Indian Geotechnical Journal, 31 (3), 2001

Stability of Embankments on Nonhomogeneous Clay of Finite Depth

N. Sabhahit*, M.R. Madhav† and P.K. Basudhar‡

Introduction

Several methods are available for the analysis of stability of embankments constructed on soft clay foundations, such as, the limit equilibrium method (Jewell, 1986; Low, 1989), the limit analysis (Yeo and Woodrow, 1992) and the finite element method (Sakai et al., 1990). However, the limit equilibrium method is commonly preferred for its simplicity and reasonable accuracy in determining the factor of safety. The factor of safety depends on the shape of the assumed trial slip surface, the method of stability analysis adopted, the method for minimization and the assumptions made regarding the variation of the undrained shear strength of the foundation soil with depth.

Milligan and La Rochelle (1984) present a design method for embankments over weak soils considering a translational failure mechanism, when the desiccated zone is relatively thin and recommend that the analysis of fills over weak soils should be based on the least strength available in-situ, or on the large strain ultimate residual strength. Jewell (1986) has modified the slip circle analysis considering a plane Coulomb wedge in the fill to calculate the active lateral thrust and a slip circle in the foundation. Leshchinsky and Smith (1988) also have suggested to decouple the calculation of the lateral fill thrust and the search for the worst slip circle through the

* Reader, Department of Civil Engineering, MIT, Manipal – 576119, India. Email: sabha4@rediffmail.com

† Professor, Department of Civil Engineering, IIT, Kanpur – 208016, India. Email: madhav@iitk.ac.in

‡ Professor, Department of Civil Engineering, IIT, Kanpur – 208016, India. Email: pkbd@iitk.ac.in

soft foundation. Low (1989) has proposed a procedure to calculate the factor of safety for embankments constructed on soft clay, using the conventional slip circle analysis but by estimating a weighted equivalent uniform shear strength, when the undrained shear strength of the soft clay foundation varies with depth. The presently available methods assume the shape of the critical surface as planar, circular or logarithmic spiral. The critical surface is obtained using either grid pattern search or other minimization techniques such as the simplex method.

In this paper, stability of embankments constructed on soft clay foundations is analyzed using the generalized procedure of slices (GPS) (Janbu, 1973). The non-circular critical surface is obtained by the sequential unconstrained minimization technique in conjunction with Powell's method (Bhowmik and Basudhar, 1989). The proposed method considers the presence of desiccated zone and the variation of undrained shear strength of the foundation soil with depth.

The Proposed Method

The proposed method is based on the generalized procedure of slices (GPS) developed by Janbu (1973). The shape of the critical surface is not prescribed but is generated as part of the minimization technique. A number of points are selected on the initial trial slip surface at equal or unequal intervals and the factor of safety is minimized with respect to the position of these points on the slip surface using the sequential unconstrained minimization technique.

Figure 1(a) shows an embankment of height, H, slope angle, β, cohesion, c_e and angle of shearing resistance, ϕ_e, constructed on soft foundation clay layer, of thickness D_H. The trail slip surface is assumed initially to be circular and divided into n number of slices. The critical surface is obtained by varying the elevation of the points on the base of each slice (d_i) and the position of the points of intersection (end points A and B) of the failure surface with the ground surfaces, subject to certain constraints. In Fig.1 (a), d_1, d_2, ... d_i, ... d_n are the decision variables of the minimization technique.

The actual and linearized variation of the undrained shear strength of the soft foundation soil with depth is shown in Figs.1(b) and 1(c) respectively, where, D_m is the thickness of the desiccated zone, c_0 and c_m are the undrained shear strengths of soil at the top and the bottom of the desiccated zone respectively, and c_D – undrained shear strength of soil at depth D_H. The undrained shear strength is maximum at the ground surface, decreases up to a certain depth due to the effect of weathering below which it increases with depth up to the hard stratum. It is evident that the undrained strength value

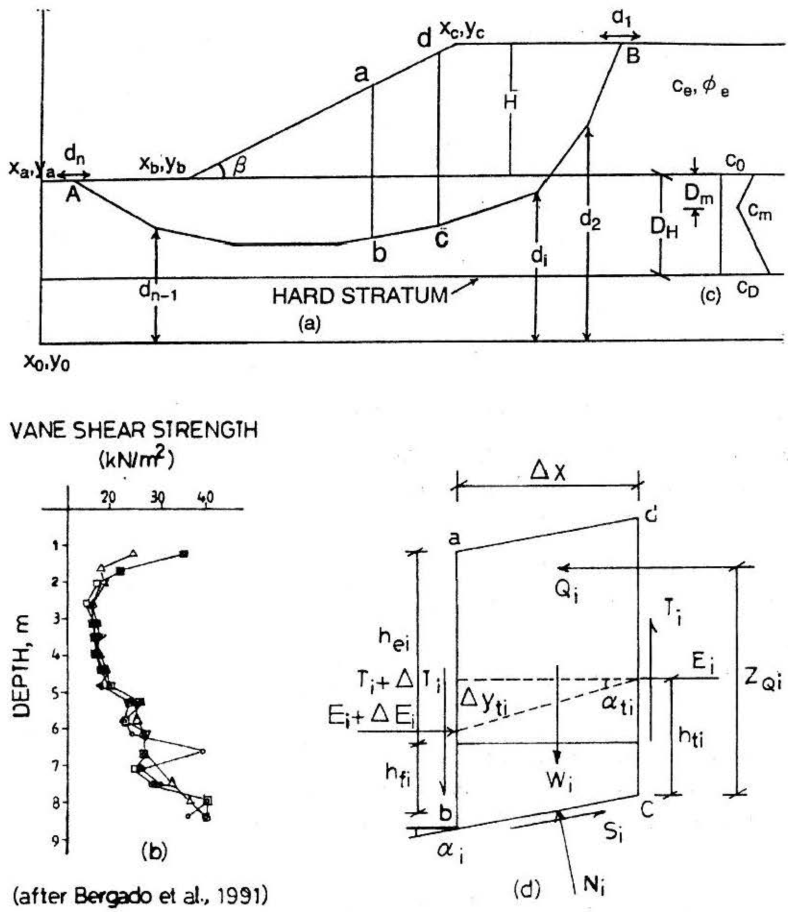

FIGURE 1 : Definition Sketch (a) Embankment on Soft Clay; (b) Variation of Undrained Strength with Depth; (c) Assumed Variation with Depth; (d) Forces on a Slice

along the base of a slice is not constant. The use of an equivalent constant shear strength or neglecting the thickness of the desiccated zone, will certainly affect the location of the critical surface and result in a poor estimation of the stability. Hence, the resistance mobilized at the base of each slice is obtained by integrating the undrained shear strength along its base.

The forces acting on a typical slice are shown in Fig.1(d), where

Δx = is the slice width,
h_i = total height of a slice,
h_{ei} = height of slice in the fill,
h_{fi} = height of slice in the foundation soil,

α_i = angle made by the base of the slice with the horizontal,
W_i = weight of the slice,
N_i and S_i = normal and shear forces along the base of the slice respectively,
h_{ti} and Z_{Qi} = locations of the thrust line and of the seismic force, Q_i, respectively,
E_i and T_i = interslice normal and shear forces, respectively.

Referring to Fig.1(b), the factor of safety, F, can be expressed (Janbu, 1973), as

$$F = \frac{\sum_{i=1}^{n} A_i}{E_a - E_b + \sum_{i=1}^{n} B_i} \tag{1}$$

where
$$A_i = A_i'/\eta_i \tag{2}$$

$$A_i' = \left[c_i + (p_i + t_i - u_i)\tan\alpha_i\right]\Delta x \tag{3}$$

$$\eta_i = \frac{1 + (1/F)\cdot\tan\phi_i\cdot\tan\alpha_i}{1+\tan^2\alpha_i} \tag{4}$$

$$p_i = \gamma_e h_e + \gamma_f h_f \tag{5}$$

c_i, ϕ_i and u_i = average cohesion, angle of shearing resistance of the soil and pore pressure at the base of the slice, respectively,

γ_e and γ_f = unit weight of embankment and foundation soil, respectively.

$$t_i = (T_i - T_{i-1})/\Delta x \tag{6}$$

$$T_i = \frac{dE_i}{dx}\cdot h_{ti} - E_i\cdot\tan\alpha_{ti} - z_{Qi}\cdot\frac{dQ_i}{dx} \tag{7}$$

$$E_i = E_a - \sum_{i=1}^{n}\Delta E_i \tag{8}$$

$$\Delta E_i = B_i - A_i/F \tag{9}$$

$$B_i = Q_i + (p_i + t_i)\Delta x \cdot \tan\alpha_i \quad (10)$$

$$\tan\alpha_{ti} = \text{slope of line of thrust} = (h_{ti} - h_{ti-1})/(2 \cdot \Delta x) \quad (11)$$

E_a and E_b = boundary forces.

To generalize the formulation, the cohesions of the embankment fill and the foundation soils are normalized with respect to $\gamma_e H$, and all the linear dimensions with the height of the embankment, H. The expression for factor of safety in non-dimensional form is given in Appendix I. The factor of safety, F, is minimized with respect to the locations of the points, d_i, along the failure surface, by the sequential unconstrained minimization procedure (Bhowmik and Basudhar, 1989) elaborated in Appendix I.

Results and Discussions

To check the efficacy of the proposed method, the results obtained from the proposed method are compared with some available design methods. Low (1989) presented a simple procedure to calculate the factor of safety of an embankment constructed on soft clay (Fig.2), assuming a circular slip surface and weighted equivalent uniform undrained shear strength of foundation soil. Low's method gave a factor of safety of 1.38 whereas STABR the computer program developed by Duncan and Wong (Low, 1989) gave 1.36 and 1.14 based on ordinary method of slices and Bishop's simplified method respectively. Low investigating the discrepancy between the values given by the ordinary method of slices and by Bishop's simplified method concluded

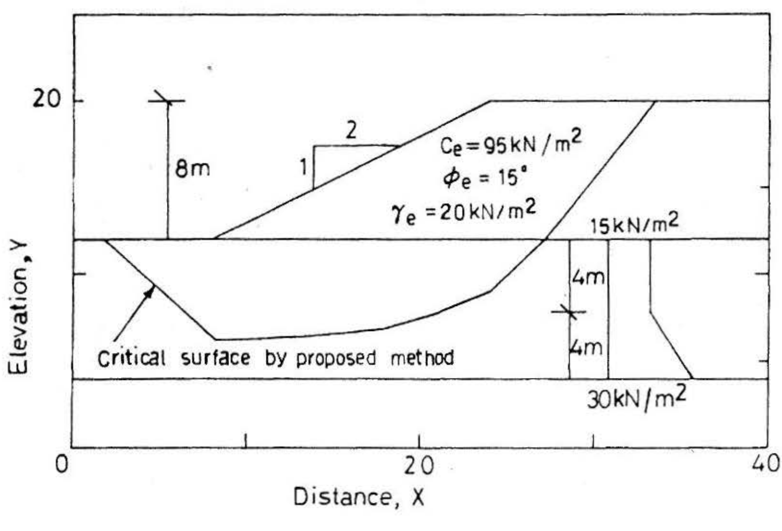

FIGURE 2 : Example Problem

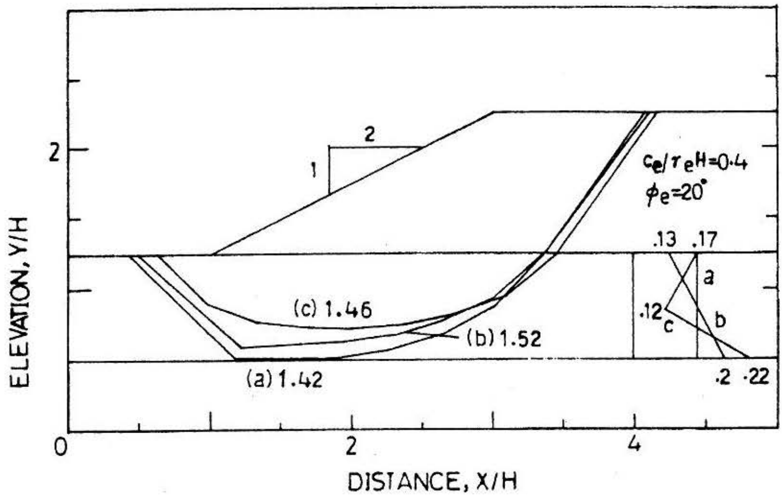

FIGURE 3 : Effect of Variation of Undrained Strength with Depth

that Bishop's simplified method is misleading when the bases of the slices are steeply inclined or when the embankment material has a high cohesion.

The stability of the same embankment is investigated using the proposed method and a factor of safety of 1.44 is obtained which is about 4.2% higher than Low's method. For $\phi_e = 0$, the factor of safety is 1.34 as against 1.31 by Low's method and by STABR (Low 1989). A higher value of the factor of safety is obtained by the present method due to the differences in the assumptions made in Low's and Duncan and Wong's methods and the present one, regarding the slip surface and the equilibrium conditions. The method of slices and Bishop's simplified method consider only the moment equilibrium of the forces on the slices, whereas the proposed method uses a more rigorous method considering both the force and moment equilibrium of the slices leading to a slightly larger value of the factor of safety as shown by Morgenstern and Price (1965). The slip surface shown in Fig.2 is non-circular and is obtained from a more powerful search technique.

The effect of assuming three different types of variations of undrained shear strength of foundation clay with depth, on the factor of safety of an embankment constructed over it, is studied. Figure 3 shows the critical surfaces for an embankment of slope $\cot\beta = 2$, $c_e/\gamma_e H = 0.4$, and $\phi_e = 20°$, constructed over soft clay of normalized thickness, $D_H/H = 0.75$, for constant (case a), linearly increasing (case b) and a bilinear (linearly decreasing and then increasing with depth) (case c), variations of undrained shear strength with depth. In case (a), the normalized undrained shear strength is constant and is 0.17. The critical surface touches the hard stratum and the computed factor of safety is 1.42. In case (b), the normalized undrained shear strength increases linearly with depth from 0.13 to 0.2 but with a weighted average value of 0.17. Since c_u is maximum at the bottom of the clay layer the surface gets shifted upward, giving a higher factor of safety of 1.52. In case (c), where the variation of c_u with depth is similar to the one observed in practice for normally consolidated soil with a desiccated layer on top (Figs. 1b and 1c), but with the same weighted average value of 0.17, the critical surface is higher and passes through the zone of minimum shear strength, and gives a factor of safety of 1.46, which is about 2.7% higher than in case (a). Thus the assumption of constant or uniform shear strength for the foundation soil when in fact it either increases linearly or bilinearly with depth, leads to a lower factor of safety and a deeper location of the critical failure surface than the corresponding values and critical surfaces for the true variations.

A parametric study has been carried out, in order to quantify the effects of thicknesses of the clay layer and the desiccated zone, and the variation with depth of undrained shear strength of the foundation soil, on the factor of safety of an embankment constructed over it. The normalized embankment shear strength, $c_e/\gamma_e H = 0.4$ and $\phi_e = 20°$, are kept constant while studying the effects of the above parameters. The results are presented for slopes with $\cot\beta = 2, 3$ and 4. The influence of these parameters on the critical surfaces are also studied. Finally the proposed method is compared with the available ones.

The thickness of the clay layer (D_H/H) is one of the important parameters (Tayler, 1948) which affects the location of the critical surface and hence the stability of the embankment built over soft clays. The variation of the factor of safety (F) with D_H/H, is shown in Fig.4, for $D_m/H = 0.375$, $c_0/\gamma_e H = 0.15$, $c_m/\gamma_e H = 0.1$ and $c_D/\gamma_e H = 0.2$. For $\cot\beta = 4$, the factor of safety decreases from 1.887 to 1.402 when the thickness of the clay layer increases from 0.5 to 1.0. As the slopes become steeper the rate of decrease in the factor of safety reduces with D_H/H. For example, for $\cot\beta = 2$, the change in the factor of safety is only 0.22, when the normalized thickness of the clay layer increases from 0.5 to 1.0. For flatter slopes, the critical surface is at a greater depth and hence the thickness of the clay layer has more influence on them than on the steeper ones. The slopes of the factor of safety versus the normalized thickness of the clay layer curves, decrease with increasing thickness indicating a diminishing influence of the latter.

Due to natural weathering, the undrained strength of the clay is usually relatively high at the ground surface and decreases with depth to a minimum at the base of the crust (Fig.1b). Below this zone, the undrained strength of the soil increases linearly in case of normally consolidated soils. When an

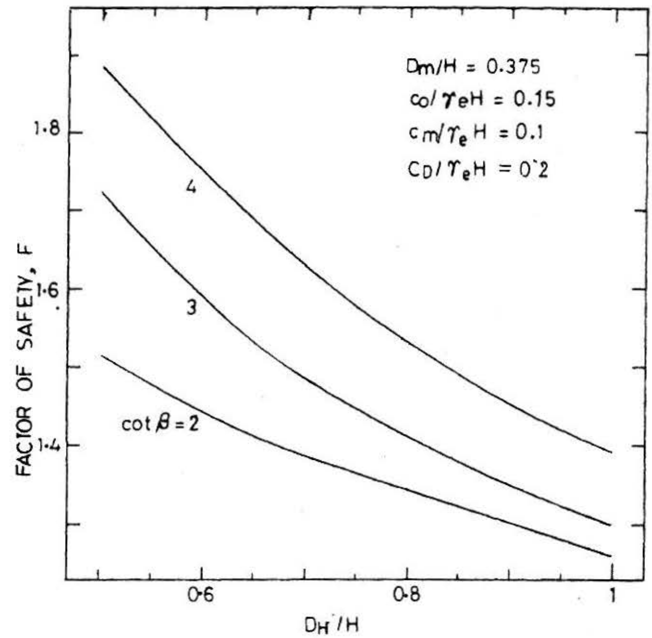

FIGURE 4 : Effect of Thickness of Soft Clay

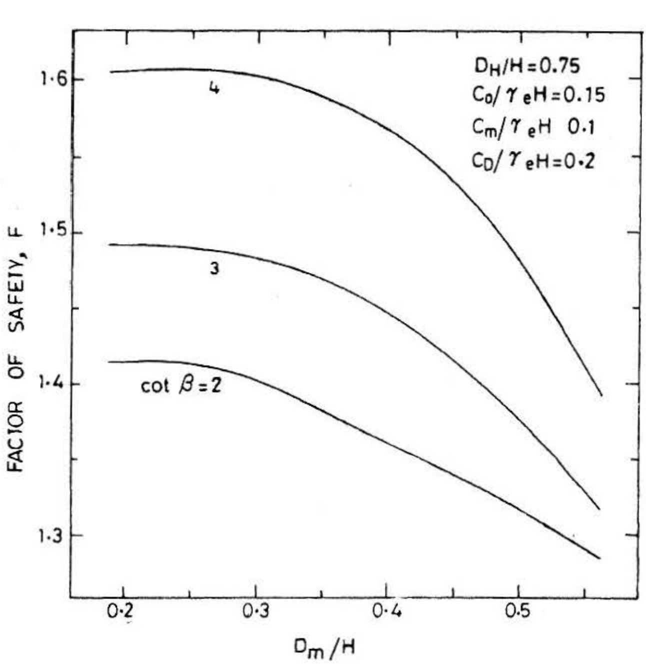

FIGURE 5 : Effect of Thickness of Desiccated Zone

embankment is constructed on such a clay deposit, one has to consider the thickness of the desiccated zone while determining its stability. Fig. 5 shows the variation of the factor of safety for the embankments with the normalized thickness of the desiccated zone D_m/H, for $D_H/H = 0.75$, $c_0/\gamma_e H = 0.15$, $c_m/\gamma_e H = 0.1$ and $c_D/\gamma_e H = 0.2$. For $\cot\beta = 4$, the factor of safety is nearly constant when the thickness of the desiccated zone is small ($D_m/H < 0.3$). It reduces significantly with D_m/H for higher values of D_m/H. Flatter slopes usually have deep-seated failure surfaces mobilizing the strength available there. If the thickness of the desiccated zone is small, the strength mobilized is beneath it and the factor of safety is nearly independent of D_m. In case D_m is larger, the contribution of the minimum soil strength from the desiccated zone is more and the factor of safety is smaller. The range of D_m/H for which the factor of safety is constant with D_m, decreases with increasing slope angles of the embankment. Since for steeper embankment slopes, the critical failure surface is shallow, the factor of safety is affected by the strength of the soil especially the minimum value in the desiccated zone, and the corresponding curves exhibit flatter slopes.

The undrained strength of the soil at the top of the desiccated zone depends on the amount of weathering to which the soil was subjected to in the past. Fig.6 shows the variation of the factor of safety with $c_0/\gamma_e H$ for

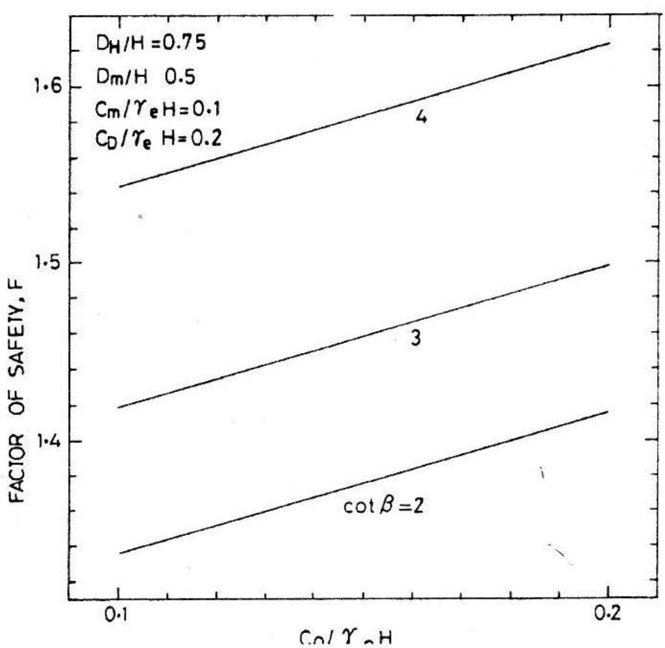

FIGURE 6 : Effect of Cohesion at Ground Surface

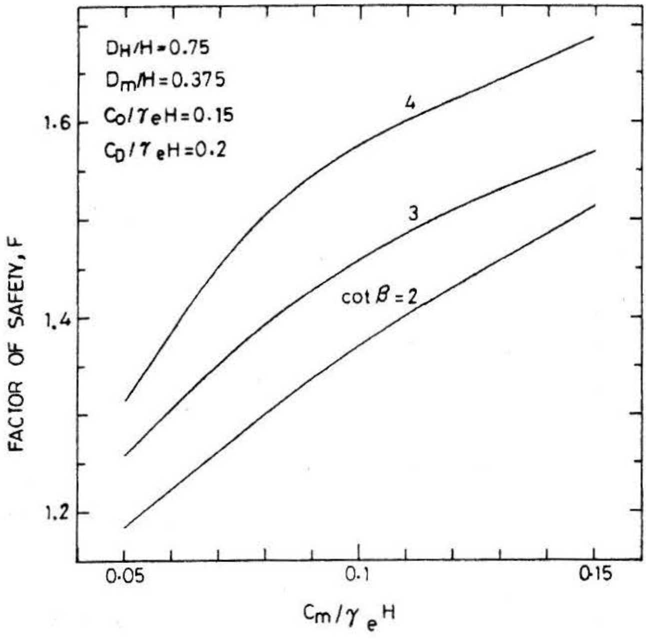

FIGURE 7 : Effect of Minimum Value of Cohesion

$D_H/H = 0.75$, $D_m/H = 0.375$, $c_m/\gamma_e H = 0.1$ and $c_D/\gamma_e H = 0.2$. The factor of safety increases linearly with $c_0/\gamma_e H$ for a given slope angle. The rate of increase in the factor of safety with $c_0/\gamma_e H$ appears to be independent of the slope angle. Since the critical surface passes through the foundation soil for the slopes considered, increase in c_0 value increases the stability of the slopes by a corresponding amount. The equation of the factor of safety versus $c_0/\gamma_e H$ lines can be expressed as:

$$F = 0.8 c_o/\gamma_e H + F_0 \tag{12}$$

where F_0 is the intercept on the factor of safety axis.

As stated earlier, the undrained shear strength in the desiccated zone decreases with depth and reaches a minimum value. This minimum undrained strength (c_m) affects the stability of the embankment considerably. The results presented in Fig.7 show the variation of factor of safety with $c_m/\gamma_e H$ for $D_H/H = 0.75$, $D_m/H = 0.375$, $c_0/\gamma_e H = 0.1$ and $c_D/\gamma_e H = 0.15$. For $\cot\beta = 4$, the factor of safety increases from 1.31 to 1.68 when the normalized value of c_m increases from 0.05 to 0.15. The factor of safety increases linearly for smaller values of $c_m/\gamma_e H$. The rate of increase in the factor of safety decreases with increasing values of $c_m/\gamma_e H$. For steeper

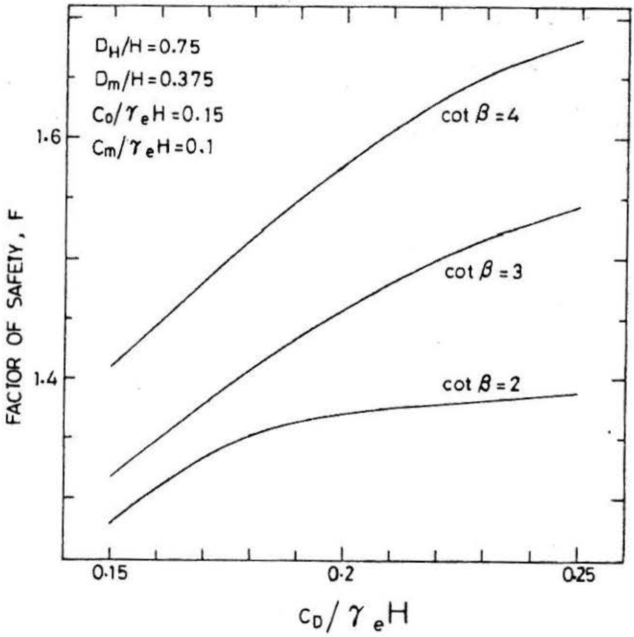

FIGURE 8 : Effect of Cohesion of Soft Clay at Depth, D_h

slopes, the range of linearity is more as compared to that of the flatter ones. For flatter slopes, the strength is mobilized along a critical surface, which is deeper, and the factor of safety is significantly affected by the minimum value of c_m of the soft soil. The rate of increase of factor of safety with $c_m/\gamma_e H$ decreases with increasing values $c_m/\gamma_e H$ for flatter slopes ($\cot\beta = 4$), while it is almost constant for steeper slopes ($\cot\beta = 2$) of the embankment. In the latter case, the critical surface being shallower, is probably unaffected by the magnitude of $c_m/\gamma_e H$, while the strength mobilized and consequently, the factor of safety increase proportionately with $c_m/\gamma_e H$.

The variation of factor of safety with the strength of foundation soil at its base, $c_D/\gamma_e H$, is shown in Fig. 8, for $D_H/H = 0.75$, $D_m/H = 0.375$, $c_0/\gamma_e H = 0.15$ and $c_m/\gamma_e H = 0.1$. For $\cot\beta = 4$, the factor of safety increases from 1.41 to 1.68 when the value of $c_D/\gamma_e H$ increases from 0.15 to 0.25. The factor of safety increases linearly with $c_D/\gamma_e H$ up to a value of $c_D/\gamma_e H = 0.23$, and less steeply for $c_D/\gamma_e H > 0.23$. The range of linearity in the factor of safety versus $c_D/\gamma_e H$ curves decreases as the slopes become steeper. For $\cot\beta = 2$, the variation of the factor of safety with $c_D/\gamma_e H$ is linear up to a value of $c_D/\gamma_e H = 0.17$ and nearly constant for higher values of $c_D/\gamma_e H$. As the value of c_D increases, the critical surface gets shifted upward and increase in the value of c_D will contribute to a

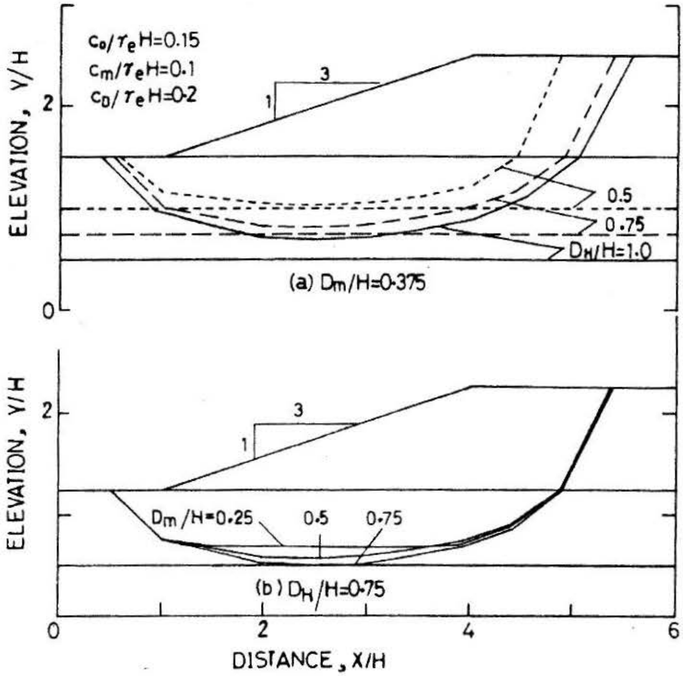

FIGURE 9 : Critical Failure Surfaces Obtained with Different Thicknesses of (a) Clay Layer; (b) Desiccated Zone

corresponding increase in the stability. This observation is particularly true for higher slope angles for which the critical surfaces are at shallow depths.

Critical Failure Surface

The location and the shape of the critical surface depends on the geometry of the slope, and the variation and magnitude of the shear strength parameters of the foundation soil. The influence of the thicknesses of the clay layer and the desiccated zone, and the undrained shear strength of foundation soil, on the location and the shape of the critical surfaces are studied for $\phi_e = 20°$, $c_e/\gamma_e H = 0.4$ and $\cot\beta = 3$.

The influence of the thickness of the clay layer on the location of the critical surface is shown in Fig.9(a), for the values of D_H/H, of 0.5, 0.75 and 1.0. For a thin clay layer, $D_H/H = 0.5$, the critical surface is located very close to the hard stratum. The mechanism of failure of the embankment appears to be a translational one. The shape of the critical surface in the embankment is analogous to the Coulomb's active failure wedge. As the thickness of the clay layer increases, the critical surface moves deeper but is much above the bottom of the soft clay layer and a rotational type of failure

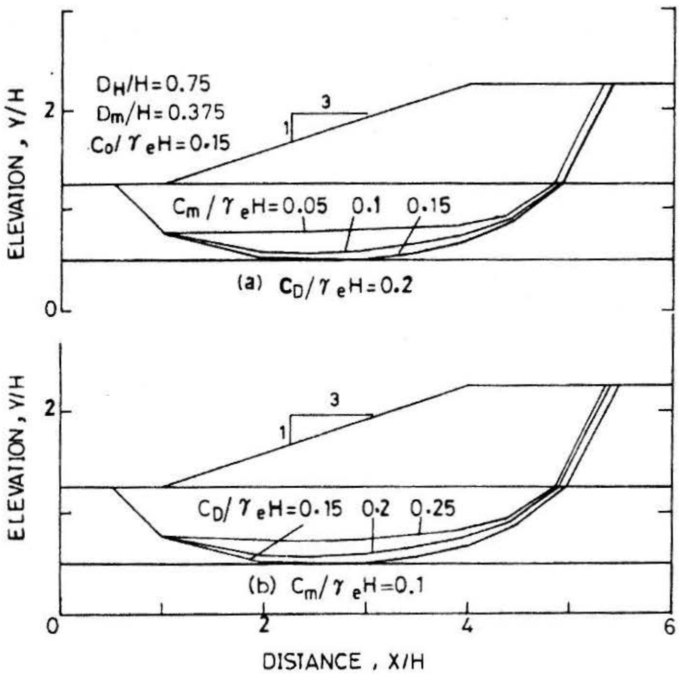

FIGURE 10 : Critical Surfaces for Different (a) c_m Value; (b) c_d Value

mechanism can be noted. As the critical surface extends to a greater depth, a relatively deep-seated failure surface with a smaller factor of safety as reported in Fig.4, is possible. The thickness of the desiccated zone has very little effect on the location of the critical surface in the embankment and near the toe (Fig.9b). However, in the foundation, the critical surface moves deeper as the thickness of the desiccated zone increases, since, with an increase in the thickness of the desiccated zone, the minimum value of c_u would be at greater depth forcing the critical surface to move downward.

The effect of c_m on the location of the critical surface is depicted in Fig.10(a), for the values of $c_m/\gamma_e H$ = 0.05, 0.1 and 0.15. For low values of minimum undrained strength ($c_m/\gamma_e H$ = 0.05), the critical surface is nearly parallel to the hard stratum and passes through the plane of minimum shear strength. As the value of $c_m/\gamma_e H$ increases the foundation soil tends towards having uniform shear strength and the critical surface moves down and touches the hard stratum. Similar kind of observation can be made on the location of the critical surface when the value of $c_D/\gamma_e H$ varies from 0.15 to 0.25 (Fig.10b). When the value of $c_D/\gamma_e H$ is small, the critical surface touches the hard stratum and is more curved. As the value of $c_D/\gamma_e H$ increases the critical surface moves upward and passes through the zone of minimum shear strength. From this study, it is clear that the location and the shape of the critical surface depend on all factors discussed above. The proposed method considers a combination of all these factors while locating the critical surface, and computes the factor of safety.

Conclusions

A method is proposed for the efficient computation of the factor of safety and the location of the critical surface for an embankment constructed on non-homogeneous clay of finite depth. The thicknesses of the clay layer and of the desiccated zone, and the variation of the undrained shear strength of foundation clay with depth, are important for determining the critical failure surface. The mechanism of failure appears to be translational for smaller thickness and rotational for higher thickness of the clay layer. The assumption of equivalent constant shear strength for the soil leads to an under prediction of the factor of safety and deeper critical failure surface than the actual ones for a non-homogeneous clay (linearly increasing c_u or of a desiccated layer over a normally consolidated soil).

References

Bergado, D.T., Shivashankar, R. Sampaco, C.L., Alfaro, M.C. and Anderson, L.R. (1991) : "Behaviour of a Welded Wire Wall with Poor Quality Cohesive-Friction Backfills on Soft Bangkok Clay: A Case Study", *Canadian Geotech. Journal*, 28, pp.860–880.

Bhowmik, S.K. and Basudhar, P.K. (1989) : "Sequential Unconstrained Minimization Technique in Slope Stability Analysis", *Indian Geotechnical Journal*, Vol.19, No.2, pp.124–146.

Fox, R.L. (1971) : *Optimization Methods for Engineering Design*, Addison-Wesley, Reading Mass.

Janbu, N. (1973) : "Slope Stability Computation", *Embankment Dam Engineering*, Casagrande Volume, Ed. R.C. Hirchfield and S.J. Poulos, John Wiley and Sons, New York, 47–86.

Jewell, R.A. (1986) : "Mechanics of Reinforced Embankments on Soft Soils", *Proc. of the Prediction Symp. on a Reinforced Embankment on Soft Ground*, London, Section 2, Article 8:1–63.

Leshchinsky, D. and Smith, D.S. (1988) : "Reinforcement against Deep-Seated Failure: Stability Analysis", *Proc. of the Int. Geotechnical Symp. on Theory and Practice of Earth Reinforcement*, I.S. Kyushu, Fukuoka, 419-424.

Low, B.K. (1989) : "Stability Analysis of Embankments on Soft Ground" *J. of Geotech. Engg.*, Vol.115, No.2, 211-227.

Milligan, V. and La Rochelle, P. (1984) : "Design Methods for Embankments over Weak Soils", *Proc. Symp. on Polymer Grid Reinforcement in Civil Engineering*, London, 1(8)–8(8).

Morgenstern, N.R. and Price, V.E. (1965) : "The Analysis of Stability of General Slip Surfaces", *Geotechnique*, 15, 79–93.

Powell, M.J.D. (1964) : "An Efficient Method for Finding Minimum of a Function of Several Variables without Calculating Derivatives", *Computers*, 7, 155–162.

Sakai, A., Aramaki, G., Miura, N. and Koga, K. (1990) : "Analysis of Test Embankments on Rokkaku River by Finite Element Method", *Proc. of the Int. Seminar on Geotechnical and Water Problems in Lowland*, Saga, 127–132.

Taylor, D.W. (1948) : *Fundamentals of Soil Mechanics*, John Wiley and Sons, New York.

Yeo, K.C. and Woodrow, L.K.R. (1992) : "Design of Embankment on Soft Ground", *Proc. of the ILT Seminar on Problems of Lowland Development*, Saga, 317–322.

Notation

The following symbols are used in this paper:

c_e	=	embankment fill cohesion;
c_u	=	undrained shear strength of foundation clay;
c_0	=	c_u at the top of the crust;
c_m	=	c_u at base of the crust;
c_D	=	c_u at bottom of the clay layer;
D_H	=	thickness of the clay layer;
D_m	=	thickness of the desiccated zone;
$\{D\}$	=	decision vector;
d_i	=	decision variables;
E_i	=	inter-slice normal force;
F	=	factor of safety;
F_0	=	intercept;
H	=	embankment height;
h_i	=	total height of the slice;
h_{ei}	=	height of the slice in the embankment fill;
h_{fi}	=	height of the slice in the foundation;
h_{ti}	=	assumed position of the thrust line;
N_i	=	normal force at the base of the slice;
n	=	number of slices;
p_i	=	normal stress;

Q_i = seismic force;

S_i = shear force at the base of the slice;

T_i = inter-slice shear force;

u_i = pore pressure at the base of the slice;

W_i = weight of slice;

Z_{Qi} = assumed position of the seismic force;

α_i = inclination of the base of slice;

α_{ti} = inclination of the line of thrust;

β = slope angle;

Δx = slice width;

η_i = coefficient;

γ_e = unit weight of embankment fill;

γ_f = unit weight of foundation soil;

ϕ_e = angle of shearing resistance of embankment soil;

Ψ = penalty function.

Appendix I

Factor of Safety in Nondimensional Form and Minimization Procedure

Dividing both the numerator and the denominator of Eqn.1 by $\gamma_e H^2$, the expression for the factor of safety is,

$$F = \frac{\sum_{i=1}^{n} \frac{A_i}{\gamma_e H^2}}{\frac{(E_a - E_b)}{\gamma_e H^2} + \sum_{i=1}^{n} \frac{B_i}{\gamma_e H^2}} \tag{A.1}$$

where

$$\frac{A_i}{\gamma_e H^2} = \frac{A_i'}{(\gamma_e H^2 \cdot \eta_i)} \tag{A.2}$$

$$\frac{A_i'}{\gamma_e H^2} = \left[\frac{c_i}{\gamma_e H} + \left\{\frac{p_i}{\gamma_e H} + \frac{t_i}{\gamma_e H} - \frac{u_i}{\gamma_e H}\right\}\right]\frac{\Delta x}{H} \tag{A.3}$$

$$\frac{p_i}{\gamma_e H} = \frac{h_e}{H} + \frac{\gamma_f}{\gamma_e} \cdot \frac{h_f}{H} \tag{A.4}$$

$$\frac{t_i}{\gamma_e H} = \frac{\Delta T}{\Delta x} \cdot \frac{1}{\gamma_e H} \tag{A.5}$$

Minimization Procedure

The factor of safety (Eqn.1) is minimized with respect to the decision vector, $\{D\}^T$, consisting of n number of decision variables, as

$$\{D\}^T = (d_1, d_2, \ldots, d_i, \ldots, d_n) \tag{A.6}$$

The objective function, F, is expressed in terms of the decision vector $\{D\}^T$ as:

$$F = f(\{D\}) = f(d_1, d_2, \ldots\ldots, d_n) \tag{A.7}$$

Constraints

To obtain an acceptable critical surface, the following design constraints are imposed on the decision variables:

1. The type of failure can be either a toe or a base failure; i.e, a face failure is not considered in the analysis. Hence, the decision variable, d_1, should be greater than x_c, and d_n should be less than x_b (Fig.1a). That is,

$$(x_c - d_i) < 0 \tag{A.8}$$

$$(d_n - x_b) < 0 \tag{A.9}$$

2. The slip surface should be located above the hard stratum and within the soil, i.e.

$$[(y_a - D_H) - d_i] < 0; \qquad i = 2, 3, \ldots\ldots, (n-1) \tag{A.10}$$

$$(d_i - y_i) < 0 \tag{A.11}$$

where y_i are the elevations (y-coordinates) of the points on the ground and along the failure surface, from a datum.

3. The curvature of the slip surface should be concave upward for an acceptable critical surface. This requires:

$$[d_i - (d_{i-1} + d_{i+1})/2] < 0 \tag{A.12}$$

4. The first and last slices should satisfy the active and passive conditions respectively. i.e.,

$$[(45 + \phi/2) - \alpha_1] < 0 \tag{A.13}$$

$$[(45 - \phi/2) - \alpha_n] < 0 \tag{A.14}$$

There are $[3(n-1)+4]$ number of constraints for n number of slices.

Mathematical Programming Problem

The problem is stated as an optimization problem as follows:

Find the decision vector {D} such that,

$$F = \min\left[f(\{D\})\right] \tag{A.15}$$

Subject to

$$g_j(\{D\}) < 0; \qquad j = 1, 2, \ldots\ldots, M \tag{A.16}$$

where $g_j(\{D\})$ and M are the inequality constraint functions and total number of constraints respectively.

There are a number of approaches for solving constrained minimization problems (Fox, 1971). In the present case, the original constrained problem is converted to an unconstrained minimization one by blending the constraints into a penalty function $\Psi(\{D\}, r_k)$ and sequential unconstrained minimization has been carried out using interior penalty function method, in combination with Powell's method (Powell 1964), as

$$\text{Minimize } \Psi(\{D\}, r_k) = f(\{D\}) - \frac{r_k}{\sum_{i=1}^{M} g_j(\{D\})} \tag{A.17}$$

The function $g_j(\{D\})$ is chosen in such a way that it should be less than zero. i.e.,

$$g_j(\{D\}) < 0 \tag{A.18}$$

and r_k is a penalty parameter, whose value is made successively smaller in order to obtain the constrained minimization of $f(\{D\})$. The sequential unconstrained minimization technique (SUMT) is very efficient for the analysis of stability of slopes (Bhowmik and Basudhar, 1989).