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Design of Reinforced Embankment by Limit Equilibrium and Finite Element Methods

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Introduction

Embankments are often constructed on soft clay deposits for various purposes such as highways and railways. Geosynthetics are used (i) in the embankment and (ii) at the interface between embankment and foundation to improve the stability of the embankment-foundation system. Herein, the latter use is considered. The design of embankments on soft soil is essentially based on limit equilibrium methods. Four modes of failure, viz. (i) bearing capacity, (ii) sliding, (iii) foundation soil squeezing and (iv) rotational failures are investigated in the analysis of the stability of reinforced embankments (Van Zanten, 1986). Kaniraj (1988) presented a design approach for an economical selection of embankment slope and the reinforcement which would make the embankment safe against the four modes of failure. Jewell (1988) and Houlsby and Jewell (1988) provided solutions from plasticity theory for the design of unreinforced and reinforced embankments on soft clay foundation.

The finite element method has also been used for the analysis of reinforced embankments considering various factors affecting the embankment-foundation system (e.g. Rowe, 1982, 1984, Rowe and Soderman, 1985, Rowe and Mylleville, 1988, Hird and Jewell, 1990, Chai and Bergado, 1993, Gnanendran and Rowe, 1994). In these studies, the behaviour of various elements in the embankment-foundation system is often characterised with simplified models such as elastic behaviour for reinforcement and Mohr-Coulomb criterion for the foundation clay.

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In the present study, additional characteristics with respect to foundation are included. The consequence of this on the rotational failure is investigated. Limit equilibrium methods depict the soil behaviour to be rigid and perfectly plastic whereas in finite element methods, realistic elastoplastic strain hardening behaviour can be considered. The effect due to the difference in modelling on the embankment-foundation system is investigated. A methodology for design based on charts is proposed.

Scope

The scope of this paper is to (i) present a comprehensive design method based on limit equilibrium including additional factors, viz., (a) variation of shear strength with foundation depth and (b) unlimited foundation depth besides other factors that are currently used, (ii) study the effect of reinforcement on the rotational failure, (iii) conduct elastoplastic finite element analysis using realistic constitutive models and to compare the results with the design based on limit equilibrium method, and (iv) develop design procedure using charts for the embankment-foundation system.

Design with Limit Equilibrium Method (LEM)

Various steps in the design are presented in the following:

Problem Definition (Step 1)

The design parameters of the embankment, the underlying foundation soil and the reinforcement as indicated in Fig. 1 are defined as follows:

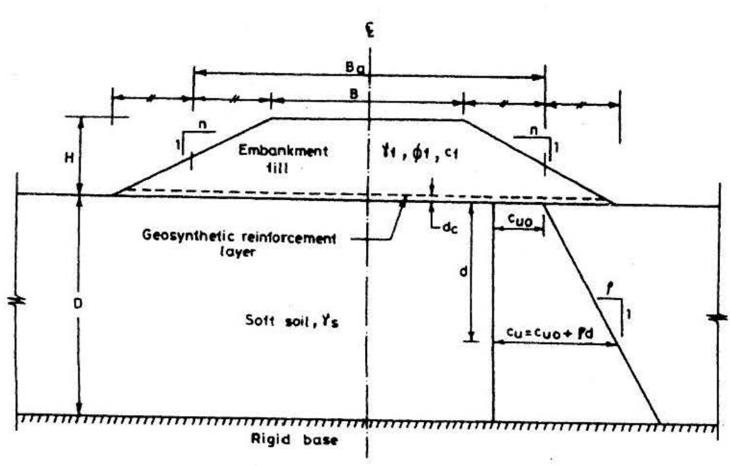

FIGURE 1 : Design parameters of Reinforced Embankment – Foundation System.

i) *Geometry*: the suggested height of the embankment (H), the foundation thickness (D) and the crest width of the embankment (B).

ii) *Embankment fill parameters*: cohesion intercept, c_f, angle of shearing resistance, ϕ_f and the unit weight, γ_f.

iii) *Foundation soil parameters*: profile of the undrained shear strength, c_u and the unit weight, γ_s.

iv) *Reinforcement parameters*: the bond coefficient between the reinforcement and the soil α, the allowable strain, ε_a, the initial estimate of the reinforcement modulus, J, and the clearance between the reinforcement and the foundation surface, d_c.

v) The required factor of safety, Fs_{req}.

Safety against Sliding Failure (Step 2)

In this case, failure involves lateral spreading of the embankment soil only (Fig. 2). Of the two mechanisms of failure shown in Fig. 2, the second one is shown to be critical. Comparing the activating force, P_1', and the resisting force, P_2', the maximum angle of slope, β, ($\tan\beta = 1/n$) that can be attained against sliding failure can be calculated from the expression as (Kaniraj, 1988, Aly, 1995)

$$\begin{aligned}(\alpha_f \sin\phi_f)^2(n^2+1)^{3/2} \\ -2\alpha_f \tan\phi_f(1+\sin^2\phi_f)(n^2+1) \\ +\cos^2\phi_f(n^2+1)^{1/2}+4\alpha_f\tan\phi_f = 0 \qquad (1)\end{aligned}$$

where ϕ_f = angle of shearing resistance of the fill material,

α_f = (tand/tanf$_f$). the bond coefficient between fill and reinforcement, a_f varies between 0 (no bond) and 1 (perfect bond), and

δ = angle of shearing resistance between the reinforcement and the embankment fill.

The requirement that P_2' should be greater than P_1' leads to the condition that $n \geq K_a'/\tan\delta$ where K_a' is active earth pressure coefficient.

Equation (1) represents cubic algebraic equation of $(n^2+1)^{1/2}$. It can

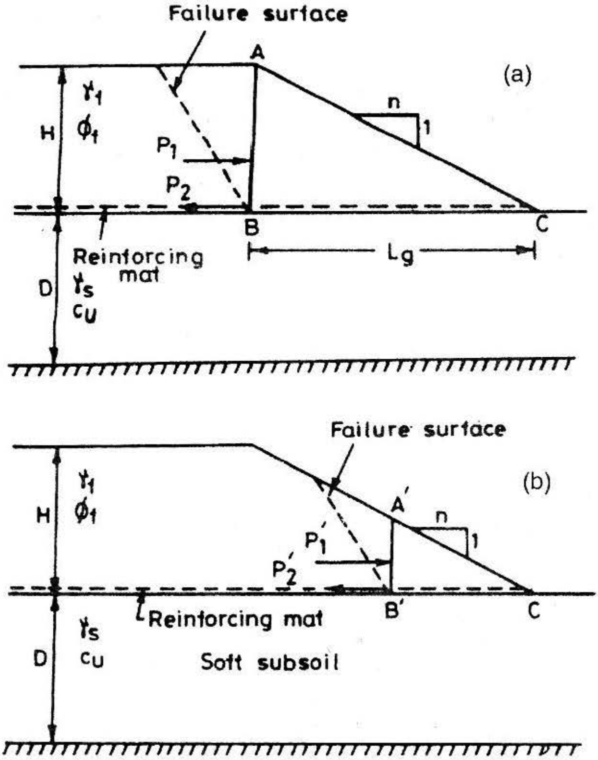

FIGURE 2 : Internal Stability Calculation (Kaniraj, 1988)

be written as,

$$a_1X^3 + a_2X^2 + a_3X + a_4 = 0 \tag{2}$$

where
$$X = \left(n^2+1\right)^{1/2}$$
$$a_1 = \left(\alpha_f \sin\phi_f\right)^2$$
$$a_2 = -2\alpha_f \tan\phi_f\left(1+\sin^2\phi_f\right)$$
$$a_3 = \cos^2\phi_f\text{, and}$$
$$a_4 = 4\alpha_f \tan\phi_f$$

This equation can be solved to obtain n. For sliding failure, n is designated as SN. The limiting value of SN based on ϕ_f is restricted to $1/\tan\phi_f$.

Safety against Squeezing Failure (Step 3)

In this case, failure involves extrusion of the foundation soil beneath an intact embankment (Figs. 3 and 4). The minimum value of n designated as FN, for safety against squeezing failure can be calculated for case of limited foundation depth with constant undrained shear strength c_u as (Van Zanten, 1986);

$$FN = \frac{D}{H(1+\alpha_f)}\left[\frac{\gamma_f H}{c_u} - 4\right] \tag{3}$$

where D = foundation thickness,

H = height of embankment,

γ_f = unit weight of embankment fill,

α_s = (c_a/c_{uo}), the bond coefficient between foundation soil and reinforcement,

c_a = adhesion of soil to the reinforcement, and

c_{uo} = undrained shear strength of the soil at the foundation surface.

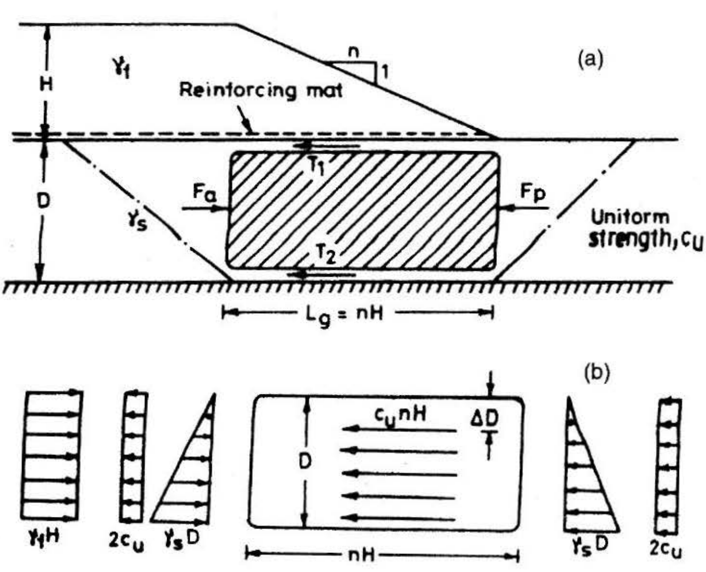

FIGURE 3 : Squeezing Failure of Foundation with Uniform Strength over a limited Depth above a Rigid Layer (after Van Zanten, 1986)

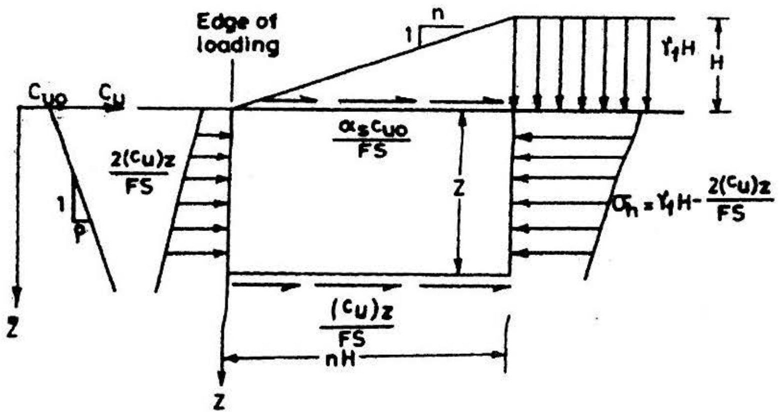

FIGURE 4 : Approximate Block Analysis for an Embankment on a Foundation with Strength Increasing with Depth (after Jewell, 1988)

In the case of unlimited foundation depth and undrained shear strength c_u increasing linearly with depth, the minimum value of n, i.e., FN can be calculated from the expression (Jewell, 1988, Aly, 1995)

$$\left[\frac{\rho H}{c_{uo}}\right]^2 n^2 - \frac{\rho H}{c_{uo}}\left[\frac{2\,FS\gamma_f H}{c_{uo}} + 8\alpha_s\right] n + \left[\frac{2\,FS\gamma_f H}{c_{uo}} - 4\right]^2 = 0 \tag{4}$$

where ρ = rate of increase of undrained strength with depth, and

FS = safety factor applied to the undrained shear strength.

Equation (4) represents a quadratic equation in terms of n and can be written as,

$$b_1 n^2 + b_2 n + b_3 = 0 \tag{5}$$

where
$$b_1 = \left[\frac{\rho H}{c_{uo}}\right]^2$$

$$b_2 = -\frac{\rho H}{c_{uo}}\left[\frac{2\,FS\gamma_f H}{c_{uo}} + 8\alpha_s\right], \text{ and}$$

$$b_3 = \left[\frac{2\,FS\gamma_f H}{c_{uo}} - 4\right]^2$$

In the case of limited foundation depth and undrained shear strength increasing linearly with depth, Eqn. (3) can be used by taking the average value of the undrained strength.

Safety against Bearing Capacity Failure (Step 4)

After determining the angle of slope β required for safety against the two modes of failure mentioned above, safety against bearing capacity failure is checked. Firstly, the suggested height of the embankment H is used to check for the bearing capacity of the foundation soil using the properties of the foundation soil as follows:

i) For the case of limited foundation depth and uniform cohesion strength, c_u,

$$\gamma_f H = c_u N_c \tag{6}$$

where N_c is the bearing capacity factor depending on the ratio R of average embankment width B_a (= crest width B + nH) to the foundation depth D ($R = B_a/D$) as shown in Fig. 5. If $R \leq 1.5$, N_c is equal to $(\pi + 2)$. For the case of $R > 1.5$, the following equation is derived based on the data plotted in Fig. 5 (Van Zanten, 1986) as,

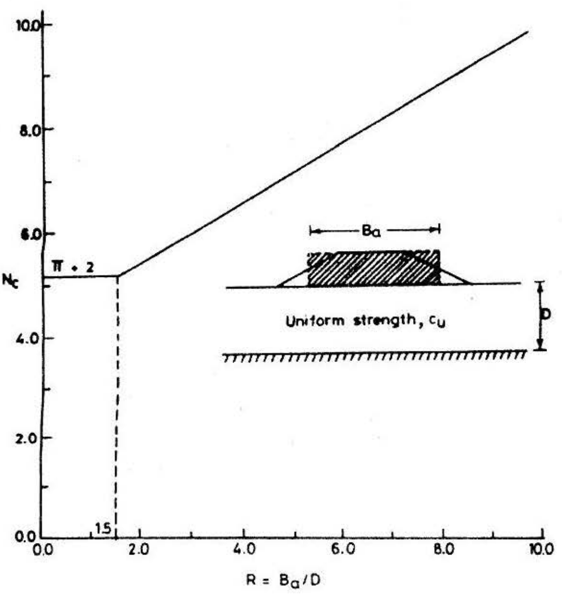

FIGURE 5 : Bearing Capacity Factor, N_c, According to Pilot (after Van Zanteen, 1986)

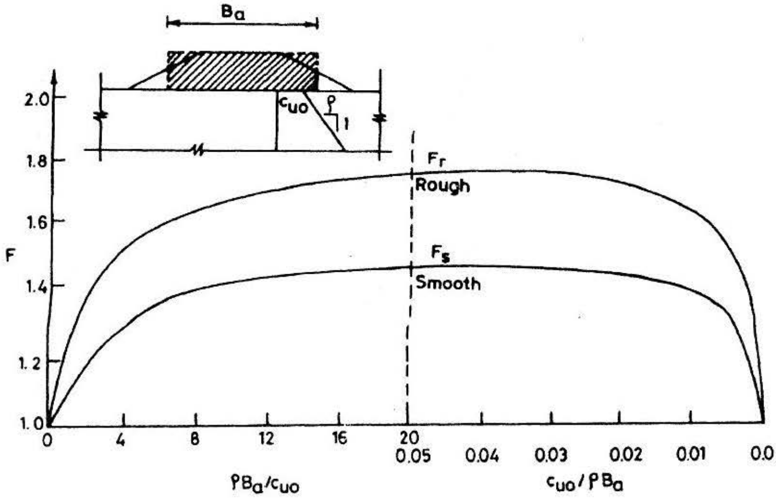

FIGURE 6 : Correction Factors for Rough and Smooth Footings (after Davis and Booker, 1973)

$$N_c = 4.4 + 0.494\,R \tag{7}$$

ii) For the case of unlimited foundation depth and undrained shear strength increasing linearly with depth (Davis and Booker, 1973),

$$\gamma_f H = F\left[(\pi + 2)c_{uo} + \rho B_a/4\right] \tag{8}$$

where F is a dimensionless factor which depends on the ratio $\rho B_a / C_{uo}$ and the footing condition (smooth or rough footing) as has been presented by Davis and Booker (1973). For the cases of unreinforced and reinforced embankments, the values of F are the same as those considered for smooth (F_s) and rough (F_r) footings respectively and are determined using Fig. 6 (Davis and Booker, 1973).

Comparing the left hand side (LHS) and the right hand side (RHS) in Eqn. (6) or Eqn. (8), the following three cases can be considered in the design:

i) If LHS $\leq$ RHS, then the suggested height of the embankment H is satisfactory. The design angle of slope, β, computed using Steps 2 and 3 is adopted and the next step in the solution followed.

As an alternative, when LHS < RHS, the required height of the embankment is computed using Eqn. (6) or Eqn. (8) and Steps 2, 3 and

4 are repeated.

ii) If LHS > RHS, the minimum value of n (designated as BN) for safety against bearing failure is determined for the case of uniform undrained shear strength and limited foundation depth as (Kaniraj, 1988, Aly, 1995)

$$BN = \frac{1}{\pi+2}\left[\frac{\gamma_f D}{c_u}\right]\left[R-\frac{B}{D}\right] \qquad \text{for } R \leq 1.5 \tag{9}$$

$$BN = \frac{D}{0.494\,H}\left[\frac{\gamma_f H}{c_u}-4.4\right]\left[\frac{B}{H}\right] \qquad \text{for } R > 1.5 \tag{10}$$

where $R = B_a/D$ and B_a is determined using the embankment crest width B, the suggested height of embankment, H, and design value of n from Step 2 or Step 3 (SN or FN). If $R \leq 1.5$, the value of BN computed from Eqn. (9) is used again to check the value of R. For the case of unlimited foundation depth and undrained shear strength increasing linearly with depth, the value of BN is computed as

$$BN = \frac{4}{\rho H}\left[\frac{\gamma_f H}{F}-(\pi+2)c_{uo}\right]-\left[\frac{B}{H}\right] \tag{11}$$

iii) If the angle of slope β ($\tan\beta = 1/BN$) for safety against bearing capacity failure is not suitable (much flatter) for the purposes of construction, then the suggested height of the embankment H must be decreased or light fill material (lower unit weight of embankment fill, g_f) must be used and the steps of design (Steps 2, 3 and 4) repeated.

Safety against Rotational Failure (Step 5)

In the rotational failure mode, the overall stability of embankment-foundation system is considered. The factor of safety is determined as the ratio of the sum of the restoring moments calculated from the soil shearing resistance to the sum of the disturbing moments. In the case of embankment reinforced at its base, it is assumed that the reinforcement force also acts in the direction along which the reinforcement is originally placed and the reinforcement force is conservatively limited to that generated by bond between the reinforcement and the foundation soil (Jewell, 1982).

For checking against the rotational failure, an extension of the simplified

Bishop's slip circle analysis (Bishop, 1955) is used to calculate the factor of safety as follows:

i) With the maximum value of n (minimum angle of slope β) among SN, FN and BN based on the safety requirements against the three modes of failure presented in the earlier steps, the embankment geometry is determined.

ii) The factor of safety for the case of unreinforced embankment (Fs_{unrf}) is calculated using the conventional method and its value compared with the required factor of safety (FS_{req}). If $Fs_{unrf} \geq FS_{req}$, then no reinforcement is required and the solution is terminated.

iii) If $Fs_{unrf} < FS_{req}$, the reinforcement force T is determined to calculate the additional restoring moment. The force T is taken as the lesser of the following two forces:

a) The force F_b developed due to the bond between the reinforcement and the foundation soil is defined as

$$F_b = \alpha_s \cdot c_{uo} \cdot L_x \tag{12}$$

in which L_x is the distance between the embankment toe and the intersection point of a slip circle with the reinforcement layer as illustrated in Fig. 7, where L_x may not exceed half the embankment base width ($B/2 + nH$). For this case, it is assumed

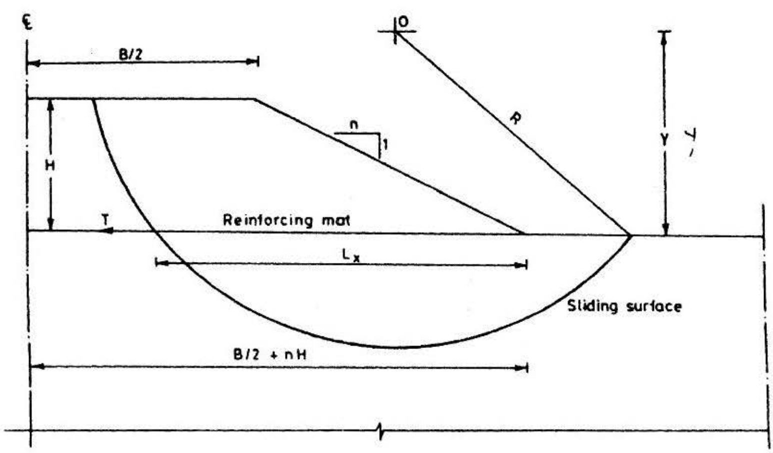

FIGURE 7 : Rotational Slope-Foundation Failure of a Typical Reinforced Embankment

that shear stress is mobilized only along the weaker bottom surface of the reinforcement (clay-reinforcement interface).

b) The force F_c developed in a reinforcement of modulus J at an allowable value of strain at failure ε_a, is calculated as

$$F_c = \varepsilon_a J \tag{13}$$

The value of ε_a for geosynthetic reinforcement ranges from 5% to 10% (Koerner et al., 1987; Bonaparte and Christopher, 1987; Tavassoli and Bakeer, 1994).

iv) The Bishop's expression for factor of safety has been modified as follows to include the effect of reinforcement (Aly, 1995)

$$FS_{rf} = \frac{\sum M_r + M_{rrf}}{\sum M_o} \tag{14}$$

where, FS_{rf} = the factor of safety for the reinforced embankment,

$\sum M_r$ = the sum of the restoring moments,

$\sum M_o$ = the sum of the overtuning moments,

M_{rrf} = the restoring moment due to the geosynthetic reinforcement equal to TY (see Fig. 7),

T = the reinforcement force (minimum of F_b and F_c), and

Y = the vertical distance between the horizontal reinforcement layer and the centre of the slip circle.

The standard procedure of using trial failure surface is adopted in finding the critical failure surface.

Selection of Geosynthetic Reinforcement (Step 6)

The determination of the desired reinforcement modulus requires an iterative procedure. The values of allowable strain ε_a and the modulus J are selected and the performance of the selected reinforcement, is checked as follows.

Case 1: $FS_{rf} \neq FS_{req}$. In this case, the reinforcement force (T_{req}) required to get the value of FS_{req} is determined for the critical slip circle by rearranging the expression for the factor of safety in Eqn. (14) as follows,

$$T_{req} = \frac{1}{Y}\left(FS_{req}\sum M_o - \sum M_r\right) \tag{15}$$

and two conditions for the value of FS_{rf} are considered.

i) If $FS_{rf} > FS_{req}$, a reinforcement *with a lower value of reinforcement* modulus J equal to T_{req}/ε_a is selected and Step 5 (procedure iii and iv) and Step 6 are repeated.

ii) If $FS_{rf} < FS_{req}$, and F_c computed from Eqn.(13) is critical (controls the selection of the force T), a reinforcement with a higher value of J equal to T_{req}/ε_a is selected and Step 5 (procedure iii and iv) and Step 6 are repeated. If interface slip governs the selection of T (i.e. if F_b computed from Eqn. 12 is critical), the required factor of safety cannot be achieved by the present geometry of the embankment. Side slopes should be flattened by increasing the design value of n as computed from Steps 2 to 4 or the value of the desired factor of safety (FS_{req}) should be reduced and Step 5 (procedure ii to iv) and Step 6 should be repeated.

Case 2: $FS_{rf} = FS_{req}$. In this case, the selected reinforcement is satisfactory. If the force F_b controls the selection of T (i.e. $F_b < F_c$), the selected reinforcement modulus may be reduced to $J = F_b/\varepsilon_a$ to prevent overdesign of the reinforcement and Step 5 (procedure iii and iv) and Step 6 are repeated. If $F_b > F_c$ (i.e. $T = F_c$), then the selected reinforcement modulus is satisfactory and the design is complete.

The allowable tensile force T_a of the chosen reinforcement should satisfy the following condition (Kaniraj, 1988),

$$T_a \geq 0.5\gamma_f K_a H^2 + c_{uo}(FN)H \tag{16}$$

where K_a = active earth pressure coefficient, and

c_{uo} = undrained shear strength of the foundation soil at the surface.

The terms on the right hand side of Eqn. (16) represent the condition of maximum sliding force and maximum squeezing force acting on the reinforcement.

A computer program has been developed incorporating all the design steps. In the next section, the design of reinforced embankment sections for two foundation conditions are presented.

Effect of Reinforcement on Failure Surface

An embankment of 6 m height and 8 m width on two foundations viz. (i) with limited depth of 4 m and (ii) with unlimited depth has been chosen. The details of the embankment, reinforcement and foundation are as follows:

Embankment fill parameters	:	$c_f = 0$, $\phi_f = 32°$ $\gamma_f = 20$ kN/m^3
Foundation, soil parameters	:	
(i) Limited depth = 4m		$c_u = 17$ kPa and $\gamma_s = 15$ kN/m^3
(ii) Unlimited depth		$c_{uo} = 14$ kPa, $\rho = 2$ kPa/m, $\gamma_s = 17$ kN/m^3
Reinforcement parameters	:	$\alpha_f = \alpha_s = 1$, $\varepsilon_a = 10\%$, Initial value of J = 2000 kN/m and reinforcement rests directly on foundation soil surface.
Required factor of safety	:	1.3

The embankment and reinforcement are first designed following various steps. In Appendix I is given the details of the design steps for the

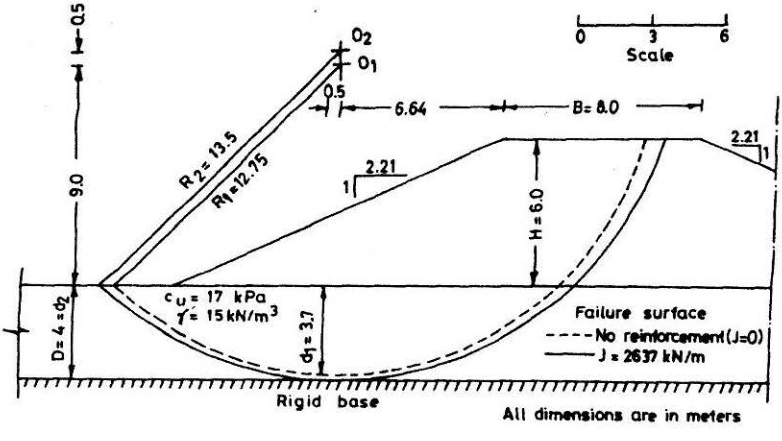

FIGURE 8 : Critical Failure Surfaces for Unreinforced and Reinforced Embankments for Limited Foundation Depth and Uniform Strength

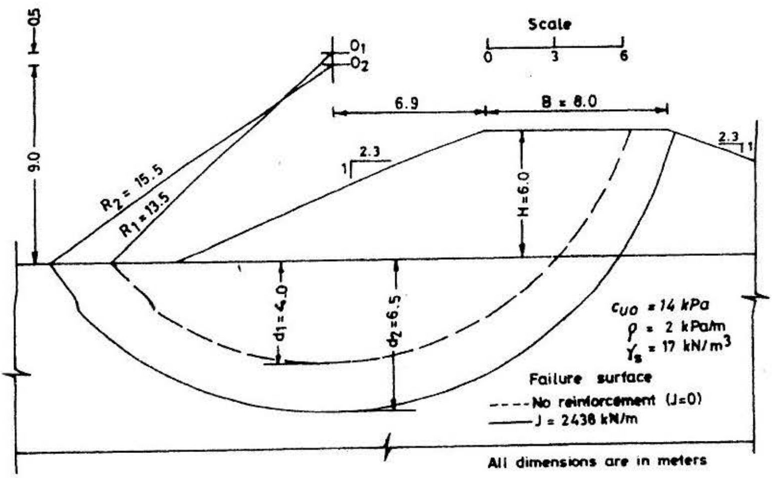

FIGURE 9 : Critical Failure Surfaces for Unreinforced and Reinforced Embankments for Unlimited Foundation Depth and Strength Linearly Increasing with Depth

embankment on foundation with limited depth. In this case, the embankment is found to fail in rotational failure (Factor of Safety = 0.917) for the slope of 2.5 : 1 as determined from other modes of failure. A reinforcement having J = 2637 kN/m is required to give the required factor of safety of 1.3.

In the case of unlimited foundation depth, the embankment is found to fail (Factor of Safety = 0.9997) for a slope angle of 2.09 : 1. With this slope, even the use of a reinforcement does not provide the required factor of safety. By flattening the slope to n = 2.3 and using a reinforcement having J = 2438 kN/m, the required factor of safety is achieved.

The critical failure surfaces for the two cases of embankment-foundation are shown in Figs. 8 and 9 with and without reinforcement. In the case of limited foundation depth, the failure surface is not very much affected by the reinforcement. But, in the second case of unlimited foundation depth, the effect of reinforcement is very significant. The reinforcement forces the failure surface downward into the stronger soil leading to improved embankment stability.

Comparison of Limit Equilibrium Method (LEM) and Finite Element Method (FEM)

Two types of clay foundation viz. Kerala Clay (Clay I) and Madras Clay (Clay II) both found in southern part of Indian subcontinent have been chosen. Clay II is relatively stronger than Clay I. The material parameters for

Table 1 : Physical Properties of the Clays Used

	Clay I Kerala Clay	Clay II Madras Clay
Water Content	99-145%	60-75%
Liquid Limit	104-150%	65-86%
Plastic Limit	31-48%	24-28%
Plasticity Index	73-102%	40-64%
Specific Gravity	2.67-2.77	2.7

these clays are derived from the laboratory test results in the published literature (Viswanathan, 1971; Narasimha Rao and Kodandaramaswamy, 1984; Narasimha Rao et al. 1988). The physical properties of the two clays are given in Table 1. A typical embankment with a crest width of 18 m and a slope of 3 : 1 has been used in the study. An elastoplastic finite element analysis has been carried out under undrained condition. Eight-noded isoparametric elements have been used for embankment and foundation. For the interfaces, six-noded zero thickness interface elements (Goodman et al., 1968) and for the reinforcement, three-noded bar elements have been used. The details are shown in Fig. 10 for the case of foundation depth = 10 m. The behaviour of the embankment and the interfaces have been characterised with Mohr-Coulomb yield criterion and fully associated flow rule. The

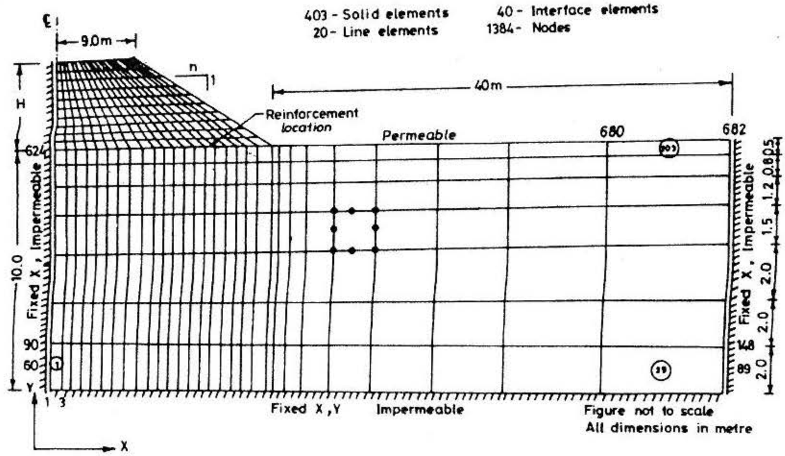

FIGURE 10 : Finite Element Discretisation of the Embankment-Foundation System

Table 2 : Soil Parameters Used for Limit Equilibrium Method

Soil	Properties
Embankment Fill	$\gamma_r = 20\ kN/m^3$ $c_r = 0$ $\phi_r = 40^o$
Kerala Clay (Clay I)	$\gamma_s = 14\ kN/m^3$ $c_{uo} = 4.8\ kPa$ $\rho = 1.5\ kPa/m$ $\phi_u = 0$
Madras Clay (Clay II)	$\gamma_s = 17\ kN/m^3$ $c_{uo} = 11.2\ kPa$ $\rho = 1.5\ kPa/m$ $\phi_u = 0$

Table 3 : Properties and Strength Parameters Used for Embankment Fill Material and Clay Foundation

Embankment Fill

Unit weight, γ_r	20 kN/m^3
Cohesion, c_r	0
Angle of internal friction, ϕ_r	40^o
Janbu's parameter, K	150
Janbu's parameter, m	0.5
Poisson's ratio, ν_r	0.35

Clay Foundation

	Kerala Clay (Clay I)	Madras Clay (Clay II)
Submerged Unit Weight, γ'	4 kN/m^3	7 kN/m^3
Angle of internal friction, ϕ'	29^o	35^o
Initial void ratio, e_o	3.92	1.9
Compressibility Index, λ	0.83	0.27
Swelling Index, κ	0.13	0.05
Young's Modulus, E'	3500 kPa	8000 kPa
Poisson's Ratio, ν'	0.3	0.3
In-situ Stress Ratio, K_o	0.52	0.58
Preconsolidation Pressure, P_{co}	40 kPa	90 kPa
For Undrained Analysis		
Apparent bulk modulus, K_a	3.5 × 10^5	8 × 10^5 kPa

Table 4 : Properties and Strength Parameters Used for Interface Elements and Reinforcement

Fill-Reinforcement Interface

Adhesion, C_a	0
Interface friction angle, $\delta_f = \phi_f$	40°
Shear Stiffness, K_s	2000 kN/m^3
Normal Stiffness, K_n	3×10^6 kN/m^3

Clay-Reinforcement Interface

	(Clay I)	(Clay II)
Adhesion, $C_a = C_{uo}$	4.8	11.2 kPa
Interface friction angle, δ_f	0	0
Shear Stiffness, K_s	2000	2000 kN/m^3
Normal Stiffness, K_n	3×10^6	3×10^6 kN/m^3

Reinforcement

Stiffness J (kN/m)	Tensile Strength δ_o (kN/m)
200	18
1000	80
2000	140
4000	260
8000	480

foundation clay behaviour has been modelled with modified cam clay model (Roscoe and Burland, 1968). The reinforcement is assumed to carry only the tensile force and its behaviour is modelled with von Mises yield criterion. The analysis is conducted under undrained condition using the coupled formulation based on Biot's theory (Sandhu and Wilson, 1969; Zienkiewicz and Humpheson, 1977). In both LEM and FEM studies, the maximum heights of embankment have been calculated with a foundation depth of 10 m. Two reinforcement moduli (J = 1000 kN/m and 4000 kN/m) have been considered. The material parameters used in LEM are given in Table 2 and those used in FEM are given in Tables 3 and 4.

In the study by FEM the embankment construction is simulated in ten lifts. In each lift the loading is applied in several increments. As many as 250 load increments have been used in each case. A number of iterations have been carried out to ensure convergence. The total number of iterations required have often added up to 1500. The failure of the embankment has been defined as

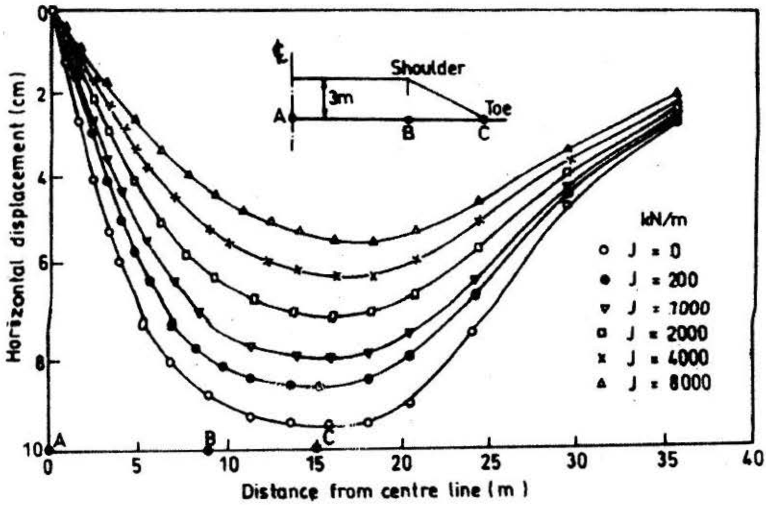

FIGURE 11 : Variation of Horizontal Displacement of the Foundation Surface for Various Reinforcements (Clay I, Foundation Depth = 10m)

the height at which the increment in vertical displacement is equal to or more than the present increment in fill thickness (Rowe and Soderman, 1987). In all the cases the maximum tensile force developed in the reinforcement has been found to be less than the tensile strength of the reinforcement. The variation of horizontal displacement and vertical displacement on the surface of the foundation is presented in Figs. 11 and 12 for an embankment height of 3 m

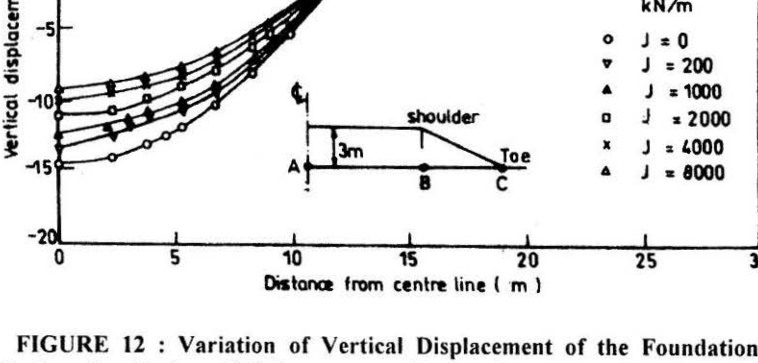

FIGURE 12 : Variation of Vertical Displacement of the Foundation Surface for Various Reinforcements (Clay I, Foundation Depth = 10m)

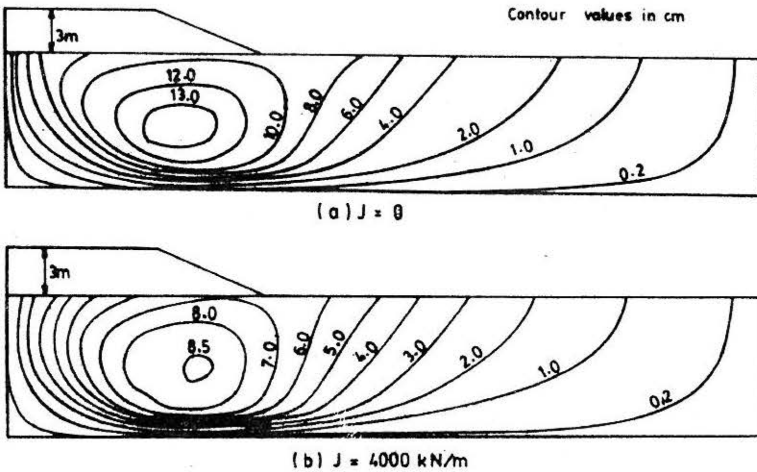

FIGURE 13 : Horizontal Displacement Contours in the Foundation (Clay I, Foundation Depth = 10m) (a) J = 0, (b) J = 4000 kN/m

and foundation depth of 10 m for various reinforcements. The effect of reinforcement on vertical and horizontal displacements in the foundation is shown in Figs. 13 and 14. The variation of reinforcement force is shown in Fig. 15. The force in the reinforcement varies from zero at the toe to maximum value at the centre of the embankment. The complete details of the finite

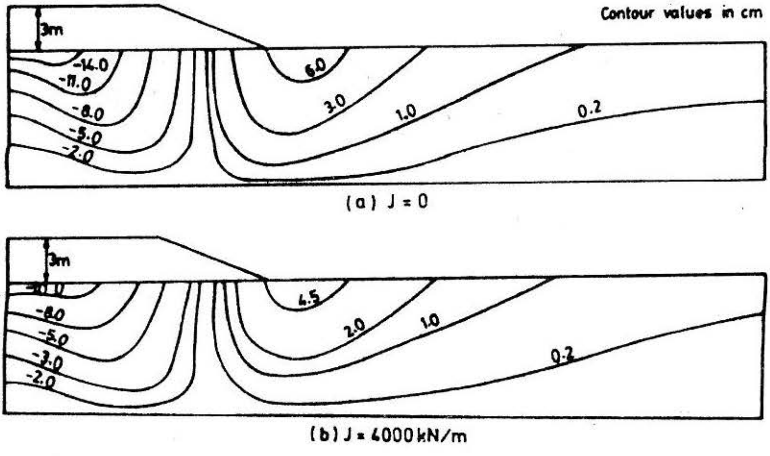

FIGURE 14 : Vertical Displacement Contours in the Foundation (Clay I, Foundation Depth = 10m) (a) J = 0, (b) J = 4000 kN/m

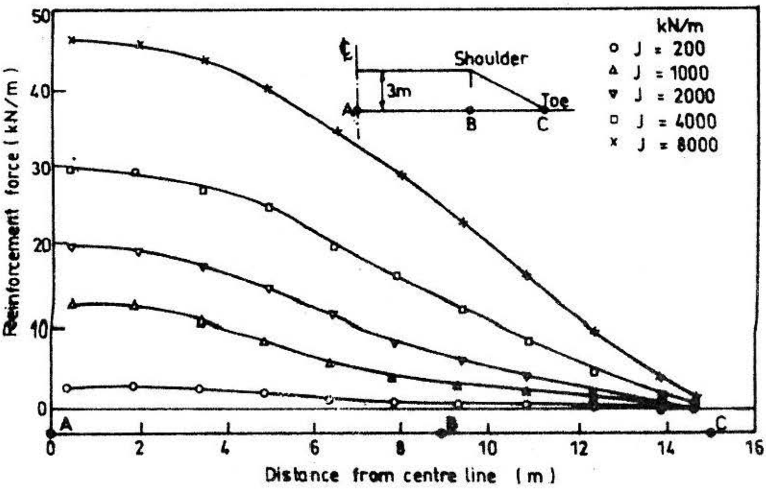

FIGURE 15 : Variation of Tensile Force in the Reinforcement for Various Reinforcements (Clay I, Foundation depth = 10m)

element analysis of reinforced embankment foundation can be found in Aly (1995) and Varadarajan et. al (1999).

In Table 5 is presented the comparison of the results obtained from FEM and LEM. The maximum strains obtained from FEM are very much related to the stiffness of the reinforcement. The heights predicted by FEM and LEM are comparable to each other (difference being less than 9%) for the two clays.

Table 5 : Comparison of Finite Element and Limit Equilibrium Results

Foundation Soil	Reinforcement Modulus, J (kN/m)	Finite Element Analysis		Limit Equilibrium Analysis	
		Height, H_f at Failure (m)	Maximum Geosynthetic Strain at Failure, ε_r (%)	Height, H_f (m) based on limiting strain ε_f of 5%	10%
Kerala Clay (Clay I)	1000	3.3	8.05	3.21	3.38
	4000	3.55	3.4	3.38	3.38
Madras Clay (Clay II)	1000	5.85	10.8	5.98	6.38
	4000	6.6	4.85	6.8	7.2

In their study of embankment on soft clay deposits with significant increase in strength with depth (ρ > 11 kPa/m) using FEM with Mohr-Coulomb failure criterion. Rowe and Mylleville (1990) have also presented similar results.

Design Charts using FEM

A series of analysis have been carried out with the clay-foundation-reinforcement system (Tables 3 and 4 and Fig. 10) with foundation depths, viz., 2.5 m, 6.0 m and 10.0 m. Maximum embankment height is obtained for each side slope and reinforcement stiffness. Contours of heights are drawn for each foundation depth for the two types of clay and presented in Figs. 16 to 21. These charts may be used for the design of reinforced-embankments for these two types of clay at the three foundation depths. The heights of embankments refer to the values at failure condition. For example, Clay II, depth of foundation = 6 m, side slope = 2 : 1, heights of embankment required are 5m and 6m. Referring to the chart (Fig. 15), the maximum height of embankment for side slope 2 : 1 with J = 0 (unreinforced) is 5.47 m. This value is more than 5 m and hence non-reinforced embankment with 5 m height is achievable. For the embankment height = 6 m, reinforcement is required and the stiffness of the reinforcement required is 2760 kN/m.

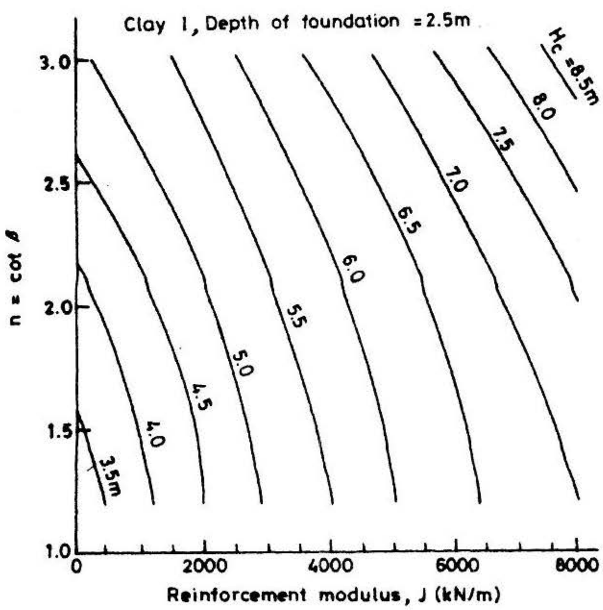

FIGURE 16 : Design Chart for Embankment with Clay I Foundation, Depth = 2.5 m

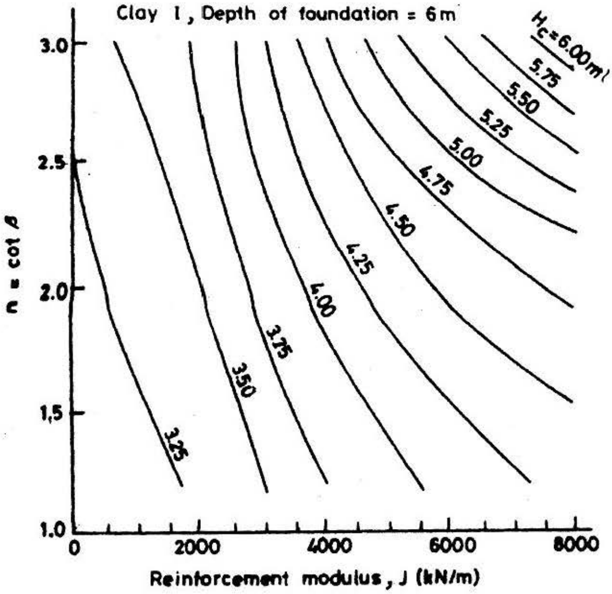

FIGURE 17 : Design Chart for Embankment with Clay I Foundation, Depth = 6.0 m

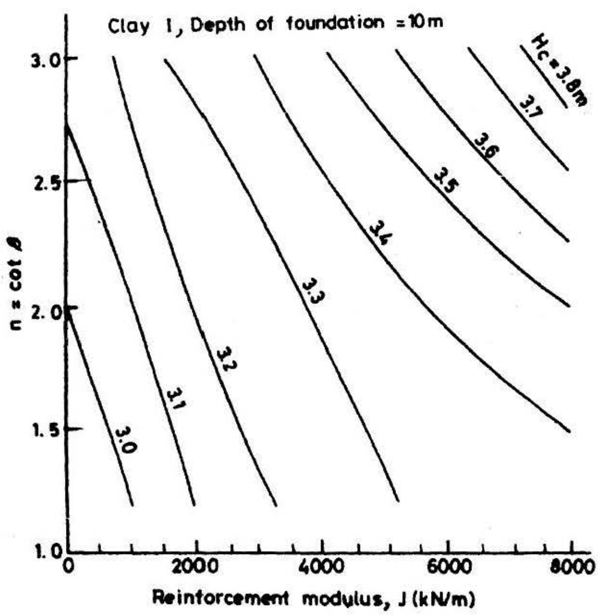

FIGURE 18 : Design Chart for Embankment with Clay I Foundation, Depth = 10.0 m

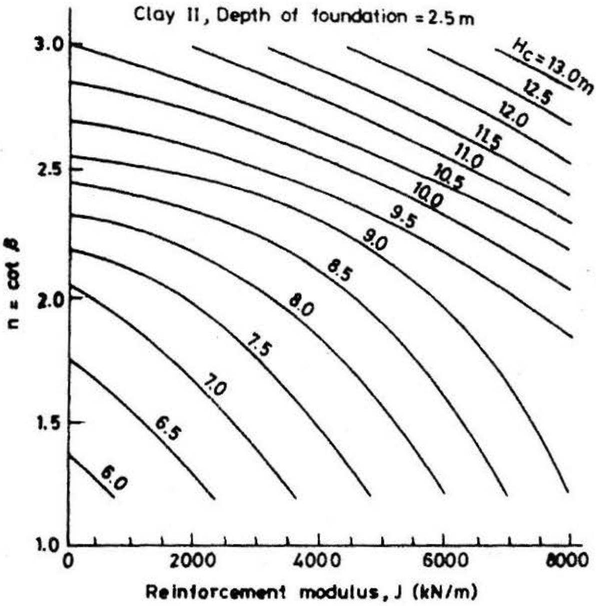

FIGURE 19 : Design Chart for Embankment with Clay II Foundation, Depth = 2.5 m

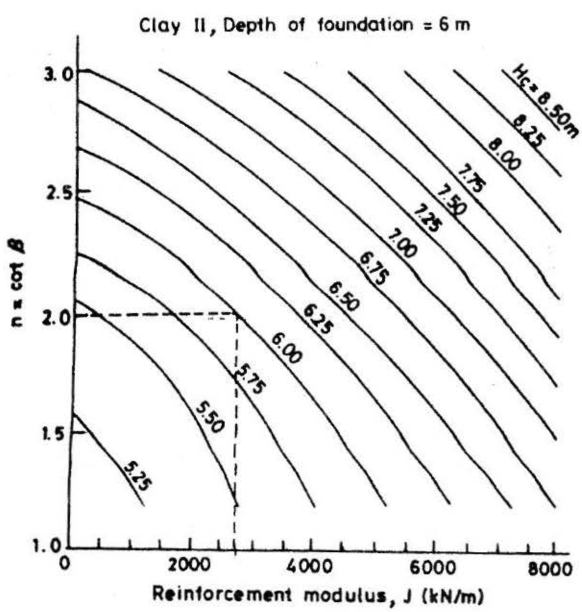

FIGURE 20 : Design Chart for Embankment with Clay II Foundation, Depth = 6.0 m

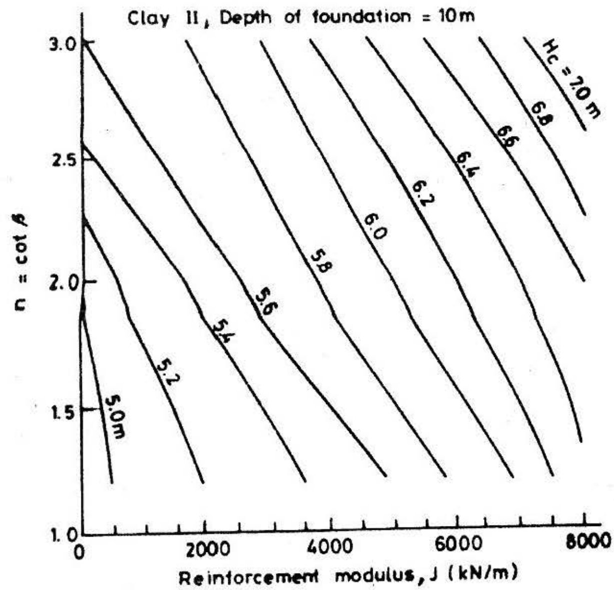

FIGURE 21 : Design Chart for Embankment with Clay II Foundation, Depth = 10.0 m

In an attempt to generalize the design procedure, the relationship between embankment height-reinforcement stiffness as obtained from the charts have been plotted for three side slopes 1.5 : 1, 2 : 1 and 2.5 : 1 for the two clays for the three foundation depths. They are presented in Figs. 22 to 24 for Clay I and Figs. 25 to 27 for Clay II. The height of the

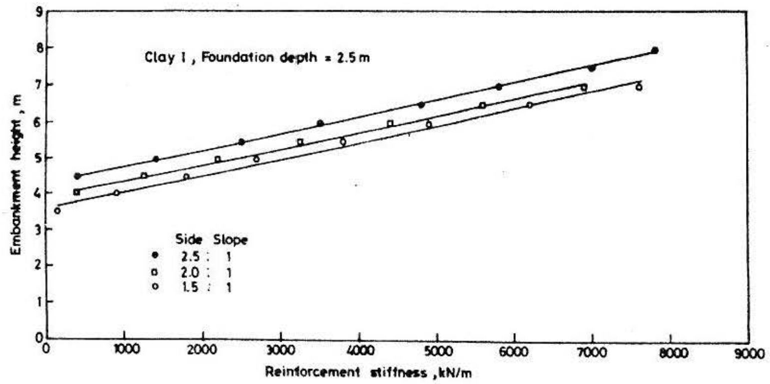

FIGURE 22 : Embankment Height-Reinforcement Stiffness Relationship for Clay I, Depth = 2.5 m

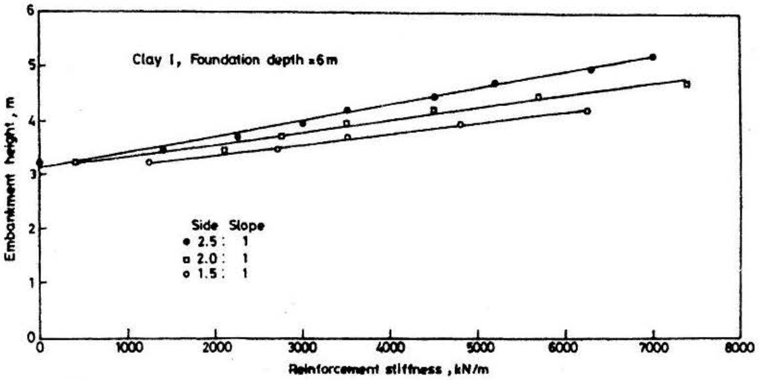

FIGURE 23 : Embankment Height-Reinforcement Stiffness Relationship for Clay I, Depth = 6.0 m

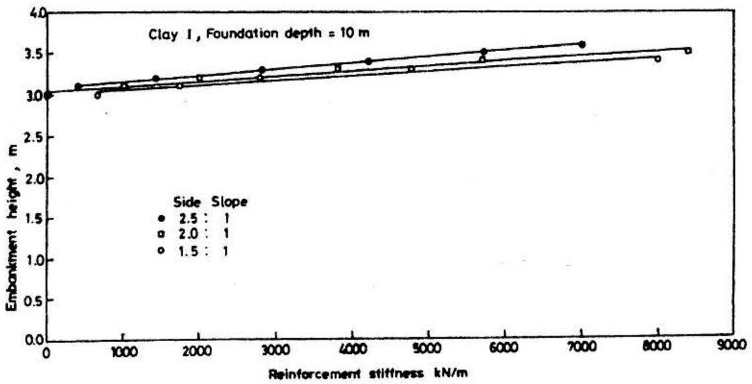

FIGURE 24 : Embankment Height-Reinforcement Stiffness Relationship for Clay I, Depth = 10.0 m

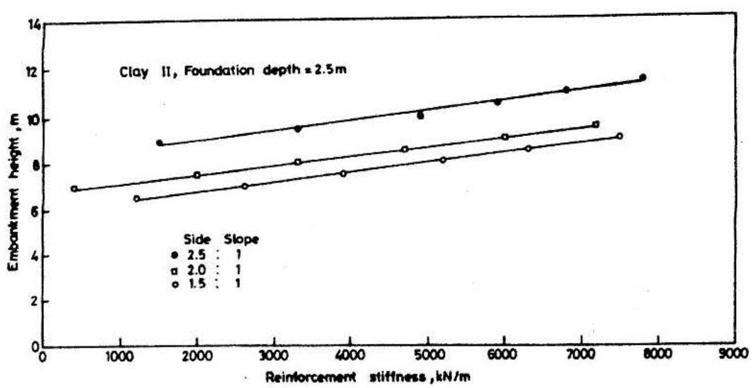

FIGURE 25 : Embankment Height-Reinforcement Stiffness Relationship for Clay II, Depth = 2.5 m

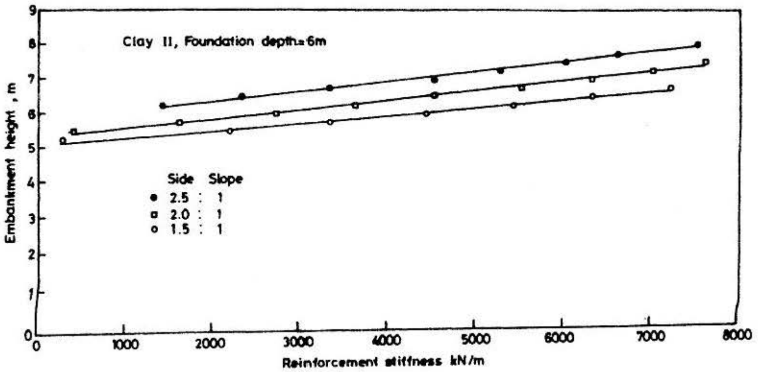

FIGURE 26 : Embankment Height-Reinforcement Stiffness Relationship for Clay II. Depth = 6.0 m

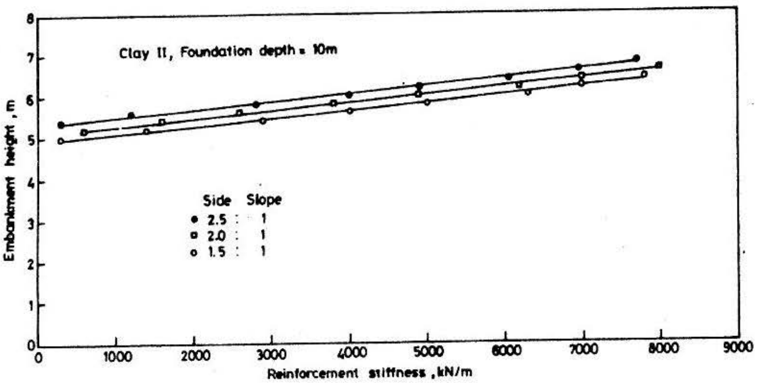

FIGURE 27 : Embankment Height-Reinforcement Stiffness Relationship for Clay II, Depth = 10.0m

embankment without reinforcement (J = 0) and the rate of increase of embankment height, DH with increase in reinforcement stiffness, ΔJ as defined by $\Delta H/\Delta J$ are obtained from these figures and are shown in Table 6 for various side slopes and foundation depths for the two clay types. Using the values in the Table 6, the height of reinforced embankment with a side slope may be predicted as

Height of reinforced embankment

$$= \text{Height of unreinforced embankment} + (\Delta H/\Delta J) \times \text{Reinforcement Stiffness Value}$$

For example, for Clay I for the foundation depth of 2.5 m, the height

Table 6 : Unreinforced Embankment Height and ΔH/ΔJ for Various Cases

Foundation Depth, m	Side Slope	Embankment Height, m with J = 0 (unreinforced)		ΔH/ΔJ m/kN/m	
		Clay I	Clay II	Clay I	Clay II
2.5m	2.5 : 1	4.34	8.25	0.0005	0.0004
	2 : 1	3.93	6.80	0.0005	0.0004
	1.5 : 1	3.61	5.98	0.0005	0.0004
6.0m	2.5 : 1	3.16	5.81	0.0003	0.0003
	2 : 1	3.14	5.30	0.0002	0.0003
	1.5 : 1	2.99	5.08	0.0002	0.0002
10.0m	2.5 : 1	3.09*	5.33	0.00007*	0.0002
	2 : 1	3.05*	5.10	0.00006*	0.0002
	1.5 : 1	3.01*	4.91	0.00005*	0.0002

* Negligible effect of reinforcement

of reinforced embankment with a side slope of 2 : 1 for the reinforcement stiffness value of 2000 kN/m may be calculated as

$$3.93 + 0.0005 \times 2000 = 4.93 \text{ m}$$

From the Figs. 22 to 24 and Table 6 it is observed that for smaller foundation depths, (i) the embankment height increases and (ii) the effectiveness of the reinforcement increases as also reported by Rowe and Soderman (1987) and Varadarajan et al. (1999). It is also noted that in the case of weaker Clay I, the effect of reinforcement is negligible for the largest foundation depth (10 m) which is more than three times the embankment height.

In Table 6, it is interesting to observe that the value of DH/DJ is nearly independent of the side slope for a foundation depth for each clay. The value of DH/DJ decreases with the increase in foundation depth.

Comparing the values of ΔH/ΔJ for the two significantly different clay types (the height of embankment for Clay II is about 1.6 to 1.9 times that for Clay I) for the foundation depths, 2.5 m and 6.0 m (where the depth is less than three times the embankment heights), it is found that the maximum difference is only 0.0001 for each depth. For this value of 0.0001, the difference in embankment height is 0.4 m for the J value of 4000 kN/m and 0.8 m for the J value of 8000 kN/m. The maximum percentage error due to this in the height of embankment for the least height of 2.99 m (Table 6) is 13.4% for J = 4000 kN/m and 26.7% for J = 8000 kN/m. From this, it appears that if the same embankment-reinforcement system is used, the value of $\Delta H/\Delta J$ obtained for one type of clay for a foundation depth may be adopted without serious error for another type of clay of the same foundation depth provided the depth of foundation is less than three times the height of embankment and the reinforcement stiffness value is less than 4000 kN/m.

From the above observations, in the present study, it may be concluded that the $\Delta H/\Delta J$ value is constant for a foundation depth and is not very much affected by variation in the side slope of the embankment and the type of clay in the foundation if same embankment fill-reinforcement system is adopted. This conclusion is applicable with the limitation in J value and the depth of the foundation.

Using the above conclusions, the steps for the design of reinforced embankment on clay foundation are suggested as follows.

Step 1: For embankments used in such projects such as railways or highways with the same embankment fill material, identify various clays, foundation depths and side slopes of embankment in the project. Determine embankment height by conducting finite element analysis using (i) any one side slope of embankment (ii) any one clay type, (iii) one depth of foundation and (iv) various values of reinforcement stiffness up to 4000 kN/m. Apply the required factor of safety to the strength parameters of fill material foundation clay and tensile strength of reinforcement before using the parameters in the analysis. Plot embankment height vs reinforcement stiffness value and determine $\Delta H/\Delta J$ value for the chosen foundation depth. Repeat the process for other foundation depths.

Step 2: By conducting finite element analysis, obtain heights of embankments without reinforcement using (i) various foundation depths and (ii) various side slopes for various clays encountered in the project. As an approximation limit equilibrium method may also be used for this purpose.

Step 3: The height of a reinforced embankment with a side slope to be constructed on a clay deposit is predicted as equal to

$$\begin{pmatrix}\text{height of the} \\ \text{unreinforced} \\ \text{embankment} \\ \text{with the same} \\ \text{side slope on} \\ \text{the same clay} \\ \text{deposit (with} \\ \text{the same deposit)} \\ \text{as determined} \\ \text{in Step 2}\end{pmatrix} + \begin{pmatrix}(\Delta H/\Delta J)\text{ as obtained} \\ \text{in Step 1 for the same} \\ \text{foundation depth}\end{pmatrix} \times \begin{pmatrix}\text{reinforcement} \\ \text{stiffness value} \\ \text{to be used}\end{pmatrix}$$

Conclusions

A comprehensive design procedure with limit equilibrium method has been presented for the design of reinforced embankments on soft soil. Included in this are all the existing procedures and a few additional features with respect to foundation.

It is found that the reinforcement significantly changes the failure surface in the case of unlimited foundation depth. This effect is marginal in the case of limited thickness of foundation.

A comparison of the height of embankment predicted by limit equilibrium and finite element methods using two significantly different clays with respect to strength in the foundation show that both the methods provide comparable results.

Design charts have been prepared using the finite element method for the two types of clays in foundation. Analysis of the design charts show that the rate of increase in embankment height with reinforcement stiffness value up to 4000 kN/m is not very much affected by the side slope of the embankment and the type of clay in the foundation provided the same embankment fill material is used and the depth of foundation is small with respect to the embankment height. On this basis a generalised procedure is presented for the prediction of the height of a reinforced embankment.

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APPENDIX – I

Embankment on foundation with limited depth and uniform shear strength

Step 1. Problem definition

i) Geometry: H = 6 m, D = 4 and B = 8

ii) Embankment fill parameters: $c_t = 0$, $\phi_f = 32°$ and $\gamma_f = 20\ kN/m^3$

iii) Foundation soil parameters: $C_u = 17$ kPa and $\gamma_s = 15$ kN/m^3

iv) Reinforcement parameters: $\alpha_f = \alpha_s = 1$, $\varepsilon_a = 10\%$, initial value of $J(J_i) = 2000$ kN/m and reinforcement rests directly on foundation soil surface ($d_c = 0$).

v) Required factor of safety $FS_{req} = 1.3$.

Step 2. Safety against sliding failure: SN = 1.6.

Step 3. Safety against squeezing failure: FN = 1.02.

Step 4. Safety against bearing capacity failure.

From Steps 2 and 3, the design value of n is 1.6. $R = B_a/D$ is 4.4 which is greater than 1.5, therefore N_c is computed from Eqn. (7) as 6.57. The left hand side (LHS) of Eqn. (6) is computed as 120.0 kPa and the right hand side (RHS) is 111.69 kPa, thus LHS > RHS. The minimum value of n for safety against bearing capacity failure (BN) is computed from Eqn. (10) as 2.25.

Step 5. Safety against rotational failure

i) From Steps 2 to 4 the maximum value of SN, FN and BN is 2.25 ($\beta = 24.0°$) which is the design value for n to determine the embankment geometry.

ii) The factor of safety for the case of unreinforced embankment FS_{unrf} is 0.917 < FS_{req}.

iii) The forces F_b and F_c are computed as 275.86 kN/m and 200 kN/m respectively, so the reinforcement force T to calculate the additional restoring moment is 200 kN/m.

iv) The factor of safety for the case of reinforced embankment FS_{rf} is 1.209 which is smaller than FS_{req}.

Step 6. Selection of geosynthetic reinforcement

i) The reinforcement force T_{req} is determined, using Eqn. (15) as 263.706 kN/m to have factor of safety FS_{rf} equal to the desired safety factor (FS_{req} = 1.3). Thus a reinforcement modulus J ($= T_{req}/\varepsilon_a$) of 2637 kN/m and ε_a of 10% are used for the second iteration.

ii) In the second iteration by repeating Step 5 (procedure iii and iv) and Step 6, the results are: F_b = 275.87 kN/m, F_c = 263.7 kN/m, so T = F_c and FS_{rf} = 1.3.

iii) Since the design is governed by the force F_c and the value of $FS_{rf} = FS_{req}$ = 1.3 after the second iteration, then the selected reinforcement is adequate to satisfy the safety requirements and the solution is complete.

The selected geosynthetic reinforcement properties are stiffness modulus J of 2637 kN/m and allowable tensile force, T_a (from Eqn. 16) of 214.7 kN/m.