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Interfacial Friction Between Cohesionless Soils and Solid Surfaces – A Review

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Introduction

A need to estimate the frictional resistance between a solid body against another material (solid, fluid or particulate) exists in many engineering disciplines like mechanical engineering, geotechnical engineering, aeronautical engineering, hydraulic engineering, powder technology, etc. The interfacial friction of concern to the geotechnical engineer is that which arises between solid surfaces and soils. The importance of friction between soils and solid materials has been recognized, as early as the 18th century by Coulomb (1776) while developing the earth pressure theories. The materials used in engineering construction may be soft or hard, and extensible or inextensible. The influence of the hardness of solid material on the interfacial friction between sand and polymer surfaces was studied in detail by O'Rourke et al. (1990) who correlated the interfacial friction angle with the hardness of the solid materials (polymers). The effect of extensibility of the solid material on the interfacial friction has also been the subject of several investigators (for example Ingold, 1984; Jewell and Wroth, 1987; Murthy et al., 1993 to name a few.) This paper is mainly concerned with the interfacial friction between cohesionless soils and hard non-deformable solid surfaces. Unless otherwise specified, the term solid material in this paper refers to non-deformable material. Several studies and recommendations are available in the literature regarding the friction/adhesion between soils and

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such solid materials. These are derived either from field observations or from laboratory experiments.

On the submicroscopic scale most solid surfaces (even carefully polished surfaces) are actually rough. Two solids will be in contact only where the high points of their surfaces (termed asperities) touch one another. In other words, the actual contact is a very small fraction of the apparent contact area and the normal stresses across these contacts will be extremely high. The high contact stresses cause the two surfaces to adhere at the points of actual contact; i.e., the two bodies are joined by chemical bonds. The adhesion theory of friction states that the shear resistance is provided by the adhesive strength of these contact points (Bowden and Tabor, 1964; Lambe and Whitman, 1969).

The 'overall' friction theory (on the macro level) examines the 'average' frictional resistance overlooking the surface details. This approach enables the utilization of experimental evidence showing that the resisting frictional force is proportional to the normal force pressing the two objects together. This simple empirical law, also known as Coulombian friction law, is routinely used to describe the shear strength of soils and interfaces. The interfacial frictional resistance (τ) is expressed as,

$$\tau = \sigma_n \tan\delta + c_a \tag{1}$$

in which σ_n = normal stress acting on the interface,

δ = friction angle and

c_a = adhesion between soil and the solid surface.

In the case of cohesionless soil-solid surface interface, the value of c_a is equal to zero.

Considerable work has been done on the interfacial friction between cohesionless soils and solid surfaces over the past five decades commencing with Meyerhof (1948). Several types of apparatus are reported in the literature to evaluate the interfacial friction and attempts have been made to identify the factors influencing it. This paper summarizes the evolution of techniques of evaluation and the design values recommended for the interfacial friction between sands and solid surfaces.

Apparatus Types Reported in the Literature

Meyerhof (1948) is one of the earliest researchers who determined the interfacial friction angle of sand on brass sections from constant rate of strain shear tests using a shear box apparatus. Kézdi (1957) used a similar type of direct shear box of 300 × 300 mm size and evaluated the δ value for sandy gravel on a concrete surface.

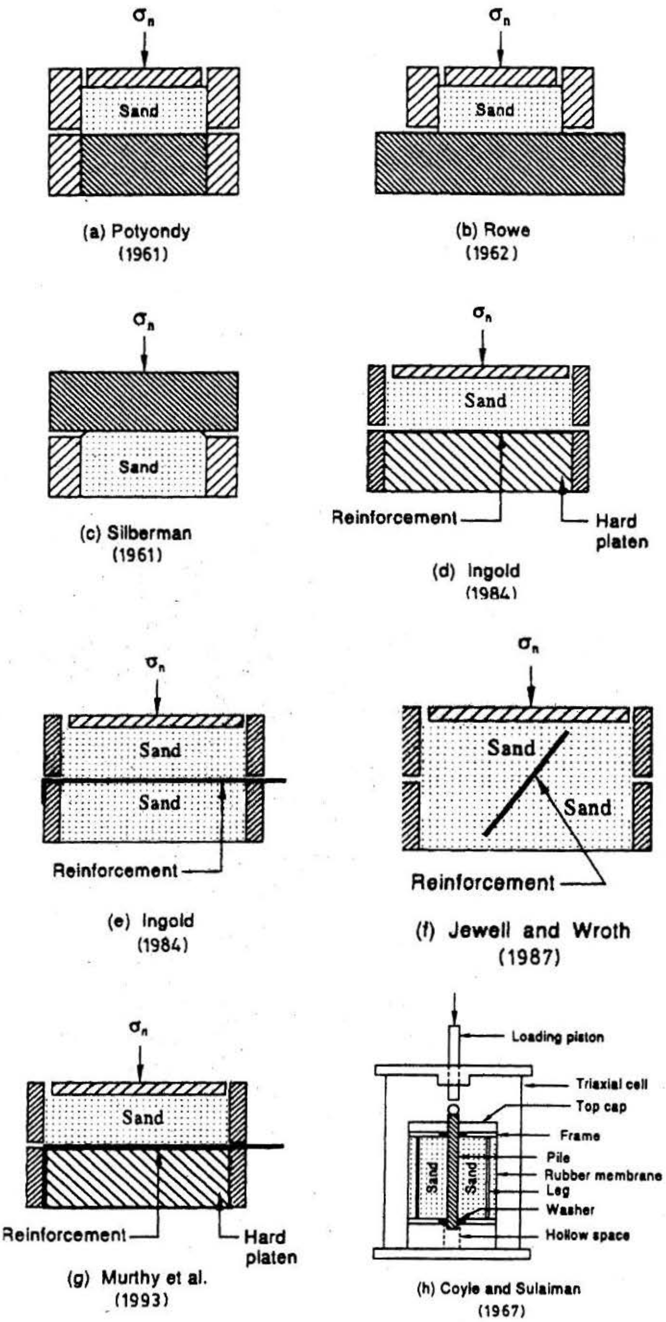
The credit for the first systematic study on skin friction between soils and various construction materials goes to Potyondy (1961). He used a strain controlled shear box of 36 cm^2 shearing area and stress controlled shear box with 80 cm^2 shearing area for the study. The specimens of construction materials were placed in the lower portion of the box and the soil in the upper box. A schematic representation of this apparatus is shown in Fig.1a. This type is very widely used for evaluating δ (O'Rourke et al., 1990; Panchanathan and Ramaswamy, 1964; Bosscher and Ortiz, 1987; Thandavamurthy, 1990 to name a few).

In the modified direct shear apparatus reported above (e.g. Potyondy, 1961), the interface area decreases during shear. Rowe (1962) rectified this drawback by replacing the bottom half of the box by a test material block of sufficiently larger size than the interface dimensions (Fig.1b). This modification was adopted by Levacher and Sieffert (1984), Tatsuoka and Haibara (1985), Kishida and Uesugi (1987), Abderrahim and Tisot (1993), Tejchman and Wu (1995) and Subba Rao et al. (1998).

Silberman (1961) utilized a shear box in which the upper box was replaced with the test material (Fig.1c). This mode is the reverse of the configuration adopted by Potyondy (1961). Subba Rao et al. (1998), Neely et al. (1973) and Noorany (1985) adopted this method of testing to obtain δ values.

The direct shear apparatus has also been adopted to evaluate the interfacial resistance of earth reinforcements (Ingold, 1984; Jewell and Wroth, 1987; Murthy et al., 1993). The interfacial resistance can also be evaluated under four types of reinforcement placing viz., (a) Modified direct shear (b) free shear (c) direct shear with inclined reinforcements and (d) modified free shear.

In the modified shear test, the reinforcement is glued to a platen so that elongation of the reinforcement is prevented and is placed in the lower half of the shear box such that the top surface of reinforcement is along the horizontal shear plane (Fig.1d). In the free shear type, the reinforcement is fixed at one end and the soil is filled in both halves of the box as shown in Fig.1e. Fig.1f shows the schematic diagram of direct shear test with inclined reinforcement. The reinforcement is placed inclined to the horizontal and extending equal lengths in both halves of the shear box. The relation between the maximum reinforcement force and the improvement in the shearing resistance of the soil mass was obtained from the equilibrium of



(a) Potyondy (1961)

(b) Rowe (1962)

(c) Silberman (1961)

(d) Ingold (1984)

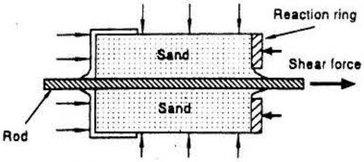
(e) Ingold (1984)

(f) Jewell and Wroth (1987)

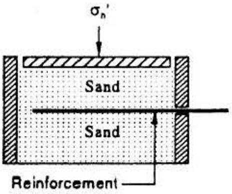
(g) Murthy et al. (1993)

(h) Coyle and Sulaiman (1967)

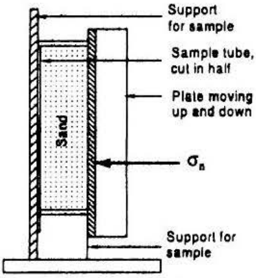
FIGURE 1 : Schematic Diagram of Types of Apparatus Used to Evaluate δ



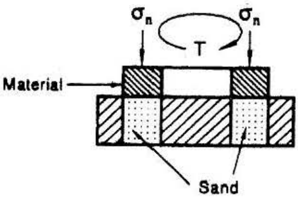
(i) Brumund and Leonards
(1973)



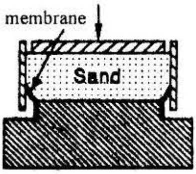
(j) Ingold
(1984)



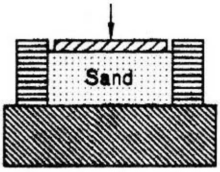
(k) Heerema
(1979)



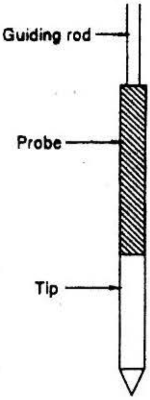
(l) Yoshimi and Kishida
(1981)



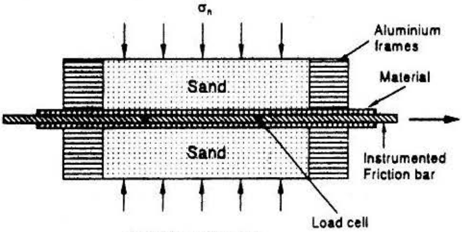
(m) Desai et al.
(1985)



(n) Uesugi and Kishida
(1986)

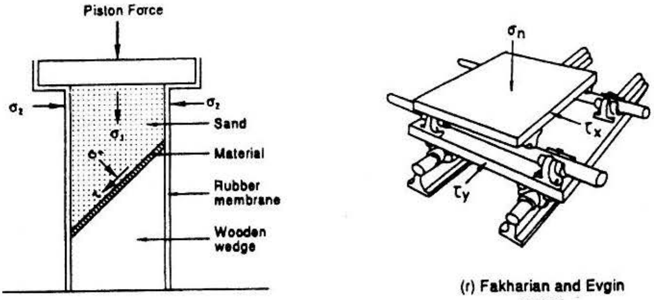


(o) Abderrahim and Tisot
(1993)



(p) Paikowsky et al.
(1995)

FIGURE 1 : Continues



(q) Tejchman and Wu (1995)

(r) Fakharian and Evgin (1996)

FIGURE 1 : Continues

forces in the soil. In the fourth type of test the reinforcement is fixed at one end and rests on a very smooth hard surface and is free to elongate during shear (Fig.1g).

While the above studies used the direct shear apparatus, Coyle and Sulaiman (1967) designed a miniature pile testing apparatus by modifying a large triaxial cell (Fig.1h). Though the idea of designing this apparatus was to get a constant interface area, the δ values obtained from this apparatus may be more realistic for pile-sand interfaces than those determined from a direct shear set-up. The apparatus consists of a four-legged steel frame, which transmits the vertical pressure (due to cell pressure) on the top cap to the base without inducing stress in the soil. The cell pressure is transferred to the granular soil in the lateral direction through a rubber membrane, which acts as a normal stress. The miniature pile, surrounded by the sand, can be either pushed-in or pulled-out after applying the normal pressure and the stress-movement relationship can be established. This device was also adopted by Thandavamurthy (1990) and Venkatesh (1989) to obtain the interfacial friction between pile material and sand.

Brumund and Leonards (1973) used a cylindrical device to investigate static and dynamic friction between sands and rods of various construction materials varying from polished steel to graphite coated steel and from smooth mortar to graphite coated mortar and rough mortar. It consists of a cylinder of sand encased in a rubber membrane with the rod located along its axis (Fig.1i). The normal stress is applied by evacuating air from within the membrane and the maximum value of normal stress is limited to atmospheric pressure. Stress controlled tests were conducted to evaluate the coefficient of friction.

Since under operational conditions the reinforcing earth reinforcement is subjected to pull-out it has been argued that soil-reinforcement interface strength should be determined using the pull-out test (Holtz, 1977). In pull-out tests, the reinforcement material is buried in the soil and the required normal stress is applied (Fig.1j). The reinforcement is pulled out and the load-deformation data is collected for evaluating δ. Many researchers (Ingold, 1984; Murthy et al., 1993; Bacot et al., 1978; Schlosser and Elias, 1978; to name a few) have conducted pull-out tests to study the effects of various parameters. Palmeria and Milligan (1989) have studied the influence of boundary conditions in terms of size of the box and the roughness of the front wall on the pull-out test results and Murthy et al. (1993) have suggested a method of estimating the boundary effect.

Heerema (1979) devised a system using a steel tube that was cut in half length-wise (Fig.1k). The half-specimen has a face of 50 mm, and its height is 150 mm. The tube is placed in a vertical position in a half cylindrical support frame. A flat steel plate (test material) is pressed against the soil sample contained in this split tube by applying normal stress. The test material was moved up and down and all the forces and displacements were continuously recorded.

Yoshimi and Kishida (1981) utilized a ring torsion apparatus, in which a ring shaped metal specimen was placed on the prepared sand in an annular container in order to have an unlimited circumferential deformation (Fig.1l). A constant vertical load was applied through the metal specimen and static torque (T) was applied so that the metal surface moved at a rate of about 0.6 mm/min in circumferential direction. The deformation of sand and slippage at the soil-metal contact were measured using X-ray radiography. Abderrahim and Tisot (1993), Lemos (1986) and Lahane et al. (1993) have used identical apparatus in their investigation.

For finding out the interfacial resistance of geomembranes using the ring torsion apparatus the geomembrane is glued or fastened to a ring shaped platen (Negussey et al., 1989; Evans and Fennick, 1995).

Desai et al. (1985) designed an apparatus, which was named as the Cyclic-Multi-Degree-Of-Freedom (CYMDOF) shear device. This apparatus essentially resembles a direct shear apparatus (Fig.1m) but differs in that a rubber membrane, about 16 mm thick is clamped between the bottom solid sample and the sides of the box as shown in Fig.1m. This modification prevents leakage of sand during cyclic testing.

Uesugi and Kishida (1986a) used a simple shear apparatus for evaluating δ (Fig.1n). With this apparatus, the sliding displacement at the sand-steel contact surface can be obtained in distinction from the displacement due to the shear deformation of sand mass. This apparatus was made from a container consisting of stacked aluminium plates of 2 mm thick with internal openings of 400 × 100 mm. The surface of each aluminium plate was lubricated to reduce the frictional resistance. The rectangular container was kept over the test material and sand was filled. A constant normal load was applied to the interface through the sand and the solid material was forced to slide. The authors have done extensive investigations using this type of apparatus (Kishida and Uesugi, 1987; Uesugi and Kishida, 1986b; Uesugi et al., 1989, 1990).

Abderrahim and Tisot (1993) used a mini pressure meter probe (Fig.1o), which was protected by an outer steel sheath with longitudinal slits like in a Chinese lantern. This sheath forms the material surface. The normal stress at the interface is calculated according to the internal pressure and the stiffness of the probe. The probe is gradually extracted at a constant speed of 0.65 mm/sec and the load-displacement data is collected.

Paikowsky et al. (1995) developed a dual interface shear apparatus to evaluate the distribution and magnitude of friction between granular materials and solid inextensible surfaces. The apparatus consists of a shear box with two compartments separated by an instrumented friction bar (Fig.1p). Six interchangeable plates constituting the solid surfaces are mounted on the instrumented bar, three on each side. The plate in the central section is assumed to be free from boundary effects. A constant total area of 1000 cm^2 (500 cm^2 per side) exists between the soil and the solid surface. Normal stress is applied to the top and the bottom sand specimens. A constant rate of displacement is applied to the instrumented friction bar. The total load and the load transferred along the interface are measured.

Tejchman and Wu (1995) used a modified plane strain apparatus (Biaxial apparatus) (Fig.1q) for measurement of friction between sand specimen and steel plate. The cross-section of the sand specimen was chosen to be 40 × 80 mm. A wooden wedge with an angle of 67.5° against the horizontal was covered with a steel plate (solid material). The wooden wedge was placed at the bottom of the sand mould. The specimen was prepared by pluviating dry sand in the mould. The wedge and the sand specimen were then enclosed by a rubber membrane. The sand specimen and the wedge were confined between two rigid platens and the whole set-up was placed into a pressure chamber. A constant cell pressure σ_2 was applied and the loading piston (which applies σ_1) was moved at a rate of 0.42 mm/min. Knowing σ_1 and σ_2, δ can be evaluated.

An automated apparatus for three-dimensional monotonic and cyclic testing of interface was developed by Fakharian and Evgin (1996). In this apparatus the interface can be subjected to a normal stress, σ_n, and two shear stresses, τ_x and τ_y, acting simultaneously on the interface plane. The shear stresses were applied to the interface plane through a X-Y loading table

Table 1 : Review of Granular Material/Solid Interfacial Friction Testing Apparatus*

Testing apparatus (1)	Reference (2)	Advantages (3)	Disadvantages (4)
Direct shear	Potyondy (1961) Kishida and Uesugi (1987) Desai *et al.* (1985) Uesugi and Kishida (1986b) Kulhawy and Peterson (1979) Acar *et al.* (1982).	• Commonly available device • Simple sample preparation and operation • Solid surface can be either above or below soil sample	• Physical boundaries markedly influence results • Displacement components cannot be independently identified • Interface area may change during shear (set up dependent)
Axisymmetric Loading	Coyle and Sulaiman (1967)	• Geometrical configuration resembles skin friction of piles • Common triaxial device can be modified and employed • Constant interface area	• Stress concentration at the ends • Normal stress on interface unknown • Method and direction of soil placement around the bar may markedly affect the soil/solid interaction • Displacement components cannot be independently identified
Cylindrical device	Brumund and Leonards (1973)	• Geometrically similar to skin friction of piles and friction of steel reinforcements	• Normal stress on interface unknown • Stress concentration at ends
Ring Torsion	Abderahim & Tisot (1993) Yoshimi and Kishida (1981) Lemos (1986) Lahane *et al.* (1993)	• No end effects, 'endless' constant interface area	• Complicated experimental system, sample preparation and procedure • Displacement gradient across the interface, and as a result, shear strain variation in the sample • Solid overlaying the soil

* Modified after (Kishida & Uesugi 1987 and Paikowsky *et al.* 1995)

Table 1 : Continued

(1)	(2)	(3)	(4)
Simple shear	Kishida & Uesugi (1987) Uesugi and Kishida (1986a, b) Uesugi *et al.* (1989, 1990)	• Simple sample preparation and operation • Displacement components can be measured independently • Constant interface area	• Stress concentration at the ends
Mini pressuremeter	Abderrahim and Tisot (1993)	• May be used to obtain the skin friction in the field.	• Preparation of test material is very difficult.
Dual shear	Paikowsky *et al.* (1995)	• Simple or direct shear • No boundary effects with respect to central segment • Single (up or down) or dual interface for the same bar • Interchangeable solid surfaces • Constant interface area • Displacement components can be measured independently under simple shear conditions	• Requires instrumentation to enable load measurement along the interface • Interface needs to be long enough to enable measurements at locations away from the non-uniform zones at the ends
Plane strain apparatus	Tejchman and Wu (1995)	• All the stress components in the plane of shearing can be determined.	• Stress concentration at the ends • Shear area decreases during deformation.

(Fig.1r) which can move both in X and Y directions simultaneously without any rotation about the vertical axis. The soil is contained in a 25 mm thick hollow aluminium box, with inside dimensions of 100 × 100 mm, which is placed on a plate of the structural material with a size of 300 × 300 mm so that the contact area remains the same during sliding. The plate (solid surface), that carries the soil container, is fixed to the top of the X-Y loading table. Normal stress is applied through a loading platen from the Z-direction. Tests can be conducted, either displacement or load controlled.

Table 1 summarizes the advantages and disadvantages of different apparatus used to estimate interfacial friction between soils and solid surfaces.

Factors Influencing Interfacial Friction

Several studies were reported in the literature, which have examined the influence of various soil parameters such as grain size and shape, density of the granular soil and the solid material properties such as surface roughness and hardness. The influence of other factors like normal stress, deformation rate, type of apparatus and size effects have also been the focus of studies reported in the literature. Not infrequently, the results of studies on the same factor have differed. The reports of several studies on each of these factors influencing δ can be carefully reinterpreted so that unambiguous conclusions can be drawn regarding the role of each factor.

Density

It is well demonstrated in the literature that the angle of internal friction (φ_p) of sand increases with density (Burmister, 1948; Bolton, 1986). Attempts to study the influence of density on the interfacial friction angle δ between sands and solid surfaces have yielded contradictory conclusions.

The studies of Noorany (1985), Yoshimi and Kishida (1981) and Broms (1963) indicated that the friction angle δ is independent of the sand density. Typical results of Noorany (1985) are given in Table 2. On the other hand, O'Rourke et al. (1990), Levacher and Sieffert (1984), Tejchman and Wu (1995), Coyley and Sulaiman (1967), Lahane et al. (1993), Desai et al. (1985), Uesugi et al. (1990), McClelland (1974), Kulhawy and Peterson (1979), Acar et al. (1982), Subba Rao and Venkatesh (1985) and Bagdadi et al. (1991) showed that the friction angle δ increases as the density of sand increases. Typically, data from Acar et al. (1982) is reproduced in Fig.2.

Subba Rao et al. (1996) after critically examining the above results reconciled the seemingly contradictory variation of δ with density. They showed that the test configuration adopted for evaluation of δ has an important bearing on the influence of density on δ. They classified the

Table 2 : Results of Triaxial and Soil-Steel Friction Tests (after Noorany, 1985)

Soil type	Soil condition	$\phi°$	$\delta°$
Silica sand	loose	35	21
	dense	40	20
Calcareous sand from Guam	loose	46	18
	dense	49	18
	loose, crushed	46	21
	loose, ground	46	—
	dense, crushed	48	22
Calcareous sand from Florida	loose	44	20
	medium	45	20
	dense	47	23
	medium, crushed	45	23
	medium, ground	45	—
	dense, crushed	49	23

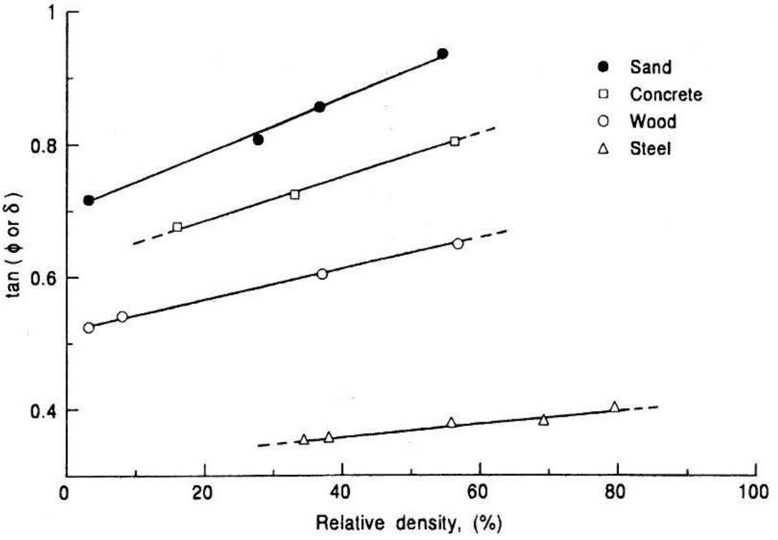


FIGURE 2 : Influence of Density on Friction Angle Obtained from Direct Shear Apparatus (after Acar et al., 1982)

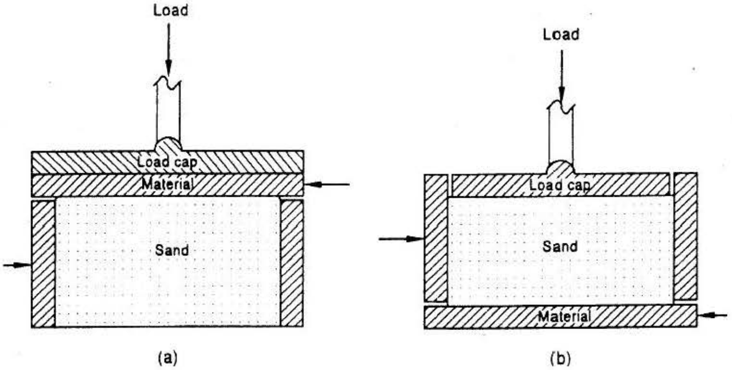


FIGURE 3 : Schematic Diagram of (a) Type A Apparatus and (b) Type B Apparatus

apparatus configuration reported in the literature into two categories as type A (solid material over sand) and type B (sand over solid material). A schematic diagram outlining the main differences between type A and type B apparatus is shown in Fig.3. In type A situation δ is independent of density while δ increases with sand density in type B situation. In type B situation it was observed that the ratio of the peak friction angle δ_{pB} to the peak angle of internal friction of sand ϕ_p is practically independent of sand density (Subba Rao et al., 1998; Acar et al., 1982).

Normal stress

The peak angle of internal friction ϕ_p of granular material is generally assumed to be constant over the stress range of interest in geotechnical engineering (Taylor, 1948; Bishop and Eldin, 1953). The failure envelope for granular materials is in fact curved and hence ϕ_p decreases with the increase in normal stress (Bolton, 1986). Similar results have been reported by Acar et al. (1982) on uniform quartz sand for stress levels in the range 100 to 300 kPa. However, an assumption of linear behaviour is justifiable at low stress levels as has been confirmed by O'Rourke et al. (1990) from tests on Ottawa sand at stress levels less than 70 kPa.

The influence of normal stress on ϕ_p depends on the material characteristics of the granular material. For all particulate materials the deviation from a simple frictional behaviour (constant ϕ_p) is due to crushing and breaking of grains (Lambe and Whitman, 1969). Hence at low stress levels (with less crushing) ϕ_p is constant and at higher stress levels ϕ_p decreases due to crushing. From this it can be inferred that δ, which derives from the characteristics of both granular material and solid surfaces can be influenced by the normal stress level used for its evaluation.

Table 3 : Friction Angle Values (*after* Potyondy, 1961)

Dry sand [Dense, w = 0.8%; Relative density = 66%]

Material	Normal load 48.7 kPa			Normal load 146 kPa		
	ϕ°	δ°	(δ/ϕ)	ϕ°	δ°	(δ/ϕ)
Smooth steel	44°30'	24°10'	0.543	43°30'	24°00'	0.47
Rough steel		34°00'	0.765		33°40'	0.70
Wood parallel to grain		35°00'	0.790		33°20'	0.69
Wood at right angles to grain		39°10'	0.880		38°30'	0.84
Smooth concrete		39°30'	0.890		38°30'	0.84
Rough concrete		44°00'	0.990		42°30'	0.97

Saturated sand [Dense]

Material	Normal load 48.7 kPa			Normal load 146 kPa		
	ϕ°	δ°	(δ/ϕ)	ϕ°	δ°	(δ/ϕ)
Smooth steel	39°30'	24°50'	0.64	37°00'	23°30'	0.64
Wood parallel to grain		33°20'	0.85		33°00'	0.89
Wood at right angles to grain		34°30'	0.89		34°30'	0.93
Smooth concrete		34°40'	0.89		33°20'	0.90

Potyondy (1961) observed that the friction angle δ decreases with the increase in normal stress in the range 48.7 to 146 kPa (Table 3). But the ratio (δ/ϕ_p) is not very different for both the loads. Acar et al. (1982) and Schultz and Horn (1967) corroborated the results of Potyondy (1961). Panchanathan and Ramaswamy (1964) also concluded that δ decreases with the increase in normal stress for most of the surfaces tested. But for soft materials like soft wood they found that the sand grains bite into the fibers of wood resulting in increase in the friction angle with normal stress increase. Everton (1991) also observed that δ was strongly affected by the normal stress level (Jardine et al., 1993).

Tejchman and Wu (1995) observed that the effect of normal stress on δ values was most pronounced at low normal stress levels ($\sigma_n < 100$ kPa). In this range the friction angle was seen to decrease with increase in normal stress. The normal stress has only a minor effect on the δ values in the range 100 to 400 kPa. However, O'Rourke et al. (1990), Abderrahim and Tisot (1993), Heerema (1979) and Uesugi and Kishida (1986a, b) concluded that the normal stress has no influence on the δ values.

In summary, the influence of normal stress (within the normal engineering range) on δ values can either be to decrease the δ values or to have no influence. In the case of soft solid surfaces δ can increase with the increase in normal stress. Thus the influence of normal stress depends on the properties of sand (mainly its tendency to crushing), stress level and the material hardness. Some studies (Potyondy, 1961; Acar et al., 1982) show that expressing the interfacial test results as (δ/ϕ_p) nullifies the effect of normal stress. Clearly, more research is required into this aspect of interface behaviour (Jardine et al., 1993).

Deformation rate

Of the few studies, which considered the rate of deformation as an influencing factor, the results of Heerema (1979) and Lemos (1986) are significant. Heerema (1979) varied the deformation rate from 0.7 to 600 mm/sec on a steel-sand interface. He observed that δ was independent of deformation rate and obtained a constant δ value of 25° for all deformation rates. Lemos (1986) sheared sand against a steel surface in a ring shear apparatus. A test was carried out along a displacement of 273 mm in six stages in which the deformation rate ranged from 0.0038 to 133 mm/min. The ultimate shear resistance at all stages was approximately constant and was not affected by the deformation rate. These studies show that the deformation rate has no influence on the measured friction angle.

Size of apparatus

Laboratory studies on interfacial friction use relatively small size apparatus. It is essential to know the influence of size of apparatus. The tests performed by Brumund and Leonards (1973) using a 51 mm diameter rod (contact area $\approx$ 400 cm^2) showed no appreciable difference in the computed values of coefficient of friction compared to the values obtained using a 28 mm diameter rod (area $\approx$ 225 cm^2). Uesugi and Kishida (1986b) compared the results obtained from the simple shear apparatus with interface areas of 40 cm^2 and 400 cm^2. The results showed a reasonable agreement with each other showing that the size of apparatus has no influence on the δ values. Jewell and Wroth (1987) suggested that a ratio of shear box length (L_b) to average particle size (D_{50}) in the range 50 – 300 would be most likely yielding the same results. However, Palmeria (1988) reported that the mobilized friction angle was not significantly affected by the (L_b/D_{50}) ratio over the range 38 to 1280. O'Rourke et al. (1990) have concluded from the results on Ottawa sand-HDPE interface using direct shear box of sizes ranging from 60 × 60 mm to 305 × 305 mm that the effect of size of apparatus on δ is insignificant.

Paikowsky et al. (1995) compared the friction angle obtained from the dual shear apparatus (Fig.1p) having an interface area of 500 cm^2 per side with those obtained from direct shear apparatus having an interface area of 36 cm^2. The interface resistance of smooth (normalized roughness R_n, will be defined later, ≤ 0.02) and intermediate ($0.02 \leq R_n \leq 0.5$) surfaces obtained from direct shear apparatus are approximately 50% and 20%, respectively, higher than those obtained along the unrestricted area at the centre of the dual shear apparatus. They concluded that the small size direct shear box of 60 mm square is inadequate for interfacial friction measurements. It may be noted that the comparisons are based on measurements due to interaction on both sides of the solid material in the dual shear apparatus with those measured on one side using the direct shear box. Further, the comparison does not involve dual shear apparatus of different dimensions. Moreover, the referred study focusses on the influence of boundary effects on the δ values rather than size effects.

It may be concluded that size of apparatus, with a minimum size of 60 mm × 60 mm, has no influence on δ values obtained between sands (particle size < 2 mm) and solid surfaces. For particle sizes more than sand size the size of apparatus has to be chosen considering the particle size of the granular material as is specified for evaluation of angle of internal friction (BS: 1377, 1990).

Grain size and shape

Rowe (1962) showed that the friction angle between quartz sand and quartz block decreases as the particle size of the sand increases. This phenomena was attributed to the larger particles being able to roll more easily than the smaller particles, perhaps as a result of their centre of gravity being further away from the plane of shear (Lambe and Whitman, 1969). Hence the measured interfacial friction angle, which involves both rolling and sliding components, is smaller for the larger particles. Rowe's observation was further confirmed by Kishida and Uesugi (1987), Uesugi and Kishida (1986b), Jardine and Lahane (1994) and Fioravante et al. (1995). The result of Rowe (1962) is reproduced in Fig.4.

Uesugi and Kishida (1986b) observed that the coefficient of friction between steel and sand for angular particles was higher than that for round particles. The test results of Brumund and Leonards (1973) support this observation. O'Rourke et al. (1990) also concluded the same from the direct shear test results on sand-polymer interfaces. Paikowsky et al. (1995) observed that Ottawa sand yielded higher friction angle than glass beads. This increase in δ values for the sand with respect to the glass beads was attributed to the effect of grain shape.

To sum up it can be said that as the particle size of sand decreases δ

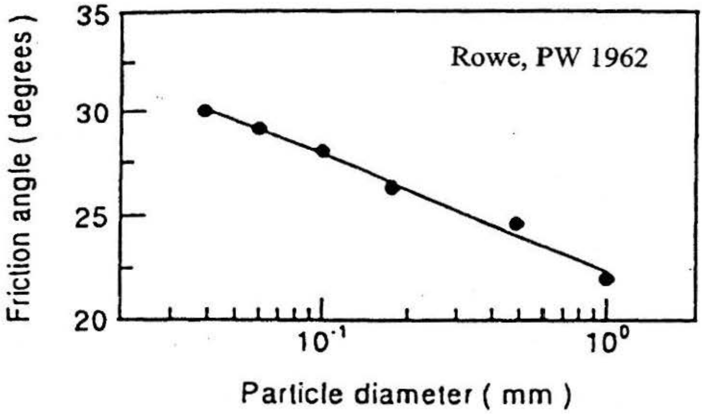


FIGURE 4 : Influence of Particle Size on δ (after Rowe, 1962)

value increases. The interfacial friction angle values for angular particles are higher than those for rounded particles.

Type of apparatus

It was brought out earlier that several types of apparatus are available to evaluate the friction angle. There have been some attempts to study the influence of type of apparatus on the measured δ values. In these studies the major influencing factors such as surface roughness, sand density, sand type, normal stress etc. were kept the same with the type of apparatus different. It was seen that the δ values obtained were different even though the interface conditions remained the same.

Kishida and Uesugi (1987) compared the coefficient of friction values obtained from direct shear apparatus with simple shear apparatus. They noted that the values obtained from simple shear apparatus and direct shear apparatus are essentially the same. The end restraints (rigid or deformable boundary) appeared to have no influence on the δ values.

Abderrahim and Tisot (1993) compared δ values obtained from the direct shear apparatus, the ring shear apparatus and a mini pressuremeter probe. Divergent results were obtained with the three apparatus. The friction at the interface measured with the mini pressure meter probe was greater than the value measured with the ring shear apparatus and less than that measured with direct shear apparatus.

Tejchman and Wu (1995) conducted interface shear tests between sand and steel from three types of apparatus viz; plane strain apparatus, parallely guided direct shear apparatus and a model silo. The results showed that the coefficient of friction and the corresponding displacements are different.

The results mentioned above show that the δ values are strongly influenced by the type of apparatus used for their evaluation. This suggests that the friction angle values for design should not be selected without considering the type of field situation emulated by the apparatus type for the determination of δ.

However, Jardine et al. (1993) observed that the δ values obtained in the laboratory direct shear tests were virtually identical to those measured in the field instrumented pile tests. Jardine and Chow (1996) recommended that the operational interfacial resistance should be measured in interface direct shear tests involving same surface roughness and hardness as the pile material and under the same normal stress in the field.

Quantification of Interface Roughness and Empirical Correlations

The surface roughness of the solid material has a very significant influence on the friction angle (δ). In the earlier studies the type of construction material was considered to be important. Potyondy (1961) determined the friction angle between sand and smooth and rough surfaces of the then commonly used construction materials steel, wood and concrete. The test results were expressed in the form of interfacial-to-internal friction angle ratio (δ/ϕ_p) as can be seen in Table 3. These values are commonly used in design (NAVDOCKS, 1962). This practice of assigning values to δ arbitrary fractions of the angle of internal friction ϕ_p of the soil, irrespective of the soil-solid material interface roughness, is currently being critically reviewed.

Esashi et al. (1966) showed that skin friction between sands and construction materials could be correlated to the quantified surface roughness regardless of the type of the solid material (Yoshimi and Kishida 1981).

Yoshimi and Kishida (1981) measured the interfacial shear resistance between solid surfaces (steel, brass, aluminium, wood and concrete) possessing a wide range of surface roughness and sands of different types (Toyoura sand, Tonegawa sand, Nigigata sand and Soma sand). They correlated the coefficient of friction with the surface roughness. The surface roughness was quantified by using $R_{max}(L = 2.5\text{ mm})$, defined as the vertical distance between the highest peak and the lowest trough along a gauge length L of 2.5 mm of the surface profile (Fig.5).

$R_{max}(L = 2.5\text{ mm})$ is a parameter of surface roughness only and does not involve grain size. Allowing for the effect of grain size on the friction

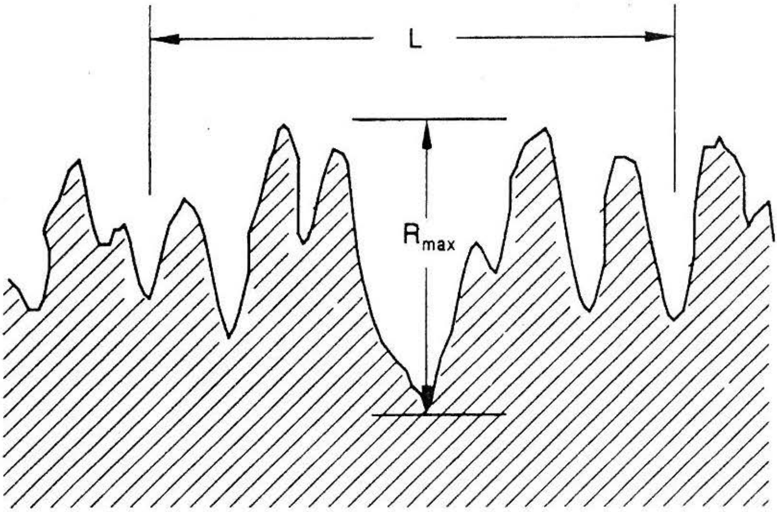


FIGURE 5 : Typical Surface Profile Showing R_{max}

angle Kishida and Uesugi (1987) defined a Normalized roughness (R_n). R_n is defined as,

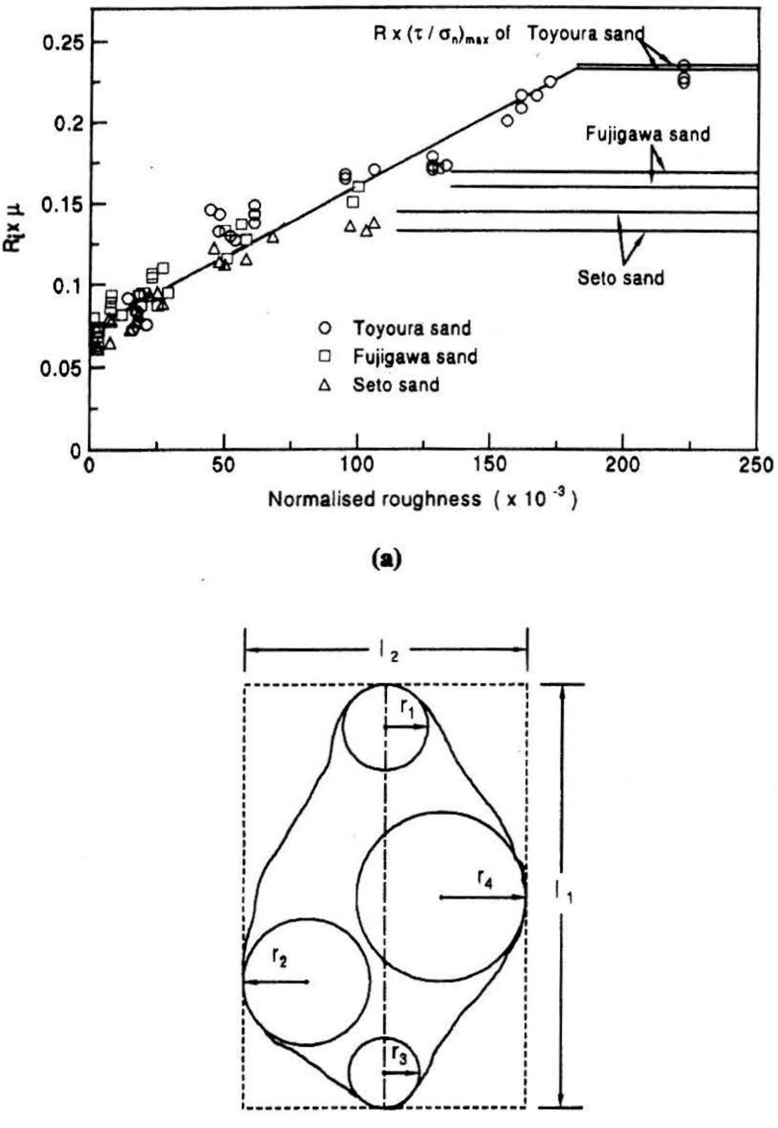
$$R_n = \frac{R_{max}\left(L = D_{50}\right)}{D_{50}} \tag{2}$$

where, $R_{max}(L = D_{50})$ is the relative height between the highest peak and the lowest valley along a surface profile over a gauge length L equal to D_{50} size of sand (D_{50} is the size at which 50% of the particles are finer by weight). A bilinear correlation was obtained between coefficient of friction of dense Toyoura sand and normalized roughness over a wide range of sand diameters.

Uesugi and Kishida (1986b) estimated the coefficient of friction between sands of different angularity and steel as a function of modified roundness and normalized roughness (R_n) as shown in Fig.6a. The modified roundness of a particle is defined as,

$$R_i = \frac{1}{2}\left(\frac{r_2 + r_4}{l_1} + \frac{r_1 + r_3}{l_2}\right. \tag{3}$$

where, r_1, r_2, r_3, r_4, l_1 and l_2 are defined in Fig.6b. The modified roundness values of Toyoura, Fujigava and Seto sand were estimated to be 0.27, 0.19 and 0.17, respectively.



(a)

(b)

FIGURE 6 : (a) Coefficient of Friction Multiplied by Modified Roundness as a Function of Normalised Roughness; (b) : Modified Roundness (after Uesugi et al., 1986b)

Bosscher and Ortiz (1987) obtained the interfacial friction angle between sand at a relative density of 67% and various construction materials such as sand stone, limestone, granite, smooth concrete and rough concrete. The δ values were measured in the modified direct shear apparatus resembling type B mode (sand over the solid material). A relationship between the centre line average roughness (R_a) of the construction materials and δ values was obtained.

O'Rourke et al. (1990) proposed the following correlation for the evaluation of interfacial frictional strength of sand in contact with plastic piping, geomembranes, soil strip reinforcement and a variety of other soil-polymer systems.

$$\frac{\delta}{\phi_{ds}} = 1.15 - 0.0088\,H_D \tag{4}$$

where ϕ_{ds} = direct shear angle of internal friction and
H_D = shore D Hardness.

The ratio (δ/ϕ_{ds}) was found to be relatively constant at 0.55 - 0.65 for different types of sands (rounded, sub-rounded and sub-angular) placed at a variety of densities.

Dove et al. (1997) proposed a surface roughness parameter, R_s, (defined as the ratio of actual surface area to the projected surface area) based on the three-dimensional characteristics of a surface, to quantify the surface roughness of geomembranes. This approach is quite interesting and its applicability to solid surfaces merits investigation.

Recently, Subba Rao et al. (1998) quantified the interface roughness using a parameter called Relative roughness (R). The relative roughness is defined as the ratio of the average roughness (R_a) of the surface to the average particle size (D_{av}) of sand. They correlated (δ_{pB}/ϕ_p) with R as shown in Fig.7 where the results of tests using surfaces of different average roughness and sands of different average sizes are shown. The following empirical correlation has been proposed for the estimation of friction angle in type B situation.

$$\frac{\delta_{pB}}{\phi_p} = 1.0 - 0.80\exp\left(-15R^{0.54}\right) \tag{5}$$

where δ_{pB} = peak friction angle from type B apparatus and
ϕ_p = peak angle of internal friction of sand

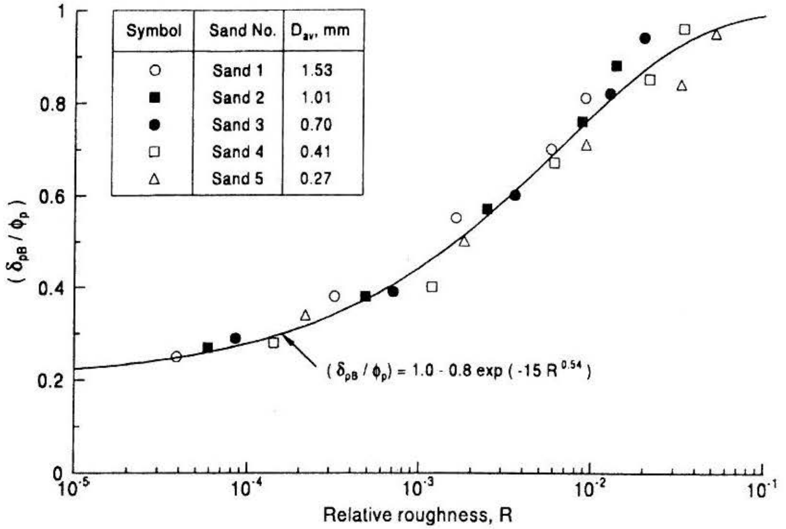


FIGURE 7 : Variation of (d_{pB}/ϕ) with Relative Roughness R

They have observed that the critical state friction angle δ_{cvB} obtained from type B apparatus is practically the same as δ_{cvA} obtained from type A apparatus. The variation of δ_{cvA} with R is shown in Fig.8. The following empirical relationship has been proposed for the estimation of the friction angle in type A situations.

$$\delta_{cvA} = 33^\circ - 21^\circ \exp\left(-160\,R^{0.86}\right) \tag{6}$$

The current position is that it is possible to estimate the friction angle between sands and solid surfaces from the knowledge of the sand properties (grain size and angle of internal friction) and the surface roughness of the solid material. However, the quantification of the effect of grain shape on ϕ_p and δ merits further investigation.

Limiting Values of δ

The dependence of δ on surface roughness has been well demonstrated (Potyondy, 1961; Subba Rao et al., 1998; Yoshimi and Kishida, 1981; Uesugi and Kishida 1986a, b, to name a few). As the roughness increases, δ values increase and tend to a constant value.

Potyondy (1961) reported a value of δ approximately equal to ϕ_p ($\delta = 0.99\,\phi_p$) for rough concrete-sand interface. Similarly Panchanathan and Ramaswamy (1964) observed that for very rough surfaces the friction angle

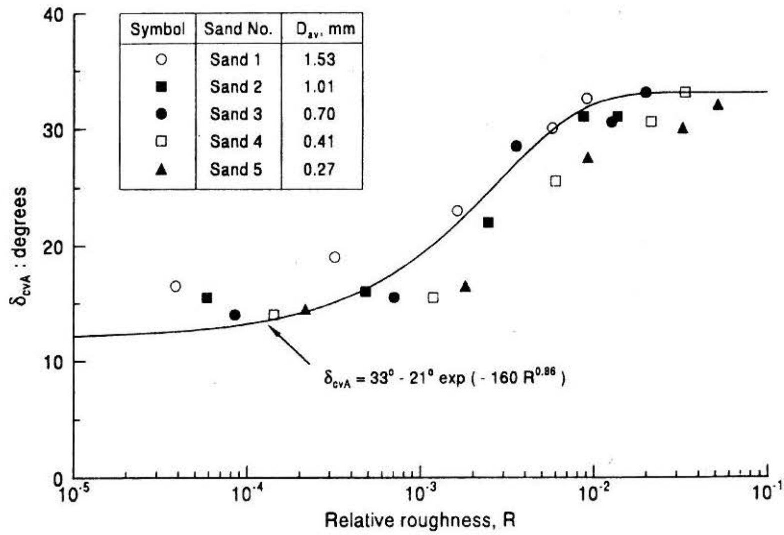


FIGURE 8 : Variation of d_{cvA} with Relative Roughness R

δ is as much as the ϕ_p value of sand. Recent studies using simple shear apparatus also show that the limiting maximum value of δ (δ_{lim}) is the peak angle of internal friction of the sand (Kishida and Uesugi, 1987; Uesugi and Kishida, 1986a, b; Uesugi et al., 1990). Everton (1991) also observed from direct shear tests that $\delta_{lim} = \delta_p$ (Jardine et al., 1993).

While the above mentioned studies report that δ_{lim} is the peak angle of internal friction of sand, the studies of Yoshimi and Kishida (1981) using ring torsion apparatus show that it is the critical state angle of internal friction of sand ϕ_{cv}.

It is seen that there is a contradiction in the conclusion regarding the maximum limiting value of δ. The limiting maximum value can either be the peak angle of internal friction ϕ_p or the critical state friction angle ϕ_{cv} of sand. Subba Rao et al. (1996) explained the possible reason for this discrepancy based on the mode of shear (type A and type B). In type A mode (solid material over sand) the maximum limiting value of δ is the critical state angle of internal friction of sand, ϕ_{cv}. But in type B (sand over solid material) mode d can be as much as the peak angle of internal friction of sand ϕ_p.

Theoretically, one may assume that for very smooth surfaces, with the roughness equal to zero, the friction angle is equal to zero. But experimental studies on interfacial friction between highly polished surfaces and sand show that the coefficient of friction for very smooth surfaces is not equal to zero.

Table 4 : Minimum Values of δ Reported by Various Authors

Interface	δ°	Source
Sand-material		
Sand-smooth surface	$\phi_\mu{}^*$	Lambe and Whitman (1969)
Sand-smooth material	$< 0.5\ \phi_\mu$	Yoshimi and Kishida (1981)
Sand-normal glass	7 – 10	Tatsuoka and Haibara (1985)
Sand-pyrex glass	5 – 6	Tatsuoka and Haibara (1985)
Sand-Stainless steel	7	Tatsuoka and Haibara (1985)
Sand-steel	$\approx \tan^{-1}(0.07/R_i)^\S$	Uesugi and Kishida (1986b)
Sand-steel	$\approx 0.5\ \phi_\mu$	Tejchman and Wu (1995)
Glass beads-steel	≈ 5	Paikowsky *et al.* (1995)
Material-Material		
Diamond-diamond	3	Bowden and Tabor (1986)
Sapphire-sapphire	11	Bowden and Tabor (1986)
Metal-diamond	3	Bowden and Tabor (1986)
Steel-sapphire	7	Bowden and Tabor (1986)

Notes : * ϕ_μ Particle-to-particle friction angle

§ R_i Modified roundness

In material-material friction studies on highly polished surfaces δ was found to have a value greater than zero (Bowden and Tabor 1986). The minimum values observed by various authors on sand-material friction and material-material friction studies are summarized in Table 4.

Recommended Design δ Values

Recommendations in the literature (Acar et al., 1982; NAVDOCKS, 1962; Peck, 1958; Meyerhof, 1959, 1962; Broms, 1966; Terzaghi and Peck, 1967; Perlof and Baran, 1976; Kulhawy, 1984; Bowles, 1988 and Nayak, 1996) largely do not provide for the effect of the important parameters affecting the interfacial friction like particle size and shape of granular soil, density, normal stress, roughness and hardness of the solid material, etc.

For example Terzaghi and Peck (1967) have given values of $\tan\delta = 0.55$ for concrete with clean sand and 0.35 – 0.45 for concrete with fine sand, respectively. Lambe and Whitman (1969) defined the limiting values of δ, between sand and smooth or rough surfaces, as the particle-to-particle friction angle ϕ_μ or the critical state friction angle δ_{cv}, respectively. Broms

Table 5 : Values of Friction Angle for Various Interface Conditions (after Kulhawy, 1984)

Pile/soil interface condition	δ
Smooth (coated) steel/sand	0.5ϕ to 0.7ϕ
Rough (corrugated) steel/sand	0.7ϕ to 0.9ϕ
Precast concrete/sand	0.8ϕ to 1.0ϕ
Cast-in-place concrete/sand	1.0ϕ
Timber/sand	0.8ϕ to 0.9ϕ

(1966) suggested a δ value of 20° for smooth steel-sand interface and for cast-in-place concrete-sand interface and timber-sand interface the recommended values are 0.75ϕ and 0.7ϕ, respectively. Acar et al. (1982) obtained (δ/ϕ_p) values of 0.40, 0.75 and 0.85 – 0.95 for steel, wood and concrete surfaces, respectively. Values of the angle of pile to soil friction for various interface conditions suggested by Kulhawy (1984) are given in Table 5.

Most of these recommendations are based on the results of Potyondy (1961), reported in Table 3. While it is interesting to note that a decrease in δ value to account for the interfacial stress levels (Bowles 1988) has been suggested no corrections to δ to account for other factors exists. These recommendations are limited only to the interaction of hard grained granular materials with hard solid surfaces and need to be revised to account for the influence of the several important factors affecting the interfacial friction.

Looking Forward

From the literature it is seen that the phenomenon of interfacial friction has developed from a mere consideration of only the type of solid material (steel, wood or concrete) to a complex function involving the roughness characteristics of both the solid surface (roughness and hardness) and the sand (like size, shape, gradation, density, etc.).

The available practices and recommendations for the choice of δ need to be more comprehensive than at present and should incorporate the characteristics of interface and type of field situation.

Where the interface consists of solid concrete or steel and the field situation involves small deformations at the interface as in the case of gravity retaining walls, Eqn.5 will be appropriate. Similarly, when interfacial deformations are large, as in the case of piles, use of Eqn.6 will be appropriate. Where the interface is a material of lower hardness than the cohesionless soil and relative deformations are large as in the case of reinforced soil applications, Eqn.4 can be employed to arrive at a design value.

Acknowledgements

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Notation

c_a = adhesion between soils and solid surfaces

D_{av} = weighted average particle size of sand

D_{50} = particle size at which 50% of the particles are finer by weight

H_D = shore D hardness

L = gauge length

L_b = length of shear box

R = relative roughness

R_i = modified roundness

R_n = normalized roughness

$R_{max}(L = 2.5\text{ mm})$ = roughness, defined as the relative height between the highest peak and the lowest trough over a gauge length (L) of 2.5 mm.

R_s = surface roughness parameter

δ = friction angle

δ_{cvA} = friction angle from type A apparatus

δ_{cvB} = critical state friction angle from type B apparatus

δ_{pB} = peak friction angle from type B apparatus

δ_{lim} = limiting maximum value of δ

μ = coefficient of friction

τ = interfacial frictional resistance

σ_n = normal stress

σ_1 = major principal stress

σ_2 = minor principal stress

ϕ_p = peak angle of internal friction of sand

ϕ_{cv} = critical state angle of internal friction of sand.

ϕ_μ = particle-to-particle friction angle

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