$$w_h = \frac{8bq(1+\mu)}{E}F(h/b, h/b, a/b) \tag{14}$$

$$\begin{aligned} w_s &= \frac{8bq(1+\mu)}{E}\left[F(h/b, h/b, a/b) - F(h/b, z/b, a/b)\right] \\ &= \frac{8bq(1+\mu)}{E}F_s \end{aligned} \tag{15}$$

where $$F_s = F(h/b, h/b, a/b) - F(h/b, z/b, a/b)$$

Settlement of rectangular footing embedded into half space can be obtained either from Eqn.14 or from Eqn.15 with value of z/b as ∞ whereas settlement of rectangular footing embedded into stratum can be obtained only from Eqn.15. Settlement beyond a depth z/b is assumed to be zero as it is assumed in the circular footing.

Using Eqn.15 settlement factor, F_s for different embedment depths, h/b , different stratum depths, z/b and different half-length to half width ratios, a/b are obtained. Finally, $1/F_s$ versus h/b are plotted for different z/b and a/b. Fig. 5 presents $1/F_s$ versus h/b for stratum depths of 2, 3, 4, 5, 6, 8, 10 and ∞ and for a/b ratio of 1. It can be seen from the above figure that with the increase in depth of embedment, $1/F_s$ increases i.e. F_s decreases resulting in decrease in settlement. For example, values of F_s are found 0.177 and 0.102 respectively for embedment depth of 0 and 2.0 of a square footing on a stratum of depth, z/b = *10*. Further, it can be seen that with the increase in stratum thickness, $1/F_s$ decreases for a particular depth of embedment resulting in increase in settlement. For example, settlement factor, F_s increases from 0.112 to 0.196 due to increase of

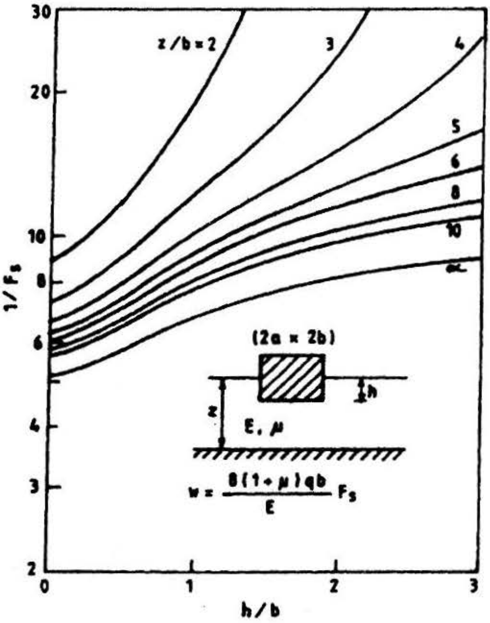

FIGURE 5 : Settlement Factor, $1/F_s$ Versus Non-Dimensional Depth of Embedment, h/b for Square Footing (μ = 0.3)

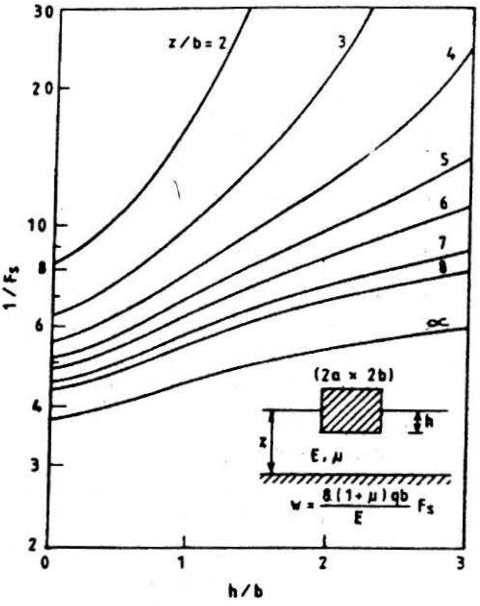

FIGURE 6 : Settlement Factor, $1/F_s$ Versus Non-Dimensional Depth of Embedment, h/b for Rectangular Footing with $a/b = 2$ ($\mu = 0.3$)

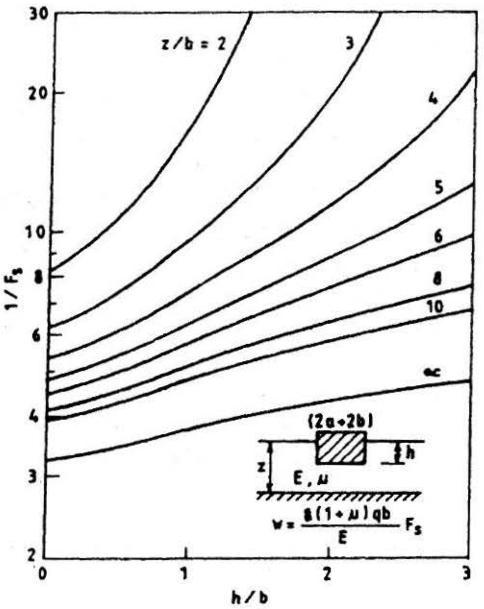

FIGURE 7 : Settlement Factor, $1/F_s$ Versus Non-Dimensional Depth of Embedment, h/b for Rectangular Footing with $a/b = 3$ ($\mu = 0.3$)

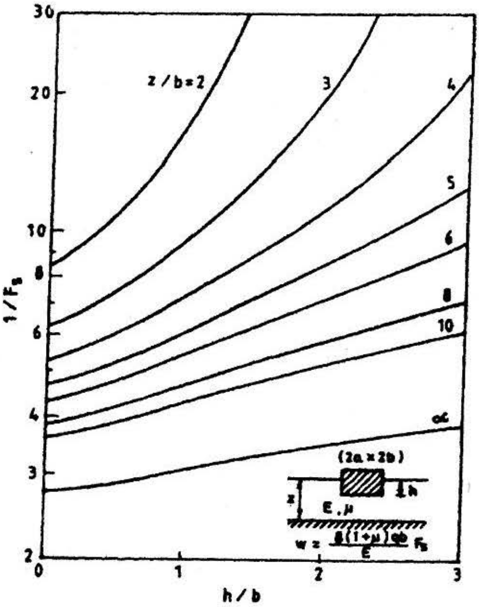

FIGURE 8 : Settlement Factor, $1/F_s$ Versus Non-Dimensional Depth of Embedment, h/b for Rectangular Footing with $a/b = 5$ ($\mu = 0.3$)

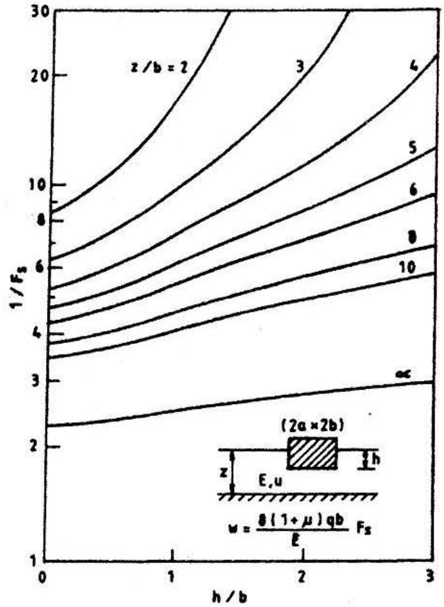

FIGURE 9 : Settlement Factor, $1/F_s$ Versus Non-Dimensional Depth of Embedment, h/b for Rectangular Footing with $a/b = 10$ ($\mu = 0.3$)

z/b from 2 to ∞ for a square surface footing. Hence, settlement of a footing on a half space is more than that on a finite stratum. Maximum settlement can be observed for the footing resting on the surface of the elastic half space (as obtained in Eqns.12 and 13 for square and rectangular footings respectively). Figs.6 to 9 present similar results for a/b ratio of 2, 3, 5 and 10 respectively. Poisson's ratio, μ of the soil is assumed as 0.3 for the entire calculation.

Effects of embedment and stratum thickness on settlement are presented in detail in Tables 1 and 2 respectively. Table 1 presents reduction of settlement of footing due to embedment into elastic half space. For example, settlements of a circular footing embedded into half space at a depth $h/r_0 = 1.0$ and 2 are 0.73 and 0.61 times the settlement of the same footing on the surface. Similarly, settlements of a rectangular footing of $a/b = 2.0$ embedded into the half space at a depth $h/r_0 = 1$ and 2 are 0.82 and 0.69 times the settlement of the same footing on the surface respectively. Settlement reduction factor for different depth of embedments and different footings can be seen in detail from Table 1.

Table 2 presents reduction of settlement due to change in thickness of stratum i.e. the ratio of settlement on finite stratum and half space. For example, settlement of a circular footing on the surface of a stratum of depth $z/r_0 = 4$ is 0.79 times the settlement of the same footing on the surface of a half space. Similarly, settlement of a rectangular footing with $a/b = 2$ on the surface of a stratum of depth $z/r_0 = 4$ is 0.66 times the

TABLE 1 : Settlement Reduction due to Embedment into Half Space ($\mu = 0.3$)

Embedment depth h/r_0 or h/b	Settlement Reduction Factor due to Embedment*					
	Circular Footing	Rectangular Footing with Ratio of a/b				
		1.0	2.0	3.0	5.0	10.0
0.0	1.00	1.00	1.00	1.00	1.00	1.00
0.5	0.86	0.88	0.92	0.93	0.93	0.95
1.0	0.73	0.75	0.82	0.85	0.88	0.90
1.5	0.65	0.68	0.74	0.78	0.82	0.85
2.0	0.61	0.63	0.69	0.73	0.77	0.82
3.0	0.56	0.57	0.62	0.66	0.71	0.76

* Reduction Factor = Settlement of the Footing on the Surface of the Stratum / Settlement of the same footing on the Surface of the Hald Space

TABLE 2 : Settlement Reduction due to Change of Finite Stratum (μ = 0.3)

Stratum depth z/r_0 or z/b	Settlement Reduction Factor for Finite Stratum*					
	Circular Footing	Rectangular Footing with Ratio of a/b				
		1.0	2.0	3.0	5.0	10.0
2.0	0.61	0.57	0.47	0.39	0.33	0.27
4.0	0.79	0.77	0.66	0.61	0.52	0.43
6.0	0.86	0.84	0.76	0.72	0.64	0.53
8.0	0.89	0.88	0.82	0.80	0.71	0.60
10.0	0.92	0.91	0.85	0.82	0.76	0.65
∞	1.00	1.00	1.00	1.00	1.00	1.00

* Reduction Factor = Settlement of the Footing on the Surface of the Stratum / Settlement of the same footing on the Surface of the Hald Space

settlement of the same footing on the half space. Settlement reduction factor for other stratum depths and other footings can be seen in detail from Table 2. On the whole it can be seen from Tables 1 and 2 that the settlement of footing reduces due to increase of embedment depth and decrease of stratum depth.

Effects of embedment into stratum and half space are compared in Table 3 for circular and rectangular footings (a/b = 1 and 5). For example, settlement of a circular footing embedded into the half space at a depth h/r_0 = 2.0 is 0.61 times the settlement of the same foundation on the surface. Whereas the settlement of the same circular footing embedded into a stratum (embedment depth is the same) of depth z/r_0 = 6 is 0.52 times the settlement of the same footing on the surface of the stratum. Reduction of settlement due to embedment into the stratum is more than the reduction due to embedment into the half space. Difference between these two factors becomes more significant with the increase of embedment depth for a constant stratum depth. Reduction in settlement considering the effect of both factors i.e., embedment and stratum depth can be obtained from the Figs.4 to 9 or from Eqns.11 and 15 for circular and rectangular footings respectively. Eqn.15 gives the settlement at the center of the rectangular footing of size $2a \times 2b$. Settlement at the corner of the same footing is ¼ times the settlement obtained by substituting $b = 2b = B$, $h/b = h/B$ and $z/b = z/B$ in Eqn.15.

TABLE 3 : Comparison between Settlement Reduction Factor due to Embedment into Half Space and Embedment into Stratum (μ = 0.3)

Depth of Embedment h/b or h/r_0	Settlement Reduction due to Embedment into					
	Half Space*			Stratum of Depth $z/b = 6^\$$		
	Circular	a/b = 1.0	a/b = 5.0	Circular	a/b = 1.0	a/b = 5.0
0.0	1.00	1.00	1.00	1.00	1.00	1.00
0.5	0.86	0.88	0.93	0.83	0.85	0.88
1.0	0.73	0.75	0.88	0.68	0.69	0.77
1.5	0.65	0.68	0.82	0.58	0.59	0.66
2.0	0.61	0.63	0.77	0.52	0.52	0.57
3.0	0.56	0.57	0.71	0.44	0.42	0.42

* Reduction Factor = Settlement of the Footing Embedded into Half Space / Settlement of the Same Footing on the Surface of the Half Space

$ Reduction Factor = Settlement of the Footing Embedded into Stratum / Settlement of the Same Footing on the Surface of the Stratum

Other Shapes

Results are presented for circular, square and rectangular footings in the preceding sections. Settlement of other shape of footings can be obtained approximately converting the actual footing into an equivalent circle and then using the result of circular footing. Radius of equivalent circle, r_{eqv} can be obtained as $\sqrt{(A/\pi)}$, where A is the actual area of contact of the footing with the soil.

Validity of the equivalent circle approximation is examined comparing the results of rectangular surface footing from actual analysis with the results obtained by the equivalent circle method for the same footing. Table 4 presents the comparisons. It can be seen from Table 4 that the equivalent circle method overestimates the settlement and order of overestimation increases with increasing a/b ratios. Estimated settlements by the equivalent circle method are 1.005 times the settlement of square footing (a/b = 1) by actual analysis. Similarly, settlements of rectangular footings of a/b ratio 2, 3, 4, 5 and 10 by equivalent circle method are respectively 1.096, 1.148, 1.197 and 1.402 times the settlements obtained by actual analysis. Hence, settlement of square or nearly squared (forming square circumscribing actual shape) shape footings can be obtained accurately with the equivalent circle method. Also settlement of rectangular to nearly rectangular (forming rectangle circumscribing actual shape) footing with a/b ratio up to 5 can be

TABLE 4 : Comparison of Results of Actual Analysis with Results by Equivalent Circle Analysis (μ = 0.3)

Methods of Analysis	Values of k^* of Rectangular Footings for a/b of					
	1.0	2.0	3.0	4.0	5.0	10.0
Actual analysis	2.04	2.79	3.24	3.57	3.83	4.63
Equivalent Circle analysis	2.05	2.90	3.55	4.10	4.59	6.49

$$w = k^* \frac{bq}{E}$$

estimated satisfactorily by the equivalent circle method in absence of accurate analysis for such foundation shapes.

Examination on a Number of Settlement Cases

A number of settlement cases from the literature, which were used by Bowles (1987), are used for examining the applicability of the present solution. Thickness of the compressible strata and elastic modulus as suggested by him for each case are also used for the settlement estimation by the present method. Table 5 presents the comparison. There are total 12 cases out of which 7 (serial no. 1, 2, 3, 5, 6, 7 and 9) are embedded footings. Out of seven embedded footings there are slight differences between the estimated values by two methods in three cases. For serial no. (2) and (3) reduction factors for embedment were estimated as 0.87 and 0.75 respectively using Fox's method (which is independent of stratum thickness) by Bowles (1987). However, in preceding section it was shown that it also depends on stratum thickness (Table 3). Reduction factors for those cases are estimated by the present method as 0.75 and 0.65 respectively. If these values are used in Bowles's method, it estimate the settlement as 1.25 and 0.58 for serial no. (2) and (3) respectively, which are very close to the estimated values by the present method. In serial no. (7) stratum thickness is very less (less than B). Neither the Steinbrenner's solution nor the present solution gives good results for stratum thickness less than B. Estimated value was found closed to the observed by both the methods because of adjustment in the elastic modulus. The merit of such adjustment was to know the degree of adjustment required in the modulus value for some typical foundation situations. In other cases (1, 4, 5, 6 and 9) depth of embedments are low and stratum thickness are high. In such cases possibilities of having differences between two methods are very less and it is found so. Serial nos. 4, 8, 10, 11 and 12 are surface footings. Estimated values for the above five cases are found almost same by both the methods. If h/b or h/r_0 substituted as zero in the Mindlin's solution, it becomes identical to the Steinbrenner's solution. As a result both the methods yield same result. On the whole, it

TABLE 5 : Examination of Settlement Cases (Source: Bowles 1987)

References	Stratum depth, z (ft)	Footing width, B (ft)	Side ratio a/b	Embedment depth h/B	Elastic Modulus E (ksf)	μ	q (ksf)	Settlement (in)		
								Measured	Bowles (1987)	Present
1. D'Appolonia, et al. (1968)	4B	12.5	1.6	0.5	1200	0.33	3.4	0.3-0.4	0.33	0.33
Schmertmann (1970)										
2. Case 1	5B	8.5	8.8	0.78	310	0.40	3.74	1.02-1.53	1.45	1.27
3. Case 2	5B	9.8	4.2	1.00	620	0.30	3.34	0.8-0.9	0.67	0.59
4. Case 5	5B	62.0	1.0	0.00	350	0.45	1.56	2.48	2.64	2.65
5. Case 6	B	87.0	2.2	0.10	230	0.30	4.14	10.6	11.7	11.18
6. Case 8	5B	2.0	1.0	0.55	110	0.30	2.28	0.27	0.35	0.32
7. Tchebotarioff (1951)	0.8B	90.0	1.1	0.10	270	0.30	7.20	3.9	3.9	3.5[s]
8. Davisson & Salley (1972)	90	124.0	1.0	0.00	390	0.30	3.14	5.3	6.6	6.65
9. Fisher, et al.(1972)	1700	500.0	1.0	0.20	58200	0.45	7.00	0.50	0.5	0.48
10. Webb and Melvill (1971)	150	177.0	1.0	0.00	1100	0.30	4.50	1.50	1.27	1.27[*]
11. Swiger (1974)	4B	32.0	1.0	0.00	3900	0.30	2.75	0.24	0.24	0.24
12. Kantey (1965)	3.5B	20.0	1.0	0.00	2600	0.30	4.00	3.25	3.25	3.25

[s] Corner settlement

[*] Settlement at the corner converting the circle into an equivalent square as it was done by Bowles (1987)

Note: 1 ft = 0.3048 m, 1 ksf = 50 kPa and 1 in = 25.4 mm can be seen from Table 5 that the present method predicts the settlements for different cases quite satisfactorily.

Conclusions

Closed form solutions for the estimation of elastic settlement of circular, square and rectangular footings embedded into elastic stratum or half space are developed based on Mindlin's solution. Results obtained by the present solution is compared with other elastic solutions for similar conditions (surface footings on half space) and found it to be identical. Effects of stratum thickness (z/r_0 or z/b), shape (circular, squire, rectangular) and size (a/b ratio), and depth of embedment (h/r_0 or h/b) on elastic settlement are investigated using the present solution. Design charts to estimate elastic settlements considering the effects of embedment and stratum thickness together are presented for circular and rectangular footing of a/b ratios 1, 2, 3, 5 and 10. However, use of charts to estimate settlement is not essential and it can also be estimated using equation. Use of the equivalent circle method to estimate elastic settlement of other foundation shape is investigated and found to be satisfactory up to a/b ratio of 5. A number of settlement cases are examined by the present method and found good results. Further, suggestions given by Bowles (1987) for estimating elastic modulus and estimating appropriate stratum depths are found to be encouraging. Advantages of the present solution over the existing methods are

(1) it presents the settlement in the form of simple algebraic equation and in terms of various affecting parameters namely, h/r_0, h/b, z/r_0, z/b, a/b, etc.

(2) settlement of any specific type of foundation (surface footing on half space/stratum, footing embedded into half space/stratum and square/ rectangular footing) can be obtained very easily modifying the general solution suitably.

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Notations

a = Half length of the rectangular footing

B = Width of the rectangular footing = 2a

b = Half width of the rectangular footing

E = Elastic modulus of the soil

F_s = Settlement factor

h = Embedment depth

Q = Total load on the footing

q = Load intensity on the footing

r and z = Axes of polar coordinates

r_0 = Radius of the circular footing

w_h = Settlement of footing on half space

w_s = Settlement of footing on stratum

x, y, z = Axes in rectangular coordinate system

z = Stratum depth

μ = Poisson's ratio of soil