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Reliability Analysis of Landfill Slopes

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Introduction

Solid wastes comprise all the discarded wastes in solid form arising from human activities in various ways and include residues from processing of wastes or recovery of conversion products. Of the wastes, municipal wastes, often, termed Municipal Solid Waste (MSW) are significant in quantities. As waste generation of solid wastes continues to increase, the capacity to handle wastes is decreasing. Incinerators have been of limited use and at the same time, it is difficult to locate new disposal sites. Of the waste disposal methods such as incineration, usage and land filling, land filling will continue to be the only option.

The stability of landfills assumes considerable importance, as landfills need to be considered for increased capacity by building landfills to greater heights or steeper slopes. In this regard, evaluation of shear strength properties of municipal solid wastes (MSW) and their role in influencing the slope stability needs a detailed examination. In this paper, studies on shear strength and stability of slopes of MSW are examined. The results are analyzed using the principles of reliability and guidelines on the assessment of reliability of MSW slopes are provided.

Material Properties and Stability

The crucial aspect of evaluation of slope stability is the identification of appropriate values for material parameters such as unit weight and shear strength parameters, viz., cohesion and friction angle. Many uncertainties are involved in sampling, testing, analysis and characterization of waste materials and in many cases, the uncertainties associated with material properties

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outweigh the uncertainties in the method of analysis for failure conditions (Mitchell and Mitchell, 1992).

Unit Weight

The unit weight of landfill material depends on composition, climatic conditions, compaction effort, moisture content, overburden pressure and other factors. Landva and Clark (1990) reported that the typical values are in the range of 7.2 to 14.4 kN/m^3. Fassett (1993) and Fassett et al. (1994) compiled information on unit weight and other properties of MSW with unit weights ranging from 3 to 16 kN/m^3 and attributed the wide range to the diversity of waste material, variable amounts of daily cover and moisture content. As the compaction effort increased, variation in unit weights was less. With depth, increase in unit weight was observed. However, with time, there was no discernible difference in unit weights as the effects of biodegradation compensated the resulting settlements.

Shear Strength Parameters

A number of researchers worked on shear strength properties of MSW and their role in influencing slope stability (Singh and Murphy, (1990), Landva and Clark (1990), Byrne et al. (1992), Gifford et al. (1992), Howland and Landva (1992), Mitchell and Mitchell (1992), Mitchell et al. (1992), Fassett (1993), Fassett et al. (1994), Jessberger (1994) and Jessberger and Kockel (1995)). The shear strength parameters are estimated based on three approaches; 1) direct laboratory and field testing, 2) indirect in-situ testing and 3) back calculations from failures and load tests. Direct shear testing on reconstituted or disturbed samples, triaxial testing on samples obtained from Shelby and drive samples and unconfined strength and tensile testing of bailed waste were reported in literature (Fang et al., 1977, Jessberger, 1994). In-situ field tests using Standard Penetration Tests (SPT) and vane tests and cone penetration tests (Oakley, 1990) to determine the field shear strength of waste material were also reported. Back calculations of failures and load tests were performed at many sites. Back calculated strength data has also been obtained observing satisfactory performance of landfills in California, during earthquakes. Since landfill slopes survived the events, back calculated values of cohesion and friction were obtained by assuming a factor of safety equal to 1.0. These values are conservative as the available strength is underestimated by an unknown order. Singh and Murphy (1990) recommended that the strength parameters chosen for stability should be interpreted judgmentally in favor of least conservatism, since moderately steep landfill slopes are known to exist with few signs of distress. Howland and Landva (1992) and Gabr and Valero (1995) observed that the results of back analysis indicate strengths lesser than those obtained from direct measurements and laboratory measurements. Mitchell and Mitchell (1992) suggest that though,

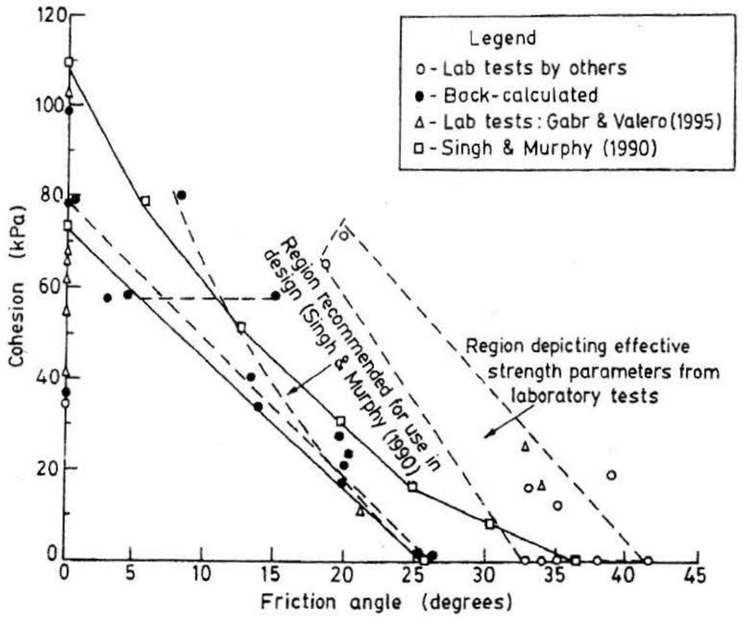


FIGURE 1 : Summary of Shear Strength Data (after Gabr and Valero, 1995)

the cohesive nature of refuse has not been adequately characterized so far, cohesion component that results from interlocking or overlapping of waste constituents needs to be included in the stability analysis. This is also supported by the common observation that vertical cuts in a refuse fill can stand unsupported to considerable heights (Mitchell and Mitchell; 1992 and Leonards, 1995). Jessberger (1994) and Jessberger and Kockel (1995) suggest that the high strength characteristics of waste materials may be explained by the interlocking effect of fibrous components comparable to a reinforcing effect, which is homogeneously distributed throughout the waste body.

The above discussion shows that the test results in general were highly variable and were affected by different placement, stress, boundary and environmental conditions. Considering the uncertainties and explanations on MSW shear strength, the summary of information that exists today is shown in Fig. 1 (Gabr and Valero, 1995).

Slope Stability

Howland and Landva (1992) examined the overall stability of vertical expansion of an existing MSW landfill in New York. Eyrne et al. (1992) used three dimensional stability analysis to examine the failure of Kettleman

Hills facility and concluded that low liner interface strengths within the landfill along with application of conventional analysis of stability and testing methods were responsible for failure. Mitchell and Mitchell (1992) and Mitchell et al. (1995) delineated the various mechanisms and types of failure in landfills. They reasoned that the critical aspect is the shearing resistance of MSW along interfaces within liner and cover systems. It appears that a general agreement on either the properties of MSW or the methods of analysis is still not available. Under these circumstances, reliability engineering principles could offer guidance in understanding the role of uncertainties and their influence in design of landfill slopes. Landrum et al. (1995) examined the reliability of geomembrane lined slopes of landfill covers and liners and showed that the uncertainties in interface friction angles and adhesion can lead to low values of slope reliability. So far, no attempt has been made to examine the effect of variations of material properties of landfill and it's relationship with reliability of landfill slope. Hence, in this paper, an attempt is made in this direction.

Reliability of Landfill Slopes

Judgment versus Reliability

Many investigating agencies comply with the regulations, determine the material parameters necessary and evaluate the stability of slopes but are left with a few nagging questions on the parameters chosen and the methods of analysis adopted. After an extensive laboratory and field testing, the selection of design strength parameters is based on judgment, since numerous combinations of unit weight, cohesion and friction parameters would produce similar safety factors. For example, 10 sets of the above required parameters along the potential failure surface will give 120 combinations to assist us in evaluating stability. Uncertainties of this nature always confronted geotechnical engineers. No doubt, engineering judgment has played a significant role in the identification of avoidable and unavoidable possibilities. However, in an attempt to assure safety, engineers often chose judiciously conservative values of the geotechnical properties. It is necessary that the uncertainties are treated in a systematic manner and quantified to examine the reliability of the design of the structure. It is also important that land filling and closure plans be developed in such a way that an adequate factor of safety is always maintained for all heights and geometries. In the following sections, a brief review of reliability engineering principles necessary for evaluation of landfill slope stability is presented and examined with reference to a typical problem.

Probability and Reliability

Geotechnical engineers have recognized the role of uncertainties in slope stability quite a few years back (Wu and Kraft, 1970; Alonso, 1976; Li and

Lumb, 1987) but have been slow on implementing them in analysis and design and to assess the probability of success or failure of a structure. Christian et al. (1992) suggest that the effective applications of probability and reliability principles lie in identifying the relative probabilities of failure or in which the effects of uncertainties on design are clearly brought out.

In the assessment of reliability, material parameters affecting the design are considered as random variables. In order to obtain information about the performance function of the random variables, a number of methods are available and are divided into three categories. Detailed description of methods is provided by Harr (1987). The first category called exact methods require the probability distribution functions of all component variables. Monte Carlo simulation method belongs to this category. The second category, called First Order, Second Moment (FOSM) method simplifies the implied relationship. Taylor's series involving expected values, standard deviations of variables and higher moments form the basis of the method. However, the method becomes complex, when the variables are correlated and involve complicated functions (Harr, 1987).

The third method called Point Estimate Method (PEM) proposed by Rosenblueth (1975) is a simple convenient tool (Harr, 1987) to arrive at the resulting function distribution and is considered in the present analysis. The point estimate method requires the knowledge of mean and standard deviation of each variable and the correlation coefficient between them. The mean and the standard deviation of the design function are the inputs that help in identifying the probability distribution of the design function. For two random variables, Rosenblueth considered the probability distribution to be analogous to a distributed load acting over a rigid plate supported at four points p_{++}, p_{+-}, p_{-+}, p_{--} as shown in Fig. 2. The terms p_{++}, p_{+-}, p_{-+}, p_{--} represent the corresponding probabilities. For this case, for example, the first moment of functional relationship is given by

$$E(y) = p_{++}y_{++} + p_{+-}y_{+-} + p_{-+}y_{-+} + p_{--}y_{--} \tag{1}$$

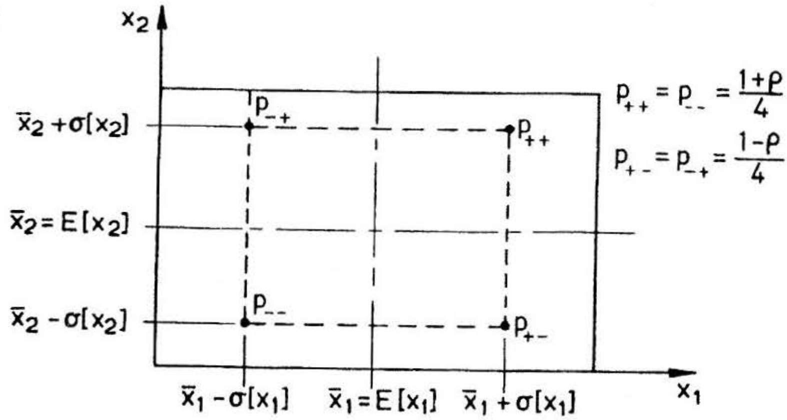
$$y_{\pm\pm} = y(x_1 \pm \sigma[x_1], x_2 \pm \sigma[x_2]) \tag{2}$$

$$p_{++} = p_{--} = (1+\rho)/4 \tag{3}$$

$$p_{+-} = p_{-+} = (1-\rho)/4 \tag{4}$$

In these relationships, ρ is the correlation coefficient between the random variables x_1 and x_2 In the present case, cohesion and friction angle are the design variables and factor of safety of slope is the required performance function. The shear strength data shown in Fig. 1 and unit weight data from Fassett (1993) are examined and coefficients of variation are

(a)



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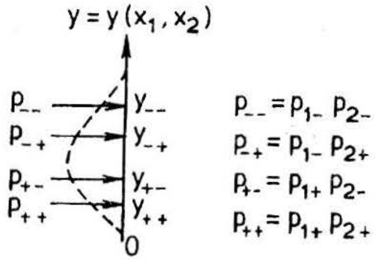

FIGURE 2 : (a) Representation of Two Variables as :oading on Rigid Plate (b) Generated Joint Probability Distribution Function

obtained. The corresponding values for soils (Harr, 1987) are included for comparison. Coefficient of variation (CV) is a measure of reliability of central tendency. The higher the standard deviation (SD) and hence the coefficient of variation, greater is the scatter and hence less reliable. The results in Table 1 show that MSW properties are highly variable even when compared to natural soils that are inherently highly variable.

In order to keep the analysis simple and to illustrate the use of reliability principles, a landfill slope formed of the MSW materials is considered and Bishop's simplified method is used for stability analysis. Cohesion, friction angle and unit weights are taken as 50 kPa, 15° and 10 kN/m3 respectively. Stability computations were performed for two heights of slopes (20m and 30m). The slope angle is varied from 18.4° (3:1) to

TABLE 1 : Statistical information for MSW and soils

Material Parameter	Statistical parameters for MSW				CV(%) for soils
	range	mean	SD	CV%	
Unit weight, kN/m^3	3 to 16	8.6	3.0	35	3
Cohesion, kPa	1 to 103	46.6	28.9	62	40
Friction angle	0 to 34°	11.6	11.6	100	12

90°(vertical). The slope is assumed to be dry and resting on a waste deposit of 15m thick of identical properties.

Results and Discussion

In the present investigation, cohesion and friction are considered as random variables with mean values of 50 kPa and 15° and coefficients of variation of 50% and 33.3% respectively. Correlation coefficient of -0.84 is obtained for the data of friction angle and cohesion values given in Fig.1. Unit weight is taken as 10 kN/m3 and no variation is assumed. Point estimate method is used to calculate the expected factor of safety, E(FS), and coefficient of variation, CV(FS). Probability of failure is calculated corresponding to factor of safety less than unity. Reliability or the probability of success is defined as (1- probability of failure). To estimate the probability of failure, the use of more general distribution such as beta distribution is preferable as cohesion and friction angle and the desired design variable, factor of safety have definite ranges (Harr, 1987). For example, factor of safety that can be targeted for design in the practical range is upto 3. Hence, probability of failure is estimated from beta distribution. The results are shown in Table 2. Table 2 shows that as the slope angle is increased, expected factor of safety as well as reliability decreases. It also shows that the slope becomes less reliable as the height of the slope is increased. The results show that the normally recommended and adopted slope (of 3:1) during landfilling for waste cells and for final landfill slopes is very reliable considering the variations in shear strength properties of MSW.

Table 3 shows the influence of coefficient of variation on reliability and expected factor of safety. The results from normal distribution are also presented for comparison. The results show that for the same expected factor of safety, depending on the coefficient of variation, reliability is different. It is also difficult to achieve higher reliability, if the coefficient of variation is high.

Table 2. Results of reliability analysis of landfill slope

Slope angle (H:V)	Expected FS		SD (FS)		CV (FS)		Reliability	
	20m	30m	20m	30m	20 m	30 m	20m	30m
3:1 (18.4°)	3.11	2.61	0.69	0.62	0.22	0.24	1.0	1.0
2:1 (26.5°)	2.65	1.99	0.74	0.46	0.28	0.23	0.99	0.99
1:1 (45°)	2.21	1.56	0.70	0.47	0.32	0.30	0.96	0.89
1:2 (63.4°)	2.04	1.35	0.75	0.42	0.37	0.31	0.92	0.79
vertical (90°)	1.90	1.06	0.51	0.29	0.27	0.27	0.96	0.66

TABLE 3 : Reliability vs expected factor of safety (FS) and coefficient of variation (CV)

E(FS)	CV = 0.10		CV = 0.20		CV = 0.30		CV = 0.40	
	beta	nor	beta	nor	beta	nor	beta	nor
1	0.49	0.50	0.48	0.5	0.47	0.50	0.46	0.50
1.5	1.00	1.00	0.96	0.95	0.87	0.87	0.78	0.80
2	1.00	1.00	0.99	0.99	0.95	0.95	0.89	0.89
2.5	1.00	1.00	1.00	0.99	0.98	0.98	0.93	0.94
3.0	1.00	1.00	1.00	1.00	0.99	0.99	0.93	0.95

However, this factor can be advantageously used in the construction stage by attempting to achieve lower values of CV. The CV value in unit weight could be reduced considerably by proper compaction. For example, the coefficient of variation in unit weights in the case of poorly compacted MSW is 48%. It can be reduced to 8% for MSW compacted by moderate and medium effort (Fasset, 1993) which also leads to lesser CV values in cohesion and friction. Proper placement of waste also helps in reducing the CV values. Properly baled and compacted waste can exhibit lesser CV values in cohesion, friction and bulk density so that higher factor of safety and reliability could be achieved.

Concluding Remarks

In this paper, the reliability of landfill slopes is examined. To understand the implications of variations in material properties used in landfill slope design, point estimate method is used. The results show that the normally recommended and adopted slope (of 3: 1) during landfilling for waste cells and for final landfill slopes is very reliable for the heights and the variations in shear strength properties of MSW considered. Factors such as compaction effort, stress, climatic conditions, laboratory and field tests, etc. affect the input material parameters. It is also essential that the parameters for analysis should be representative of actual stress, environmental conditions as well as their long term variations due to biodegradation and other effects.

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