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Estimation of Rock Participation in Pressure Shaft Liners

N. Kumar Pitchumani* and B. Dasgupta†

Introduction

Lining of tunnels to support the roof and walls have been in practice from the time tunnels were used to mine natural resources. Timber and logs of wood have paved the way for steel and concrete as support measures in tunnels. The use of liners becomes all the more essential in the case of tunnels which carry water under pressure as in the case of pressure shafts. This is mainly to restrict the pressure transferred to the rock so as to prevent failure of the rock mass.

The design of liners for pressure shafts considers mainly two aspects, (i) when the pressure shaft is empty, i.e. for the external pressure resulting from the excavation and (ii) when the pressure shaft carries water under pressure, i.e. for the internal pressure.

The buckling of thin steel liners under external pressure has been studied by Amstutz (1970). Patterson et al. (1957) presented a relation to compute a parameter λ which is defined as the proportion of pressure taken outside the steel liner. The formula has been derived based on the assumption that the whole concrete and a region of the rock until a radius 'd' are cracked such that the tangential stresses in these regions are zero. The sound rock beyond the fissured zone is assumed to behave as an infinitely thick shell. A gap between the steel liner and the concrete is considered in the formula. It is assumed that the radial fissures in rock extend to the point where the natural compressive stresses in the rock are just exceeded by the

* Formerly Assistant Professor, Department of Civil Engineering, Indian Institute of Technology, Chennai, India.

† Advanced Technology and Engineering Services, A-8, Naimex House, Sarita Vihar, New Delhi – 110044, India.

tensile stresses caused by the internal pressure in the pressure shaft. The formula to calculate λ includes the distance of the fissured zone 'd'. The authors do not suggest a direct procedure to calculate this distance. Since d is a function of the pressure transferred from the concrete to the immediate rock, it is improper to assume this value. It hence becomes difficult to evaluate the Rock Participation Factor using Patterson's formula. The role of concrete in transferring the pressure to the rock is also not considered as it is assumed to be cracked and no tangential stresses are developed in it.

The IS 4880 (Part VII, 1975) gives a relation for the Rock Participation Factor which is dimensionally unbalanced. Although this relation, which is based on Patterson's formula can be corrected, it suffers from all the limitations of Patterson's formula. Further, the relation considers the allowable stress in the steel liner which implies that the steel liner is stressed to its allowable strength. This may not be always true.

Gupta and Dhawan (1995) have suggested a method of design of pressure tunnels. The stresses around a circular pressure shaft are estimated by taking the rock participation into consideration. The rock surrounding the liner is simulated by a Winkler model. An effective modulus of subgrade reaction for the Winkler model has been considered based on the modulii of subgrade reaction for the fissured and intact rock. The radius to the end of the fissures in rock has been assumed.

In this paper, an analytical approach using the thick cylinder theory is presented for obtaining the stresses in the steel and concrete liners and the Rock Participation Factor. The paper also gives equations to evaluate the extent of fissured zone in rock. In evaluating this zone, it is initially ascertained whether the fissures are due to tension or shear. A parametric study is carried out to study the effects of the thickness of the liners, gaps at the interfaces developed due to shrinkage of concrete, the internal pressure in the shaft and the moduli of concrete and rock on the Rock Participation Factor and stresses developed in the liners. Design charts are provided to design the thicknesses of the liners so that the stresses in the liners and the rock are restricted to allowable limits.

The advantage of this analysis and the formulae and charts provided are as follows:

(a) All materials are assumed to be elastic and hence their contribution in transferring the stresses to the rock are considered.

(b) The gaps at the interfaces are considered. The IS 4880 (Part VII, 1975) allows for some minimum gap resulting from the shrinkage of concrete.

(c) The formulae and charts can be used without difficulty by the design engineer.

Problem Definition

In pressure shafts lined by steel and concrete, shrinkage gaps are formed at the concrete-rock and steel-concrete interfaces. In such cases the proportioning of the internal pressure between steel, concrete and rock will depend on the amount of these gaps. When the pressure shaft becomes operational, water under pressure flows through it and the liners deform radially. The steel liner, while deforming closes the gap at the steel–concrete interface and transfers stresses to the concrete. The concrete liner, in turn deforms radially, closes the gap at the concrete-rock interface and transfers stresses to the rock. The proportion of the internal pressure transferred to the rock is defined as the Rock Participation Factor. If the stresses transferred to the rock exceed the strength of the rock then fissures develop.

The problem is considered to be axi-symmetric in nature and all materials, viz., steel, concrete and rock are assumed to behave elastically. The analysis suggests the estimation of the following:

a) Rock Participation factor
b) Hoop Stresses in the steel liner
c) Maximum tensile tangential stresses in concrete liner
d) Extent of the fissured zone in rock

Formulation

The problem consists of a circular pressure shaft lined by a thick concrete liner and a thin steel liner (Fig. la). The flowing water exerts a uniform pressure, p_i on the steel liner. Uniform gaps y_0 and y_1 are assumed at the steel-concrete and concrete-rock interfaces respectively. It is intended to evaluate the stresses developed in the liners, the stresses transferred to the rock and eventually the extent of the fissured zone in rock. In this analysis it is assumed the rock face is circular. The parameters considered in the analysis are

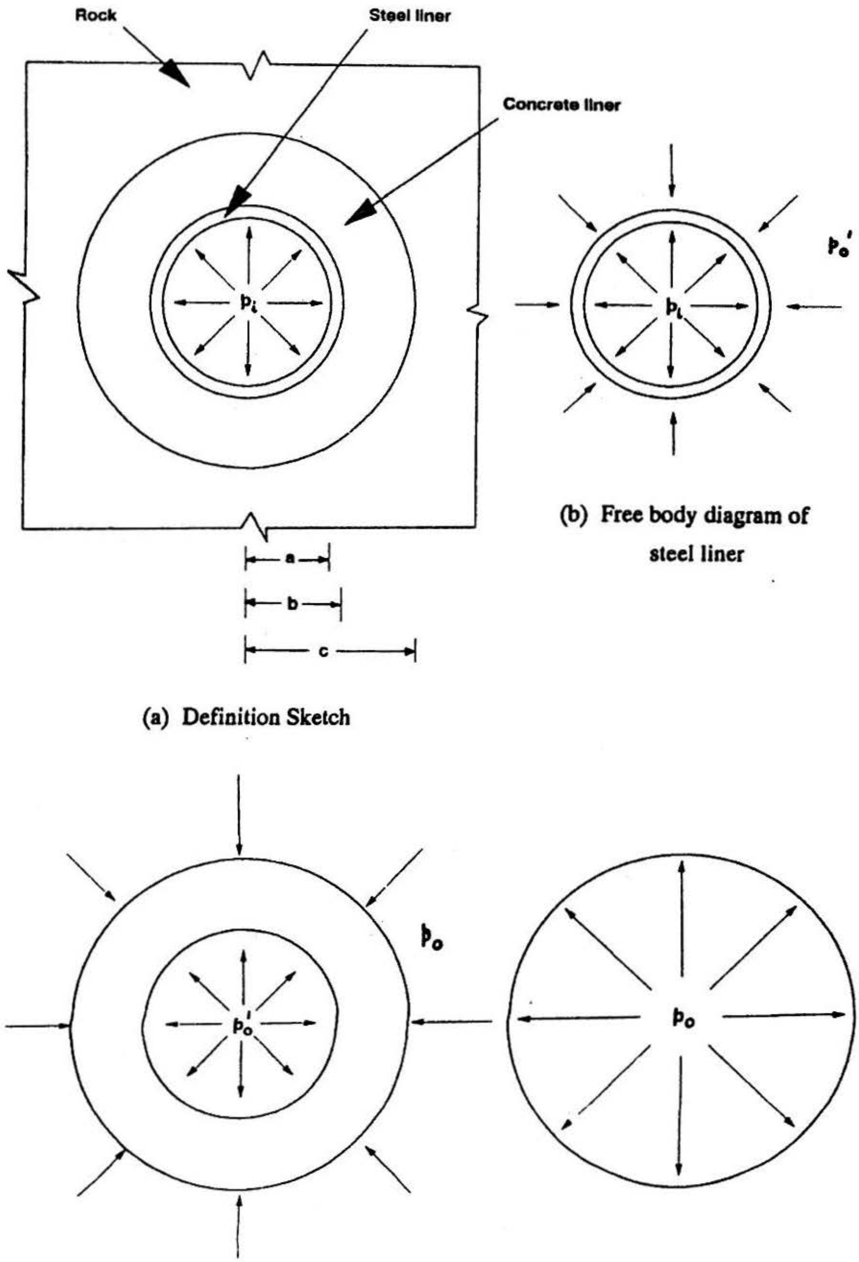
a = radius to the inside of steel liner

b = radius to the outside of steel liner

t = thickness of steel liner = $b - a$

c = radius to the outside of surrounding concrete radius of tunnel

y_0 = gap between steel and concrete



(a) Definition Sketch

(b) Free body diagram of steel liner

(c) Free body diagram of concrete liner

(d) Free body diagram of rock

FIGURE 1 : Problem Definition

y_1 = gap between concrete and rock

p_i = internal pressure in the pressure shaft

E_s = Young's Modulus of Steel

E_c = Young's Modulus of Concrete

E_r = Young's Modulus of Rock

v_c = Poisson's ratio of concrete

v_r = Poisson's ratio of rock

The problem is axi-symmetric in nature and hence the steel liner, the concrete liner and the rock are assumed to behave as cylinders. All cylinders deform radially. Since the internal pressure in the steel liner is uniform, it will deform uniformly in the radial direction, close the gap y_0 at the steel-concrete interface and exert a uniform radial pressure p_0' on the inner face of the concrete (Fig. 1b)

Due to the uniform pressure p_0' on the inner face of the concrete liner, it will deform uniformly in the radial direction, close the gap y_1 at the concrete-rock interface and exert a uniform radial pressure p_0 on the rock face (Fig. 1c). The free body diagram for the rock is shown in Fig. 1d.

The steel liner being thin behaves as a thin cylinder, the concrete liner as a thick cylinder and the rock as an infinitely thick cylinder. These assumptions are valid for an isotropic stress condition in rock, i.e. when the horizontal stress equals the vertical stress. In this formulation it is assumed that the concrete is not cracked.

It is intended to evaluate the pressures p_0 and p_0'. To evaluate these two stresses it is required to solve the two equations which are obtained by satisfying the displacement compatibility and equilibrium at the two interfaces. The displacement compatibility at the steel-concrete interface is given by

$$u_{s(r=b)} - u_{c(r=b)} = y_0 \tag{1}$$

where $u_{s(r=b)}$ = Radial displacement of outer face of steel liner.

$u_{c(r=b)}$ = Radial displacement of inner face of concrete liner

The displacement compatibility condition at the concrete rock interface is given by

$$u_{c(r=c)} - u_{R(r=c)} = y_1 \tag{2}$$

where $u_{c(r=c)}$ = Radial displacement of outer face of concrete liner.

$u_{R(r=c)}$ = Radial displacement of the rock face

These radial displacements are obtained from the expressions for the radial displacements in a thick cylinder. For a thick cylinder, under plane strain conditions with internal radius a_1 and external radius b_1, acted upon by an uniform internal pressure p_1 and a uniform external pressure p_2 and having an elastic modulus E and Poisson's ratio n, the radial displacement at any radius r within the cylinder is given as (Timoshenko and Goodier, 1987)

$$u_r = -\frac{(1+\nu)}{E}\frac{a_1^2 b_1^2 (p_2 - p_1)}{(b_1^2 - a_1^2) r} + \frac{(1-\nu-2\nu^2)}{E}\frac{p_1 a_1^2 - p_2 b_1^2}{b_1^2 - a_1^2} \tag{3}$$

For the steel liner assumed to behave as a thin cylinder, the radial displacement at its outer face is given by

$$u_{s(r=b)} = \frac{(p_1 - p_0')a^2}{E_s t} \tag{4}$$

The radial displacement at the inner face of the concrete liner is given by

$$u_{c(r=b)} = \frac{b}{E_c(c^2-b^2)}\left[\begin{aligned}(1-\nu_c)c^2 p_0' - 2(1-\nu_c^2)c^2 p_0 \\ +(1-\nu_c-2\nu_c^2)b^2 p_0'\end{aligned}\right] \tag{5}$$

The radial displacement at the outer face of the concrete liner is given by

$$u_{c(r=c)} = \frac{c}{E_c(c^2-b^2)}\left[\begin{aligned}-(1+\nu_c)b^2 p_0 + 2(1-\nu_c^2)b^2 p_0' \\ -(1-\nu_c-2\nu_c^2)c^2 p_0\end{aligned}\right] \tag{6}$$

The radial displacement at the rock face is given by

$$u_{R(r=c)} = \frac{(1+\nu_r)}{E_r} p_0 c \tag{7}$$

Substituting Eqns. 4 through 7 in Eqns. 1 and 2 and solving simultaneously, one obtains the radial pressures p_0 (transferred from the concrete to the rock) and p_0' (transferred from the steel liner to the concrete) as

$$p_0 = \frac{y_0 E_s E_c t\left(c^2 - b^2\right) - \dfrac{y_1 E_c\left(c^2 - b^2\right)}{2\left(1-\nu_c^2\right)b^2 c} L - E_c\left(c^2 - b^2\right)a^2 p_i}{XL + 2\left(1-\nu_c^2\right)E_s c^2 t\, b} \tag{8}$$

and

$$p_0' = \frac{y_1 E_c\left(c^2 - b^2\right)}{2\left(1-\nu_c^2\right)b^2 c} + X p_0 \tag{9}$$

where

$$L = a^2 E_c\left(c^2 - b^2\right) - E_s t\, b\left[\left(1+\nu_c\right)c^2 + \left(1-\nu_c - 2\nu_c^2\right)b^2\right]$$

$$X = \frac{1}{2\left(1-\nu_c\right)}\left[\left(1+\nu_c\right) + \left(1-\nu_c - 2\nu_c^2\right)\frac{c^2}{b^2} + \left(1+\nu_r\right)\frac{E_c}{E_r}\frac{\left(c^2 - b^2\right)}{b^2}\right]$$

p_0 should be positive. A negative p_0 indicates no transfer of stresses to rock indicating that the gaps, either y_0 or y_1 are too large to close for the specified internal pressure.

Stresses in Liners

Tangential stresses are developed in the liners which are tensile in nature. For a thick cylinder with internal and external radii a_1 and b_1 acted upon by uniform internal and external pressures, p_1 and p_2 the tangential stresses at any radius r within the cylinder are given by

$$\sigma_t = -\left[-\frac{a_1^2 b_1^2\left(p_2 - p_1\right)}{\left(b_1^2 - a_1^2\right)r^2} + \frac{p_1 a_1^2 - p_2 a_1^2}{\left(b_1^2 - a_1^2\right)}\right] \tag{10}$$

The hoop stress in the steel liner assumed to behave as a thin cylinder is given by

$$\sigma_s = \frac{\left(p_i - p_0'\right)a}{t} \tag{11}$$

The maximum tensile stress in concrete at the inner face i.e. at r = b as obtained from Eqn. 10 is given by

$$\sigma_t = -\frac{p_0'\left(c^2+b^2\right)-2c^2p_0}{c^2-b^2} \tag{12}$$

The negative sign in Eqns. 10 to 12 indicates tensile stresses. These stresses should be less than the tensile strength of the respective materials.

Proportioning of Internal Pressure

The proportion of the internal pressure taken outside the steel liner to the concrete is expressed as

$$\lambda_c = \frac{p_0'}{p_i} \tag{13}$$

and the Rock Participation Factor defined as the proportion of internal pressure transferred outside the concrete to rock is expressed as

$$\lambda_R = \frac{p_0}{p_i} \tag{14}$$

Determination of Fissured Zone in Rock

The radial pressure p_0 acting on the rock face induces tensile tangential stresses and compressive radial stresses in the rock. The final state of stress in the rock will now be a net effect of the stresses caused by the pressure p_0 and the induced stresses from the excavation. This state of stress may cause the immediate rock to fracture, either in tension or in shear.

In considering the extent of fissured zone the general equation for the stresses induced around an excavation given by Kirsch (Hoek and Brown, 1980) are considered. This equation involves the in-situ stress ratio k which is the ratio of the horizontal in-situ stress to the vertical in-situ stress. The problem seizes to be axisymmetric in nature when $k \neq 1$. As such, when Kirsch's solution is used for the present analysis which is based on an axisymmetric problem, we would be erring on the conservative side. By doing so the solution is not restricted to a hydrostatic stress field but is valid for any biaxial stress field.

The minimum tangential stress in rock due to excavation is obtained from Kirsch solution as

$$\sigma_{\theta 1} = \frac{1}{2}p_z\left[(1+k)\left(1+\frac{c^2}{r^2}\right)-(1-k)\left(1+\frac{3c^4}{r^4}\right)\right] \qquad (15)$$

where p_z = in-situ vertical stress

r = radial distance from centre of the opening.

The tangential tensile stress in rock due to pressure p_0 on the rock face is given by

$$\sigma_{\theta 2} = -p_0\left(\frac{c^2}{r^2}\right) \qquad (16)$$

The negative sign indicates tension.

The rock may rupture either in tension or in shear depending on whether the summation of the two tangential stresses given by Eqns. 15 and 16 is tensile or compressive. This check can be made at the face of the rock, i.e. at r = c. If $\sigma_{\theta 1} + \sigma_{\theta 2} < 0$ at r = c then the net tangential stress is tensile and if $\sigma_{\theta 1} + \sigma_{\theta 2} > 0$ at r = c then the net tangential stress is compressive.

Fissures due to Tension

If the net tangential stress in rock is tensile, the fissures will develop due to tension if 'the these stresses exceed the tensile strength of the rock. The region of the fractured zone will extend up to a definite distance into the rock where the algebraic sum of the induced compressive tangential stress due to excavation ($\sigma_{\theta 1}$) and the tensile stress ($\sigma_{\theta 2}$) due to pressure p_0 equals the tensile strength of the rock mass (σ_{tmass}). This implies

$$\sigma_{\theta 1}+\sigma_{\theta 2} = -\sigma_{tmass} \qquad (17)$$

If $\sigma_{\theta 1}+\sigma_{\theta 2} > -\sigma_{tmass}$ at the face, i.e. at r = c then the tensile stresses in the rock are less than or equal to the tensile strength of the rock at the face itself and hence no fissured zone develops.

Substituting Eqns. 15 and 16 in Eqn. 17 and obtaining the roots of the resulting fourth degree equation one obtains the distance of the fissured zone as

$$d = \sqrt{\frac{-B+\sqrt{B^2-4D}}{2}} \tag{18}$$

where $$B = \frac{\frac{p_z}{2}(1+k)-p_0}{p_z k+\sigma_{tmass}} c^2$$

and $$D = \frac{p_z(1-k)}{p_z k+\sigma_{tmass}} \frac{3}{2} c^4$$

Only the real root is to be considered. If k = 1 then

$$d = c\sqrt{\frac{p_0-p_z}{p_z+\sigma_{tmass}}} \tag{19}$$

Fissures due to Shear

If the net tangential stress in rock is compressive then fissures may develop due to shear depending on the combination of the tangential and radial stresses. The extent of the fissured zone based on an elasto-plastic analysis of rock is given by (Brady and Brown, 1993)

$$d = c\left[\frac{2p_z-c_0}{(1+m)p_0}\right]^{1/(n-1)} \tag{20}$$

where $$c_0 = \frac{2c'\cos\phi}{1-\sin\phi}, \quad m = \frac{(1+\sin\phi)}{(1-\sin\phi)}, \quad n = \frac{(1+\sin\phi^f)}{(1-\sin\phi^f)}$$

wherein c' and ϕ are the cohesion and angle of internal friction of the unfailed rock, and ϕ^f is the angle of friction of the fractured rock. The elasto-plastic analysis is based on the principle that the fractured rock exhibits a purely frictional behaviour.

If $d < c$ it implies that the combination of the tangential and radial stresses does not cause the rock to shear and hence no fissured zones develop.

Results

A parametric study was carried out to study the effect of the thickness and radius of the steel liner, internal pressure, gaps at the interfaces and the modulii of concrete and rock on the maximum tangential stresses developed in the steel and concrete liners and the Rock Participation Factor. All length parameters were normalized with the radius of the shaft, c.

The range of parameters considered in the analysis are as follows:

Normalized thickness of steel liner, t/c	= 0.001, 0.0025, 0.005 and 0.01
Normalized gap at steel-concrete interface, y_0/c	= 0.0002 and 0
Modulus of rock, E_r	= 1 GPa, 5 GPa, 10 GPa, 25 GPa and 50 GPa
Modulus of concrete, E_c	= 10 GPa, 20 GPa, 30 GPa and 40 GPa.

It is seen from Eqns. 8 and 9 that if there are no gaps at the steel-concrete or concrete-rock interfaces, then the interfacial normal stresses can be normalized with the internal pressure p_i. Consequently one can obtain a maximum normalized tangential stress in the steel liner, σ_s/p_i and a maximum normalized tangential stress in concrete, σ_t/p_i. Hence these tangential stresses are directly proportional to the internal pressure, p_i. On the other hand if the gaps, y_0 or y_1 or both exist then the stresses cannot be normalized with the internal pressure and hence cannot be directly related to the internal pressure.

In order to study the variation of the maximum tangential stress in steel and concrete due to the parameters mentioned, two cases were considered. In the first case, it was assumed that there are no gaps at the interfaces, i.e. $y_0/c = 0$ and $y_1/c = 0$. This is the extreme case and it is imperative that the Rock Participation would be the maximum for this case. Consequently, the design would be more conservative. In the second case, a gap $y_0/c = 0.0002$ was considered. This value of gap represents a typical gap of 1 mm in a 10 m diameter tunnel. For both cases an a/c of 0.85 was considered.

Figures 2 and 3 show the variation of the maximum normalized tangential stress σ_s/p_i in steel and σ_t/p_i in concrete respectively with the normalized thickness of the steel liner t/c for various values of E_r and E_c. It is observed from these figures that the stresses decrease with increase in t/c. The rate of decrease decreases with increase in E_r and E_c. The maximum value of σ_s/p_i is -81.1 for $t/c = 0.001$, $E_r = 1$ GPa and $E_c = 10$ GPa while the corresponding value of σ_t/p_i is -3.79. The maximum value of σ_t/p_i is -5.38 for $t/c = 0.001$, $E_r = 1$ GPa and $E_c = 40$ GPa. The stresses for higher E_r values are small and are not affected appreciably by variation in t/c.

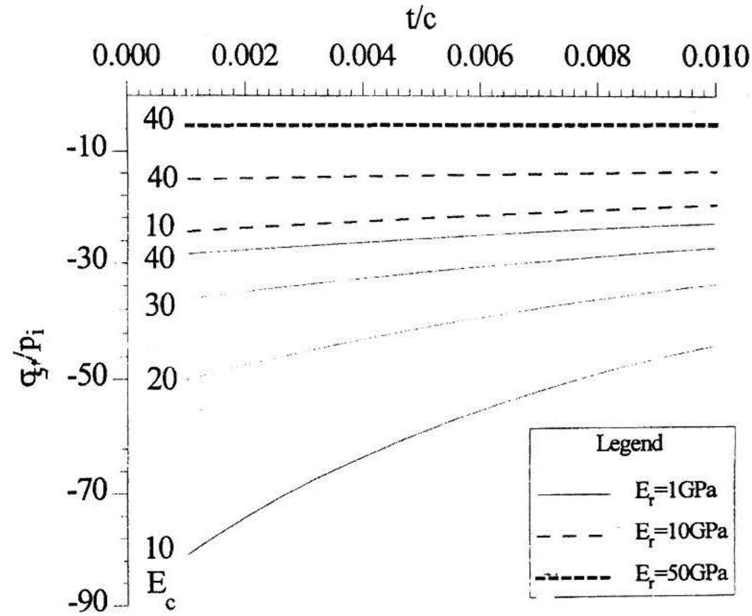


FIGURE 2 : Variation of Normalized Stress in Steel with t/c for different E_r and E_c Values

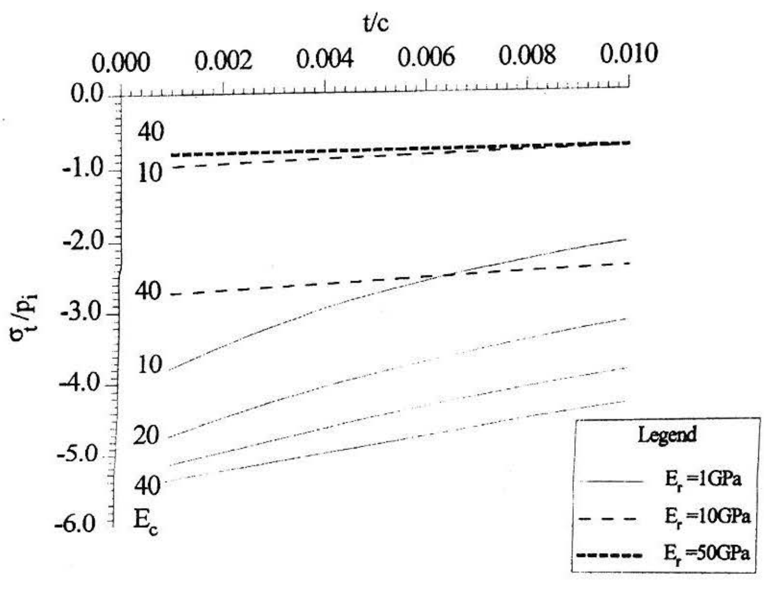


FIGURE 3 : Variation of Normalized Concrete in Steel with t/c for different E_r and E_c Values

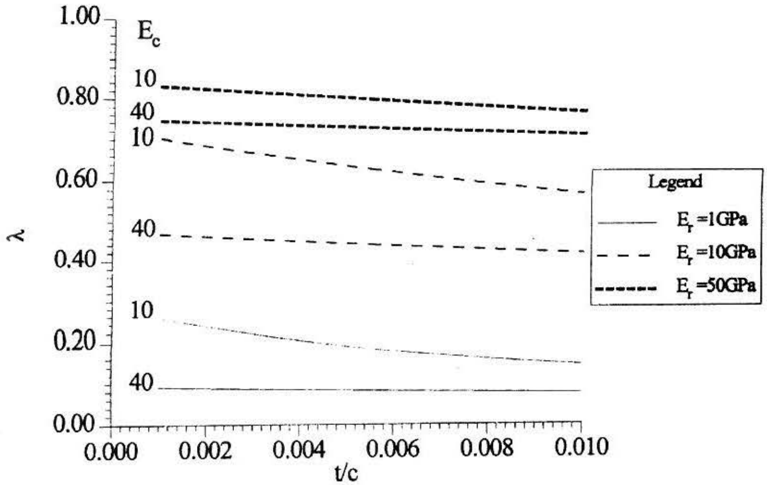


FIGURE 4 : Variation of Rock Participation Factor with t/c for different E_r and E_c Values

Figure 4 shows the variation of the rock participation factor λ with t/c for various values of E_r and E_c. λ decreases gradually with increase in t/c and E_c and decrease in E_r. The maximum λ of 0.83 is observed for $t/c = 0.001$, $E_r = 50$ GPa and $E_c = 10$ GPa. It is thus seen that the maximum proportion of the internal pressure gets transferred to rock when the stiffness of rock is high and that of concrete is low.

Figures 5 through 13 show the variation of tangential stresses in steel and concrete, and for the second case where a gap of $y_0/c = 0.0002$ is considered at the steel-concrete interface. In these figures the absolute stresses are considered for various values of internal pressure p_i. Values of p_i considered are 0.5 MPa, 1 MPa and 2 MPa which correspond to pressure shafts at shallow to deep depths. Figures 5 and 6 show the variation of σ_s and σ_t with t/c for $p_i = 0.5$ MPa and various values of E_c and E_r. It is seen that the stresses decrease with increase in t/c and all curves converge to a point at $t/c = 0.0086$. At this point $\sigma_s = -49.4$ MPa and $\sigma_t = 0$. This indicates that beyond a thickness of steel liner of $t/c = 0.0086$, the gap y_0 does not close and hence no stresses are transferred to either the concrete or rock. This corresponds to the case of a thin steel liner subjected to only an internal pressure, where the hoop stress is $p_1 \cdot a/t$ which works out to -49.4 MPa. A maximum stress σ_s, of -85.25 MPa is observed for $t/c = 0.001$, $E_r = 1$ GPa and $E_c = 10$ GPa. Maximum value of -2.37 MPa is observed for $t/c = 0.001$, $E_r = 1$ GPa and $E_c = 40$ GPa. The rock participation factor λ as depicted in Fig. 7 decreases rapidly with increase in t/c and is equal to 0 for a t/c of 0.0086 indicating no transfer of stresses to rock. This fact is also corroborated from Fig.6 where $\sigma_t = 0$ for $t/c = 0.0086$. The rate of

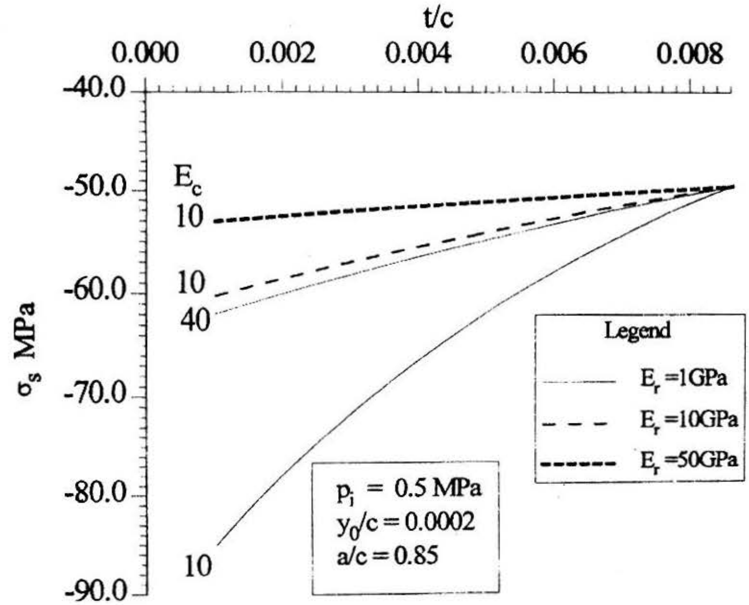


FIGURE 5 : Variation of Hoop Stress in Steel with t/c for different E_r and E_c Values

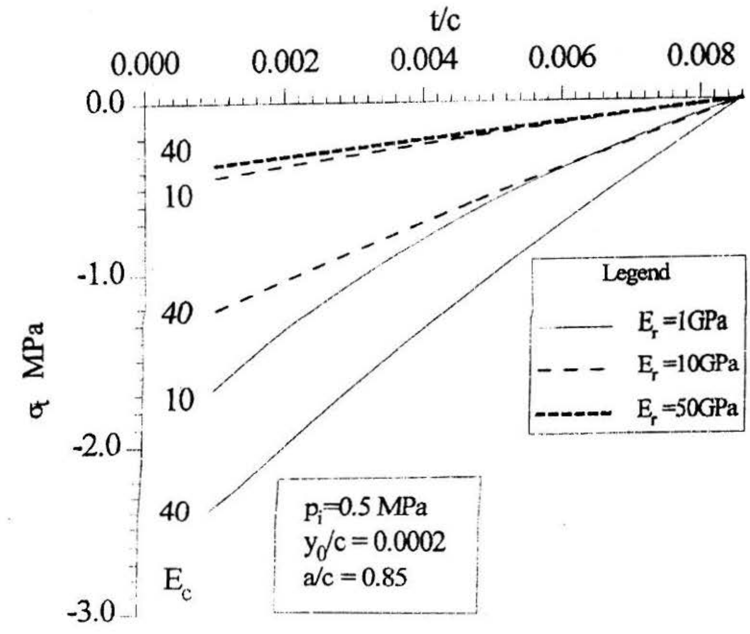


FIGURE 6 : Variation of Stress in Concrete with t/c for different E_r and E_c Values

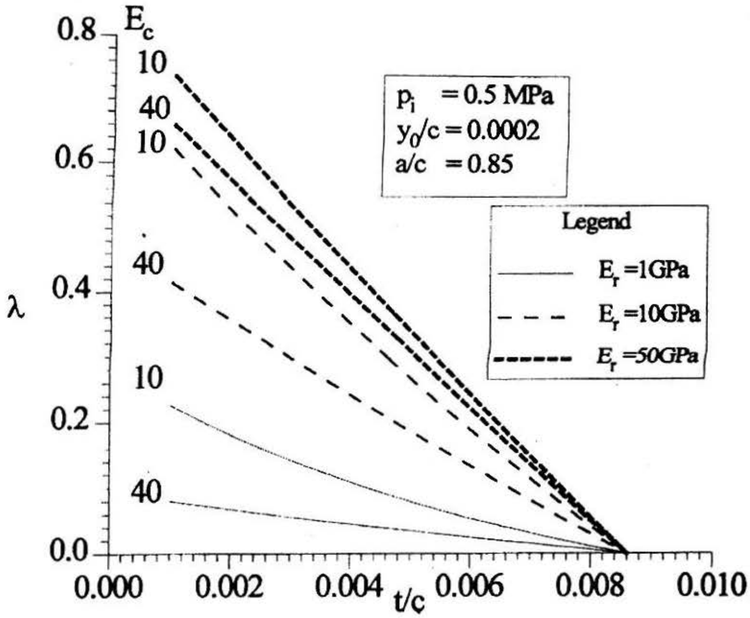


FIGURE 7 : Variation of Rock Participation Factor with t/c for different E_r and E_c Values

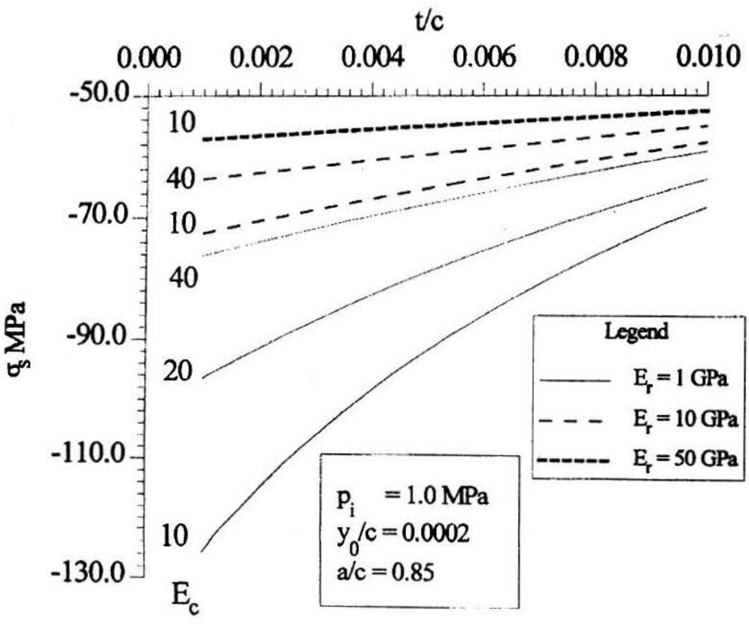


FIGURE 8 : Variation of Hoop Stress in Steel with t/c for different E_r and E_c Values

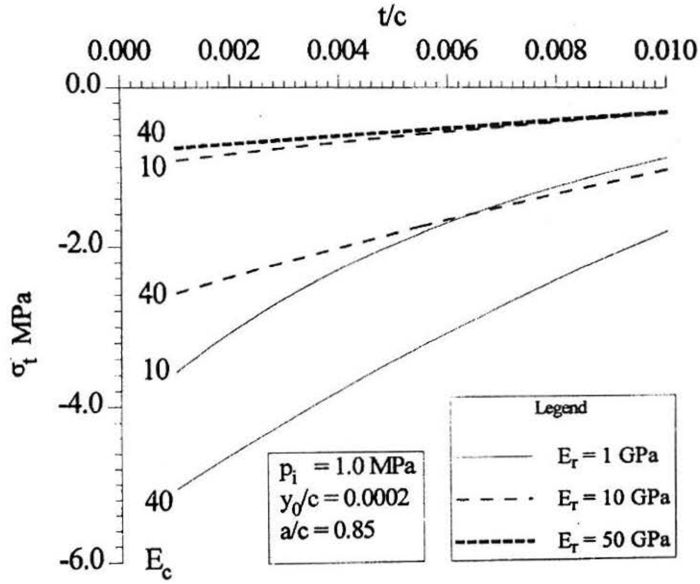


FIGURE 9 : Variation of Stress in Concrete with t/c for different E_r and E_c Values

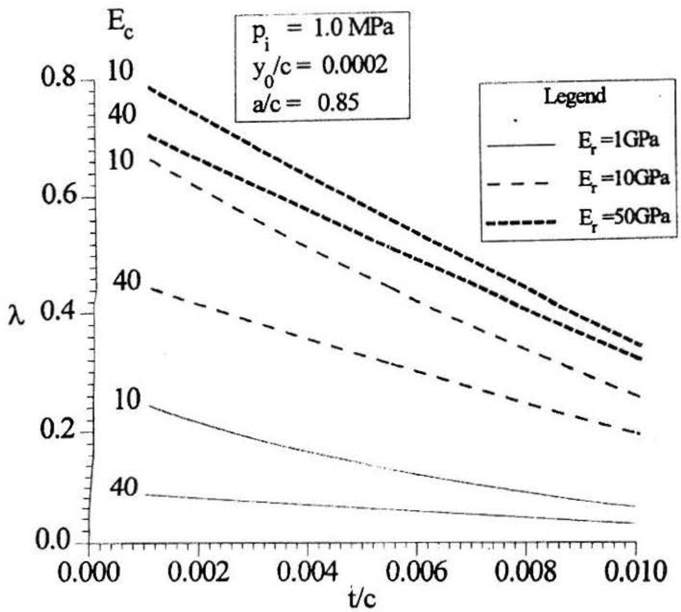


FIGURE 10 : Variation of Rock Participation Factor with t/c for different E_r and E_c Values

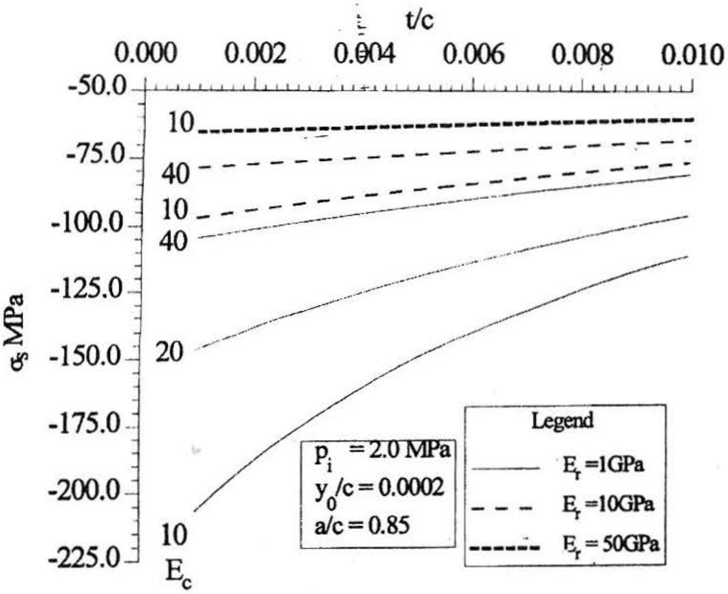


FIGURE 11 : Variation of Hoop Stress in Steel with t/c for different E_r and E_c Values

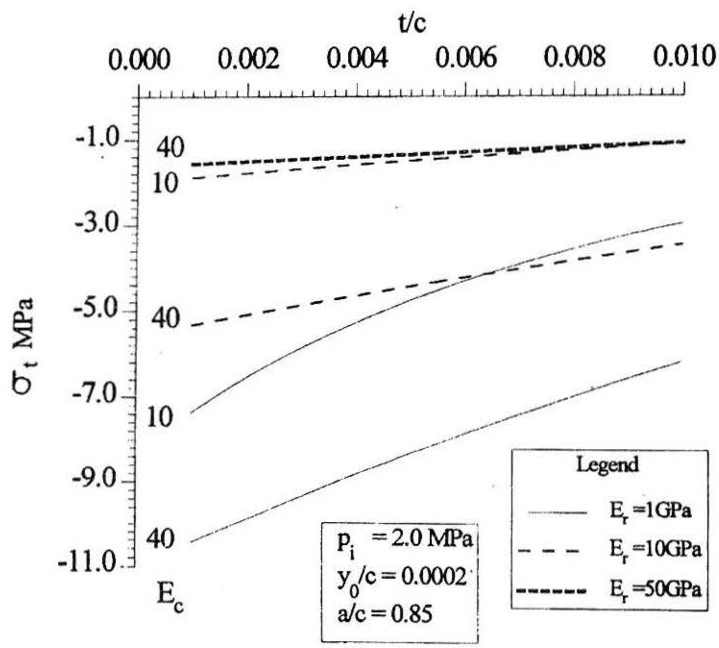


FIGURE 12 : Variation of Stress in Concrete with t/c for different E_r and E_c Values

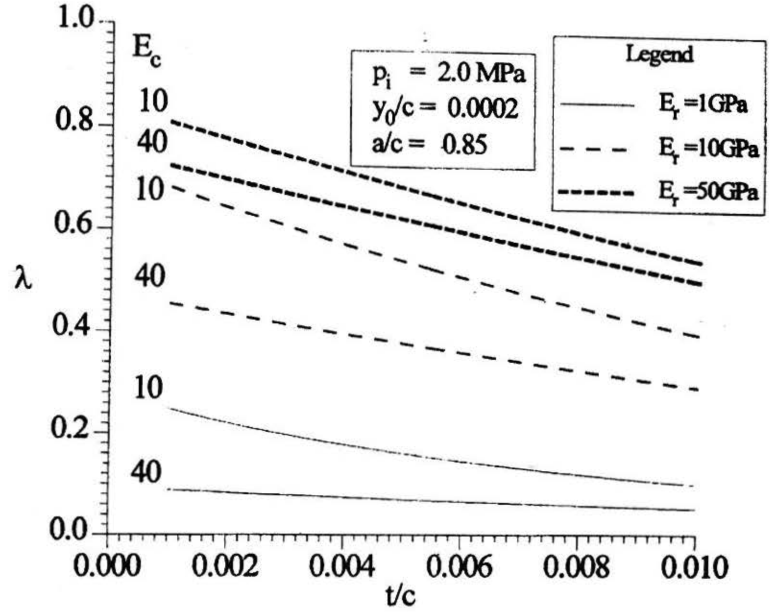


FIGURE 13 : Variation of Rock Participation Factor with t/c for different E_r and E_c Values

decrease is much higher with increase in stiffness of the rock.

Figures 8, 9 and 10 depict the variation of σ_s, σ_t and λ for p_i = 1 MPa. The variation of stresses is similar to Figs. 5, 6 and 7. The maximum σ_s = -125.8 MPa and maximum σ_t = -5.07 MPa for the same parameters as those for p_i = 0.5 MPa. For no pressure transfer the thickness of steel liner required would be 0.0172c which was checked for and matched exactly in the analysis.

Similar plots are provided for p_i = 2 MPa in Figs. 11, 12 and 13.

The variations of rock participation factor λ with t/c for various values of p_i, E_r and E_c show that the decrease in λ is more rapid for low internal pressures as compared to high p_i values. For p_i = 0.5, l = 0 at t/c = 0.0086. *Similarly, λ would be 0 for t/c = 0.0172 and 0.0344 for p_i = 1 MPa and 2 MPa respectively. It is also seen that λ is higher for higher p_i when all* other parameters are kept constant.

Conclusions

It is seen from the plots that if the rock is stiff then the major proportion of the internal pressure in the pressure shaft gets transferred to the rock. The steel liner and concrete share a lesser amount of pressure. As the stiffness of the rock begins to decrease, more and more stresses are shared by the steel and concrete liners and hence are subjected to higher tensile tangential stresses. Hoop stresses in steel and concrete are higher when the thickness of steel liner is small. The aspect that steel can take a high tensile stress can be effectively utilized in selecting the thickness of the steel liner such that the stress developed in the steel liner reaches its allowable strength. This may not be always practicable because this may lead to a very thin steel section. Hence the design should consider a more pragmatic thickness of the steel liner, such that tensile stresses in the concrete do not exceed its tensile strength.

References

AMSTUTZ, E. (1970) : "Buckling of Pressure Shafts and Steel Linings", *Water Power*.

BRADY, B.H.G. and BROWN, E.T., (1993) : *Rock Mechanics for Underground Mining*, Chapman and Hall, London, pp.214-215.

GUPTA, S.L. and DHAWAN, A.K. (1995) : "Design of Pressure Tunnels by Simple Analytical Method", *Proc. of Conf. on Design and Construction of Underground Structures*, New Delhi, pp.269-279.

HOEK, E. and BROWN, E.T. (1980) : "Underground Excavations in Rock", *The Institution of Mining and Metallurgy*, London, p.527.

IS 4880 (Part VII). (1975) : *Code of Practice for Design of Tunnels Carrying Water, Structural Design of Steel Linings*, Bureau of Indian Standards.

JAEGER, J.C. and COOK, N.G.W., (1976) : *Fundamentals of Rock Mechanics*, Chapman and Hall, London, p.585.

PATTERSON, F.W., CLINCH, R.L. and McCAIG, I.W. (1957) : "Design of Large Pressure Conduits in Rock", *Proc. of the ASCE Power Div.*, ASCE, Vol.83, pp.1457-1 to 1457-29

PITCHUMANI, N.K. and DASGUPTA, B. (1998) : "Analysis and Design of Pressure Shaft Liners", *Indian Geotechnical Conf.-98*, New Delhi, pp.331-334.

TIMOSHENKO, S.P. and GOODIER, J.N. (1987) : *Theory of Elasticity*, McGraw Hill, p.567.