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Effect of Relative Density on Creep Characteristics of Sands

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Introduction

Creep or time dependent behavior may be best described by rheology. Rheological models composed of linear springs in combination with linear or nonlinear dashpots and sliders are generally used for expressing mathematically the time dependent behavior of materials.

A lot of work has been carried out to study the creep behavior of clays (Abdel-Hady et al., 1966; Companella and Vaid, 1974; Christensen and Wu, 1964) but little has been done to study the creep of sands due primarily to their inconspicuous time dependency. It was supposed that the model developed for clays would explain the behavior of sands as well. Murayama (1983) had proposed an independent rheological model to explain creep characteristics of sands.

In this study, the effect of relative density on creep characteristics of local dune sands of eastern part of Saudi Arabia is studied in the light of rheological model proposed by Murayama (1983). Drained triaxial creep tests were performed at three different relative densities such as 60%, 70% and 85%. It is found that the result of these tests is consistent with the above-mentioned model. It is further noted that variation in relative density from 60% to 85% increases both failure and elastic stress of sands.

Literature Review

Past studies on creep characteristics of soils may be classified into four

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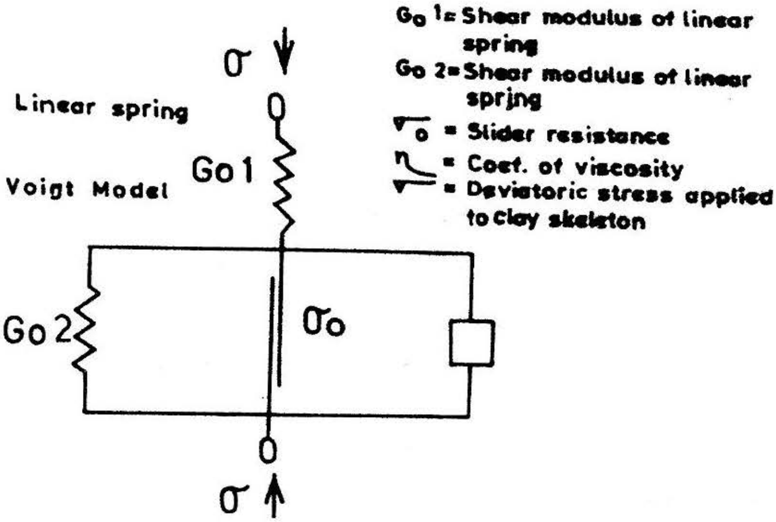

FIGURE 1 : Rheological Model with Linear Spring Elements (After Murayama and Shibata, 1964)

groups: Visco-elastic, Visco-elasto-plastic, Failure and Shear resistance characteristics. In the subsequent section, these characteristics will be discussed .

Murayama & Shibata (1958, 1961, 1964) based on creep tests on highly overconsolidated clays at different deviatoric stresses, had pointed out that if the stress applied is less than a set level of stress known as lower yield stress, the soil behaves elastically and if it is in between the lower and upper yield stresses (a higher level of stress), the soil behaves visco-elastically. A model consisting of a combination of an elastic linear spring and a modified Voigt model with three elements (Fig.1) was proposed. This model has been shown to predict the creep and stress relaxation phenomenon in highly overconsolidated clay as long as the stress is in the elastic range (Murayama, 1961).

The existence of failure strength was reported by Adachi and Okano (1974). The behavior of soils under a stress between the upper yield stress and the failure stress is considered to be visco-elastic and strain hardening. Perzyna (1963, 1971) pointed out the difference of the dynamic and static behavior of materials and define this rate sensitive behavior as visco-plastic. Adachi and Okano (1974) by combining the Roscoe's theory (Roscoe and Burland, 1968) with Perzyna's theory and introducing some experimental evidence, developed visco-elasto-plastic constitutive equations for normally consolidated clays. Sekiguchi (1977) based on time dependency of volumetric strain had developed constitutive relationships for normally consolidated clays through a visco-elastic potential. Singh and Mitchell (1968) developed the following equation, which describes satisfactorily the creep characteristics of various kinds of soils under a stress in the range of 30% to 90% of the strength.

$$\dot{\varepsilon} = A \exp(\alpha\sigma)(t_1/t)^m$$

where $\dot{\varepsilon}$ = Axial strain rate

σ = Constant deviatoric stress used for creep tests.

t = Time

t_1 = Unit time e.g. 1 minute.

A, α and m = Parameters for a given soil.

Murayama and Shibata (1958, 1961) obtained a relationship that can predict the loading time necessary to cause creep failure under undrained condition based upon the theory of rate process.

$$\log_{10} t_{of} = \log_{10}(h/KT) + (Eo/2.3KT) - (\lambda\alpha/4.6N_{bo}KT)$$

where

Eo, λ and N_{bo} = parameters of soil,

h and K = physical constants, and

T = absolute temperature.

Saito and Uezawa (1961) found experimentally the following relationship between creep strain and time period t

$$t\dot{\varepsilon} = \text{constant}$$

Similar relationship was found experimentally by Murayama et al. (1970). They reported that the above relationship is valid for any period from the time of loading to the beginning of steady stage creep, to the beginning of accelerating creep or to the creep failure.

With respect to the frictional and viscous resistances in soil, Mitchell et al. (1969) studied the mechanism of shear resistance generated in the soil based upon the Rate Process Theory and Adhesion theory of friction.

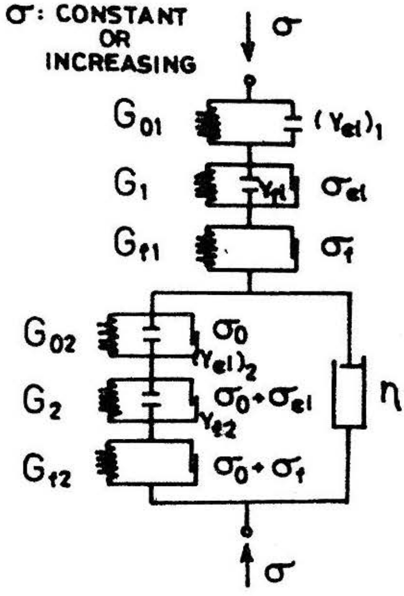

FIGURE 2 : Rheological Model for Sands. (After Murayama, 1983)

Though the Rate Process Theory has not been proven to be correct, it has been shown (Mitchell et al., 1968) that some soils behave in general conformity with the theory, provided variations of soil structure are taken into account.

It is clear from the above review that there are two kinds of approaches for constitutive equations. One is to derive constitutive equations from visco-elasto-plastic potential and the other is to obtain relationships between shearing strain and the deviatoric stress.

These earlier studies of rheological behavior of soils have been mostly on clays and later, they are generalized in sands, as sands have also shown the same rheological characteristics. However, in sands, no physico-chemical bonding forces exist at junction of particles, which is very much pronounced in clayey soils. Thus, for sands, the effective bonding stress is zero (Murayama, 1969). So, one can not apply the same approach for sands as for clays. Thus, in order to explain the creep characteristics of sands, Murayama (1983) has proposed a new rheological model. The detail of this model is given elsewhere. Now, considering the two possible cases:

1. When the applied stress difference is greater than but smaller than or equal to elastic limit stress σ_{el}, then the strain behavior of sand is visco-elastic and shearing strain can be expressed by

$$\log\gamma = A_o + B_o \log t_o \tag{1}$$

where γ = Shearing Strain

A_o and B_o = Intercepts and slopes respectively

t_o = Time

2. When the applied stress difference is between elastic stress (σ_{el}) and failure stress (σ_f), then there are two possible cases:

(a) When time at the beginning of creep is less than or equal to time at which stress is less than or equal to elastic stress (σ_{el}) i.e. $t_o < t_{el}$ or $\gamma_2 < (\gamma_{el})_2$, then

$$\log\gamma = A_p + B_p \log t \tag{2}$$

where $t = t_o - t_{el}$

(b) When time at the beginning of creep is less than or equal to time at which stress is greater than or equal to elastic stress (σ_{el}) i.e. $t_o > t_{el}$ or $\gamma_2 > (\gamma_{el})_2$, then

$$\log\gamma = A_1 + B_1 \log t \tag{3}$$

where $t = t_o - t_{el}$

It is evident from Eqns. 1, 2 and 3 that the $\log\gamma - \log t$ relationship is expressed by the straight line of Eqn. 1 from the beginning of creep until t_{el}, but that after t_{el} it is expressed by the straight line of Eqn. 2 or 3, whose inclination is different from that of the former.

Test Material

Silica sand obtained from local dunes of eastern province of Saudi Arabia was the main source for the test performed. The grain size distribution of the selected sand is shown in Fig.3. The properties of the sand were as follows: effective diameter (D_{10}) = 0.149; Medium grain size (D_{50}) = 0.250; Uniformity coefficient (C_U) = 3.0 and coefficient of curvature (C_c) = 0.422. The sand may be described as uniform based on grain size distribution. The maximum and minimum densities were determined by the procedure recommended by ASTMSTP No. 523(1971) and found to be 1.83 gm/cm^3 and 1.56 gm/cm^3.

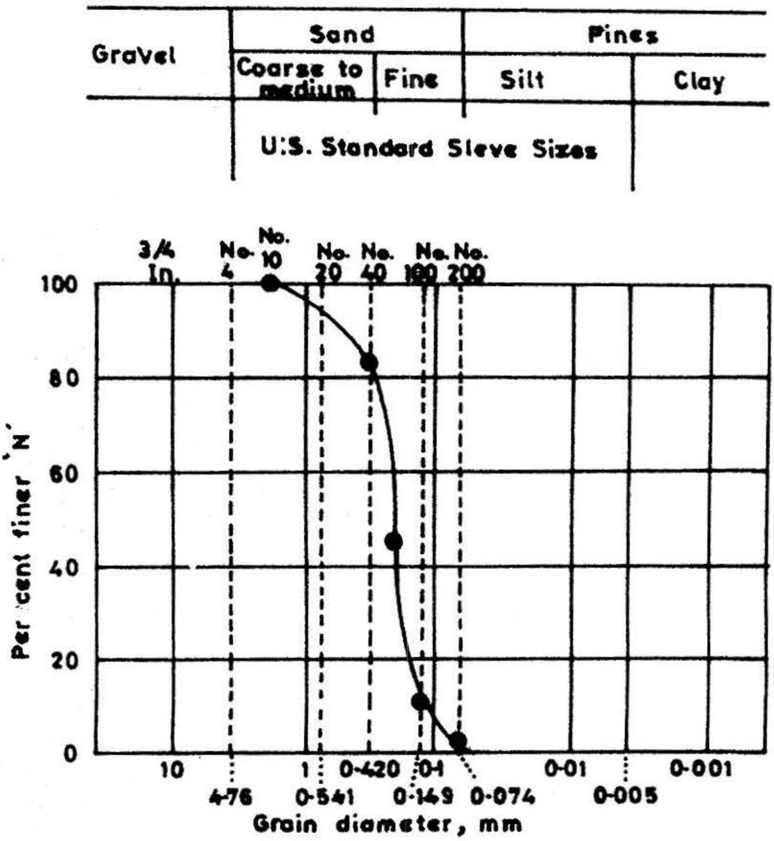

FIGURE 3 : Grain Size Distribution Curve

Sample Preparation and Experimental Setup

Based on the study performed by Mulilis et al. (1975), it was decided to prepare the sand specimens for drained triaxial creep tests at different relative densities by pluviation compaction through air technique. Dune sand specimens for drained creep test were prepared by the above technique, so that all specimens could attain a relative density of 85 ± 2 percent. Confining pressure applied using pressured water, was gradually introduced to the cell with the slow removal of vacuum applied during sample preparation. A back pressure of 310 kPa applied from bottom drainage line of the triaxial cell, matched equally with a confining pressure on the outside of the specimen, preventing any differential stress, to ensure full saturation of the test specimen. After ensuring the full saturation (pore pressure parameter B $\geq$ 95%), the sample was allowed to consolidate under the selected confining pressure for about one hour by opening the top drainage line.

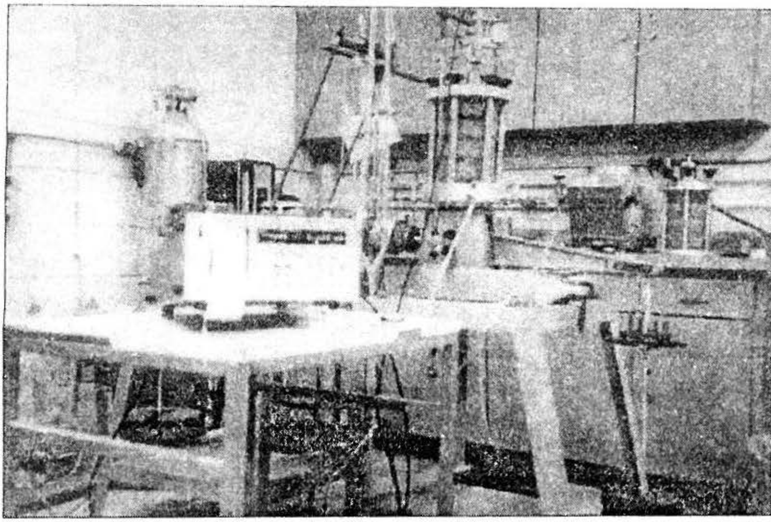

PLATE 1 : Creep Test Setup

The creep testing system (CTS) consisted of a table top consolidation device that has been converted to accommodate 70 mm, triaxial cell (Plate 1). A lever arm ratio of ten to one permits application of larger loads, using relatively small dead weight with no shock loading on the sample. Cell pressure was measured by a pressure gauge of precision 0.68 kN/m. The volumetric change of the specimen was measured by the volume of the water drained into a burette with 0.02 graduation and axial strain was measured by 0.002 mm deformation gauge. For high-pressure tests, a transducer (Model - LDP-10 B; Make - Tokyo Sokki Kenkyujo Co. Ltd., Japan) with a precision of 0.001 mm was used. A 10-channel data logger system (Model - TDS-301; Make - Tokyo Sokki Kenkyujo Co Ltd., Japan) was used to record creep strain under high stress test.

Creep tests were run on saturated sand specimen. In these tests, in order to analyze the creep behavior due to principal stress difference (σ'), the mean principal stress (σ'_m) was kept constant for all the tests of the same test series, where

$$\sigma'_m = (1/3)(\sigma'_1 + 2\sigma'_3) \qquad \text{and} \qquad \sigma' = (\sigma'_1 - \sigma'_3)$$

The maximum shearing strain (γ) was calculated using the following formula

$$\gamma = \varepsilon_1 - \varepsilon_3$$

where $\varepsilon_3 = (1/2)(\varepsilon_v - \varepsilon_1)$ = radial strain,

ε_1 = axial strain, and

ε_v = volumetric strain.

Three different relative densities are used in this study e.g. 60%, 75% and 85%. Drained triaxial compression creep tests were performed under different constant mean effective principal stress (σ'_m) for a period ranging from few minutes to several days, depending upon the applied stress level. In these tests, samples are consolidated under a constant confining pressure referred here as the precompression stress (σ'_{pc}) and then tested at constant mean effective pressure (σ'_m), keeping confining pressure constant during the test.

Test Results and Discussions

The logarithmic relationships between shearing strain (γ') and time t_0 at relative density (D_r = 60%), under various intensities of principal stress (σ') and mean effective principal stress (σ'_m) of 1.3 kgf/cm^2 are shown in Fig.4. Similar relationships at relative densities of 75% and 85% are obtained. These relationships are in accordance with the relationships proposed vide Eqns. 1, 2 and 3. Similar relationships are also observed at higher mean effective principal stress (σ'_m) i.e. 1.5 kgf/cm^2 and 2.0 kgf/cm^2 at relative densities of 60%, 75% and 85%. Linear relationships are observed between intercepts obtained from above mentioned relationships and principal stress (σ'), as appended here as Fig.5. The variation of creep strain rate

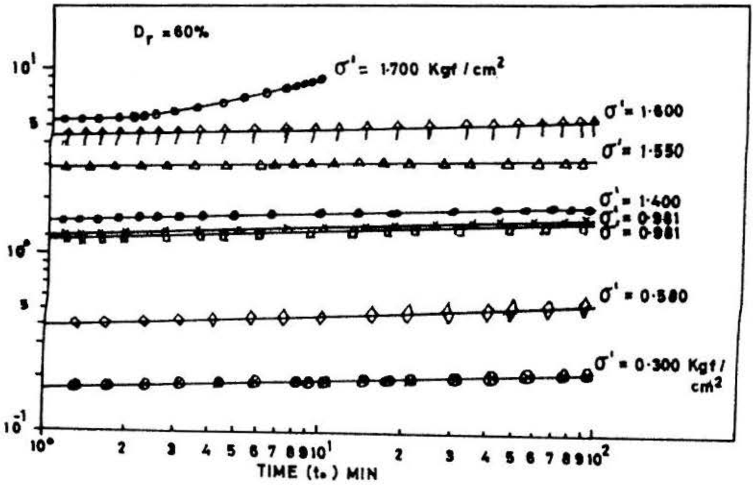

FIGURE 4 : $\log\gamma - \log t_o$ Relationships under Various Levels of Stress Difference at D_r = 60% and = 1.3 kgf/cm^2

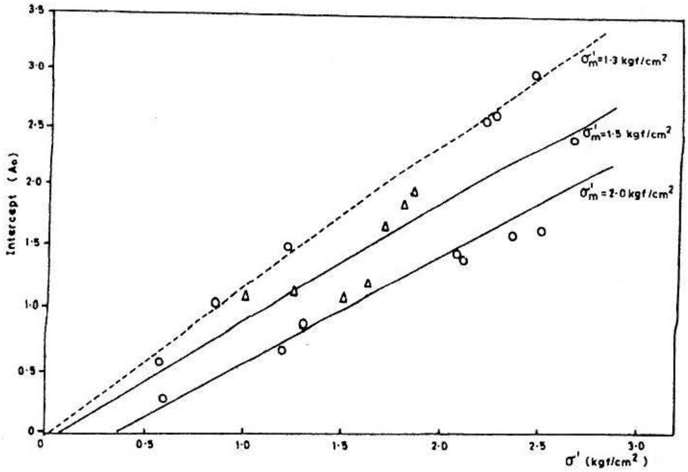

FIGURE 5 : Relationship Between Intercept and Deviatoric Stress at D_r = 60%

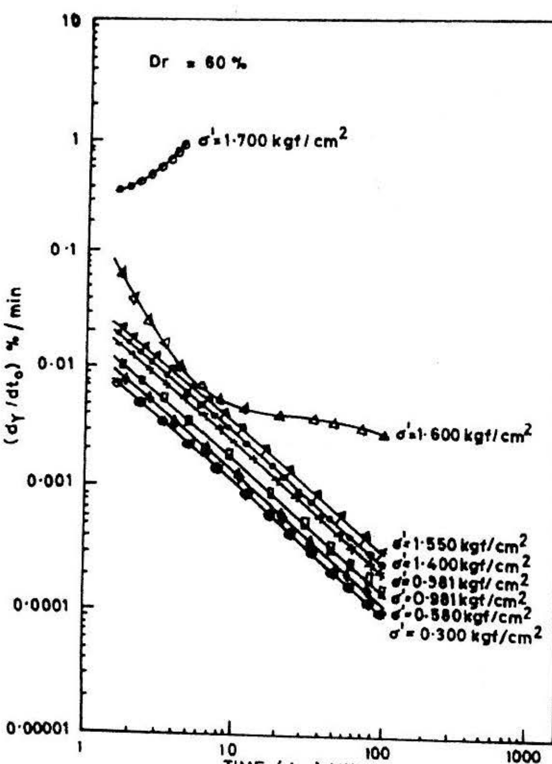

FIGURE 6 : $\log(d\gamma/dt_o) - \log t_o$ **Relationships under Various Levels of Stress Difference at D_r = 60% and σ'_m = 1.3 kgf/cm²**

with time under various principal stress (σ'), at mean effective stress (σ'_m) of 1.3 kgf/cm^2 is shown in Fig.6. It is observed that this relation is expressed by parallel lines, except at principal stress (σ') exceeding the critical stress. In this case, the descending curves are in a concave downward shape. The uppermost curves in these figures have a distinct point of minimum creep strain rates and after passing the minimum points, they rise at an accelerating rate towards creep failure. Similar trends were observed at higher mean effective stresses (σ'_m) at relative densities such as 60%, 75% and 85%. Similar behavior is reported by Murayama et al. (1984) as reproduced in Fig.7.

There are two critical limit stresses for sands. One is the elastic limit stress (σ_{el}), which is defined as the stress up to which sand behaves elastically and the other stress is failure stress (σ_f). The concept of flow diagram (Murayama et al., 1984) is used for finding the lower yield stress (σ_o) and elastic limit stress (σ_{el}) at a certain elapsed time. Fig.8 shows the

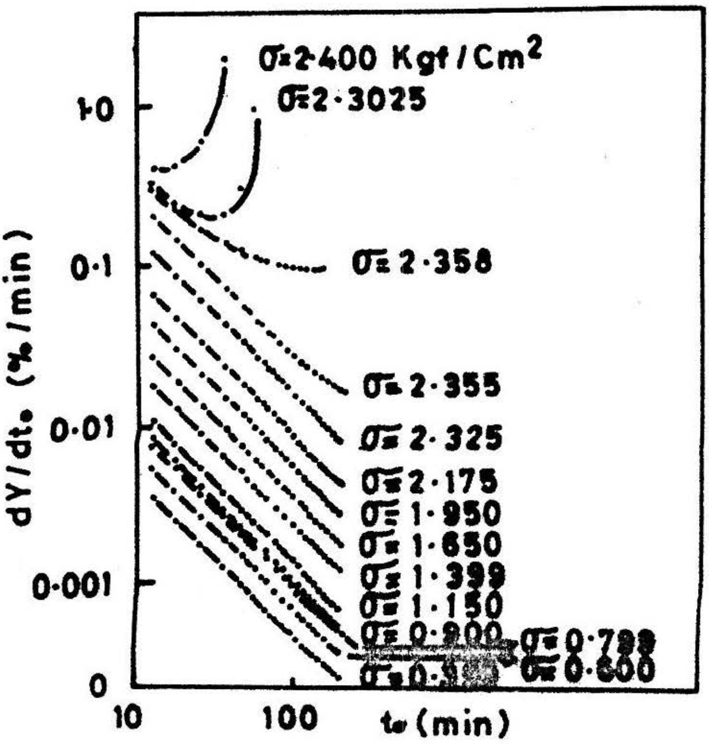

FIGURE 7 : $\log(d\gamma/dt_o) - \log t_o$ Relationships under Various Levels of Stress Difference at D_r = 60% and σ'_m = 1.3 kgf/cm^2 (After Muryama et al., 1984)

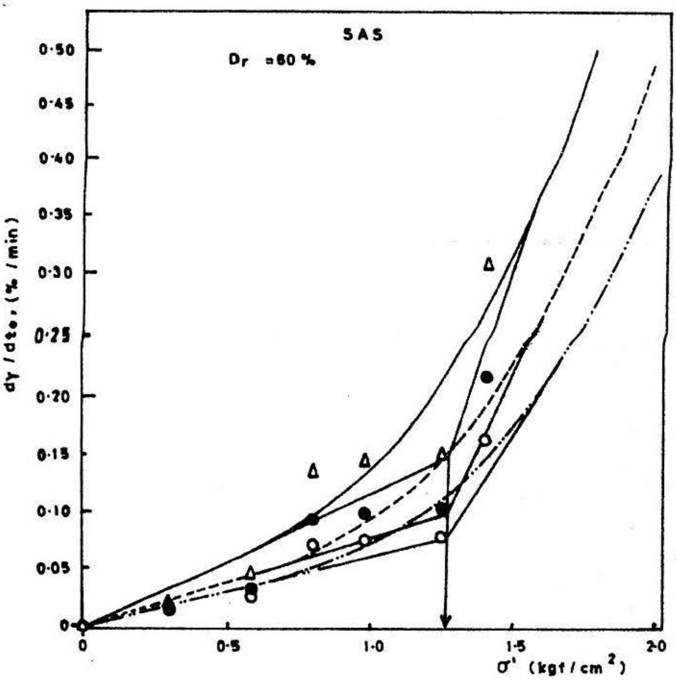

FIGURE 8 : $\log(d\gamma/dt_o) - \sigma'$ Relationships at D_r = 60% and σ'_m = 1.3 kgf/cm^2

TABLE 1 : Failure and Elastic Stress of Soil at Different Density

S.No.	Relative Density D_r %	Mean effective Pressure, (σ'_m) kgf/cm^2	Elastic limit stress (σ_{el}) kgf/cm^2	Failure stress (σ_f) kgf/cm^2
1	60	1.30	1.25	1.55
		1.50	1.25	1.70
		2.00	1.25	1.90
2	75	1.30	1.55	1.86
		1.50	1.55	2.25
		2.00	1.55	2.47
3	85	1.30	1.65	2.18
		1.50	1.65	2.55
		2.00	1.65	2.65

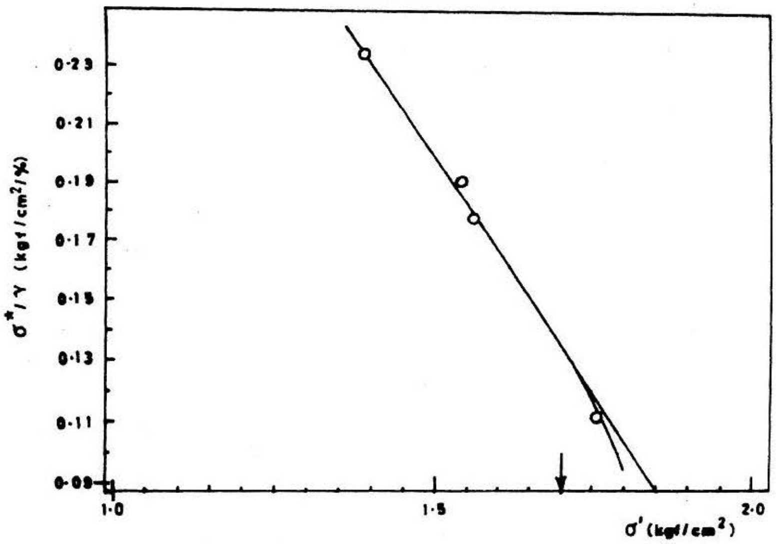

FIGURE 9 : Relationship between Stress-Strain Ratio $\left(\sigma^{*}/\gamma\right)$ and σ' at D_r = 60% and σ'_m = 1.3 kgf/cm^2 [Here, $\sigma^{*} = \sigma - \sigma_{el}$]

flow diagram at mean effective stress $\left(\sigma'_m\right)$ of 1.3 kgf/cm^2 at 60% relative density. Values of elastic limit stress at different relative densities are tabulated in Table 1.

A plot between stress-strain ratio $\left[\left(\sigma - \sigma_{el}\right)/\gamma\right]$ and principal stress (σ') shows a linear relationship as depicted in Fig.9. Thus, the failure strength $\left(\sigma_f\right)$ is determined as the inflection point where the linearity of relationship deviates (Murayama et al., 1984). Using the above mentioned approach, failure stress values are found out at different relative densities (Table 1.)

A linear relationship is observed between elastic limit stress $\left(\sigma_{el}\right)$ and relative density (D_r) as shown in Fig.10. Similarly, a linear relationship between mean effective principal stress $\left(\sigma'_m\right)$ and failure stress $\left(\sigma_f\right)$ is shown in Fig.11. The above relationship may be represented by equation

$$\sigma_f = P + \sigma'_m \tan Q$$

where P and Q are intercept and slope respectively. These parameters depend upon various properties such as gradation, soil density, and condition of laboratory testing or loading in the field. However, the above mentioned relationship needs further investigation. It is found that slopes of these curves increase with the relative density. This may be due to fact that density contributes hardness, which is responsible for an increase in failure stress $\left(\sigma_f\right)$.

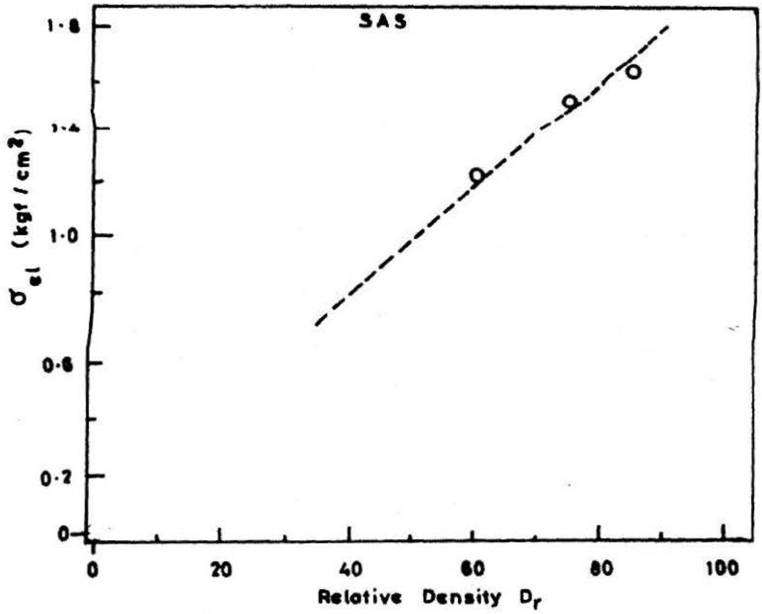

FIGURE 10 : Relationship between Relative Density (D_r) and Elastic Limit Stress (σ_{el})

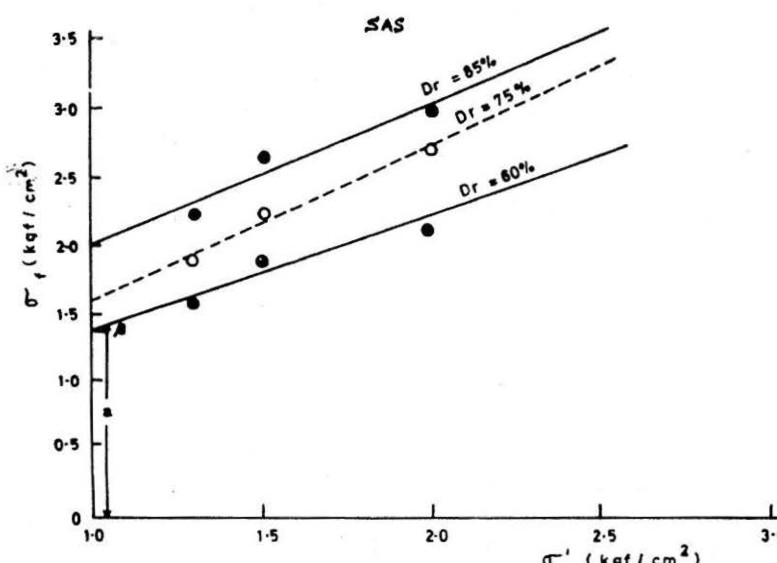

FIGURE 11 : Relationship between Failure Stress (σ_f) and Mean Effective Pressure (σ'_m) at Different Relative Density (D_r)

Conclusion

Based on this study, the following conclusions are drawn:

1. The creep behavior of tested dune sand may be represented by the following logarithmic relationships:

$$\log\gamma = A_p + B_p \log t \qquad (\sigma' < \sigma_{el})$$

$$\log\gamma = A_l + B_l \log t \qquad (\sigma_{el} < \sigma' < \sigma_r)$$

where $t = t_o - t_{el}$

The intercept of above relationships appears to increase linearly with deviatoric stress but decreases with mean effective stress.

2. The creep strain rate is nominal up to a specific stress ratio, beyond which it increases significantly.

3. The rheological model proposed by Murayama (1983) appears to be valid for dune sands.

4. It is found that elastic limit stress (σ_{el}) and failure stress (σ_f) increase linearly with relative density (D_r).

5. A linear relationship found between failure stress (σ_f) and mean effective pressure (σ'_m), is given by:

$$\sigma_f = P + \sigma'_m \tan Q$$

where P and Q are intercept and slope respectively.

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Notation

ε_1	=	Axial strain
ε_v	=	Volumetric strain
ε_3	=	Radial strain.
γ	=	Shearing Strain
t_o	=	Time
σ'_m	=	Mean principal stress
σ'	=	Effective Stress
σ_{el}	=	Elastic limit stress
σ_f	=	Failure stress
B	=	Skempton pore pressure parameter
D_r	=	Relative Density