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Hyperbolic Stress-Strain and Pore Pressure Response of Sensitive Clays

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Introduction

The explicit nature of stress-strain response of the naturally cemented soil mostly depends on, fabric and nature of bonding in addition to the usual factors such as current state, stress history, stress path and drainage conditions. Despite the availability of a good number of constitutive relations concerning the behaviour of clays (ISSMFE, 1985), there are still a large number of problems which have not been satisfactorily tackled. The analyses of the slides which occurred in the sensitive Champlain clay deposits of the Saint-Laurent lowlands (Lefebvre and La Rochelle, 1974) have shown that the use of peak strength parameters led to a gross over-estimate of the factors of safety; the residual strength parameters, however gave a fairly accurate assessment of the stability, the calculated factor of safety being close to unity. Spectacular land slides like the Saint-Jean-Vianney (Locat and Leroueil, 1988), South Nation river (Eden et al., 1971) and Rissa (Gregerson, 1981) have illustrated the great mobility resulting in long-run out distances which were actually related to low shear strength of the soil after failure (post-peak). The above failures are due to strain softening nature of these materials which lead to redistribution of stresses and progressive failure. It is therefore necessary to model the stress-strain behaviour between the peak strength and residual strength, in order to have efficient practical approach to the analysis of the stability problems.

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Most often sensitive soils are treated as overconsolidated soils because they also exhibit similar features like strain softening, higher initial stiffness, etc. But a closer study of the test results of different sensitive clays found in literature would reveal that the behaviour of these soils in undrained shear is very much different from that of uncemented overconsolidated soils. Strain softening is observed in relation to sensitive clays under all confining pressures even far greater than quasi preconsolidation pressure. Most important difference is that softening here is associated with continued positive pore pressures whereas softening in overconsolidated soils is associated with negative pore pressures. Probably, this is because there is an additional component of resistance from cementation bonds. The principles of continuum mechanics form the basis for several approaches postulated in the past to describe the strain softening behaviour. Many researchers have explained this behaviour by using the theory of plasticity (Nayak and Zienkiewicz, 1972; Prevost and Hoeg, 1975; Bannerjee and Stipho, 1979; Dragon and Mroz, 1979; Kawahara et al., 1981; Matsumoto and Ko, 1982). All the above models are basically developed for uncemented over consolidated soils and do not address to the special problem of softening in cemented soils. However, more recently Oka et al. (1989) proposed a constitutive model for natural soft clay with strain softening by extending Oka and Adachi's (1985) approach. Vatsala (1989) and Nagendraprasad (1996) while proposing a constitutive model for soft cemented clays, hypothesised that the load carrying capacity of the cemented soils can be split into two components one an uncemented or remoulded resistance and the other cementation bond resistance. However, the deformation of soil is essentially due to changes in stress increments on equivalent unbonded soil skeleton. Gens and Nova (1993), Adachi and Oka (1995) pointed out that for modelling the behaviour of cemented soils it is necessary to consider the behaviour of an equivalent uncemented state. Most of the constitutive variables involved in these plasticity based models are intricate, needless to say that evaluation of these parameters requires elaborate experimental programme. Thus there is a need for development of a realistic and simple model comprising of easily determinable constitutive parameters which are capable of capturing the most important aspects such as strain softening behaviour of soft clays.

Though it is desirable to have a comprehensive model based on sound principles of continuum mechanics, capable of describing the soil behaviour under any type of loading, most often such a generalised model may not be required. Depending on the specific field situation, it may be possible to analyse the problem with much simpler models. The hyperbolic nonlinear elastic models developed for cohesive soils (Kondner, 1963), which are widely used in finite element applications (Duncan and Chang, 1970), even to this date, because of their simplicity is one such example. However, Kondner's approach does not address the strain softening behaviour, which is typical of sensitive clays.

Table 1
Properties of Clays

Property	Saint Espirit clay	Gloucester clay (Bozozuk, 1984)	Osaka clay (Adachi et al., 1989)
Natural water content (%)	84	89	65-72
Liquid limit	75	53	69-75
Plastic limit	28	24	24-27
Liquidity index	1.19	2.24	0.91-0.94
Anisotropic yield stress (kPa)	130	87	94
Sensitivity	15	10	14.

The objective of this paper is to present a simple practical procedure for representing the nonlinear, stress dependent stress-strain behaviour of sensitive soil during undrained shear. Accordingly, the relationship described has been developed in such a way that values of the required parameters may be derived from the results of the standard laboratory triaxial tests. The formulations are proposed within the frame work of a hyperbolic response of stress ratio$\left(q'/p'\right)$ and also of effective mean principal stress$\left(p'\right)$ with strain. The formulation incorporates stress dependency, nonlinearity, strain softening aspects of the behaviour quite effectively. It provides a means to interpret the laboratory results in a form which may be used conveniently in non linear elastic finite element analysis. It must be pointed out that this is an attempt to present a simple methodology to circumvent the difficulty associated with the choice of complicated plasticity based constitutive models while dealing with undrained field situations.

Experimental Study

The clay studied has been imported from Saint Esprit, eastern Canada and is a part of Champlain sea deposits. The properties of the Saint Espirit clay along with Osaka and Gloucester clays are presented in Table 1. The samples have been extracted from block samples to a size of 37.4 mm dia. and 76 mm long. The GDS triaxial system which uses hydraulic triaxial cell of Bishop and Wesley type has been used to ensure precise measurements. Enough care has been exercised to minimise the sample disturbance. All test variables have been electronically monitored using pressure controllers.

Analysis of Test Data

The test results are analysed using the effective mean normal and deviatoric stress parameters p' and q' as given by:

$$p' = \frac{\sigma_1' + 2\sigma_3'}{3} \tag{1}$$

$$q' = \sigma_1' - \sigma_3' \tag{2}$$

where σ_1' = axial effective stresses on a cylindrical sample.

σ_3' = radial effective stresses on a cylindrical sample.

The deviatoric strain is expressed by:

$$\varepsilon_s = \frac{2}{3}(\varepsilon_1 - 2\varepsilon_3) \tag{3a}$$

$$\varepsilon_v = \varepsilon_1 + 2\varepsilon_3 \tag{3b}$$

where ε_1 = axial strains

ε_3 = radial strains.

For undrained tests $\varepsilon_v = 0$ and hence $\varepsilon_s = \varepsilon_1$. Axial strains were measured externally and the deviatoric stresses were calculated from the readings of the pressure controller and the current sample area using conventional area correction (Bishop and Henkel, 1962).

Figure 1 shows the stress-strain response of the sensitive clay for the confining pressures ranging from 50 to 800 kPa. The figure shows strain softening for the entire range of confining pressures (p_0') shown. It is well known that it is not possible to get a unique plot of q'/p_0' vs. ε_s for sensitive soils, while it is possible in the case of normally consolidated clays. This may be attributed to the fact that the evolution of cementation bond resistance and subsequent softening during deformation process is not proportional to the confining pressures, there by being more predominant for low confining pressures in comparison to the equivalent unbonded response.

Effective stress paths of the results are presented in Fig. 2 for the range of confining pressures tested. It may also be seen that the stress paths tend to converge along failure envelope corresponding to critical state (M line) gradually as the debonding occurs upon shearing. It turns

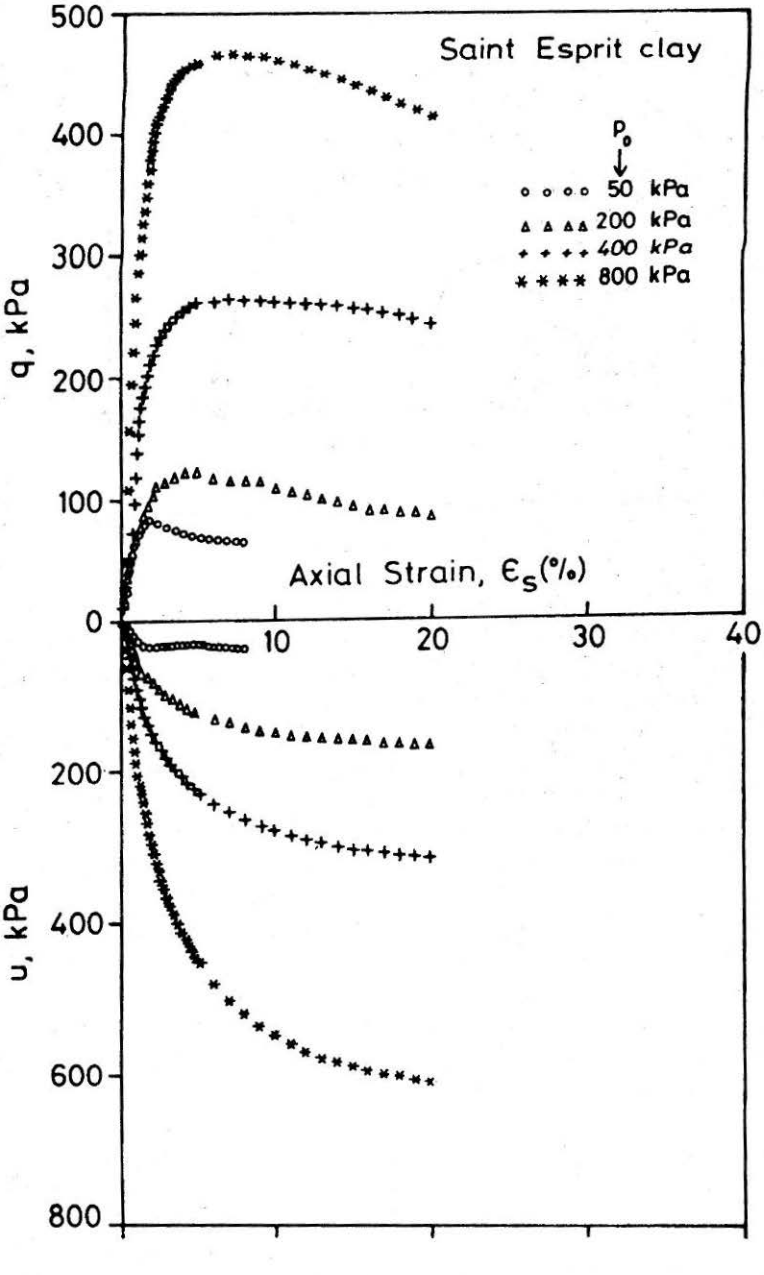

FIGURE 1 : Stress–Strain–Pore Pressure Behaviour of Saint Esprit Clay

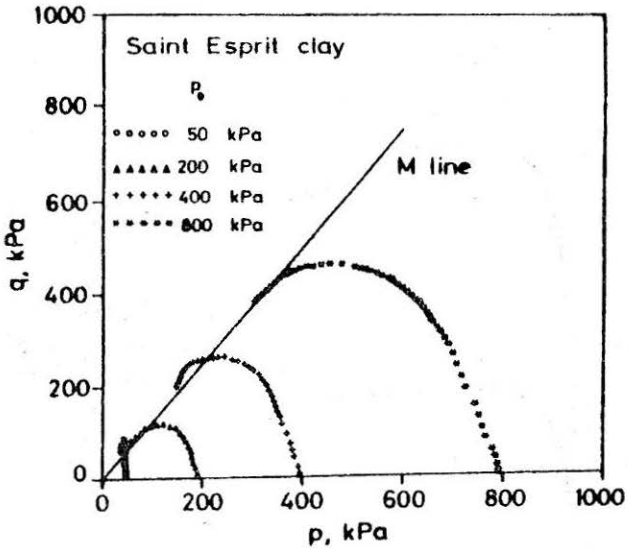

FIGURE 2 : Effective Stress Paths of Saint Esprit Clay

out that the critical state is approached only slowly at large strains. The results, naturally, indicate that it is the type of soil that determines the critical state parameters and not the initial state or cementation bonding. However, appearance of rupture zone, most often, makes it difficult to identify the critical states in case of sensitive soils. The results show that the value of stress ratio $(\eta = q'/p')$ upon reaching respective peak values remains nearly constant for all the confining pressures as indicated in Fig. 2. It may be further observed from Fig. 1 that the pore water pressure continues to be positive even in the softening region indicating that the behaviour is not akin to that of over consolidated soils as is frequently reported. The strain softening associated with positive pore pressures is perhaps peculiar feature concerning the behaviour of sensitive soils. This may be ascribed to the additional stress transfer on to the pore pressure as a consequence of debonding with progress in shearing. This stress transfer seems to occur in such a way that the value of η remains fairly a constant with distortional strain.

An examination of the data obtained from experimental results of Saint Esprit clay indicates that the p' vs. ε_s (Fig. 3) and η vs. ε_s (Fig. 4) relations are hyperbolic. These observations form the basis for the formulations proposed in this paper. The two constant hyperbolic relation has been utilized with advantage to analyse the consolidated undrained test results. The variations of stress ratio (q'/p') and the mean principal stress (p') with the deviatoric strain in terms of hyperbolic relation take the form as

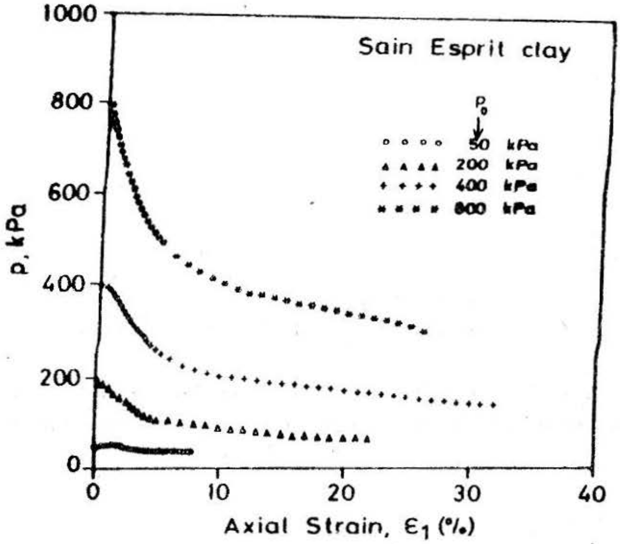

FIGURE 3 : Mean Principle Stress–Strain Behaviour of Saint Esprit Clay

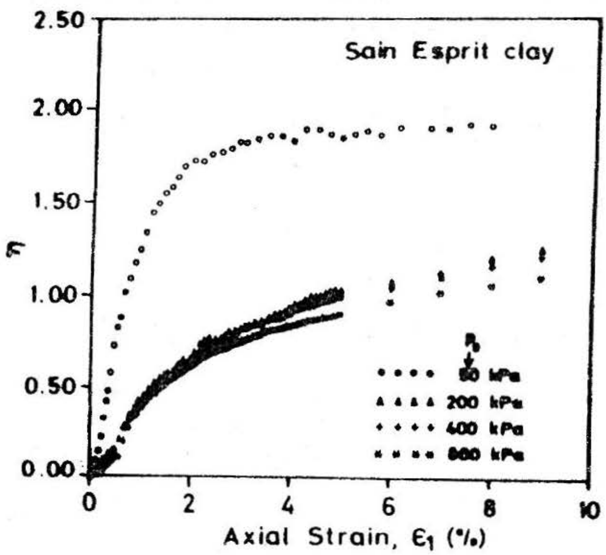

FIGURE 4 : Stress Ratio – Strain Behaviour of Saint Esprit Clay

$$\eta = \frac{\varepsilon_s}{a_2 + b_2 \times \varepsilon_s} \tag{4}$$

$$(p'_0 - p') = \frac{\varepsilon_s}{a_3 + b_3 \times \varepsilon_s} \tag{5}$$

where $\eta = q'/p'$

ε_s = shear strain

p'_0 = initial effective mean principal stress

Equations 4 and 5 can be transformed into the linear form as presented below in order to be able to make them suitable for experimental verification.

$$\frac{\varepsilon_s}{\eta} = a_2 + b_2\,\varepsilon_s \tag{6}$$

$$\frac{\varepsilon_s}{(p'_0 - p')} = a_3 + b_3\,\varepsilon_s \tag{7}$$

The experimental data of Fig. 1 is plotted in the form represented by Eqns. 6 and 7 and are shown in Figs. 5 and 6. A good straight line can be fitted to the experimental data between $\varepsilon_s/(p'_0 - p')$ vs. ε_s and ε_s/η vs. ε_s for all the selected confining pressures. This is a good indication of the applicability of the form proposed to represent the stress-strain response of the sensitive clays.

Elimination of ε_s in Eqns. 4 and 5 yields:

$$\frac{a_3(p'_0 - p')}{1 - b_3(p'_0 - p')} = \frac{a_2\eta}{1 - b_2} \tag{8}$$

which describes the undrained stress path of the sensitive clay.

For meaningful application of the relations proposed it is necessary to determine the exact nature of the parameters a_2, b_2, a_3, and b_3 in relation to the confining pressure. Figures 7 and 8 show the variation of these constants with the confining pressure.

Experimental results indicate that it is convenient to express a_2 and b_2 in the form of linear relationship with confining pressure on log scale and

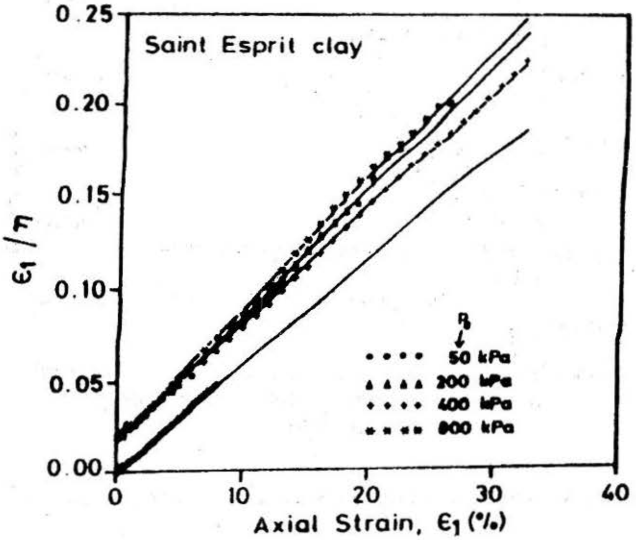

FIGURE 5 : Transformed Stress Ratio – Strain Curve of Saint Esprit Clay

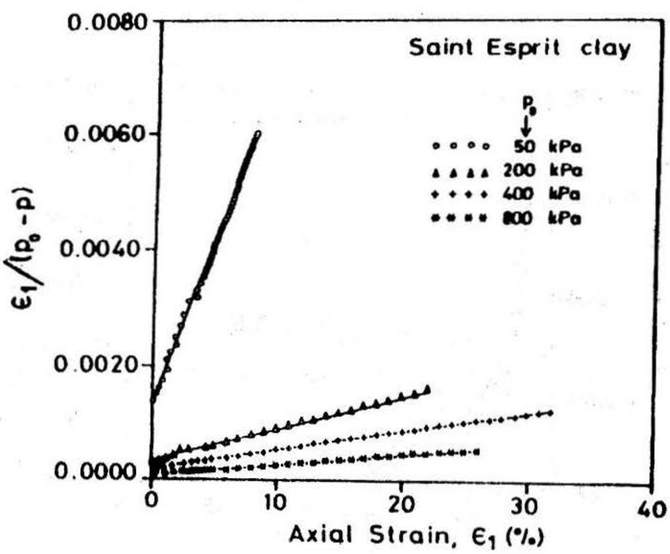

FIGURE 6 : Transformed Mean Principle Stress – Strain Curve of Saint Esprit Clay

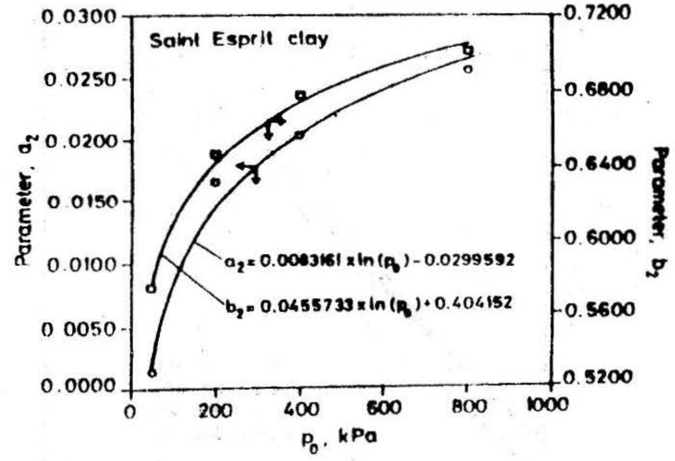

FIGURE 7 : Variation of Parameters a_2 and b_2 with Confining Pressure

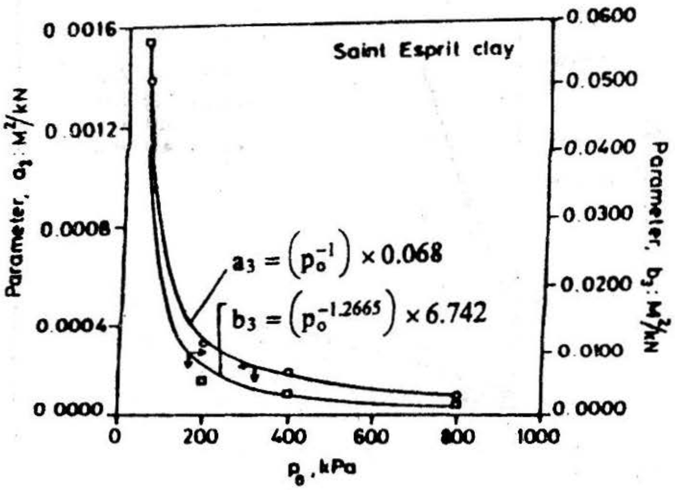

FIGURE 8 : Variation of Parameters a_3 and b_3 with Confining Pressure

parameters a_3 and b_3 in terms of initial confining pressures in the form of a power function (Eqns. 9 and 10).

$$a_2 = 0.0083 \times \ln(p'_0) - 0.03 \tag{9a}$$

$$b_2 = 0.0456 \times \ln(p'_0) - 0.404 \tag{9b}$$

$$a_3 = (p'_0)^{-1} \times 0.068 \tag{10a}$$

$$b_3 = (p'_0)^{-1.2665} \times 6.742 \tag{10b}$$

The hyperbolic constants a_2, b_2, a_3 and b_3 as obtained by the equations mentioned above are used to compute the stress-strain response. The computed and the observed plots of q vs. ε_s, q vs. p' are presented in Figs. 9 and 10 respectively. The close agreement between the computed and experimental results seems to confirm the applicability of the hyperbolic model for the sensitive clay in undrained shear. Initial drained test results indicate that these formulations also seem to be valid for drained shear tests on sensitive clays. However, these formulations may not be applicable for very low confining pressures where the pore pressure response does not follow a hyperbolic variation with strain.

Application to the Other Experimental Investigations

It is desirable to consider the proposed mathematical form in relation to other published literature in order to assess its general applicability. The test data of Gloucester clay (Bozozuk, 1984) is examined in this connection. The properties of the clay are presented in Table 1. The experimental data indicate that p' vs. ε_s (Fig 11) and η vs. ε_s (Fig 12) relations are hyperbolic. The transformed plots with $\varepsilon_s/(p'_0 - p')$ and ε_s/η on y-axis and deviatoric strain on x-axis and are presented in Figs. 13 and 14. It is of interest to note that there is a very good correlation between the fitted straight line and respective experimental results. The trend in variation of parameters a_2, b_2, a_3 and b_3 with respect to confining pressure as shown in Figs. 15 and 16 is seen to be similar to that of Saint Esprit clay. As the test data is available for a wide range of confining pressures the variation of hyperbolic parameters calibrated from the tests at certain confining pressures is used to predict the response for other confining pressures. Accordingly, the variation of these hyperbolic parameters is evaluated for confining pressures of 30 kPa, 60 kPa, 125 kPa and 220 kPa. The equations representing the variation for the above confining pressures are used to predict the stress-strain characteristics for the

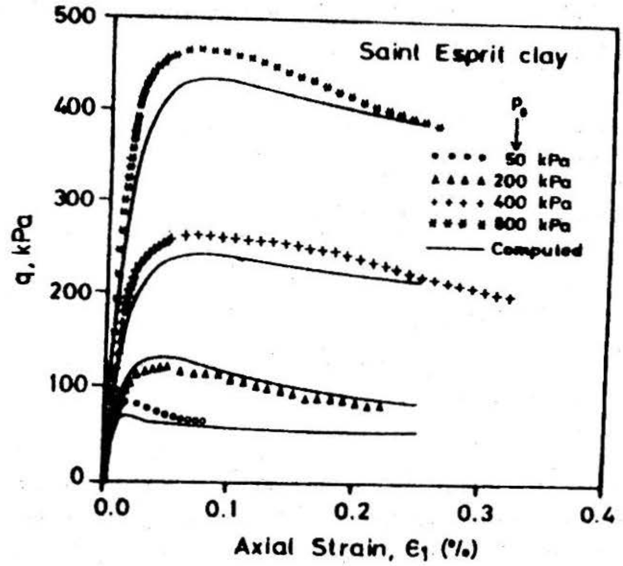

FIGURE 9 : Calculated and Experimental Stress – Strain Curves for Saint Esprit Clay

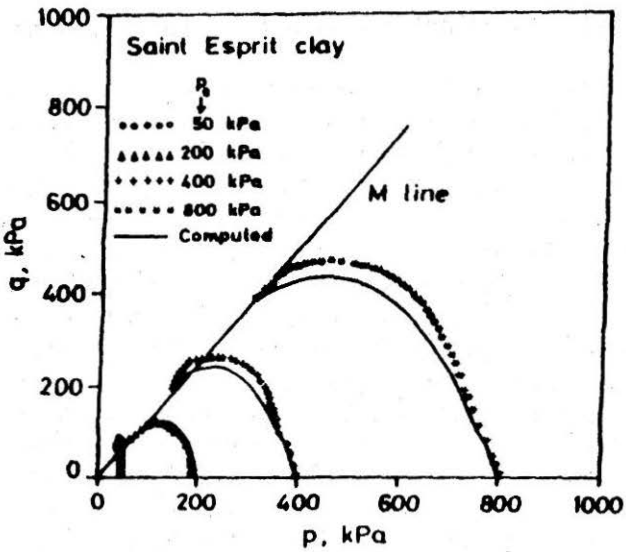

FIGURE 10 : Calculated and Experimental Stress Paths for Saint Esprit Clay

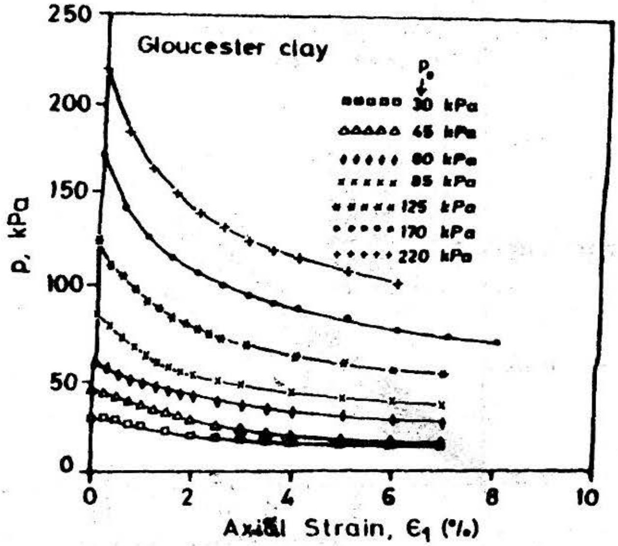

FIGURE 11 : Mean Principle Stress – Strain Behaviour of Gloucester Clay

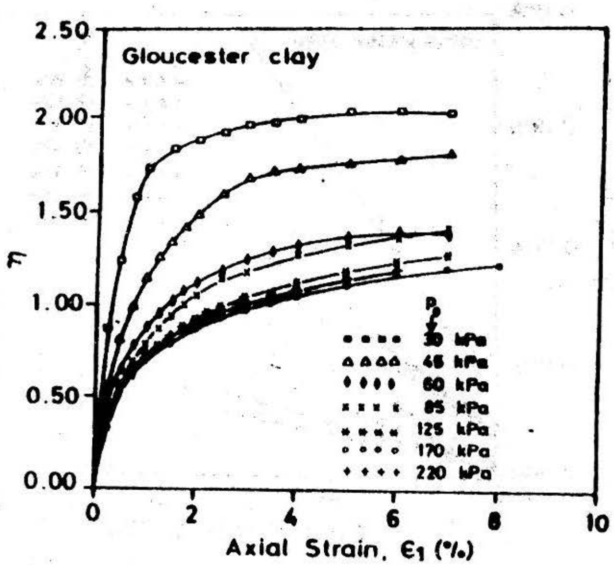

FIGURE 12 : Stress Ratio – Strain Behaviour of Gloucester Clay

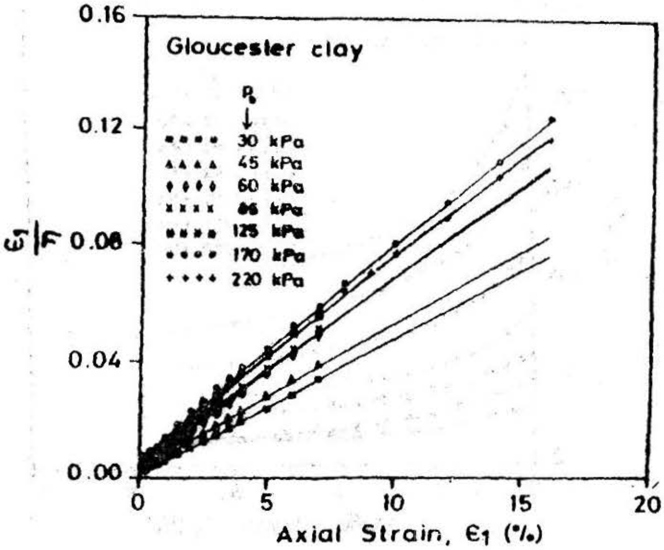

FIGURE 13 : Transformed Stress Ratio – Strain Curves for Gloucester Clay

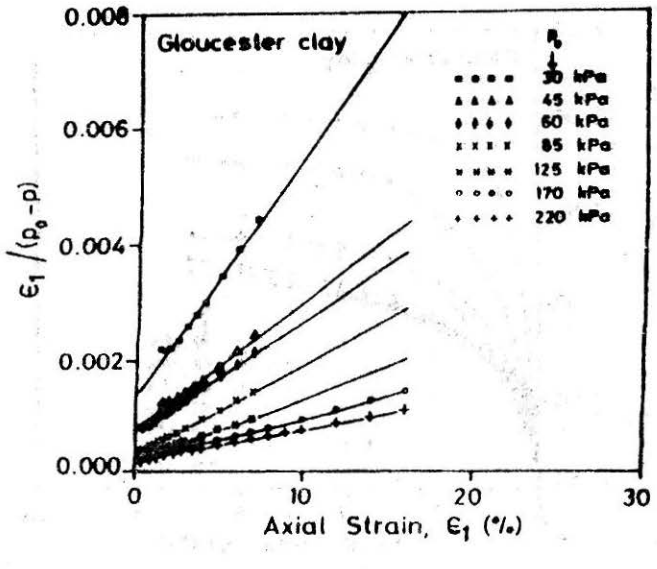

FIGURE 14 : Transformed Mean Principle Stress – Strain Curves for Gloucester Clay

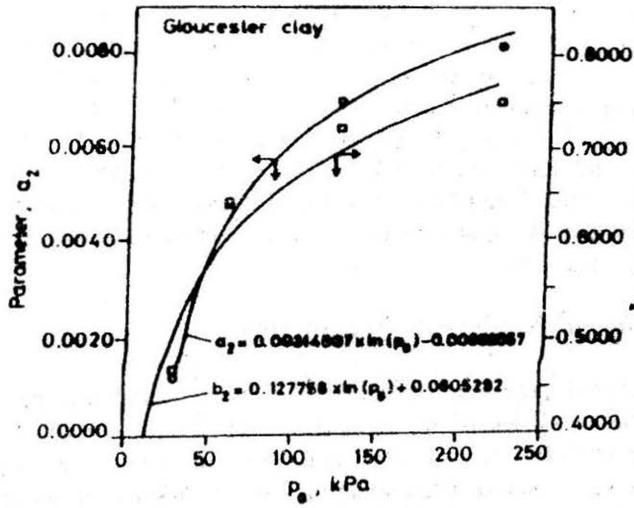

FIGURE 15 : Variation of Parameters a_2 and b_2 with Confining Pressure for Gloucester Clay

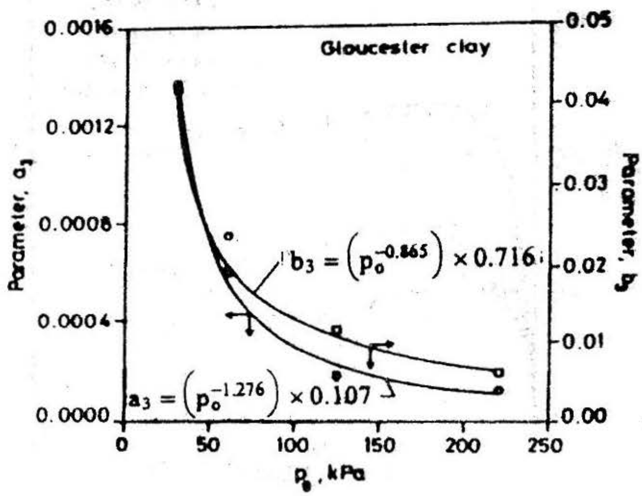

FIGURE 16 : Variation of Parameters a_3 and b_3 with Confining Pressure for Gloucester Clay

confining pressures of 45 kPa, 85 kPa and 125 kPa. The close agreement between predicted and experimental values is once again well demonstrated by comparative plots shown in Figs. 17 and 18. The results are equally encouraging in relation to the test data of Osaka clay (Adachi et al., 1989) as may be seen from Figs. 19 and 20. This turns out that selected test results reported by different investigators for different soils also exhibit similar characteristic features indicating that the nonlinear stress-strain response of sensitive clay can be approximated by a combination of hyperbolae with a high degree of accuracy.

Estimation of Nonlinear Parameters

A detailed procedure of the assessment of nonlinear parameters to represent the stress dependent stress-strain behaviour is presented in Duncan and Chang (1970). The important nonlinear parameters used are modulus number (k), the exponent determining the rate of variation of tangent modulus with confining pressure (n), and the ratio of stress difference at failure to the asymptotic value of the stress difference (R_f). In respect of sensitive clays, the confining stress dependent modulus of stress ratio (called as transformed modulus) and initial rate of mean principal stress are evaluated in accordance with the above procedure.

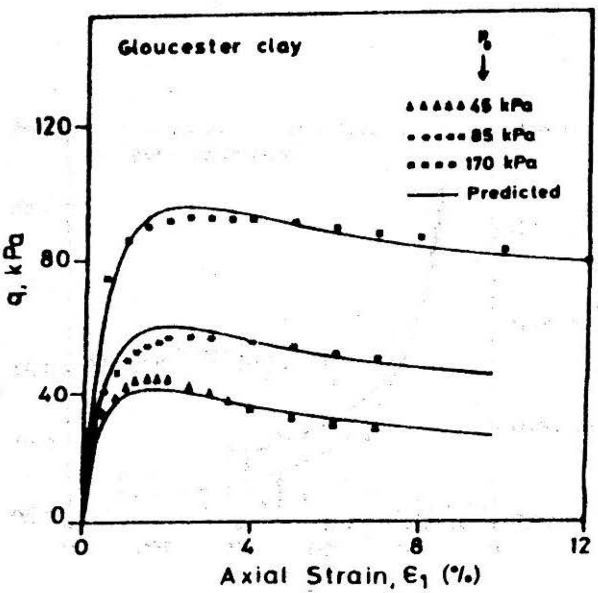

FIGURE 17 : Predicted and Experimental Stress – Strain Curves of Gloucester Clay

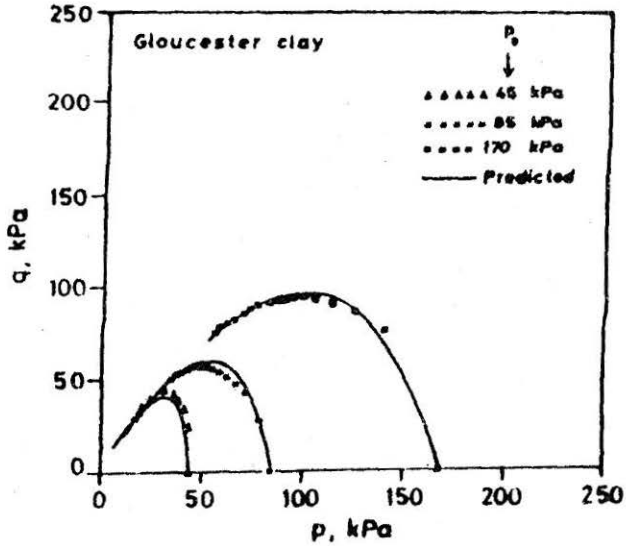

FIGURE 18 : Predicted and Experimental Stress Paths of Gloucester Clay

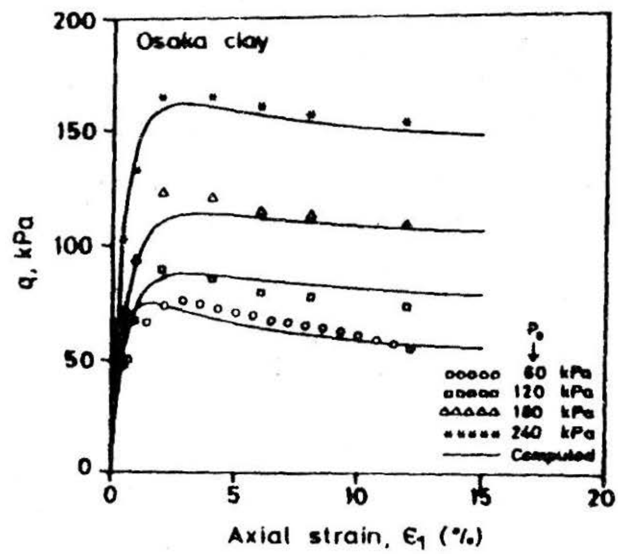

FIGURE 19 : Calculated and Experimental Stress – Strain Curves of Osaka Clay

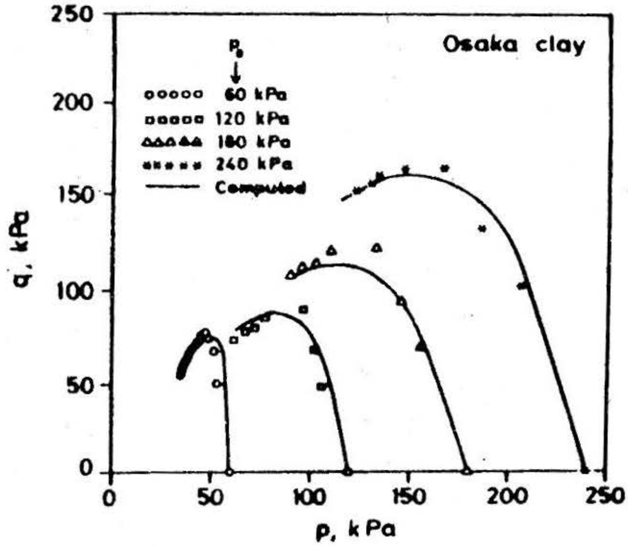

FIGURE 20 : Calculated and Experimental Stress Paths of Osaka Clay

The initial transformed modulus with respect to stress ratio as a function of confining pressure is given by:

$$E_i = k\left[\frac{p'_0}{p_a}\right]^n \tag{11}$$

where E_i = Initial transformed modulus with respect to stress ratio (slope of stress ratio vs. strain)

p_a = atmospheric pressure

k = modulus number

n = exponent

The initial rate of variation of mean principal stress with the confining pressure in a way represents the rate of variation of pore pressure which is given by:

$$\frac{B_i}{p_a} = k_p\left(\frac{p'_0}{p_a}\right) - p \tag{12}$$

where B_i = Initial rate of variation of mean principal stress with strain

p_a = atmospheric pressure

k_p = pore pressure number

p_k = pore pressure constant

The value of R_f is determined by:

$$R_f = \left(\frac{\eta_f}{\eta_{ult}}\right) \tag{13}$$

where η_f = stress ratio at failure

η_{ult} = assymptotic value of stress ratio.

A summary of these parameters for the above presented sensitive clays is given in Table 2. Use of these parameters to predict the nonlinear behaviour in incremental analysis is similar to the procedure indicated by Duncan and Chang (1970). In the conventional hyperbolic model the stiffness is obtained after normalising to take into account the different initial states. However, in the proposed approach the stiffness is evaluated after normalising with regard to the current state of stress. This procedure thus models the softening behaviour with intrinsic hyperbolic nature of the stresses.

Summary and Conclusions

The objective of this paper is to present a simple readily applicable mathematical form to represent the behaviour of sensitive clay in undrained conditions within the framework of hyperbolic response of stress-strain behaviour. The hyperbolic behaviour of stress ratio (η) as well as of effective mean principal stress (p') is introduced to represent the nonlinear, stress dependent, inelastic stress-strain behaviour. Despite limitation of the proposed formulation, it is shown that the strain softening can be described by the inherent merits of the hyperbolic form. The model consists of only five parameters which control the constitutive behaviour of sensitive clay in undrained situation. These parameters are easily determinable from standard triaxial tests.

The formulations obtained are compared with the results of different sensitive clays reported in literature by other investigators. Close agreement between observed and predicted response indicates that the nonlinear undrained stress-strain response of sensitive clays can be approximated by a combination of hyperbolae with a reasonable degree of accuracy.

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