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Passive Earth Pressure Coefficients by Generalized Procedure of Slices

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Introduction

Determination of the passive earth pressure acting on a rigid retaining wall is one of the most extensively studied stability problems in geotechnical engineering. Most methods of calculating passive pressures consider the soil mass to be in limiting equilibrium and differ only in assumptions regarding the shape of the failure surface and the application of statics. Due to its versatility in accounting for any shape of failure surface and variable soil properties, the method of slices has all along attracted the attention of the researchers (Janbu, 1957; Shields and Tolunay, 1973; Basudhar and Madhav, 1980) and promises to be extremely useful to the practicing engineers to determine the earth pressure on a retaining wall. In this note, a simple approach using the method of slices has been proposed for estimating passive earth pressure coefficients.

Formulation

In this study the failure surface has been considered to be of general or arbitrary shape and an expression for the passive thrust in a general c-ϕ soil has been derived. Figure 1 shows the soil-wall geometry with a potential slip surface of arbitrary shape and the potential sliding mass divided into n number of vertical slices. For a given geometry of the soil-wall system and the material properties, the passive earth pressure is a function of the shape

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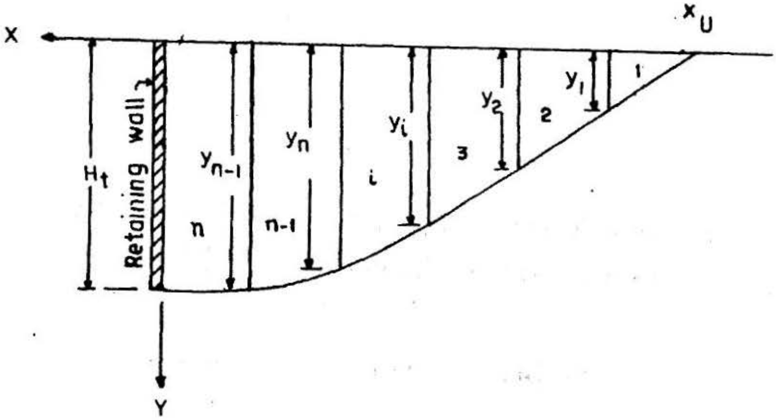

FIGURE 1 : Slice Discretization for a Trial General Slip Surface

and location of the potential slip surface. With the co-ordinate axes chosen as in Fig.1, let x_u be the x co-ordinate of the upper intersection point of the slip surface with the ground surface. Then, the shape and location of the potential slip surface is completely defined by y_1, y_2, y_3, ..., y_l, ..., y_{n-1} and x_u and, therefore, the passive earth pressure can be expressed as a function of the above co-ordinates. In the proposed procedure an attempt has been made to determine a set of values of the above co-ordinates for which the passive earth pressure is the minimum; or, in other words, the co-ordinates defining the critical slip surface which, for the condition of critical equilibrium (F = 1), is the failure surface.

Objective Function, Decision Variables and Constraints

For the evaluation of passive earth pressure the method developed by Janbu (1957) has been used in the present study. For the sake of completeness the salient working formulae are reproduced here. Referring to the Figs. 2(a) and 2(b), the horizontal (normal) component of the passive earth pressure can be expressed as:

$$P_p = E_b = -\left[\sum_{i=1}^{n} A_i + \sum_{i=1}^{n} B_i\right] \tag{1}$$

where

$$A_i = \frac{A_i'}{\eta_{\alpha_i}} \tag{2}$$

$$A_i' = c_i' + (p_i + t_i - u_i)\tan\phi' \tag{3}$$

$$\eta_{\alpha_i} = \frac{1 + \tan\phi' \tan\alpha_i}{1 + \tan^2\alpha_i} \tag{4}$$

$$p_i = \frac{dw}{dx} + \frac{dP}{dx} = \gamma z_i + q_i \tag{5}$$

$$t_i = \frac{dT}{dx} = \frac{(T_i - T_{i-1})}{\Delta x} \tag{6}$$

$$T_i = \frac{dE_i}{dx} - h_{t_i} - E_i \tan\alpha_{t_i} - z_i \frac{dQ_i}{dx} \tag{7}$$

$$B_i = \Delta Q + (p_i + t_i)\Delta x \tan\alpha_i \tag{8}$$

$$\tan\alpha_i = (y_{i+1} - y_i)/\Delta x \tag{9}$$

$$\tan\alpha_{t_i} = \left(h_{t_i} - h_{t_{i-1}}\right)/2\Delta x \tag{10}$$

where h_{t_i} = assumed position of thrust line

The significance of the terms has been explained in Figs. 2(a) and 2(b). The vertical component of the passive earth pressure at the contact face b-b is expressed as:

$$T_p = P_p \tan\delta \tag{11}$$

where δ = angle of wall friction.

The passive earth pressure coefficient is expressed as:

$$K_p = \frac{P_p}{0.5\gamma H^2 + q\,H} \tag{12}$$

In this paper passive earth pressure coefficients are obtained for two different cases:

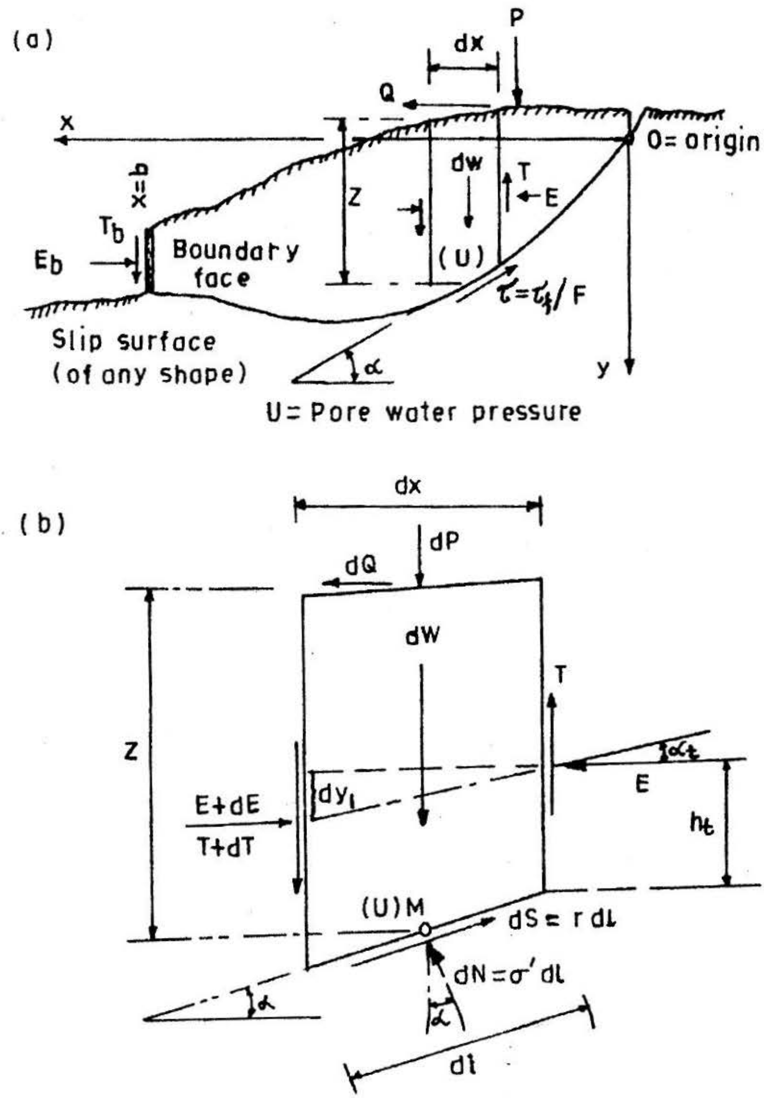

FIGURE 2 : (a) Sliding Mass with Forces Acting, (b) Forces on a Typical Slice

Case A: The lowest value of K_p will result if it is assumed that all the vertical shear forces (T) are lost close to the wall where the shear force is maximum (Shields and Tolunay, 1973). For such a case, t = 0

Case B: Passive earth pressure coefficients are also obtained without neglecting the vertical shear forces (T) at the interfaces of the slices.

To validate the developed computer program and also for the sake of comparison with existing solutions, solution has also been obtained by considering a failure surface composed of a logarithmic spiral and a straight line as suggested by Terzaghi (1943). In this case the computation becomes much simpler as the problem reduces to one of single variable unconstrained minimization (Basudhar and Madhav, 1980).

Decision Variables

From the series of equations listed above, it is clear that for a given soil-wall system the passive pressure varies only with the shape and location of the slip surface. Hence the passive pressure is minimized with respect to the co-ordinates defining the location of the slip surface. So the decision variables are the co-ordinates of the slip surface and the decision (D) can be expressed as

$$D = \left[y_1, y_2, \ldots, y_i, \ldots, y_{n-1}, x_u\right]^T \tag{13}$$

So, the problem involves n decision variables where n is the number of slices into which the sliding mass is divided.

Objective Function

The objective function is the passive pressure, which can be expressed in terms of decision vector D as:

$$P_p = f(D) = f(y_1, y_2, \ldots, y_i, \ldots, y_{n-1}, x_u) \tag{14}$$

Constraints

In order to ascertain that the shape and location of the slip surface are physically reasonable and geometrically compatible, some constraints need to be placed on the choice of the decision variables. A physically reasonable and geometrically compatible slip surface should satisfy the following constraints:

1. No part of the shear surface is permitted to lie beyond the boundary of the soil-wall system. This requires:

I. $y_i \geq 0$ or $-y_i \leq 0$

$$\text{i.e., } g_j(D) = -y_i \qquad i = 1, 2, \ldots\ldots, n-1 \tag{15}$$

and

II. $x_u \leq 0$

i.e., $g_j(D) = x_u$ (16)

2. For a general analysis it is assumed that the potential slip surface is concave upward. In the limiting case the obtained shear surface may be a plane. This requires:

$$(y_{i-1} - 2y_i + y_{i+1}) \leq 0$$

$$\text{i.e., } g_j(D) = y_{i-1} - 2y_i + y_{i+1} \qquad i = 1, 2, \ldots\ldots, n-1 \qquad (17)$$

3. The slope of the shear surfaces at its intersection with the ground surface is to satisfy the maximum obliquity condition:

$$\alpha_1 \leq \frac{\pi}{4} - \frac{\phi}{2}$$

$$\text{or, } g_i(D) = \frac{\alpha_1}{\frac{\pi}{4} - \frac{\phi}{2}} - 1 \qquad (18)$$

where α_1 = base slope of the first slice..

4. The line of thrust should be above the slip surface and within the *middle third of the interslice depths. This requires:*

$$h_{t_i} \geq 0 \qquad i = 1, 2, \ldots\ldots, n \qquad (19)$$

In the present method of analysis (Janbu, 1957), the line of thrust is pre-assigned as:

$$h_{t_i} = k z_i \qquad i = 1, 2, \ldots\ldots, n \qquad (20)$$

where k = constant whose value is normally made to lie between 0.3 and 0.4.

It appears, therefore, that there is no need to impose any constraint in this regard. However, it has been observed that for smooth convergence it is better to allow some flexibility in the position of the line of thrust during the optimization process, rather than assigning a fixed position at the start. This can be conveniently done by including the coefficient k in the design vector and imposing a reasonable limit for it as:

$$0.30 \le k \le 0.40$$

or, $$g_j(D) = -k + 0.30 \quad (21)$$

and, $$g_j(D) = k - 0.40 \quad (22)$$

Mathematical Programming Problem

To locate the critical slip surface, the factor of safety is to be minimized. The problem is stated as an optimization problem as follows:

Find the decision vector D_m such that

$$F = f(D_m) \text{ is the minimum of } f(D) \quad (23)$$

subject to

$$g_j(D) \le 0 \qquad j = 1, 2, \ldots\ldots, M \quad (24)$$

where M = total number of constraints

$g_j(D)$ = inequality constraint function.

Minimization Procedure

The problem stated above is one of multivariable constrained minimization. Solution has been attempted using the Sequential Unconstrained Minimization Technique (SUMT) popularly known as the penalty function method. Since it is difficult to obtain an initial feasible decision vector, a method, which accepts infeasible initial decision vector, is advantageous. The extended penalty function method enunciated by Kavlie (1971) has been used in the present study because of the fact that this method readily accepts infeasible decision points. A detailed description and application of the method to the stability problems are available in the literature (Bhattacharya, 1990). The developed program has the provision for using the interior penalty function method whenever an initial feasible design vector is available. The sequential unconstrained minimization has been carried out in combination with the Powell's method for multidimensional search and quadratic fit for unidimensional search.

Results and Review

Number of Slices

It has been observed that the K_p values yielded by the proposed procedure markedly vary with the number of slices used in the computation. The results show a monotonic decrease of such values with the increase in the number of slices up to a certain number of slices beyond which there is no appreciable variation. This number, again, varies with the kind of slip surface considered in the analysis. For analyses based on log spiral composite slip surfaces, this number is in the vicinity of 20, while for those based on general slip surfaces it ranges from 6 to 8. In all computations for which results are presented below, the number of slices has been chosen accordingly.

Solutions Based on Log Spiral Composite Slip Surface

Considering that the failure surface is composed of a log-spiral and a straight line, Basudhar and Madhav (1980) presented a simplified method of analysis wherein the inclinations of the interslice forces were varied linearly from the Rankine value in the Rankine zone to the wall friction angle over the length of the curved rupture surface. It has been reported that for purely frictional soils the passive earth pressure coefficients, K_p, obtained by their simplified method agree closely with those obtained using more rigorous methods (Chen, 1975; Sokolovsky, 1965; Shields and Tolunay, 1973). Keeping this in view, it has been decided to test the validity of the developed computational scheme on the basis of a comparison between the results of this paper with those of Basudhar and Madhav (1980). Table 1 presents, for a dry and purely frictional backfill with a horizontal surface, a comparison between the results of the present analysis considering a failure surface composed of a log-spiral and a straight line, with those reported by Basudhar and Madhav (1980). Both Case A and Case B have been considered. The data used for the computations are as follows: H_t = 5.0m, q = 2.0 t/m^2 and γ = 2.0 t/m^3.

Case A: It may be observed from Table 1 that for $\delta = 0$, the present solutions are identical with those of Basudhar and Madhav (1980). For higher values of δ the two solutions are in close agreement except in a few cases where the present solution gives slightly higher values; the largest variation being 1.77% corresponding to $\phi = 40°$ and $\delta = 40°$.

Table 1 : Comparison of Present Solution with Known Results

f, in degrees (1)	8, in degrees (2)	Basudhar and Madhav Case A (3)	KrishnaMurthy* (Janbu) (4)	Present Solution Case A		Basudhar and Madhav Case B (7)	Present Solution Case B	
				Composite shear surface (5)	General shear surface (6)		Composite shear surface (8)	General shear surface (9)
10	0	-	-	1.42	-	1.42	1.42	-
	5	-	1.55	1.53	1.52	1.56	1.58	1.56
	10	-	1.70	1.59	1.54	1.67	1.63	1.61
20	0	2.04	-	2.04	-	2.04	2.04	-
	10	2.43	2.60	2.43	2.40	2.56	2.56	2.50
	20	2.65	3.00	2.66	2.54	3.12	2.94	2.85
30	0	3.00	-	3.00	-	3.00	3.00	-
	15	4.13	4.50	4.13	4.04	4.64	4.57	4.35
	30	4.87	6.00	4.90	4.58	6.93	6.00	5.57
40	0	4.60	-	4.60	-	4.60	4.60	-
	20	7.82	9.00	7.87	7.52	9.56	9.38	9.28
	40	10.49	14.00	10.68	9.88	19.35	15.36	12.05

* As given in Basudhar and Madhav's Paper

In their paper Basudhar and Madhav (1980) have also reported about solutions obtained by Krishna Murthy (1972) which are based on Janbu's method (Janbu, 1957) and, therefore, can be utilized for a meaningful comparison with the present solution which are also based on the Janbu's method. For ready reference, the above results (Krishna Murthy, 1972) are also presented in Table 1. It is observed that compared to the above, the present analysis (using 20 slices) yields much lower values; the largest difference corresponding to ϕ = 40°, δ = 40° is about 30%. It is, however, interesting to note that when only a few number of slices (4 to 6) is used, the present analysis yield K_p values which fully agree with those reported by Krishna Murthy. This observation highlights the effectiveness and efficiency of the proposed procedure as also the influence of the number of slices used in the computation.

Case B: As expected, solutions corresponding to Case B yield values higher than in Case A because of the inclusion of interslice shear forces. The difference generally increases with the increase of ϕ and δ. As in Case A, for δ = 0, the present solutions are identical with those of Basudhar and Madhav (1980). For higher values of δ, the present solution generally yields lower values except for ϕ = 10°, δ = 5° in which case it is marginally higher (1.28%). In other cases, the present solutions are substantially lower; e.g., corresponding to ϕ = 30°, δ = 30°, the margin of decrease is 15.5%. For ϕ = 40°, δ = 40°, however, the present values are lower by a high margin of 25.97.

Solutions Based on General Slip Surface

The solutions based on general slip surface are also presented in Table 1. It is observed that compared to the solutions based on the log spiral composite slip surfaces, those based on general slip surfaces yield much lower K_p values. The difference increases with the increase of ϕ and δ . For Case A, the largest reduction is 8.09% corresponding to ϕ = 40°, δ = 40°. For Case B, the largest reduction is remarkably high (27.46%), also corresponding to ϕ = 40°, δ = 40°. But in other cases, however, the difference is much less; e.g., for ϕ = 30°, δ = 30° the difference is only 7.72%. As expected, Case B values are higher than those of Case A; the largest increase is about 23.4% corresponding to ϕ = 40°, δ = 20°.

Shape and Location of the Failure Surfaces

It is of some interest to determine the shape and location of the failure surface of general shapes and compare it with the classical failure surface composed of a log spiral and a straight line. With the proposed procedure, the shape and location of the failure surface (critical slip surface corresponding to F = 1) is obtained as a part of the solution along with the

K_p values. For both Case A and Case B the following observations have been made:

The obtained general failure surfaces generally differ from the corresponding classical rupture surfaces; the magnitude of difference varying with the ϕ and δ values. By and large, the variation is more pronounced for Case A solutions. For this category of solutions, difference between the two surfaces increases as ϕ value increases and, for the same ϕ value, the difference increases as δ value increases. The trend is the same for all combinations of ϕ and δ. For a short stretch near the wall the general surfaces are deeper than the corresponding log spiral composite surfaces but are located above them for the rest portion [Figs. 3(a) and 3(b)]. For the solutions for Case B, however, the trend appears to be reversed. Here the general surfaces are all along located below the corresponding composite slip surfaces and intersect the ground surface farther off from the wall. But, in comparison to the Case A, the slip surfaces are observed to be more closely located, except for $\phi = 40°$, $\delta = 20°$ and for $\phi = 40°$, $\delta = 20°$ [Figs. 4(a) and 4(b)].

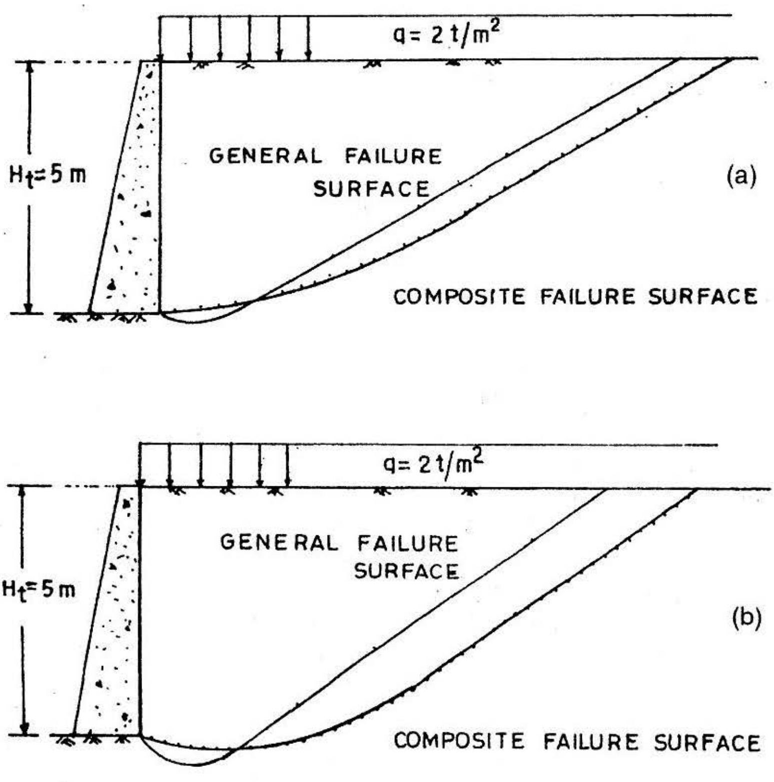

FIGURE 3 : Comparison of Failure Surface (Case A)
(a) $\phi = 30°$ and $\delta = 15°$; (b) $\phi = 30°$ and $\delta = 30°$

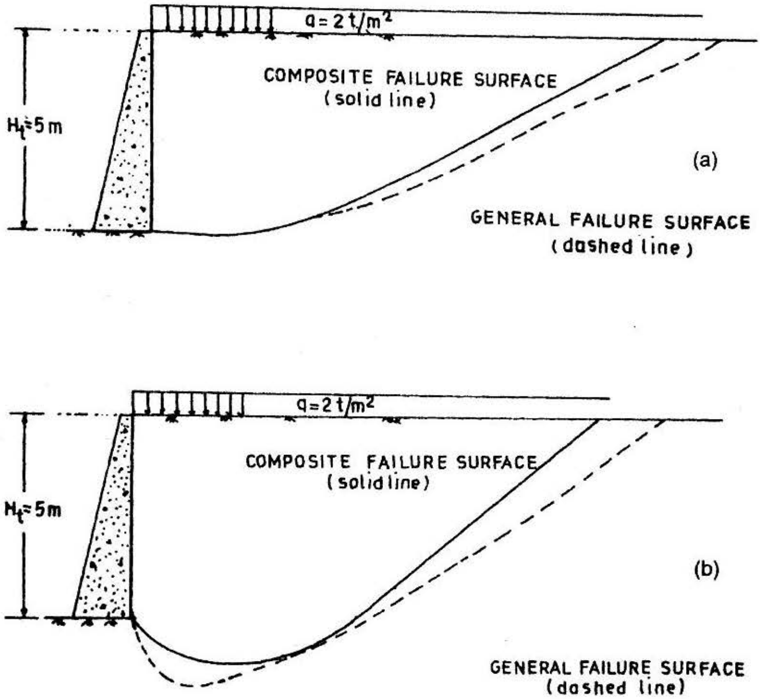

FIGURE 4 : Comparison of Failure Surface (Case B)
(a) $\phi = 40°$ and $\delta = 20°$; (b) $\phi = 40°$ and $\delta = 40°$

Concluding Remarks

On the basis of the studies contained in this paper the following concluding remarks can be made:

The developed numerical scheme, which couples Janbu's GPS procedure with the sequential unconstrained minimization technique of nonlinear programming, has proved to be an efficient and effective tool for the determination of passive earth pressure on retaining walls. In addition to the coefficient of passive earth pressure, the shape and location of the failure surface also come out as a part of the solution. The procedure does not involve any pre-assigned geometry of the shear surface and has the potential to be used for cohesive soils and also in heterogeneous soil conditions under arbitrary surcharge and earthquake type of loading. The study reveals that for purely frictional soils the passive earth pressure coefficients based on the assumption of classical log spiral rupture surface are on the nonconservative side with respect to an analysis based on general failure surface. The difference in the K_p values are, however, found to be less than 10% except for $\phi = 40°$, $\delta = 40°$ in which case the difference exceeds 25%. This is very significant. With respect to earlier solutions using method of slices (Basudhar and Madhav, 1980), the present solution appears to be on the conservative side. The difference is small (less than 6%) when the solution is obtained neglecting interslice shear forces; but when the interslice shear forces are considered the largest difference is nearly 20% for ϕ = 30°, δ = 30°. This excludes the case ϕ = 40°, δ = 40° in which case the difference is found to exceed 35%. Regarding the shape and location of the failure surfaces it has been observed that when interslice shear forces are taken into account, the obtained general failure surface is not much different from the classical log spiral straight line composite failure surface as long as ϕ value is less than 40°.

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