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A Unified Theory for Ultimate Pullout Capacity of Batter Piles

Adel Hanna* and M.R. Madhav†

Introduction

Meyerhof and Adams (1968) report a comprehensive study on uplift capacity of foundations. Hanna (1973) and Das (1990) study the influence of anchor inclination on the pull out capacity while Meyerhof (1973) reports experimental and theoretical study results on the axial pullout capacity of vertical and batter piles and anchors. Neglecting the self-weight of the pile, the net pullout resistance, Q_u, of a batter pile is expressed as:

$$Q_u = \left(c_a + \sigma_0' \cdot K_u \cdot \tan\delta\right) \cdot A_s \tag{1}$$

where K_u = uplift coefficient,

σ_0' = the average effective overburden stress,

c_a = adhesion angle

δ = wall friction angle and

A_s = pile surface area.

Meyerhof (1973 and 1982) reported that for a given angle of shearing restistance (ϕ') of the soil, the values of K_u do not differ much for moderate pile inclinations ($0 \leq \alpha \leq 40°$) where a is the inclination of the pile from the vertical. The axial uplift capacity of model piles increases with pile

* Department of Civil Engrg., Concordia University, 1455 de Maisonneuve Blvd. W., Montreal, Quebec, H3G IM8, Canada. E-mail: hanna@civil.concordia.ca

† Professor, Department of Civil Engineering, Indian Institute of Technology, Kanpur – 208016, India. E-mail: madhav@iitk.ac.in

inclination, α. The results of tests reported by Awad and Ayoub (1975) indicate however a decrease in axial pullout capacity with increasing piles inclination, α. An empirical relation suggested for the uplift coefficient, is

$$\mu_{\alpha} = \frac{Q_{u\alpha}}{Q_{u0}} = \frac{\cos\alpha}{(\cos\alpha + \tan\alpha)} \tag{2}$$

where μ_{α} is the ratio of the net axial pullout capacity of a batter pile, $Q_{u\alpha}$, with respect to that of a vertical pile, Q_{u0}.

Adams and Klym (1972) report full-scale test results for vertical and inclined circular anchors in silty sand, and model pile test results in silica sand. From their results, the axial pullout capacity for inclined piles is observed to decrease with an increase of the inclination angle, α. Results of Hanna and Afram (1986) show small variations but near constancy of the pullout capacity with batter angle up to 30°. The following empirical relation has been proposed:

$$\mu_{\alpha} = \cos(\alpha/2) \tag{3}$$

Meyerhof and Yalcin (1993) and Yalcin and Meyerhof (1994) present studies on displacements of rigid and flexible batter piles in layered soils under eccentric and inclined loads.

From the test results reported above, the following three trends of variations of the pullout capacity with pile inclination, α, are observed:

1. The pullout capacity increases with the inclination angle, α, of the pile (Meyerhof, 1973);

2. The pullout capacity decreases with increasing inclination of pile (Adams and Klym, 1972; Awad and Ayoub, 1976); and

3. The pullout capacity is a nearly constant or decreases only slightly with increasing pile inclination (Meyerhof, 1982; Hanna and Afram, 1986). A unified theory to account for these three possible variations noted above is presented in the next section.

Analysis

Figure la features an inclined pile of diameter, d, length, L, and inclined at an angle, α, with respect to the vertical. The axial pullout capacity, $Q^{*}_{u\alpha}$, can be expressed as:

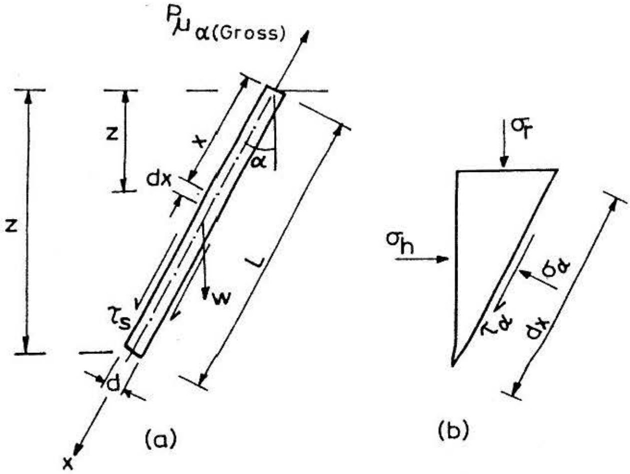

FIGURE 1 : (a) Batter Pile Subject to Axial Pullout Load and (b) Element along Pile Surface

$$Q^*_{u\alpha}(\text{gross}) = Q_{u\alpha} + W \cdot \cos\alpha \tag{4}$$

where $Q_{u\alpha}$ is the net axial pullout capacity due to shaft resistance and W is the weight of the pile. $Q_{u\alpha}$ is in turn expressed as:

$$Q_{u\alpha} = \int_0^L p_s \cdot \pi d \cdot dx \tag{5}$$

where p_s is the unit shaft resistance and x is the distance along the pile. In the case of cohesive soil,

$$p_s = c_a \tag{6}$$

where c_a is the adhesion between the pile and the soil material. Adhesion $c_a = \alpha' \cdot c_u$ where α' is the adhesion coefficient and c_u is the undrained strength of saturated soil. For anisotropic soils, adhesion, $c_{a\alpha}$, along a direction α from the vertical can be expressed as

$$c_{a\alpha} = c_{av} \cdot \cos^2\alpha + c_{ah} \cdot \sin^2\alpha$$

or $$c_{a\alpha} = c_{av}\left(\cos^2\alpha + K \cdot \sin^2\alpha\right) \tag{7}$$

where $K = c_{ah}/c_{av} = c_{uh}/c_{uv}$ is the anisotropic strength ratio in which c_{ah}

and c_{av} and c_{uh} and c_{uv} are the adhesions and undrained shear strengths along vertical and horizontal directions respectively. Combining Eqns. (2) and (5) through (7), the ratio of pullout capacity of a batter pile to that of a vertical pile, μ_α, is

$$\mu_\alpha = \frac{Q_{u\alpha}}{Q_{u0}} = \left(\cos^2\alpha + K\sin^2\alpha\right) \tag{8}$$

For normally consolidated soils

$$c_u = \beta \cdot z \tag{9}$$

where β is a constant and z is the vertical depth from the ground level. Substituting $c_a = \alpha'\beta z = \alpha'\beta \cdot x \cdot \cos\alpha$, in Eqn. 7, one gets

$$c_{a\alpha} = \alpha'\beta \cdot x \cdot \cos\alpha \cdot \left(\cos^2\alpha + K\sin^2\alpha\right) \tag{10}$$

Eqn. 8 becomes

$$\mu_\alpha = \cos\alpha \cdot \left(\cos^2\alpha + K\sin^2\alpha\right) \tag{11}$$

In case of cohesionless soil, the initial in situ state of stress at a point at a depth, z, from ground level, is

$$\sigma'_v = \gamma' \cdot z \tag{12}$$

and $$\sigma'_h = K_0 \cdot \sigma'_v \tag{13}$$

where σ'_v = in situ vertical effective stresses,

σ'_h = in situ horizontal effective stresses,

γ' = is the effective unit weight of the soil, and

K_0 = coefficient of lateral earth pressure at rest of the soil.

Along a plane inclined at an angle, α, from the vertical (Fig. 1b), the normal, σ'_α and shear, τ_α stresses are given by

$$\alpha'_\alpha = \alpha'_z \sin^2\alpha + \alpha'_h \cos^2\alpha \tag{14}$$

and $$\tau_\alpha = \left(\sigma'_z - \sigma'_h\right)\sin\alpha \cdot \cos\alpha \tag{15}$$

Substituting Eqn.(12) in Eqns. (13), (14) and (15), one gets

$$\sigma'_{\alpha} = \gamma' \cdot z\left(\sin^2\alpha + K_0 \cos^2\alpha\right) \tag{16}$$

and $$\tau_{\alpha} = \gamma' \cdot z\left(1 - K_0\right)\sin\alpha \cdot \cos\alpha \tag{17}$$

Driving of displacement piles especially into granular soils could increase the normal stress on the pile shaft due to densification. Yalcin and Meyerhof (1994) report an increase in the soil moduli due to compaction or prestressing effect of driven piles. *The post installation normal stress,* $\sigma'_{\alpha f}$, is expressed as

$$\alpha'_{\alpha f} = f \cdot \alpha'_{\alpha} \tag{18}$$

where f is a multiplication factor for the densification effect. The maximum shear resistance that can be developed along the pile shaft, is

$$\tau_{\alpha \cdot max.} = \alpha'_{\alpha f} \cdot \tan\delta \tag{19}$$

However, the available shaft resistance, τ_{av}, is the difference between the maximum resistance and the initial shear stress, τ_{α}, i.e.

$$\tau_{av} = \tau_{\alpha.max.} - \tau_{\alpha} = \alpha'_{\alpha f} \cdot \tan\delta - \tau_{\alpha} \tag{20}$$

Combining Eqns. (5), (16), (17) and (20) and noting that $z = x\cos\alpha$,

$$Q_{u\alpha} = \int_0^L \pi d \cdot \left\{ \begin{matrix} f\left(\sin^2\alpha + K_0\cos^2\alpha\right)\tan\delta \\ -\left(1-K_0\right)\sin\alpha\cdot\cos\alpha \end{matrix} \right\} \gamma' x \cdot \cos\alpha \cdot dx \tag{21}$$

or $$Q_{u\alpha} = \pi d \cdot \left(\gamma' \frac{L^2}{2}\right) \left\{ \begin{matrix} f\left(\sin^2\alpha + K_0\cos^2\alpha\right)\tan\delta \\ -\left(1-K_0\right)\sin\alpha\cdot\cos\alpha \end{matrix} \right\} \tag{22}$$

If $\alpha = 0$, the net uplift capacity, $Q_{u0,}$ of a vertical pile is obtained as

$$Q_{u0} = \pi d \cdot \left(\gamma' \frac{L^2}{2}\right) \cdot K_0 \cdot f \cdot \tan\delta \tag{23}$$

It is noted that $K_0 \cdot f = K_u$, the lateral uplift coefficient proposed by Meyerhof (1973). The ratio, μ_{α}, is

$$\mu_{\alpha} = \frac{\left\{f\left(\sin^2\alpha + K_0\cos^2\alpha\right)\cdot\tan\delta - \left(1-K_0\right)\sin\alpha\cos\alpha\right\}\cos\alpha}{f\cdot K_0\tan\delta} \tag{24}$$

Thus the pullout capacity of an inclined pile can be estimated knowing the pullout capacity for a vertical pile and the factor, μ_{α}, provided the soil conditions are identical for the two cases.

Results

A parametric study has been carried out for the following ranges of values of the parameters:

$K = 0.3$ to 4.5;
$K_0 = 0.5$ to 2.0;

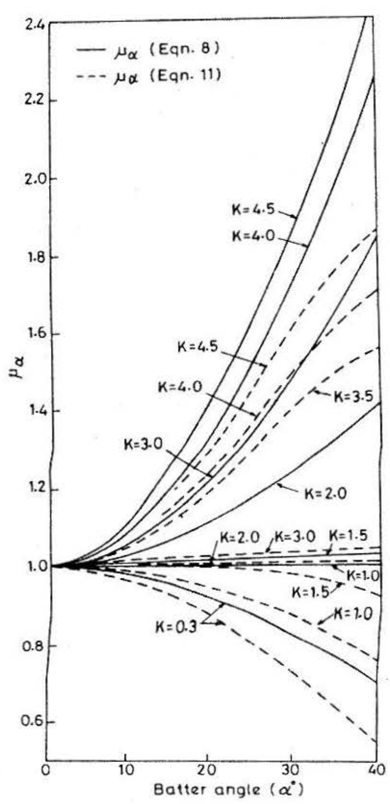

FIGURE 2 : Variation of μ_{α} with Batter Angle, α, for Cohesive Soils

$$\delta = 5^\circ \text{ to } 35^\circ;$$
$$\alpha = 0^\circ \text{ to } 40^\circ \text{ and}$$
$$f = 1.0 \text{ to } 3.0.$$

The variation of μ_α with pile inclination, α, is depicted in Fig. 2 for cohesive soils and in Figs. 3 through 5 for cohesionless soils. f = 1.0 implies no change in the state of stress in the soil due to pile driving (ideal bored pile). From these figures it may be noted that variation of μ_α with α, exhibits the three types of variations mentioned earlier, viz.

(i) The axial pullout capacity decreases *uniformly with increasing pile* inclination for low K_0 values (K_0: 0.5 - 1.0);

(ii) the axial pullout capacity is nearly constant (it increases slightly for $\alpha < 22.5^\circ$ and decreases for $\alpha > 22.5^\circ$ for medium dense or lightly over consolidated soils (K_0: 1.0 - 1.25);

(iii) the axial pullout capacity increases with a for $\alpha < 25^\circ$ in case of very dense or highly over-consolidated soils (K_0: 1.5 - 2.0).

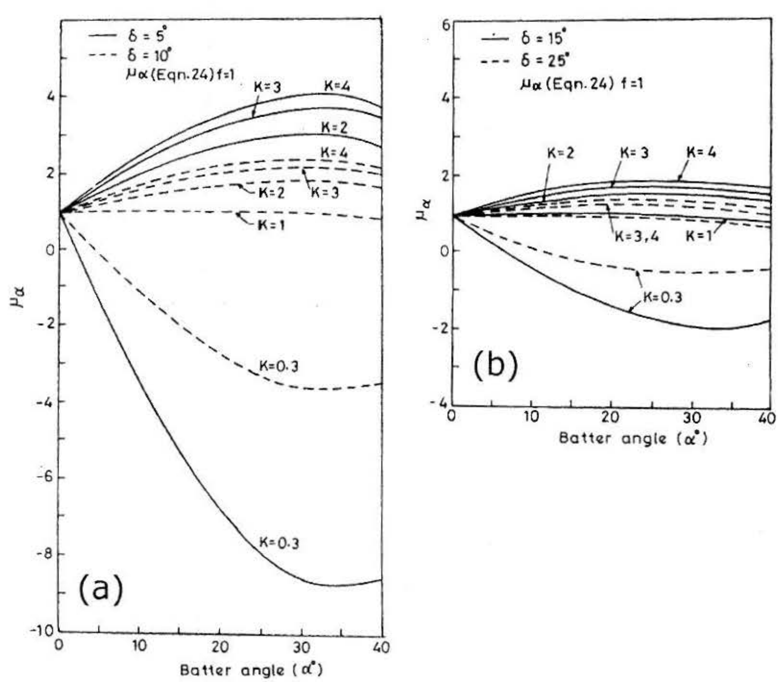

FIGURE 3 : Variation of μ_α with Batter Angle, α, for Cohesionless Soils: f = 1

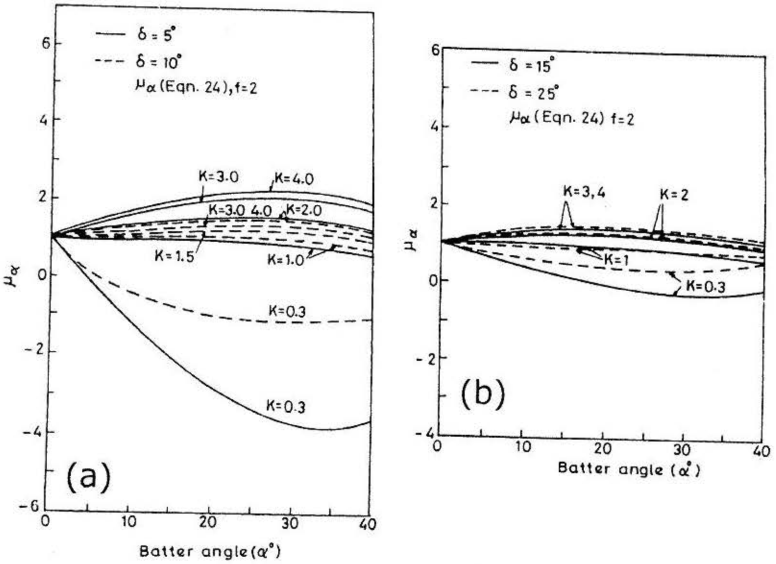

FIGURE 4 : Variation of μ_α with Batter Angle, α, for Cohesionless Soils: f = 2

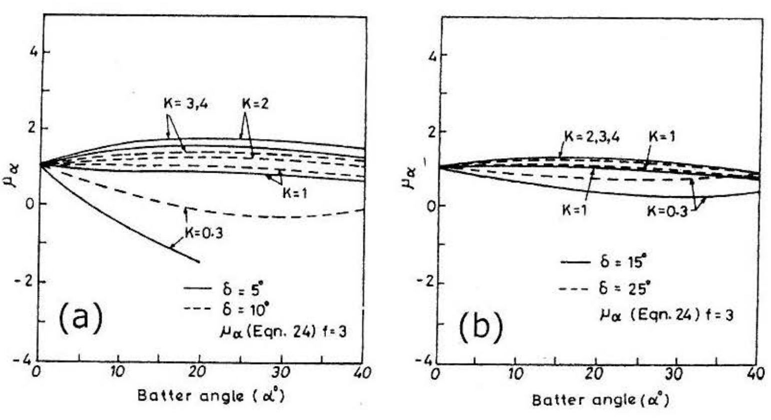

FIGURE 5 : Variation of μ_α with Batter Angle, α, for Cohesionless Soils: f = 3

Thus the three characteristic variations reported in literature are possible and are predicted by the unified theory and analysis presented above. The apparent contradictory variations can be explained if the effects of length or depth up to which a pile is driven and the in situ initial shear stresses in the soil are considered. In normally consolidated soils and cohesionless soils, the stresses and the mobilised resistance increase with depth. For a given length of a pile, a vertical pile penetrates to greater depths than those for batter piles. Furthermore the vertical depth to a point on an inclined pile decreases with

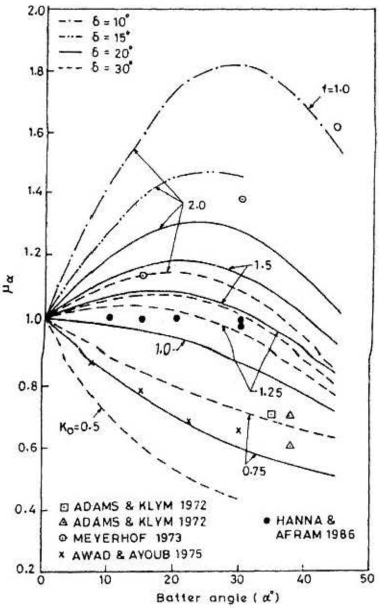

FIGURE 6 : Comparison of Experimental and Predicted Values of the Coefficient, μ_α

increasing pile inclination (Fig. 1a). Thus the effect of inclination for constant length of piles is to reduce the pullout capacity of batter piles.

The shear stresses on a plane inclined at an angle, α, are dependent on the inclination angle, α and the earth pressure at rest K_0. Further low K_0 values induce near failure stresses and hence, very little reserve strength would be available. On the contrary, high K_0 values, cause negative shear stresses, an effect similar to prestressing. A large reserve of shear resistance becomes available in very dense or heavily over-consolidated soils for which K_0 can be high. A combination of these two effects leads to the three types of variations observed in practice and predicted in this paper. It may be noted from Figs. 3 through 5, that the parameter, δ, the wall friction angle, has significant effect on μ_α values. For $K_0 < 1.0$, μ_α increases with increasing δ values. For example, for $K_0 = 0.5$, f = 1.0 and $\alpha = 30°$, μ_α values are 0.35 and 0.54, for δ equal to 20° and 30° respectively. If $K_0 = 2.0$, μ_α values decrease from 1.78 for $\delta = 5°$ to 0.87 for $\delta = 30°$. The effect of densification i.e. the factor, f, is to minimise the effects of all the other parameters. Thus the curves given in Fig. 5 are much closer to each other compared to those given in Figs. 3 and 4. The larger the value of 'f', the higher the strengths mobilised in both the cases of inclined and vertical piles. Consequently, the ratio m varies within a small range.

Comparisons with Test Results

A qualitative comparison of the test results reported in literature with the predictions based on the theory for cohesionless soils presented herein is made in Fig. 6. If the values of K_0, δ, and f are known, it would be possible to make more exact comparisons. Adams and Klym (1972) test grouted anchors in silty sand. The results are close to the theoretical values corresponding to $f = 1.0$, $K_0 = 0.75$ and $\delta = 30°$ – a likely set of values. Awad and Ayoub (1975) have used pipe piles in a soil with a relative density of only 17%. Their results lie close to a curve that corresponds to $f = 1.0$, $K_0 = 0.75$ and $\delta = 20°$. The results of Hanna and Afram (1986) correspond to a medium dense sand with $\phi = 39°$. Their results lie within the range of $K_0 = 2.0$ and $\delta = 20$ to $30°$. The high K_0 value is consistent with a value of $\phi = 43°$ reported by the authors.

Design Procedure

For the application of Eqns. 8, 11 and 24, the values of K_0 and f are needed. For normally consolidated soils, K_0 can be evaluated (Mayne and Kulhawy, 1982; Hanna and Ghaly, 1992) as

$$K_0 = (1 - \sin\phi') \tag{25}$$

where ϕ' is the angle of shearing resistance of the soil while for overconsolidated soils,

$$K_0 = (1 - \sin\phi) \cdot OCR^{\sin\phi'} \tag{26}$$

Values of K_0 may also be estimated from in situ tests such as pressuremeter test, modified Glotz earth pressure cell, dilatometer test or hydraulic fracture test (Wroth, 1975, Massarsch et al., 1975). The range of values of f is 1 to 3 and is relatively small. For non-displacement piles, $f = 1.0$, while for displacement piles in loose sands, $f = 3.0$, based on Meyerhof's K_u value ($K_u = f \cdot K_0$). For displacement piles in medium dense to dense sands $f = 1.0$. Knowing the values of K_0 and f value of μ_α can be determined from Figs. 2 to 5. Thus the pullout capacity of batter pile is defined in terms of the pullout capacity of the vertical pile and the coefficient, μ_α.

Conclusions

A new unified theory is developed to predict the pullout capacity of single batter piles as compared to that of the vertical one. The analysis accounts for the initial stress condition existing in soil prior to driving the pile in to granular soils and an-isotropic undrained strength and its variation with depth for cohesive soils. For the values of K_0 less than one, the pullout capacity decreases with increasing inclination angle, α, of a batter pile while it increases for K_0 greater than one thus explaining the contrasting trends reported in the literature.

Acknowledgement

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Notation

A_s	=	Shaft surface area
c_a	=	Adhesion between the pile and soil
c_{ah}, c_{av} and $c_{a\alpha}$	=	Adhesion along horizontal, vertical and along the pile
c_u	=	Undrained strength of the soil
c_{uh} and c_{uv}	=	Undrained strength along horizontal and vertical directions
d	=	Diameter of pile
f	=	Factor accounting for densification effect
K	=	$c_{ah}/c_{av} = c_{uh}/c_{uv}$ = Anisotropic strength ratio
K_o	=	Coefficient of earth pressure at rest
K_u	=	Meyerhof's earth pressure coefficient
L	=	Length of pile
OCR	=	Overconsolidation ratio
p_s	=	Unit shaft resistance
Q_u	=	Axial pullout capacity of a pile
Q_{u0} and $Q_{u\alpha}$	=	Axial capacity of vertical and batter piles
x	=	Distance along the pile
z	=	Depth from the ground level
α	=	Inclination of pile with vertical or batter angle
α'	=	c_a/c_u = Adhesion factor
β	=	Rate of increase of undrained strength with depth
δ	=	Wall friction angle
ϕ'	=	Angle of shearing resistance of soil
γ	=	Unit weight of the soil
μ_α	=	$Q_{u\alpha}/Q_{u0}$ = Ratio of pullout capacity of batter pile to that of the vertical pile
σ	=	Vertical stress
τ	=	Shear stress