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A New Approach of Solving the Stability Equations of General Slip Surfaces

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Introduction

There are at present numerous methods for slope stability computations based on the concept of limit equilibrium and slice discretization. Among these, only a few methods belong to a category which satisfies all conditions of equilibrium and is valid for general shear surfaces. All these methods use a model of formulation which leads to a pair of simultaneous nonlinear equations. The solution of this pair of slope stability equations yields the factor of safety associated with a given potential shear surface. Due to the nonlinearly of the simultaneous equations, it becomes necessary to use an elaborate solution technique to compute the factor of safety. As the determination of the critical shear surface requires analysis of a number of trial shear surfaces, a sound and efficient solution procedure is needed. The study reported here is specifically concerned with the technique of solving the slope stability equations.

A method of solution based on a two-variable Newton Raphson technique has been proposed by Morgenstern and Price (1967). In this method, it is necessary to put a number of intermediate controls on the variables to guard against nonconvergence and premature termination. However, intermediate controls may not be effective in all cases and their provision might reduce the efficiency of the numerical scheme. Furthermore, reservations have been expressed regarding the usefulness of the Newton Raphson technique in finding the zeroes of a function (Hamming, 1962), especially where functions of an unknown nature are involved. Wright (1969)

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has proposed a solution scheme which is also based on the Newton Raphson technique.

The simplest method of solution is presented by Spencer (1967) wherein each factor of safety equation is solved independently and plotted against an indicator of the interslice slope inclination. Thus, this is a numerical-graphical procedure. Subsequently Spencer (1973) has suggested outlines of an iterative scheme which is essentially the same as his previous procedure but can be used without graphical plotting.

The scheme involves certain prerequisites, e.g., an initial guess for the unknowns, an algorithm to iterate over each variable by turns to satisfy the equilibrium equations and an appropriate convergence or termination criterion. Some guidelines for the selection of these have been suggested earlier (Bhattacharya and Basudhar, 1992). An essential feature of Spencer's scheme is that at each iterative step it requires the satisfaction of a physical condition, corresponding to either force equilibrium or moment equilibrium. This restriction might lead to nonconvergence of the numerical scheme in cases where the choice of the initial guess is unreasonable in respect of equilibrium requirements. Such an unforeseen situation can arise in any intermediate cycle of iteration.

With this in view, a new approach is proposed in this paper for solving the factor of safety equations associated with methods of analysis satisfying complete equilibrium conditions such as the Spencer method (1973). The problem of finding the two unknowns from the two nonlinear equations has been formulated as a mathematical programming problem. In contrast to Spencer's solution technique, in the proposed formulation the numerical scheme is not forced to satisfy physical conditions like the force and the moment equilibrium conditions at each step during the progress of minimization The solution procedure has been developed with particular reference to the Spencer method of analysis (1973). With a little modification, the same can be coupled with other methods having the same model of formulation (e.g. Morgenstern and Price method, 1965) and the GLE method (Fredlund et al., 1981).

Formulation of the Slope Stability Equations

In the present study, Spencer's (1973) method of analysis has been extended to include the effect of external forces. As shown in Fig. 1, an external concentrated force acting anywhere on the ith slice is replaced by force components P_{e_i} and Q_{e_i} together with a moment M_{e_i}. The external force can be either a surface force or a body force and is assumed to act at a height h_{e_i} from the midpoint of the slice base.

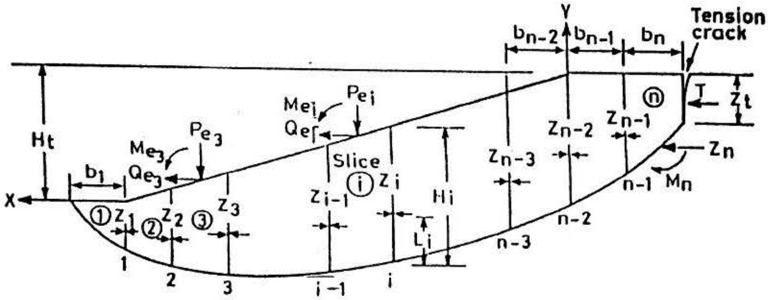

FIGURE 1 : Definition Sketch

For any method of analysis similar to the Spencer (1973) method, which satisfies all conditions of equilibrium, the formulation may be summarized as follows:

Find F and q which satisfy the following requirements

Force equilibrium

$$Z_n(F, \theta) = 0 \tag{1}$$

Moment equilibrium

$$M_n(F, \theta) = 0 \tag{2}$$

where referring to Fig. 1, Z_n and M_n are the external balancing force and moment respectively. Both Z_n and M_n are functions of the variables (F, θ) The first variable is the value of overall factor of safety F; the second is the interslice force characteristic angle θ which, together with the coefficients k_i, determines the slopes, δ_i, of the interslice forces in accordance with the following expression

$$\tan\delta_i = k_i \tan\theta \tag{3}$$

where the suffix i denotes the ith interslice boundary (Fig. 1) . The coefficient k in the Spencer method is equivalent to the interslice force function f(x) in the Morgenstern and Price method. If n is the number of slices, (n−1) values are chosen or prescribed by the user for k; e.g., if k is taken to be unity throughout, then the interslice forces will be parallel and their slope δ_i relative to the horizontal will be each equal to θ.

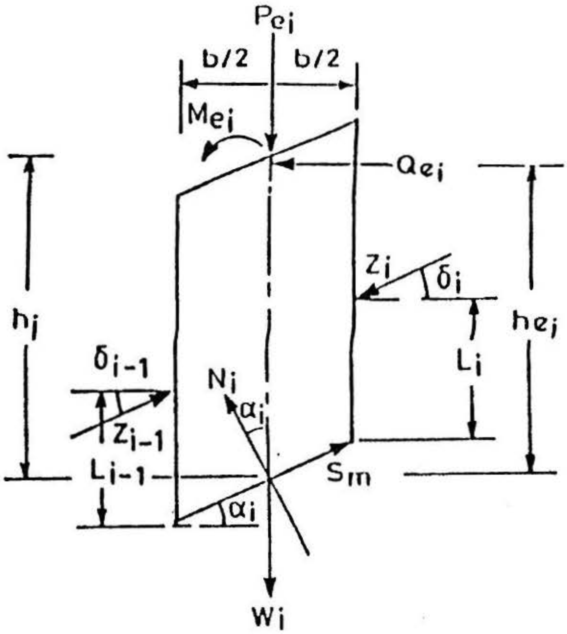

FIGURE 2 : Forces on a Typical Slicce

For the expressions for Z_n and M_n the original work of Spencer (1973) may be referred. More detailed expressions including external forces and moments are also available (Bhattacharya and Basudhar, 1992). In the Spencer method of analysis the positions of the points of action of the interslice forces (and hence the line of thrust) are obtained as part of the solution. Referring to Figs. 1 and 2, the positions of the lines of thrust for total stress and for effective stress have been given by Spencer (1973) and Bhattacharya (1990).

Proposed Method of Solution

Problem Formulation

Since, for a given shear surface, it is required to find F and θ such that Z_n and M_n are both zeros or small enough to be considered negligible, the problem may be formulated as one of nonlinear constrained optimization as follows:

$$\text{Find } D = [F, \theta]^T \tag{4}$$

Such that

$$f(D) = Z_n^2 + M_n^2 \rightarrow \text{Min.} \quad (5)$$

Subject to the side constraints

$$g_1(D) = -F + F_0 \leq 0 \quad (6a)$$

$$g_2(D) = -\theta + \theta_0 \leq 0 \quad (6b)$$

where, D = the vector of decision variables, F and q; these may be normalized by dividing them by their initial values.

f(D) = the objective function,

$g_j(D)$ = the jth inequality constraint function.

F_0 = appropriate lower bounds on F.

q_0 = appropriate lower bounds on q.

Choice of the Objective Function

The logic behind taking $\left(Z_n^2 + M_n^2\right)$ as the objective function is as under:

(i) The minimum of a sum of squares is zero.

(ii) The sum of the squares of these terms will be zero only when the terms are individually zero.

Thus, by minimizing $\left(Z_n^2 + M_n^2\right)$ both the equilibrium conditions, viz., $Z_n = 0$ and $M_n = 0$ can be satisfied. The optimal decision vector will give the factor of safety F and the interslice force characteristic angle θ associated with a given shear surface.

Scale Factor

Because of the difference in the order of magnitude of Z_n and M_n, the objective function $\left(Z_n^2 + M_n^2\right)$ may become considerably eccentric and as such it may take large number of iterations to converge. In such cases it is generally advisable to introduce a scale factor to reduce the eccentricity (Fox, 1971). Therefore, a quantity S_f has been introduced such that in the modified objective function defined as

$$f(D) = Z_n^2 + S_f M_n^2 \tag{7}$$

The contributions of the terms Z_n^2 and $S_f M_n^2$ are of the same order of magnitude. The same scale factor as used by Morgenstern and Price (1967) has been adopted in the present formulation and is given by

$$S_f = \frac{\left[\frac{\partial Z_n}{\partial F}\right]^2 + \left[\frac{\partial Z_n}{\partial \theta}\right]^2}{\left[\frac{\partial M_n}{\partial F}\right]^2 + \left[\frac{\partial M_n}{\partial \theta}\right]^2} \tag{8}$$

Further, the objective function as a whole may be normalized as follows in order to make the magnitudes of the objective function and the constraints of nearly the same order.

$$f(D) = \frac{Z_n^2 + S_f M_n^2}{\left[\gamma b_{av} H_t\right]^2} \tag{9}$$

Constraints

The inequality constraints in equations (6a) and (6b) are imposed to avoid search for negative values of F and θ apart from saving a lot of unnecessary computations by restricting the search zone. Similarly, upper bounds on F and θ may also be put as constraints if a prior assessment can be made. For example, the inclination (β) of the steepest part of the slope may be considered as an upper bound on θ.

Alternatively, an upper bound on θ may be obtained based on the requirements that F should be less than the least of the calculated values of factors of safety along vertical interfaces for the entire sliding mass. For slopes acted upon by a system of arbitrary external forces, however, the inclinations of the interslice forces δ_i (Eqn. 3) may not be all positive. Besides this, for some typical system of loading the restriction $\theta \leq \beta$ may not hold good. In such situations the imposition of the constraints on θ may be avoided.

Based on the authors' experience in solving a variety of problems it may be stated that the imposition of the constraints on θ may not be necessary at all for convergence. But the imposition of these constraints helps in restricting the search zone and generally results in less computational efforts. It may be pointed out that the proposed procedure is quite flexible in respect of imposition or removal of one or more constraints from the general formulation presented above. Such changes can be incorporated at will without affecting the flow of the numerical scheme. This flexibility is rather common in any penalty function formulation.

Method of Solution

The problem formulated above is solved by using the sequential unconstrained minimization technique also known as the penalty function method. In this method the constrained minimization problem is transformed into a sequence of unconstrained minimization problems.

The interior penalty function method (Fiacco and McCormick, 1968) is applicable here because a feasible starting point is readily available. The Powell method (Powell, 1964) of conjugate directions for multidimensional search and the Quadratic interpolation method for unidimensional search have been adopted. The Powell method is very stable and quadratically convergent. In most cases the Interior Penalty Function method is the most efficient means of solving a problem. However, in a number of cases it is preferred because of its simplicity rather than its efficiency. These methods are available in any standard text book on optimization (Fox, 1971; Rao, 1984).

Initial Feasible Decision Vector

Earlier studies (Bhattacharya and Basudhar, 1992) have shown that an initial guess for F may be chosen as the factor of safety given by the Ordinary Method of Slices while an initial guess for θ may be chosen as half the inclination of the slope surface in radians.

Computer Programs

Computer programs have been developed by the authors for the proposed solution scheme as well as for Spencer's scheme. For the latter scheme, as mentioned before, an algorithm is required to adjust F and θ such that Z_n and M_n become negligible. In the program which has been developed, there is provision for two algorithms, namely, (i) the Method of Bisection and (ii) the Modified Regula Falsi technique. The computed values have been obtained by using DEC-1090 and HP-9000/800 computer systems.

Illustrative Examples

To demonstrate the power and efficiency of the proposed equation

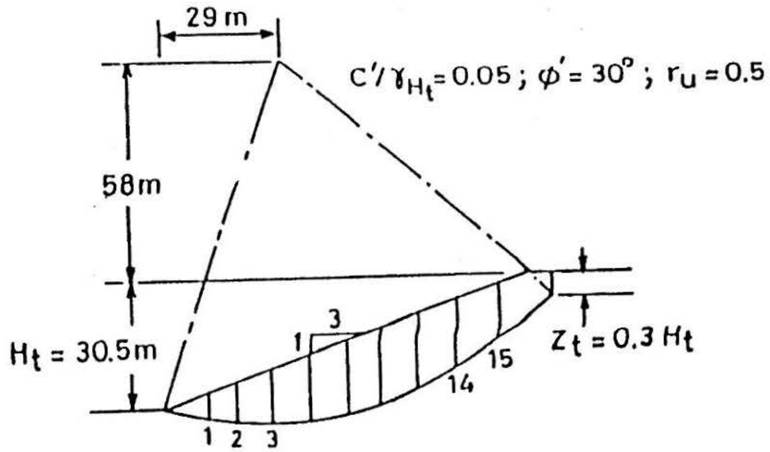

FIGURE 3 : Slope Section in Example Problem 1

solver, three example problems, a simple slope in homogeneous soil conditions, a zoned dam section resting on a thin shear zone and an embankment on soft clay have been taken up for study.

Example Problem 1 : A Homogeneous Slope

Figure 3 shows a homogeneous slope with a given potential shear surface. The shear surface is circular for the major portion but it has to be considered a general surface as it terminates at the bottom of a vertical tension crack of depth $z_t = 0.3H_t$ and runs parallel to the crest with water pressure acting on it. The same problem was earlier solved by Spencer (1973). This example, therefore, can be utilized as a test case to validate the proposed scheme. Following Spencer (1973), the number of slices has been taken as 16 and the interslice forces have been assumed to be all parallel i.e., $k_i = 1$ for all the interslice boundaries. The decision variables F and θ have been normalized by dividing these by their respective initial values. An initial step length of 0.1 has been used in the unidirectional search by the quadratic fit method. Following some guidelines (Bhattacharya and Basudhar, 1992) the starting value of F has been chosen as 1.30 which is close to the value 1.34 determined by Ordinary Method of Slices and the starting value for θ has been chosen as 0.15 which is nearly equal to the half of slope inclination.

In Table 1 detailed results are presented. The decision vector, the objective function and the composite function values and the values of the constraint function at the beginning and at the end of minimization are also presented along with the solution. Rounded off to two decimal places, the solution has been obtained as $F = 1.45$ and $\theta = 0.26$. Corresponding to this

Table 1 : Detailed Results for Example Problem 1

Item	Starting Point	Optimal Point
Factor of safety, F	1.30	1.45491 (1.46)
Interslice force angle, θ	0.15	0.257302 (0.26)
Normalized design vector, D	1.0, 1.0	1.1192, 1.7153
Constraints, g_j	−1.30, −0.15, −0.17	−1.4549, −0.2573, −0.0827
Balancing force, Z_n	0.5765×10^{3}	-0.6982×10^{-4}
Balancing moment, M_n	0.2989×10^{5}	-0.4761×10^{-3}
Scale factor, S_f	0.373×10^{-0}	0.3956×10^{-2}
Objective function, f	0.2886	0.4540×10^{-15}
Penalty parameter, r	1.0×10^{-3}	1.0×10^{-5}
Composite function, ψ	0.3091	1.6663×10^{-5}

Note : $g_1 = -F$; $g_2 = -\theta$; $g_3 = \theta - \beta$
Z_n and M_n are in kN/m and kN-m/m respectively.

θ value $\tan\theta = 0.2631$. The F and tan q values are in close agreement with those reported by Spencer (1973) as given in parentheses in Table 1.

Results obtained from the present analysis regarding the lines of thrust (Fig. 4) with respect to total stress (L/H), the lines of thrust with respect to effective stress (L'/H) as well as the normalized values of the normal components of the resultant interslice forces for effective stress (Z') had been computed and compared with those reported by Spencer (1973). However, for the sake of brevity these are not presented here. It is seen that the line of thrust for total stress (L/H) and the normalized Z_i' values are

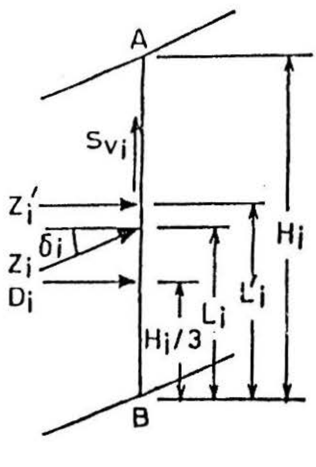

FIGURE 4 : Forces on an Interslice Boundary

Table 2 : Comparison of Time of Computation (Example Problem 1)

Method of Solution	Obtained Solution		CPU Time DEC 1090 (Sec.)
	F	θ (rad)	
Spencer's scheme using the Method of Bisection	1.45487	0.257289	4.36
Spencer's scheme using the Modified Regula Falsi Technique	1.45499	0.257290	2.40
Proposed scheme	1.45491	0.257302	1.60

identical to those reported by Spencer (1973); the line of thrust for effective stress (L'/H) differs only at three interslice boundaries near the top end of the slip surface by a small margin (3 - 4%).

In order to test the efficiency of the proposed scheme with regard to the time of computation, the same problem has also been solved by employing the program developed for Spencer's scheme. Two algorithms have been tried, namely, the Method of Bisection and the Modified Regula Falsi Technique. The results obtained in both cases are almost identical with those obtained by using the proposed technique (Table 2); the time of computation in each case has also been presented in the same Table. It is observed that the proposed scheme is the fastest of all the adopted schemes. The scheme has consumed only 37% of the CPU time required by the Spencer scheme using the method of Bisection and 67% of the same when the Modified Regula Falsi Technique is used. This difference assumes significance in the search for critical shear surface which involves a large number of such function evaluations particularly when the computations are performed on personal computers.

Example Problem 2 : A Zoned Dam with a Thin Shear Zone in the Foundation

Figure 5 shows the downstream slope of a zoned dam section with a given shear surface, an assumed phreatic line and the material properties for the different zones. The special feature of the problem is that the dam is founded on a thin shear zone whose shear resistance is at the residual state. For this slope, it can be expected that the major portion of the critical slip surface would lie along the weak shear zone. Based on this judgment, a trial slip surface as shown in Fig. 5 has been selected. The assumed phreatic line has been treated as a piezometric surface for the calculation of pore water pressures at different slice bases.

It is worth mentioning that the attempt to solve this problem by using

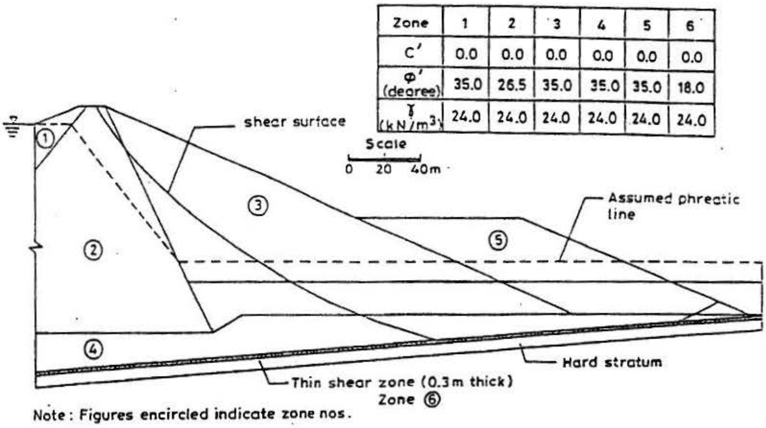

FIGURE 5 : Slope Section in Example Problem 2

the program SSOPT (Baker, 1979), which employs Spencer's scheme to evaluate a trial slip surface, has not been successful because of nonconvergence of the iterative scheme.

The results obtained by using the proposed scheme with a total of 15 slices and parallel inter slice forces are presented in Table 3. To investigate how far the results are sensitive to the initial guess, two sets of initial guesses for F and θ have been tried. in the first set a value of 1.6 for F (which is somewhat close to the factor of safety for the given surface obtained by using the Ordinary Method of Slices) and a value of 0.1 for θ (which is of the order of slope angle for the outermost sloping boundary of the dam section) have been chosen. In the second set F and θ have been arbitrarily chosen as 1.0 and 0.5 respectively. Both the solutions have converged to identical values. The Z_n and M_n values at the starting point and at the optimal point are also presented in Table 3. Thus it is evident that the final solution is independent at the starting point design vector. The associated line of thrust positions for total stress have been found to be reasonable although no

Table 3 Results of Example Problem 2

Initial Values				Final Values			
F	θ	Z_n	M_n	F	θ	Z_n	M_n
1.6	0.1	0.280287×10^{4}	0.262730×10^{6}	2.34126	0.244166	-0.16235×10^{-1}	0.222778×10^{-2}
1.0	0.5	0.21878×10^{5}	-0.16589×10^{7}	2.34126	0.244166	-0.16236×10^{-1}	0.222771×10^{-2}

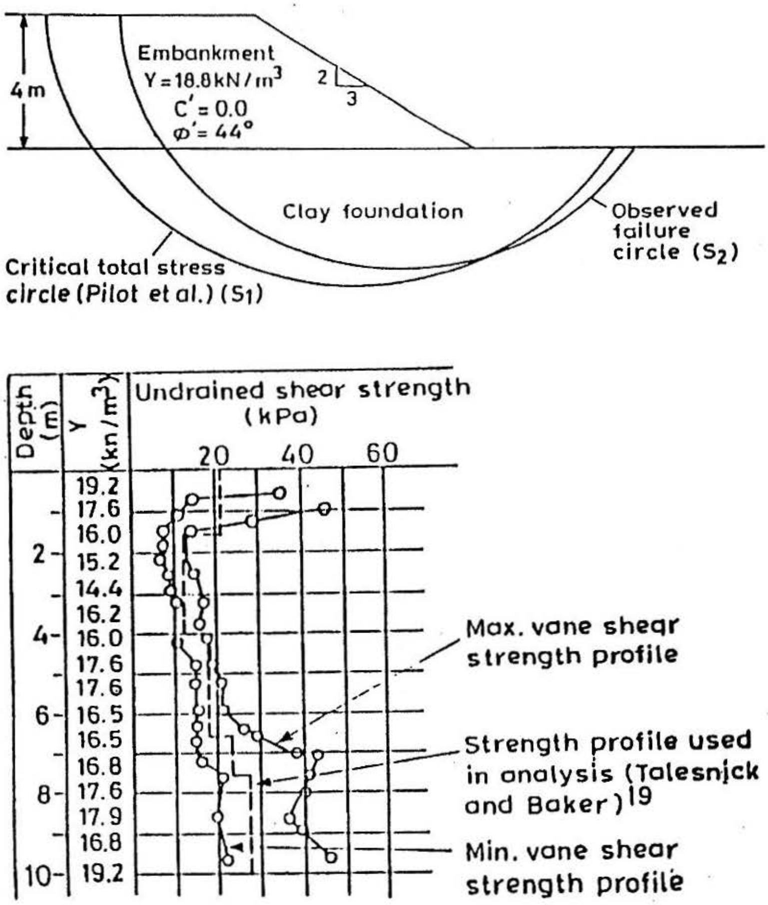

FIGURE 6 : Slope Section in Example Problem 3

special effort was directed in the formulation towards obtaining an acceptable line of thrust. Detailed studies regarding this study are not presented here for reasons of space and brevity.

Example Problem 3 : Embankment on Soft Clay

While Example Problems 1 and 2 presented above demonstrated the power of the proposed scheme to solve a homogeneous case and a typical nonhomogeneous case, Example Problem 3 illustrates a situation where Spencer's scheme encountered a lot of convergence difficulties but the proposed scheme showed smooth convergence.

Figure 6 shows a section of the Saint Alban test embankment founded on soft clay together with the undrained soil properties, the observed failure surface marked S_2 and another potential shear surface marked S_1, both S_1 and S_2 being circular in shape. This embankment, alongwith three other embankments, was brought to failure as part of a research program. Pilot et al. (1982) presented details of this research program and reported about the observed failure surface S_2. With a view to comparing the observed and the calculated failure surfaces, they carried out a total stress analysis using the Bishop Simplified Method and arrived at a critical slip surface S_1. Later on, for a similar investigation, Talesnick and Baker (1984) reanalyzed the embankment using the program SSOPT (Baker, 1979). They approximated the field vane shear strength profile by a system of layers with uniform strength as shown in the figure. The purpose of the present study is to evaluate the factor of safety of both the surfaces S_1 and S_2 using the Spencer-suggested scheme as well as the proposed scheme and thus to study the relative merits and demerits of the two methods. In each analysis the sliding mass has been divided into 20 slices.

Case I : Reanalysis of the Surface S_1

Table 4 presents the results of the studies conducted. With the Spencer-suggested scheme, three trials were made, corresponding to three different initial guesses for F and θ till convergence was achieved at the third trial. Both trials 1 and 2 were observed to fail in the said scheme during the first iteration cycle. Detailed investigation revealed that in the process of adjusting itself to make M_n negligible, the parameter θ began to assume negative values, leading to divergence. To guard against this, an intermediate control was

Table 4 : Results of the Reanalysis of the Surfaces S_1 and S_2 (Example Problem 3)

Surface Mark	Solution Scheme	Trial No.	Initial Guess		Solution		Convergence
			F	q	F	q	
S_1	Spencer's Iterative	1	1.50	0.25	—	—	No
		2	1.25	0.15	—	—	No
		3	1.20	0.025	1.1657	0.0531	Yes
	Proposed	1	1.50	0.25	1.1659	0.0530	Yes
S_2	Spencer's Iterative	1	1.50	0.25	—	—	No
	Iterative	2	1.20	0.25	—	—	No
	Scheme	3	1.10	0.25	—	—	No
	Proposed	1	1.50	0.25	1.2091	0.0426	Yes

incorporated in the program such that whenever θ become negative it was set to zero and the iteration would restart. However, such a control did not meet with success in this case. Convergence was finally achieved in the third trial for which the initial guess (F = 1.20, θ = 0.025) was closest to the solution (F = 1.1657, θ = 0.0531).

In sharp contrast to the above, using the proposed scheme it has been possible to achieve convergence in the very first trial even though the corresponding initial guess (F = 1.50, θ = 0.25) was farthest from the solution (F = 1.1659, θ = 0.0531). The superiority of the proposed scheme is thus evident at least in the respect that the solution is not dependent on the starting point. For the same surface, a factor of safety of 1.20 has been reported by Pilot et al. (1982) as well as Talesnick and Baker 1984). The small difference in the present solution (F = 1.17) may be attributed to a possible error in reproducing the geometry of the slip surface and the subsoil strength profile. The number of slices used in the reported analyses are also not known. Another reason may be the differences in the adopted techniques.

Case II : Reanalysis of the Surface S_2

The results of the reanalysis of the surface marked S_2 are also presented in Table 4. As evident from the table, even with each of the three trials taking a different initial guess, the Spencer-suggested scheme fails to converge to a solution. Trials 1 and 2 fall during the first iteration cycle, as was observed in the previous analysis of the surface S_1. In the third trial, however, the iteration proceeds in a different manner. The first iteration cycle is completed giving F = 1.18 and θ = 0.066. However, the second cycle fails also in the same fashion as before and hence no convergence is achieved.

On this occasion too, in the very first trial with a remote initial guess (F = 1.50, θ = 0.25), the proposed scheme yielded a convergent solution (F = 1.2091 ~ 1.21, θ = 0.0426). Talesnick and Baker (1984) have reported a value of F = 1.28 for the same surface. The small difference in this case may also be attributed to the factors mentioned in Case I.

Conclusions

Based on the studies conducted the following conclusions are drawn.

- The proposed numerical method coupling Spencer's method of analysis with the nonlinear programming technique of optimization has proved to be efficient and powerful in solving the stability equations for general slip surfaces. The method works successfully even in those cases where the commonly used solution scheme falls to converge.

- For the problems studied, starting with initial guesses for the factor of safety of the order of the value as determined by the Ordinary Method of Slices and the interslice force characteristic angle of the order of the slope inclination, the proposed equation solver has been found to yield unique solutions.

References

BAKER, R. (1979) : "SSOPT : A Computer Program for Determination of the Critical Slip Surface in Slope Stability Computations - User Manual", *Faculty Publication No. 253*, Israel. Instt. of Technology, Haifa, Israel.

BHATTACHARYA, G. (1990) : "Sequential Unconstrained Minimization Technique in Slope Stability Analysis", *Ph.D. Thesis*, Indian Institute of Technology, Kanpur, India.

BHATTACHARYA, G. and BASUDHAR, P.K. (1992) : "Soil Factors Involved in Solving the Stability Equations foı a General Shear Surface", *Geotechnique Today*, Vol.1, pp.453-456, *Proceedings Indian Geotechnical Conference*, Calcutta, 1992, Wiley Eastern Limited.

FIACCO, A.V. and McCORMICK, G.P. (1968) : *Nonlinear Programming Sequential Unconstrained Minimization Technique*, Wiley, New York.

FOX, R.L. (1971) : *Optimization Methods for Engineering Design*, Addison-Wesley, Reading, Mass.

FREDLUND, D.G., KRAHN, J. and PUFAHI, D.E. (1981) : "The Relationship between Limit Equilibrium Slope Stability Methods", *Proc. 10th Int. Conf. on Soil Mech.*, Stockholm, 3, 409-416.

HAMMING, R.W. (1962) : *Numerical Methods for Scientists and Engineers*, McGraw Hill, New York.

MORGENSTERN, N.R. and PRICE, V.E. (1965) : "The Analysis of the Stability of General Slip Surfaces", *Geotechnique*, 15, 79-93.

MORGENSTERN, N.R. and PRICE, V.E. (1967) : "A Numerical Method for Solving the Equations of Stability of General Slip Surfaces", *The Computer Journal*, 9, 388-393.

PILOT, G., TRAK, B. and La ROCHELLE, P. (1982) : "Effective Stress Analysis of the Stability of Embankments on Soft Soils", *Canadian Geotechnical Journal*, 19, 433-450.

POWELL, M.J.D. (1964) : "An Efficient Method for Finding the Minimum of a Function of Several Variables Without Calculating Derivatives", *Computer Journal*, Vol.7, No.4, 303-307.

PAO, S.S. (1984) : *Optimization Theory and Application*, Wiley Eastern Limited.

SORIANO, A. (1976) : "Iterative Schemes for Slope Stability Analysis", *Proc. Numer. Methods Geomech.*, 11, 713-724.

SPENCER, E. (1967) : "A Method of Analysis of the Stability of Embankments Assuming Parallel Interslice Forces", *Geotechnique*, 17, 11-26.

SPENCER, E. (1973) : "The Thrust Line Criterion in Embankment Stability Analysis", *Geotechnique*, 23, 85-100.

TALESNICK, M. and BAKER, R. (1984) : "Comparison of Observed and Calculated Slip Surface in Slope Stability Computations", *Canadian Geotechnical Journal*, 21, 713-719.

WRIGHT, S.G. (1969) : "A Study of Slope Stability and the Undrained Shear Strength of Clay Shales", *Ph.D. Dissertation*, University of California, Berkley, CA.

Notation

b	=	width of slice
b_{av}	=	average width of slices
C	=	Cohesion, in general
c'	=	effective Cohesion intercept under drained condition
c_m	=	C'/F = mobilized effective cohesion
c_u	=	Cohesion Intercept under undralned condition
D	=	force due to pore water pressure on inter-slice boundary
D	=	design or decision vector
F	=	overall or average factor of safety
$f(D)$	=	objective function
$g_j(D)$	=	Inequality constraint function
H	=	height of the inter-slice boundary
H_t	=	height of the embankment
h	=	mean height of slice
k	=	coefficient for slope of Inter-slice force
L	=	height of Inter-slice force for total stress above slip surface
L'	=	height of inter-slice force for effective stress above slip surface
M	=	Moment, in general
M_n	=	external stabilizing moment
n	=	number of slices
P_e	=	external vertical force on a slice

Q_e	=	external horizontal force on a slice
M_e	=	external moment on a slice
S_u	=	undrained shear strength
T	=	force due to water pressure in tension crack
u	=	pore water pressure
W	=	weight of slice
z_t	=	depth of tension crack
Z	=	interslice force
Z_n	=	external stabilizing force
Z'	=	force due to effective stress normal to inter-slice boundary
a	=	slope of base of slice
b	=	slope angle
g	=	bulk unit weight
g_w	=	unit weight of water
q	=	angle determining slope of inter-slice forces
d	=	slope of interslice force
f_u	=	angle of shearing resistance under undrained conditions
f'	=	angle of shearing resistance with respect to effective stress