# A Graphical Technique for Analysis of Slopes in c- $\phi$ Soils 

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## Introduction

Popularized by Taylor (1937), the friction-circle method is a graphical procedure that can be used for analyzing the stability of homogeneous slopes. In this method, a circular failure surface is assumed and the stability of the entire sliding mass is considered as a whole. The factor of safety against slope instability is estimated graphically but requires trial and error.

In this paper, a simplified version of the friction-circle method is presented. By introducing a few simplifying assumptions, the original frictioncircle method can be modified to provide a simpler and more user-friendly technique for slope stability analysis of finite slopes. This graphical technique can also be extended to analyze infinite slopes, discussed later in this paper. The advantages of this method include: (1) the stability of a slope can be analyzed directly without trial and error. It can be computed without a computer, without slope stability charts (Janbu, 1968) and without the need to memorize any formula for slope stability thus making it suitable for performing stability analysis in the field; and (2) it can be used for analyzing slopes with no seepage and slopes subjected to various seepage conditions.

Both toe circles and slope circles that daylight above the toe can be analyzed using this technique. Base circles are less critical unless the friction angle of the soil is less than 3 degrees (Terzaghi and Peck, 1948). Therefore, this method is most appropriate for analyzing slopes in $\mathrm{c}-\phi$ soils.

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## The Graphical Technique

The main features of the method are (refer to Fig.1a):

1. A force polygon is drawn directly below the sliding mass using the following force vectors: the total weight of the slope, W, the shear force required for stability, $S_{r}$, and the normal force, $N$. The force, $S_{r}$, consists of a cohesion force component and a friction force component. Also plotted within the force polygon is the resultant


FIGURE 1 : Essentials of the Graphical Procedure (a) No Seepage, (b) With Steady State Seepage
force of the intergranular stress, P , which is the vector sum of two forces: the resultant of the available shear force due to friction along the base of the slip circle, $\mathrm{F}_{\mathrm{a}}$, and N , which acts towards the center of rotation.
2. The main assumptions of the method are: (a) to locate the force polygon at a specific location (Point 1 in Figs.1a and 1b) to orient $F_{a}$ parallel to $\mathrm{S}_{\mathrm{r}}$.
3. The factor of safety is defined in terms of sliding and resisting forces (Terzaghi, 1943).
4. The method can be used to analyze slopes with and without seepage. The method for analyzing slopes without seepage and with steady state seepage is illustrated in Figs.1a and 1b, respectively.

## Analysis of Slopes Without Seepage

Steps to perform the graphical analysis for a slope without seepage are as follows (Fig.1a):

1. Draw the slope geometry and the failure arc to scale.
2. Draw a line $O S$ of length $R$ from the center of rotation that bisects the central angle, $2 \theta$, where $R$ equals $r L_{a} / L_{c}$, $r$ is the radius of the slip circle, $L_{a}$ is the length of arc $A B$, and $L_{c}$ is the length of chord $A B$. The ratio $L_{a} / L_{c}$ can be estimated from geometry based on the central angle within the failure arc as follows:

$$
\begin{equation*}
\frac{\mathrm{L}_{\mathrm{a}}}{\mathrm{~L}_{\mathrm{c}}}=\frac{\theta}{\sin \theta} \tag{1}
\end{equation*}
$$

where $\theta$ is in radians.
3. Estimate the weight of the soil mass within the slip circle, W, and its line of action. One quick way of estimating W is to divide the failure mass into a triangle and a segment of circle as shown in Fig.1a. The area of the circle segment, $A_{\text {seg }}$, is equal to:

$$
\begin{equation*}
A_{\text {seg }}=r^{2}(\theta-0.5 \sin 2 \theta) \tag{2}
\end{equation*}
$$

where $\theta$ is in radians. However, it is sufficiently accurate for practical
ranges of the central angle to approximate the area of the circle segment as follows:

$$
\begin{equation*}
\mathrm{A}_{\mathrm{seg}}=0.7 \mathrm{~L}_{\mathrm{c}} \Delta \tag{3}
\end{equation*}
$$

where $\Delta$ is the maximum height of the circle segment. The line of action of W acts through the center of gravity of the slide, which is located between the centroid of the triangle and the centroid of the circle segment with a weighted bias towards the centroid of the heavier of the two soil masses. The centroid of the circle segment is approximately $0.4 \Delta$ from the middle of the chord.
4. Construct an arc with a radius equal to R from the center of rotation till it intersects with the line of action of W at Point 1 .
5. Draw W to scale below Point 1 .
6. Construct the line of action of the resultant normal force, N. N must pass through Point 1 and the center of rotation of the slip circle.
7. Complete the force polygon by drawing a line perpendicular to N from the bottom of the weight vector, W, till it intersects with N . The length of this line represents the magnitude of the shear force required for the slope to be stable, $\mathrm{S}_{\mathrm{r}}$. (A key assumption of this method is in the inclination of $\mathrm{F}_{\mathrm{a}} \cdot \mathrm{F}_{\mathrm{a}}$ is assumed to be parallel to $\mathrm{S}_{\mathrm{r}}$ and tangent to the arc at Point 1 , which is not parallel to the chord. The importance of this assumption is discussed later. In reality, the cohesive force acts parallel to the chord with a moment arm equal to R . Therefore, locating the cohesive force tangent to this arc results in the correct moment about the center of rotation.)
8. Draw vector P through Point 1 at an angle $\phi$ from $N$, where $\phi$ is the friction angle of the soil. The length of the shear force vector, $\mathrm{S}_{\mathrm{r}}$, to the right of P is the resultant of available shear force due to friction along the base of the slip circle, $\mathrm{F}_{\mathrm{a}}$.
9. Estimate the available cohesion, $\mathrm{C}_{\mathrm{a}}$, which is equal to $\mathrm{cL}_{\mathrm{c}}$, where c is the cohesion of the soil.
10. Estimate the factor of safety, FS, with respect to sliding as follows (Terzaghi, 1943):

$$
\begin{equation*}
F S=\frac{C_{a}+F_{a}}{S_{r}} \tag{4}
\end{equation*}
$$



FIGURE 2 : Computations for a Particular Slip Circle (a) Dry Slope, (b) Submerged Slope, (c) Rapid Drawdown, (d) Steady Seepage, $r_{u}=1 / 6$.

## Example

The method is illustrated with the aid of an example of a 9.14 m high slope without seepage (Fig.2a). The soil has a unit weight of $19.6 \mathrm{kN} / \mathrm{m}^{3}$, a cohesion of 12 kPa and a friction angle of 35 degrees. For the slip circle with a radius $r=16.5 \mathrm{~m}$ and a central angle of 47 degrees, $L_{a} / L_{c}$ equals 1.03 and $\mathrm{R}=\mathrm{rL}_{\mathrm{a}} / \mathrm{L}_{\mathrm{c}}=16.9 \mathrm{~m}$. The weight of the soil within the slip circle is $474 \mathrm{kN} / \mathrm{m}$ ( $32.5 \mathrm{kips} / \mathrm{ft}$ ). From the force polygon, $\mathrm{S}_{\mathrm{r}}$ is scaled to be $309 \mathrm{kN} / \mathrm{m}$. The right hand portion of vector $\mathrm{S}_{\mathrm{r}}$ to the right of P is $\mathrm{F}_{\mathrm{a}}=251$ $\mathrm{kN} / \mathrm{m}$. The available force due to cohesion, $\mathrm{C}_{\mathrm{a}}=12 \mathrm{kPa} \times 13.1 \mathrm{~m}=157$ $\mathrm{kN} / \mathrm{m}$. The factor of safety, FS, is then computed as

$$
\frac{(157 \mathrm{kN} / \mathrm{m}+251 \mathrm{kN} / \mathrm{m})}{309 \mathrm{kN} / \mathrm{m}}=1.32
$$

In comparison, the factor of safety estimated using Bishop's (1955) modified method with the aid of the computer program STABGM (Duncan et al., 1985) is 1.30 .

Factors of safety for three other slopes having the same height, same soil properties but different slope steepnesses have been analyzed using the same slip circle. The factors of safety for all four slopes are summarized in Table 1.

## Completely Submerged Slopes

A completely submerged slope does not have any seepage forces. Therefore, it can be solved using the procedure described above for the no seepage case just by changing the unit weight from total to buoyant. An alternative approach is to revise the values of $F_{a}$ and $S_{r}$ to account for

Table 1 : Comparison of Factors of Safety from the Graphical Method and Bishop's Modified Method for Several Seepage Conditions in Four Slopes

| Condition | Slope | Factor of Safety, FS ${ }^{1}$ |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Graphical Procedure <br> (3) | Modified Bishop's Method <br> (4) | Difference (5) |
| No | 1V: 1 H | 1.72 | 1.72 | 0\% |
| Seepage | 1V: $3 / 4 \mathrm{H}$ | 1.32 | 1.30 | +2\% |
|  | IV: $1 / 2 \mathrm{H}$ | 1.24 | 1.20 | +3\% |
|  | 1V: 1/4H | 1.25 | 1.21 | +3\% |
| Submerged <br> Condition | 1V: 1H | 2.61 | 2.63 | -1\% |
|  | 1V: $3 / 4 \mathrm{H}$ | 1.82 | 1.80 | +1\% |
|  | IV : $1 / 2 \mathrm{H}$ | 1.60 | 1.56 | +3\% |
|  | IV: 1/4H | 1.55 | 1.49 | +4\% |
| Rapid <br> Drawdown | 1V: 1H | 1.31 | 1.35 | -3\% |
|  | 1V:3/4H | 0.91 | 0.92 | -1\% |
|  | 1V: $1 / 2 \mathrm{H}$ | 0.80 | 0.75 | +7\% |
|  | 1V: $1 / 4 \mathrm{H}$ | 0.77 | 0.61 | +26\% |
| Steady Seepage <br> with $\mathrm{r}_{\mathrm{u}}=1 / 6$ | IV: 1 H | 1.58 | 1.59 | -1\% |
|  | 1V: $3 / 4 \mathrm{H}$ | 1.18 | 1.17 | +1\% |
|  | 1V: $1 / 2 \mathrm{H}$ | 1.09 | 1.07 | +2\% |
|  | IV : $1 / 4 \mathrm{H}$ | 1.09 | 1.05 | +4\% |

Note 1.: The factors of safety in this table are for the same slip circle for all four slopes.
buoyancy and recomputing FS using Eqn.4. This is achieved by dividing the weight vector into two components: the effective weight of the soil within the sliding mass and the weight of the water within the sliding mass. Submerged slopes can be analyzed by performing the steps described in Table 2 in addition to Steps 1 through 9 for the no seepage case.

Using the graphical technique, the factor of safety for the completely submerged slope in Fig.2b is estimated to be 1.82 . The factor of safety based on Bishop's modified method is 1.80 . The factors of safety for three other completely submerged slopes are summarized in Table 1 for the same slip circle.

Table 2:Additional Steps for Analyzing Completely Submerged Slopes, Slopes Subjected to Rapid Drawdown and Slopes with Steady State Seepage

## Completely Submerged Slopes (Fig.2b)

a) Estimate the pore pressure ratio, $r_{u}$, as follows:

$$
\mathrm{r}_{\mathrm{u}}=\frac{\text { Area of sliding mass below the pheratic surface } \times \gamma_{\mathrm{w}}}{\text { Total area of sliding mass } \times \gamma}
$$

In the general case, $\gamma$ is the moist unit weight $\left(\gamma_{m}\right)$ for the soil above the phreatic surface and saturated unit weight $\left(\gamma_{t}\right)$ for the soil below the phreatic surface. For a completely sumerged slope, $r_{0}=\gamma_{w} / \gamma_{t}$.
b) From the bottom of vector $W$ (Point 2), scale upwards the vertical distance $W r_{u}$. The upper portion of the weight vector, $W\left(1-r_{u}\right)$ represents the effective weight of the soil, while the lower portion, $\mathrm{Wr}_{u}$, represents the unit weight of the water times the area between the phreatic surface and the slip surface.
c) From the top of $\mathrm{Wr}_{\mathrm{u}}$, draw a line perpendicular to N and scale the magnitude of the shear force required for stability, $S_{r}$, equal to the length of this line.
d) Scale the portion of the line drawn in Step $c$ to the right of its intersection with $P$. This is the resultant of the available shear force due to friction, $F_{a}$.
e) Compute FS using Eqn. 4 .

## Slopes Subjected to Rapid Drawdown (Fig.2c)

a) Estimate the pore pressure ratio, $r_{u}$.
b) From the bottom of vector $W$ (Point 2), scale upwards the vertical distance $\mathrm{Wr}_{\mathrm{u}}$.
c) From the top of $\mathrm{Wr}_{\mathrm{u}}$, draw a line perpendicular to N and scale the portion of this line to the right of its intersection with P . This is the resultant of the available shear force due to friction, $\mathrm{F}_{\mathrm{a}}$.
d) Compute FS using Equation 4, where $S_{r}$ is obtained from Step 7 for the no seepage case.

## Slopes with Steady State Seepage (Fig.1b)

Steps are identical to those for slopes subjected to rapid drawdown.

## Analysis of Slopes with Seepage

The graphical technique can be extended to include slopes with seepage including rapid drawdown and steady state seepage (Figs.2c and 2d, respectively).

## Rapid Drawdown

The worst-case rapid drawdown scenario occurs when the standing pool is drained very quickly leaving the soil no time to drain. This case is equivalent to the submerged slope without the standing pool (Fig.2c). It is analyzed simply by performing the steps described in Table 2 in addition to Steps 1 through 9 for the no seepage case.

Using the graphical technique, the factor of safety for the slope subjected to rapid drawdown in Fig.2c is estimated to be 0.91 . The factor of safety obtained from Bishop's modified method is 0.92 . These values of factors of safety of less than unity indicate that the slope is not stable in the event of rapid drawdown. Summarized in Table 1 are the factors of safety for four different slopes analyzed for rapid drawdown using the same slip circle.

## Steady State Seepage

For the case where there is no standing pool, the factor of safety for a slope with steady state seepage can be approximated very quickly for any prescribed phreatic surface using the pore pressure ratio. A slope experiencing steady state seepage (Fig.1b) is analyzed by performing the steps described in Table 2 in addition to Steps 1 through 9 for the no seepage case.

Using the graphical technique, the factor of safety for the slope experiencing steady state seepage with $r_{u}=1 / 6$ (Fig. 2d) is estimated to be 1.18. This is in good agreement with the factor of safety of 1.17 obtained from Bishop's modified method. Factors of safety for four slopes experiencing steady state seepage with $r_{u}=1 / 6$ are summarized in Table 1 for both the graphical technique and Bishop's modified method. Again, the same slip circle is analyzed for all four slopes.

## Backcalculation of Geomechanical Parameters

The method can also be used to backcalculate geomechanical parameters such as $\mathrm{c}, \phi, \gamma$ or $\mathrm{r}_{\mathrm{u}}$ for failed slopes by assuming a factor of safety of unity. Backcalculation of $r_{u}$ is described below but the same principles apply when backcalculating c, $\phi$ or $\gamma$. When the moist and total unit weights are similar, the maximum pore pressure ratio, $\mathrm{r}_{\mathrm{u}}$, that corresponds to a factor of safety of
unity can be determined by graphical construction very rapidly for the slope shown in Fig.2a as follows:
a. After performing Steps 1 through 9 for the no seepage case, scale along vector $\mathrm{S}_{\mathrm{r}}$ from the bottom of W , a distance equal to $\mathrm{C}_{\mathrm{a}}$.
b. From the point established in Step a, draw the neutral force vector, $U$, parallel to N till it intersects with P .
c. From the intersection point from Step b, draw a line perpendicular to N till it intersects with W .
d. Scale the distance from the intersection point in Step c to the bottom of W . This is the value of $\mathrm{Wr}_{\mathrm{u}}$ that corresponds to a factor of safety of 1.0 .
e. Divide $\mathrm{Wr}_{\mathrm{u}}$ by W to obtain the value of $\mathrm{r}_{\mathrm{u}}$ when the factor of safety is 1.0 .

For the slope shown in Fig.2a, graphical construction yields a maximum allowable pore pressure ratio of 0.39 , i.e., the slope will be barely stable when $39 \%$ of the area of the sliding mass lies below the phreatic surface. The same result is obtained for this slope using Bishop's modified method. The results from such an analysis will allow the engineer to design an effective drainage system in the field for on-the-spot remediation of imminent slides.

## Infinite Slopes

The graphical procedure can also be extended to infinite slopes without seepage or with seepage flowing parallel to and down the slope. Only the case with seepage is described below. The no-seepage case can be analyzed by setting $r_{u}$ equal 0 . Consider a $c-\phi$ soil layer with uniform vertical thickness, D , on an infinite slope inclined at an angle $\beta$ with respect to the horizontal (Fig.3). If the depth to the top of the flow is $D_{w}$, the following expression for the factor of safety of the infinite slope can be derived:

$$
\begin{equation*}
\mathrm{FS}=\frac{\mathrm{c}+\left[\gamma_{\mathrm{m}} \mathrm{D}_{\mathrm{w}}+\left(\gamma_{\mathrm{t}}-\gamma_{\mathrm{w}}\right)\left(\mathrm{D}-\mathrm{D}_{\mathrm{w}}\right)\right] \cos ^{2} \beta \tan \phi}{\left[\gamma_{\mathrm{m}} \mathrm{D}_{\mathrm{w}}+\gamma_{\mathrm{t}}\left(\mathrm{D}-\mathrm{D}_{\mathrm{w}}\right)\right] \cos \beta \tan \phi} \tag{5}
\end{equation*}
$$

It is not easy for an engineer out in the field to remember Eqn.5. However, it is simple to estimate the factor of safety graphically for infinite slopes with seepage flowing parallel to and down the slope by considering a column of


FIGURE 3 : Application of the Graphical Method to Infinite Slopes
soil of unit width horizontally and of unit length in the third dimension as follows:

1. Calculate the weight of the soil per unit width, W , and draw W to scale as a vertical line.
2. Draw the normal force vector, N , from the top of W perpendicular to the infinite slope.
3. Draw a line from the bottom of W parallel to the infinite slope till it intersects with N . The length of this line between W and N is the shear force required for stability, $S_{r}$.
4. From the intersection of W and N , draw a line at an angle $\phi$ from N . This is the line of action of the resultant of the intergranular force, P .
5. Compute $r_{u}$ and calculate $\mathrm{Wr}_{\mathrm{u}}$. From the bottom of W , scale vertically upwards a distance of $\mathrm{Wr}_{\mathrm{u}}$.
6. From the point established in Step 5, draw a line parallel to the infinite slope. Scale the distance between vectors N and P on this line. This is $\mathrm{F}_{\mathrm{a}}$.
7. Calculate the available force due to cohesion, $\mathrm{C}_{\mathrm{a}}$, equal to cL where L is the length parallel to the slope of the unit width column of soil equal to $1 / \cos \beta$.
8. Calculate FS using Eqn.4.

It can be easily shown that factors of safety for infinite slopes estimated using Eqn. 5 and the graphical method are identical.

## Limitations of the Method

For the engineer in the field with no access to a computer or stability charts, the graphical method presented herein provides a very useful tool for performing simple stability computations of known slides or imminent slides. As a result of its simplicity, there are several limitations on the use of the method. They include the following:

1. It is applicable only to homogeneous slopes.
2. Failure surfaces are restricted to toe circles and slope circles.
3. The steady seepage condition is analyzed using the pore pressure ratio concept, which provides a simplistic methodology to account for seepage effects, yielding results that are quite reasonable as a first approximation. However, if a more accurate representation of the effects of seepage forces for a given phreatic surface is required, then a more rigorous slope stability analysis should be performed especially if the phreatic surface is known or can be predicted with a high degree of confidence.
4. The procedure suggested for rapid drawdown is an effective stress approach that assumes a conservative set of pore pressures. It provides an approximation of the factor of safety using a single stage analysis in the field. More accurate two-stage-analyses have been developed by the Corp of Engineers (1970), Lowe and Karafiath (1960), and Wright and Duncan (1987) for slopes subjected to rapid drawdown but these procedures are less amenable to hand computations.
5. For finite slopes, the angle of inclination of the friction force, $\mathrm{F}_{\mathrm{a}}$, with respect to the horizontal is assumed in the graphical technique and may not represent the actual inclination. However, values of factor of safety calculated using the graphical technique are generally in good agreement with Bishop's modified method for all four slopes. Based on the analyses performed, values of factor of safety are typically within $4 \%$ for dry and submerged slopes and within about $7 \%$ for rapid drawdown and steady seepage. The one exception is for the very steep $1 \mathrm{~V}: 1 / 4 \mathrm{H}$ slope during rapid drawdown. The discrepancy in the factor of safety for this nearvertical slope was always the largest for all seepage conditions. This limitation, however, does not apply to infinite slopes. In the infinite slope analysis, the line of action of $F_{a}$ is assumed to be parallel to the slope, which is consistent with the true line of action of $\mathrm{F}_{\mathrm{a}}$.

## Discussion of the Results

For finite slopes, a key simplifying assumption was introduced to eliminate the trial and error procedure required in the original friction-circle method. This assumption requires that $\mathrm{F}_{\mathrm{a}}$, acts parallel to the prescribed direction of $\mathrm{S}_{\mathrm{r}}$. In the completely submerged, rapid drawdown and no seepage cases, the true direction of $\mathrm{F}_{\mathrm{a}}$ with respect to the horizontal for the $1 \mathrm{~V}: 1 \mathrm{H}$, $1 \mathrm{~V}: 3 / 4 \mathrm{H}, 1 \mathrm{~V}: 1 / 2 \mathrm{H}$ and $1 \mathrm{~V}: 1 / 4 \mathrm{H}$ slopes were estimated to be $43,43,41$ and 39 degrees, compared to the inclinations assumed in the graphical procedure of $41,41,39$ and 36 degrees, respectively. The corresponding errors in the inclinations are about $4.7 \%, 4.7 \%, 4.9 \%$ and $7.7 \%$, respectively. The percent error in the inclination of $\mathrm{E}_{R}$ is largest with the $1 \mathrm{~V}: 1 / 4 \mathrm{H}$ slope, consistent with the magnitude of the discrepancies in the factor of safety. It should be noted that the slip circle used to analyze these four slopes was the same arbitrary slip circle and does not represent the critical slip circle for any of the slopes.

Factors of safety computed using the graphical technique for critical slip circles of several slopes with no seepage are summarized in Table 3 (Taylor, 1937). Taylor defined the critical slip circle in terms of the inclination of the chord with respect to the horizontal, $\eta$, and half the central angle, $\theta$. These angles vary depending on the slope inclination and the soil parameters. The soil parameters, including unit weight, friction angle and cohesion, are defined in terms of a stability number, $\mathrm{c} /(\mathrm{FS} \cdot \gamma \cdot \mathrm{H})$, where H is the height of the slope. By setting H equal to 9.14 m and $\gamma$ equal to $19.6 \mathrm{kN} / \mathrm{m}^{3}$, the values of cohesion for each slope is then calculated for the case when the slopes are barely stable, i.e., when the factor of safety is 1.0 . Values of factor of safety for these slopes computed using the graphical method and their deviation from unity are summarized in Table 3. It can be seen that the agreement is excellent with all values within $4 \%$ of unity.

Also shown in Table 3 are the angles of inclination of the resultant friction force with respect to the horizontal assumed in the graphical analysis. In one case, the assumed inclination of the resultant friction force is as low as $52 \%$ of the true value. Yet, the facior of safety computed using the graphical method is within $3 \%$ of unity. Therefore, the error in the factor of safety is relatively insignificant when the method is used to analyze critical circles.

## Summary and Conclusions

The proposed method provides a simple graphical tool that can be used for analyzing homogeneous finite and infinite slopes in $\mathrm{c}-\phi$ soils. By plotting the force polygon directly below the slip surface, the influence of cohesion, friction angle and pore water on the factor of safety can be readily visualized.

Table 3 : Analyses of Critical Slip Surfaces for Slopes with $H=9.14 \mathrm{~m}, \gamma=19.6 \mathrm{kN} / \mathrm{m}^{\mathbf{3}}$ and $\mathrm{FS}=1.0$

| Slope Angle ${ }^{1}$ <br> $\beta$ (deg) <br> (1) | Friction Angle ${ }^{1}$ <br> $\phi$ (deg) <br> (2) | Chord Inclination ${ }^{1}$ <br> $\eta$ (deg) <br> (3) | 0.5 X <br> Central <br> Angle ${ }^{1}$ <br> $\theta$ (deg) <br> (4) | Stability Number ${ }^{1}$ $\mathrm{c} /(\mathrm{FS} \cdot \gamma \cdot \mathrm{H})$ <br> (5) | Cohesion when $\mathrm{FS}=1.0$ c ( kPa ) | FS from Simplified Friction-Circle Method | True Inclination of $\mathrm{F}_{\mathrm{a}}$ w.r.t the Horizontal <br> $\psi$ (deg) <br> (8) | Inclination of $\mathrm{F}_{\mathrm{a}}$ w.r.t. the Horizontal in Analyses <br> $\lambda$ (deg) <br> (9) | $\lambda / \psi$ <br> (\%) <br> (10) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 90 | 5 | 50.0 | 14.0 | 0.239 | 42.9 | 1.00 | 45.5 | 43.9 | 96.5 |
|  | 10 | 53.0 | 13.5 | 0.218 | 39.1 | 1.01 | 48.5 | 47.0 | 97.0 |
|  | 15 | 56.0 | 13.0 | 0.199 | 35.7 | 1.02 | 51.5 | 50.0 | 97.2 |
|  | 20 | 58.0 | 12.0 | 0.182 | 32.7 | 1.02 | 53.7 | 52.5 | 97.7 |
|  | 25 | 60.0 | 11.0 | 0.166 | 29.8 | 1.03 | 56.0 | 54.9 | 98.0 |
| 75 | 5 | 45.0 | 25.0 | 0.195 | 35.0 | 1.00 | 40.9 | 36.6 | 89.6 |
|  | 10 | 47.5 | 23.5 | 0.173 | 31.1 | 1.01 | 43.4 | 39.7 | 91.6 |
|  | 15 | 50.0 | 23.0 | 0.152 | 27.3 | 1.02 | 45.8 | 42.3 | 92.4 |
|  | 20 | 53.0 | 22.0 | 0.134 | 24.1 | 1.03 | 48.8 | 45.6 | 93.4 |
|  | 25 | 56.0 | 22.0 | 0.117 | 21.0 | 1.04 | 51.7 | 48.4 | 93.6 |

Table 3 : Continued

| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 60 | 5 | 38.5 | 34.5 | 0.162 | 29.1 | 1.00 | 37.8 | 29.7 | 78.6 |
|  | 10 | 41.0 | 33.0 | 0.138 | 24.8 | 1.02 | 39.8 | 32.6 | 81.9 |
|  | 15 | 44.0 | 31.5 | 0.116 | 20.8 | 1.02 | 42.5 | 36.1 | 84.9 |
|  | 20 | 46.5 | 30.2 | 0.097 | 17.4 | 1.03 | 44.9 | 39.0 | 86.8 |
|  | 25 | 50.0 | 30.0 | 0.079 | 14.2 | 1.04 | 48.3 | 42.3 | 87.7 |
| 45 | 5 | 31.2 | 42.1 | 0.136 | 24.4 | 1.01 | 36.9 | 23.3 | 63.1 |
|  | 10 | 34.0 | 39.7 | 0.108 | 19.4 | 1.02 | 38.2 | 26.6 | 69.6 |
|  | 15 | 36.1 | 37.2 | 0.083 | 14.9 | 1.02 | 39.5 | 29.4 | 74.5 |
|  | 20 | 38.0 | 34.5 | 0.062 | 11.1 | 1.03 | 40.7 | 32.0 | 78.5 |
|  | 25 | 40.0 | 31.0 | 0.044 | 7.9 | 1.03 | 42.1 | 35.0 | 83.1 |
| 30 | 5 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
|  | 10 | 25.0 | 44.0 | 0.075 | 13.5 | 1.03 | 37.6 | 19.5 | 51.8 |
|  | 15 | 27.0 | 39.0 | 0.046 | 8.3 | 1.03 | 36.3 | 22.3 | 61.5 |
|  | 20 | 28.0 | 31.0 | 0.025 | 4.5 | 1.03 | 33.8 | 25.0 | 74.0 |
|  | 25 | 29.0 | 25.0 | 0.009 | 1.6 | 1.03 | 32.4 | 26.9 | 83.0 |

Notes 1: The first five columns of this table are per Taylor (1937)
2 : The critical failure surface is a base circle.

This method lends itself well to back-of-the-envelope type calculations useful for backcalculating geomechanical parameters such as c, $\phi, \gamma$, and $r_{u}$ of known slides or imminent slides while the engineer is in the field without the aid of a computer or slope stability charts. It can also be used for checking the results of computer analyses.

The method can be used to analyze both finite and infinite slopes. It is especially useful for evaluating the stability of finite slopes for various stages of seepage flow from submergence to rapid drawdown to steady seepage to no seepage. With finite slopes, the method provides reliable values of factor of safety especially when used to analyze critical slip circles. For infinite slopes, values of factor of safety from the graphical technique coincide with theoretical values.

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## Notation

| $\mathrm{A}_{\text {seg }}$ | $=$ area of circular segment in sliding soil mass |
| ---: | :--- |
| $\mathrm{C}_{\mathrm{a}}$ | $=$ available resultant force due to cohesion |
| c | $=$ cohesion |
| D | $=$ vertical thickness of soil layer in an infinite slope |
| $\mathrm{D}_{\mathrm{w}}$ | $=$vertical depth to the top of the phreatic surface in <br> an infinite slope |
| $\mathrm{F}_{\mathrm{a}}$ | $=$resultant of the available shear force due to friction <br> along the failure surface |
| FS | $=$ factor of safety |
| H | $=$ height of finite slope |
| L | $=$ length parallel to infinite slope of column of soil of |
| $\mathrm{L}_{\mathrm{a}}$ | $=$ unit width = length of slip circle along its arc |
| $\mathrm{L}_{\mathrm{c}}$ | $=$ length of chord between ends of failure arc |
| N | $=$ resultant normal force on slip circle |
| P | $=$ resultant force due to the intergranular stress along |
| R | $=$ the slip circle |
| r | $=$ radius of slip circle |
| $\mathrm{r}_{\mathrm{u}}$ | $=$ pore pressure ratio |
| $\mathrm{S}_{\mathrm{r}}$ | $=$ resultant shear force due to friction and cohesion |
| U | $=$ along slip circle required for stability |
| W | $=$ resultant neutral force |
| x | $=$ total weight of the sliding soil mass |
| Z | $=$ lever arm of weight of sliding mass from center of |
| rotation |  |

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    \beta= angle of inclination of slope with respect to the
        horizontal
    \Delta = height of the circular segment in sliding soil mass
    = friction angle of the soil
    \gamma = unit weight of soil
    \gammam}=\mathrm{ unit weight of moist soil
    \mp@subsup{\gamma}{t}{}}=\mathrm{ total (saturated) unit weight of soil
\gammaw}=\mathrm{ unit weight of water
    \lambda = angle of inclination of F}\mp@subsup{\textrm{F}}{\textrm{a}}{}\mathrm{ assumed in the graphical
        method for finite slopes
    \eta = angle of chord with respect to the horizontal
    O half the central angle of a slip circle
\psi = actual inclination of }\mp@subsup{\textrm{F}}{\textrm{a}}{}\mathrm{ with respect to the
        horizontal
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